Girls' Angle Bulletin

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To Foster and Nurture Girls' Interest in Mathematics

From the Director

We hope that the first session of Girls' Angle was a fun and rewarding experience for all the members. I certainly had a lot of fun! The first session was full of surprises and lots of very interesting mathematics. During the interim, the Girls' Angle advisors and mentors will be getting together to review the first session and begin planning for the second session. (See the calendar on page 23.) We always aim to improve the quality of math education the girls receive. Please send comments and suggestions to girlsangle@gmail.com. Please tell us what you liked and disliked about the program!

In 2004, of the roughly 300 tenured mathematics professors at Berkeley, Caltech, Columbia, Harvard, Michigan, MIT, Princeton, Stanford, and Yale, only 16 were women (see [NAMS]). Indeed, Harvard has not had a single tenured woman on its math department faculty ever. But if all I knew of math education were Girls' Angle, I would walk away thinking that women must *dominate* the field of mathematics! Just browse through the Notes from the Club (p. 17) and ascertain for yourself the level of depth some of the mathematics discussions reached. Perhaps a member at Girls' Angle will grow up and become the first tenured female faculty member in the mathematics department at Harvard. I am not saying this flippantly as one of those cool sounding things that one can toss about. I really believe that it can happen.

Ken Fan
Founder and Director

[NAMS] Jackson, Allyn, *Notices of the American Mathematical Society*, **51** (2004), no. 7, pp. 776-783.
www.ams.org/notices/200407/comm-women.pdf

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Girls' Angle Bulletin
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Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin costs $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to go on and tackle any field no matter the level of mathematical sophistication required.

On the cover: The map of Cambridge used in the Treasure Hunt. **August** was the first to locate her treasure, and as you can see, she was spot on!

Mistakes: Good and Bad

Mathematicians make mistakes all the time.

I don't know a single mathematician who didn't at one time believe something to be true which turned out to be false.

The good news is that mathematics has a tendency to correct itself. If you keep making the same mistake, there is a good chance that eventually you will find yourself saying something which will clearly be false. When that happens, you know that you must have made a mistake somewhere and you can go back and search for it and try to correct the mistake.

Especially, when you are learning something new, you are bound to make mistakes. This is true with just about everything, not just mathematics. The first time you put on ice skates, you're going to fall. When you try to learn a new language, you're bound to say something mighty peculiar!

In fact, imagine how hard it would be to learn anything new if you refused to make a mistake! If you refused to fall, you probably would never learn to skate backward or to do a hockey stop because you would be too afraid to try.

So we arrive at my first point: When you learn mathematics, do not be afraid to make mistakes! Do not worry if what you are thinking might be wrong. Just try it and see!

Any mistake you make because you were trying is a good mistake. When you make such a mistake, people might laugh at you, it's true. That's human nature. But a little embarrassment is a small price to pay for greater understanding.

If you can rid yourself of a fear of mistakes and no longer be embarrassed when you make them, you will progress faster. When you have gotten to the point where you can find and correct your own mistakes, you have become your own teacher, and there's no telling how far you will go. And you will need this attitude if you someday want to solve one of the many unsolved problems that face humanity.

But this article is also about *bad* mistakes. There are mistakes that are very bad. They are so bad that these mistakes hold you back, keep you down, and limit your potential. I'll explain two really bad mistakes.

The first bad mistake you can make is to be so afraid of making a mistake that you completely give up. Suppose you had the idea that you'd like to learn how to play the cello. You borrow a cello and try to play it. It squeals and howls! It sounds terrible! Then someone who has been playing for years takes the cello out of your hand and plays a Bach cello suite from memory. Do you decide to end your cello playing aspirations at that moment just because you are now too embarrassed to make those raking squeaks? If you do quit for that reason, you are making a very bad mistake! If you desire to play the cello, don't worry about whether you sound bad. Instead, focus on trying to sound a little bit better.

In mathematics, the ability to persevere despite all the mistakes is very important. There is no limit to the difficulty level of a math problem, and there have been problems that have taken years and years to solve. Fermat's Last Theorem is a good example. If you haven't heard of Fermat's Last Theorem, you can ask about it at the club. There's also an excellent NOVA program dedicated to it, entitled "The Proof". When the now famous mathematician Andrew Wiles first announced his proof of Fermat's Last Theorem, it turned out, he had made a mistake that he had to go back and correct. To minimize embarrassment, mathematicians do try to catch their mistakes privately, but the mistake Dr. Wiles made was made in front of the entire mathematical world! Despite this very public mistake, Dr. Wiles did not give up, and he soon after supplied a corrected proof.

(By the way, to persevere does not mean to persist without taking any breaks.)

The second bad mistake you can make is to refuse to admit having made a mistake once you realize that you made one. When people make this mistake, it can be very damaging. For example, suppose you were entrusted to lock up a store, but you forgot. You get home and the store owner calls you to make sure you remembered. A sinking feeling sets in as you realize that you didn't lock up the store! The sooner you can admit that you didn't lock up the store, the sooner that problem will be solved, but if you go into denial, the store will remain unlocked and an easy target for thieves.

The first bad mistake is made by people who are afraid of embarrassment. The second bad mistake is made by people with too much pride. If you know how to laugh at yourself and are humble, you'll be immune to such mistakes.

In summary, don't worry about making mistakes. All mathematicians make mistakes. If you have an idea, but don't know if it's a good one, just try it! After you try it, if it turns out to be wrong, accept it, then just go back and try to correct it. In the process, you will have learned something and you will be far less likely to make the same mistake again.

Now, you might be wondering, what if I keep making the same mistake over and over again because I never realize that it is a mistake?

That's what the mentors are for!

But what happens if even the mentors don't recognize a persistent mistake?

Well, for one thing, that would mean you have become a very good mathematician! This situation can and does happen and I have no doubt that I harbor mistaken notions which I haven't realized yet and might, possibly, never realize. The only hope to catch these mistakes is to not only think about math, but to think about how you think about math! And, in addition, it is important to remain humble. Once you believe that you are clean of mistaken notions, that is the moment you have become most vulnerable to them.

If nobody recognizes one of your mistaken notions, but someday, you succeed in discovering one, it could very well turn out to be a breakthrough idea allowing the whole field to advance.

I can't wait to correct my next mistake!

An Interview with Lauren Williams

Lauren Williams is a Benjamin Pierce assistant professor of mathematics at Harvard University. She is a Girls' Angle advisor.

Ken: Hi Lauren, thank you for agreeing to do this interview! My first question is, what excites you about mathematics?

Lauren: Generally speaking, I enjoy mathematics because I like to understand things. And I especially like to understand things that have beautiful structure and symmetry.

Ken: Do you remember what got you first interested in mathematics?

Lauren: When I was in grade school I somehow found out about codes and ciphers, and I checked out all the books in the library that I could find about them. I was initially intrigued by the idea of being able to send secret messages, but eventually got hooked by the beautiful mathematical ideas that went into some of the codes.

Ken: Do you find mathematics really easy?

Lauren: Definitely not! As time goes on, I become more and more aware of how much there is that I don't understand. But that is what makes mathematics so interesting and exciting!

Ken: What have you found to be the best ways to study mathematics?

Lauren: There are many ways that I have found useful to study mathematics. My first approach to a problem is usually to do an example- this helps me make sure I understand the problem and maybe see some patterns. Also, reading math books and papers is important, although it takes patience- it takes much longer to read a math book than to read a novel. Finally, talking to other people is great- other people often have a different point of view which can give a new way of understanding the subject. And having someone explain something to you can be much more efficient than reading a book.

Ken: Mathematics is still a very male dominated field. Does this affect you in any way? Why do you think math is so male dominated?

Lauren: To some extent I have gotten used to being in a male dominated field. Now I don't really notice being the only woman or one of just a few women in a lecture or at a conference. But every now and then I'll be in a situation where I have to deal with the issue again. Acquaintances, friends and colleagues have at times made comments that were hurtful to me. I've heard remarks like "I wish *I* were a female mathematician- life would be so easy," coming from a man, and "Why did Harvard hire you? Did they want a woman?" coming from a woman.

I think these statements reveal the fact that some people, both men and women, have the idea that there is a lot of affirmative action out there and that women have an easier time succeeding than they should because of it. In fact, I've seen very little affirmative action, and I find peoples' assumptions about it rather frustrating. I don't want people to think that if I am successful, it is because I was given "extra credit" on account of being a woman. (Annoyingly, if I'm not successful, those same people will blame it on my gender.)

The question "Why is math so male dominated" is one that I have spent hours obsessing over. No one knows whether there are biological reasons that make women less interested in math or less good at math than men. But there are definitely a lot of sociological reasons that math is still male dominated. I grew up in a suburb of Los Angeles and attended public schools there, where most of the other kids seemed to be really superficial. Girls were supposed to be pretty, not smart. At some point during high school I actually noticed that a lot of girls acted dumb around the boys as a way of flirting; I guess they thought the boys wouldn't like them if they were too intelligent. So I'm sure that many girls shy away from pursuing math because it is "nerdy" and not "feminine."

I can think of many other sociological reasons that math is male dominated, but explaining all of these reasons would require much more time and energy than I have at the moment!

Ken: Yes, it is a complex issue! Do you have a favorite theorem that you've proved? Can you describe what it was like to prove that theorem?

Lauren: I have maybe two favorite theorems that I've proved. In one case, I struggled unsuccessfully for nine months to prove a particular conjecture. Then, after sort of giving up, I took a vacation and stopped thinking about it for a little while. When I came back to the conjecture, somehow everything came together and I was able to find the right ideas to prove it. In the other case, proving the theorem was much easier- once I had formulated the correct statement, I couldn't stop thinking about it. I soon had an insight and proved the theorem within the week.

In both cases, proving the theorem made me completely ecstatic...I don't remember so clearly but I was probably jumping up and down for awhile!

Ken: What is your daily life like as a mathematician?

Lauren: During the school year, I spend about half of my days on teaching stuff and the other half on research. Teaching activities include preparing lectures, giving lectures, talking to students in office hours, et cetera. Research activities include reading papers, talking to collaborators, and a fair bit of staring at the wall! During the summer and occasionally during the school year, I travel to conferences or visit with collaborators. All of this travel is one of the very nice perks of being in academia. At this very moment I'm at a conference at Oberwolfach, in the Black Forest of Germany (at a conference on "tropical geometry"), and next week, I'll be in Paris to work with two collaborators.

Ken: Do you have any advice for Girls' Angle members?

Lauren: I guess I would say that if you want to pursue math, then go for it! Math is challenging (for everyone), but if you have the interest and the patience then a mathematical career can be very rewarding ...And even if you don't go into math, having the logic skills that one develops by studying math will be useful for any career.

en.wikipedia.org/wiki/Arc_de_Triomphe

Ken: Thanks Lauren! Have a great time in Paris!

Tables of LCMs and GCFs

Table of Least Common Multiples

n \ m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	2	6	4	10	6	14	8	18	10	22	12	26	14	30
3	3	6	3	12	15	6	21	24	9	30	33	12	39	42	45
4	4	4	12	4	20	12	28	8	36	20	44	12	52	28	60
5	5	10	15	20	5	30	35	40	45	10	55	60	65	70	15
6	6	6	6	12	30	6	42	24	18	30	66	12	78	42	30
7	7	14	21	28	35	42	7	56	63	70	77	84	91	14	105
8	8	8	24	8	40	24	56	8	72	40	88	24	104	56	120
9	9	18	9	36	45	18	63	72	9	90	99	36	117	126	45
10	10	10	30	20	50	30	70	40	90	10	110	60	130	70	150

Table of Greatest Common Factors

n \ m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
3	1	1	3	1	1	3	1	1	3	1	1	3	1	1	3
4	1	2	1	4	1	2	1	4	1	2	1	4	1	2	1
5	1	1	1	1	5	1	1	1	1	5	1	1	1	1	5
6	1	2	3	2	1	6	1	2	3	2	1	6	1	2	3
7	1	1	1	1	1	1	7	1	1	1	1	1	1	7	1
8	1	2	1	4	1	2	1	8	1	2	1	4	1	2	1
9	1	1	3	1	1	3	1	1	9	1	1	3	1	1	3
10	1	2	1	2	5	2	1	2	1	10	1	2	1	2	5

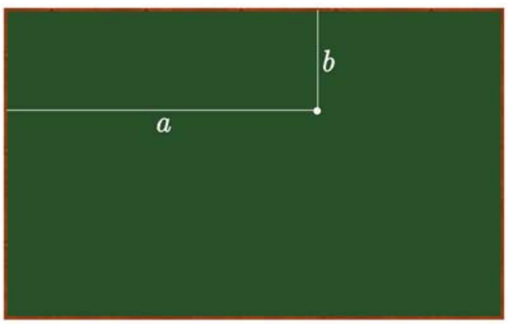
What patterns can you unearth?

Coordinate Systems

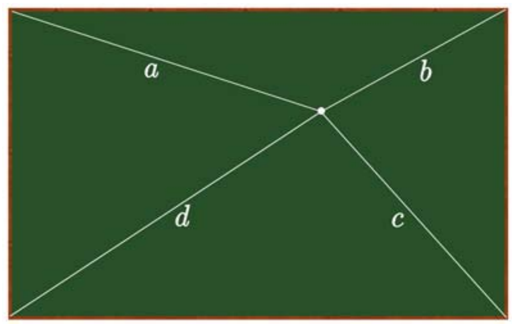
Coordinate systems are a way to systematically label points on some geometric object using numbers. There are many ways to think about coordinate systems. They can be thought of as generalizations of the number line. Recall that a number line is a line where each point is associated with a number. The association is made in such a way that many geometric properties of points on a line are reflected in properties of numbers. For example, if a, b, and c are the numbers that represent points P, Q, and R on a number line, and Q is between points P and R, then $a < b < c$.

At Girls' Angle, we began by thinking of ways to identify points on the chalkboard, which can be modeled mathematically by a rectangle.

Sylvia had the idea of identifying points on a chalkboard by measuring the distance of the point from the left edge and from the top edge of the chalkboard. **Trisscar** suggested identifying points on a chalkboard by measuring the distance of the point from the four corners.

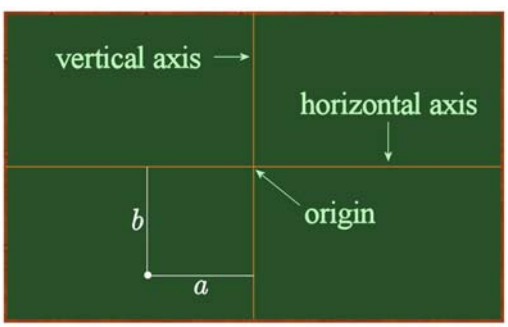


Sylvia's Coordinates

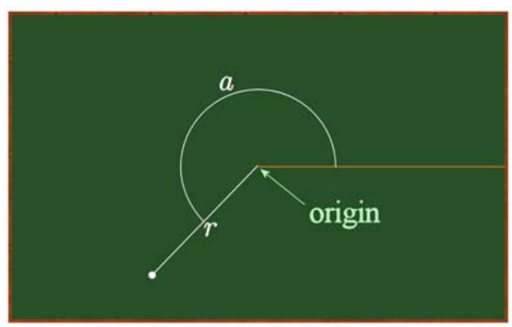


Trisscar's Coordinates

We also introduced Cartesian coordinates and polar coordinates.



Cartesian Coordinates



Polar Coordinates

In Cartesian coordinates, one horizontal and one vertical line are arbitrarily chosen. These special horizontal and vertical lines are called *axes* and the point where they intersect is called the *origin*. A point is indicated by measuring the shortest distances to each of these lines If the point is to the left of the vertical axis, then the coordinate labeled a in the figure is taken to be the negative of the distance. If the point is to the right of the vertical axis, then a is taken to be the distance. Similarly, b is taken to be the distance, or the negative of the distance, to the horizontal axis depending on whether the point is above or below the horizontal axis, respectively. Notice that if the point is on an axis, its distance to that axis is zero, so the sign doesn't matter.

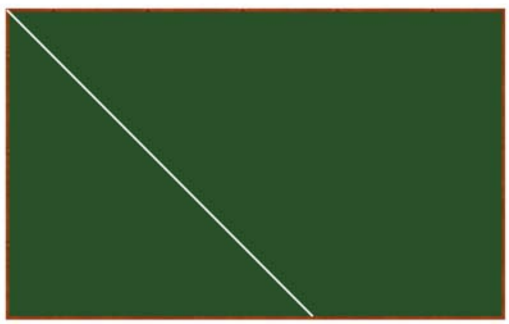
In polar coordinates, a special point called the *origin* is designated together with a ray emanating from this point. Points are designated by giving an angle and a distance. The angle is the angle that the ray must rotate through about the origin and in the counter clockwise direction so that the ray passes through the point. The distance is the distance of the point from the origin.

Because we were restricting attention to points on the chalkboard, the coordinates would never be negative in **Sylvia**'s coordinate system. Perhaps, however, you can see how to introduce negative numbers into her coordinate system by extending the reach of her coordinate system to the entire plane. With **Trisscar**'s coordinates, negative numbers are never needed, even if they are extended to cover the entire plane.

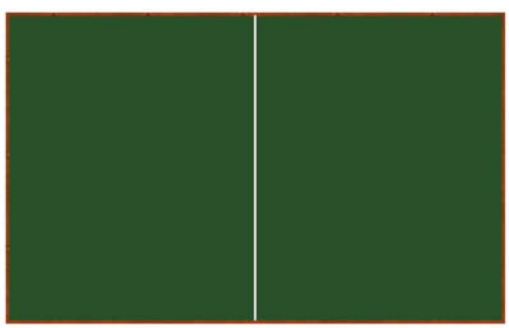
The connection with Algebra

By connecting numbers to geometry using coordinates, suddenly, solutions to algebraic equations can be given a geometric interpretation!

For example, consider the algebraic equation $a = b$. If we draw all the points whose coordinates satisfy this condition in **Sylvia**'s coordinate system, we get this picture:



If we do the same thing using **Trisscar**'s coordinates, we get this picture:



What picture do you get if you draw all the points whose coordinates satisfy $a = b$ in Cartesian coordinates? What picture do you get if you draw all the points whose coordinates satisfy $r = a$ in polar coordinates?

A digression on variable names

When I used the variables a, b, and r in the last paragraph, I was using the names that I had assigned the coordinates in the pictures of the various coordinate systems. I'll continue to do that for this article, but it is important to remember that the variables I used are just spur-of-the-moment choices and you are free to use whatever variables you wish for the various coordinates.

At school, it is very common to use the variable x for a and y for b in the Cartesian coordinate system. This naming scheme has become so widely used that there are now even teachers and textbooks that call Cartesian coordinates the "xy-coordinates". They will even call the horizontal axis the "x-axis" and the vertical axis the "y-axis". This association of variable names to the concept is too strong, but you will have to put up with it at school.

Also, in polar coordinates, I used the variable a for the angle. Typically, instead of a, the variable θ, pronounced "theta", is used. It's a Greek letter.

When you write a math paper, you have to define all the variables you are going to use so that people know what you mean when you use them. I know at least one math paper where the author used the symbols ♣, ♦, ♥, and ♠ as variables!

A special comment on Trisscar's coordinates

One feature about **Trisscar**'s coordinates that separates it from the other three we've discussed is that her coordinate system involves *four* coordinates. But, even though there are four coordinates, we are not free to specify each of them. For example, there is no point where $a = 0$, $b = 0$, $c = 0$, and $d = 0$. In the other three coordinate systems, you can assign numbers freely to the coordinates and you will be referring to some point. In **Trisscar**'s coordinates, you cannot do that.

Some questions

1. In polar coordinates, the set of points that satisfy $r = 1$ form a circle centered about the origin. Here's a big challenge: can you figure out an equation that involves the variables a and b whose solutions correspond to a circle in Cartesian coordinates? Hint: Think about the Pythagorean theorem. If you are still having trouble, read Lauren McGough's article on page 13 for the answer!

2. Can you invent coordinate systems for the surface of a sphere? the surface of a donut? for three-dimensional space?

3. The fact that you cannot arbitrarily specify the values of **Trisscar**'s coordinates means that there are constraints that those coordinates satisfy. For example, the sum $a + b$ *must be greater than or equal to the length of the chalkboard*. Do you see why? Can you think of other constraints on her coordinates?

Challenge

Draw a picture on the chalkboard and pick a coordinate system. Now see if you can write down equations whose solutions correspond to your picture.

Treasure!

The study of coordinate systems culminated in a massive treasure hunt. Girls were given big maps of Cambridge, or, in one case, the floor plan of a dancing stage. Each girl had to find the location of a treasure by solving a math problem. The solution to the problem gave the coordinates of a hidden treasure in some coordinate system on the map. On a computer, they could erase the map with a very tiny eraser. If they were erasing in the right place, the word "Yes!" would appear.

August's Treasure Hunt Puzzle

You made the cool observation that a point on the chalkboard creates four triangles, one for each side of the chalkboard. You could locate points by giving the side lengths of all of those triangles. But, we're going to tell you where your treasure is in terms of the areas of those triangles! Actually, we're not even going to tell you the areas! Instead, we'll tell you the ratio of the area of the top triangle to the area of the bottom triangle. It's one quarter. Also, we'll tell you the ratio of the area of the left triangle to the area of the right triangle. It's seven and a third.

Good luck!

This is an overview of the hunt. When devising the hunt puzzles, we tried to create puzzles that would challenge each girl and, at the same time, hint at important mathematical concepts. One cannot expect a theory to be detected from a single example, but by providing such examples, we believe that when the girls some day encounter the theory, the theory will be more easily digested. In this article, I will only go so far as to mention the theories that were hinted at.

When we were introducing coordinate systems on the chalkboard, **August** observed that each point on the chalkboard divides the chalkboard into four triangles. We used this fact to devise her puzzle. An interesting feature of **August**'s puzzle is that the puzzle solution would apply no matter the scale of the map. All that matters is that the maps be rectangles that show exactly the same region of Cambridge. **August** was the first to solve her puzzle and as you can see by the cover of this Bulletin, she unearthed the treasure with great efficiency!

Tree's Treasure Hunt Puzzle

by Lauren McGough

Your treasure may be found underneath the stage in the Wang Theater, where years ago the toy soldier and Clara could be found rehearsing for The Nutcracker. Directly above the spot of your treasure, the toy soldier and Clara would meet to do a beautiful lift. Three seconds before the lift, the toy soldier would begin ten feet away from the left wall of the stage (it is left if you are in the audience), and sixteen feet back from the front of the stage (the part of the stage closest to the audience). He would dance in a straight line to the right and move four feet to the right every second. Three seconds before the lift, Clara would be twenty five feet away from the left wall of the stage, and twenty two feet back from the front of the stage (it is a large stage). She would dance in a straight line, too, though her dance was diagonal, as every second, she would do the equivalent of dancing two feet toward the front of the stage, and one foot left (so she is running "southwest", in a sense). Find where they would meet, and find your treasure!

Good luck!

Tree loves dance. So her puzzle involved ballet, something she's very familiar with. But the math in her puzzle was probably unfamiliar to her! The movement of two dancers across a stage was indicated by giving each dancer's starting coordinates, and then indicating how these coordinates would change each second. Such a description of the motion of an object is called a *parametric* description. The equations that give the coordinates as a function of time are called *parametric equations*. Time serves as the *parameter*. Parametric equations are very useful in the physics of motion.

Several girls had to solve equations that involved two or more variables. **The Cat**'s problem essentially came down to solving this system:

$$\begin{aligned} 2a + 2b &= 31 \\ ab &= 55 \end{aligned}$$

Solving this system is essentially equivalent to solving the equation $2x^2 - 31x + 110 = 0$. This is an example of a *quadratic* equation.

Sylvia not only got a system of equations to solve, but she also had to do a weighted average. On top of that, the first puzzle I gave her was incorrect because I made a mistake when I formulated it! So **Sylvia** ended up solving two puzzles. At the following meet, **Sylvia** lamented that she got the easiest problem. On the one hand, it is nice to think that **Sylvia** thinks her problem was relatively easy! On the other, for the record, **Sylvia**, your problem was NOT the easiest problem. I wouldn't put that in writing if it weren't true. I'll also put in writing that if we ever have another Treasure Hunt, we'll probably use caffeine free treasures. **Sylvia**'s system of equations is an example of a system of *linear* equations and many colleges devote an entire course to developing *linear algebra*.

Let's see why the system of equations in **The Cat**'s puzzle is essentially equivalent to solving the quadratic equation $2x^2 - 31x + 110 = 0$. Suppose a and b are solutions to the system of equations

$$\begin{aligned} 2a + 2b &= 31 \\ ab &= 55 \end{aligned}$$

Now consider the equation $(x - a)(x - b) = 0$. This involves a product, and if the product of two numbers is zero, then at least one of the numbers must be zero. That means that if x is a solution to this equation, then $x - a = 0$ or $x - b = 0$, that is, $x = a$ or $x = b$. And if $x = a$ or $x = b$, then x will be a solution. So the solutions to $(x - a)(x - b) = 0$ are the values a and b.

If we multiply out $(x - a)(x - b)$, we get

$$\begin{aligned} (x - a)(x - b) &= (x - a)x - (x - a)b && \text{(distributive law)} \\ &= x^2 - ax - xb + ab && \text{(distributive law)} \\ &= x^2 - (a + b)x + ab && \text{(distributive law)} \\ &= x^2 - 15.5x + 55 && \text{(because } a + b = 15.5 \text{ and } ab = 55) \end{aligned}$$

So $(x - a)(x - b) = 0$ is the same equations as $x^2 - 15.5x + 55 = 0$, and the equation in the text is obtained by multiplying both sides of this last equation by 2.

Trisscar's puzzle utilized her coordinate system. With the map being a square 25 units on a side, it is impossible to have a point on the interior of the map whose Trisscarian coordinates are all whole numbers. So the solution to the equations in **Trisscar**'s puzzle involved square roots of non-perfect squares. However, **Trisscar** cleverly evaded this problem by finding the closest integer approximation to the solution and digging up a slightly larger area. Often, in mathematics, the exact solution is extraordinarily difficult to find. In such cases, finding such approximations can still be valuable, but when approximations are used, understanding how far off the approximation is from the actual solution is important. There are many, many mathematical arguments that rely on using approximations and analyzing the errors. Such an argument was used, for example, to set the value of π in your calculator to have the right digits for the degree of accuracy your calculator can handle.

Congratulations to all the girls for solving their puzzles!

We'd like to encourage the girls to invent their own puzzles. If you think of one, send it to us and we'll try to solve it!

On Being an Artist… With Equations!

by Lauren McGough

To start this article, I must admit that I am no artist in the traditional sense of the word. In fact, I can not even draw a circle free-handedly. Whenever I try to draw a circle, it comes out bumpy and lopsided, hardly like the beautiful mathematical object I want to represent!

Luckily for me, and for anybody else with this problem, I have another way of representing circles, of any size, whenever I may wish to have a circle at hand! This method of representation comes from the tool of coordinate systems we've been discussing at Girls' Angle over the past few weeks. That's right, you guessed it – I can use math to draw circles!

Before we draw circles with math, however, we have to ask ourselves, what exactly is a circle? Specifically, how can we define a circle mathematically? As you might remember from our experiments with the Banana Algorithm and the Define This Game, in math it is very important to have precise definitions of what we are working with, so that there can be no confusion regarding what we are talking about! So let's ask: What is a circle?

To get to a definition, imagine taking a pencil, placing it on a table horizontally, holding the eraser in one place, and sweeping the pointy end of the pencil all the way around in a full rotation, until it is back to where it began. What shape would the tip of the pointy end of the pencil trace out? Maybe you guessed it – it would trace out a circle! You can imagine getting a circle of any size, just by choosing differently sized pencils (assuming, of course, that a pencil can be as big or small as you want it to be). This might lead you to think of the following definition of a circle:

A circle is the set of points in a plane that are a specific distance from a specific point.

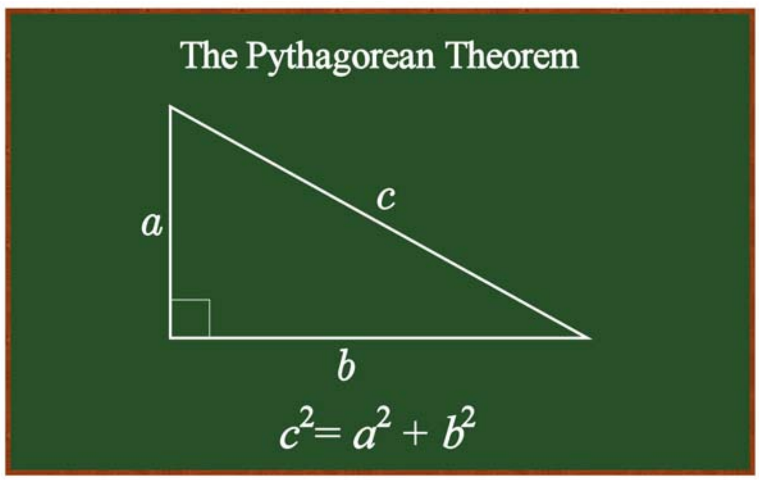
The "specific point" is just like the location of the eraser in the pencil example, while the "specific distance" is the length of the pencil. These two ideas have names: the "specific point" is called the *center* of the circle and the "specific distance" is known as the *radius* of the circle.

Now that we know a bit about what a circle actually is, let's think about how we can use equations and coordinate systems to represent circles much more accurately than my hand can. Well, there are lots of ways to represent a circle in the plane, depending on the way that we are specifying points; that is, on our choice of "coordinate system."

First, let's look at how we can represent a circle in polar coordinates. To remind you, in polar coordinates, we pick a point and call that the "origin." Then, we draw a ray emanating from the origin (usually this ray is horizontally oriented and points to the right). We specify points by giving the length of the line segment connecting the point to the origin, and the counterclockwise angle this line segment makes with the ray. Let's call the length r and the angle θ. (Recall that θ is the Greek letter "theta", often used in math for representing angles). Let's see how we can use polar coordinates to describe any circle whose center is the origin.

Well, we defined a circle as "the set of points in a plane that are a specific distance from a specific point." We already have the specified point – that is the origin. What is this specific distance? That is just the distance from the points on our circle to the origin. But wait, in our coordinate system, this is just r! So let's call our specific distance "d" – it could be any number, depending on how big of a circle we want to draw. Then an equation to represent the circle with radius d and centered at the origin in polar coordinates is just r = d! For example, if we wanted a circle of radius 5, we could use the equation $r = 5$ to specify that circle. The set of points that satisfies this equation is exactly the set of points that are 5 units away from the origin – at any angle. Taken for all the possible values of θ, this is just a circle of radius 5, centered about the origin!

Now, what if we wanted to represent a circle of radius 5 using Cartesian coordinates – that is, taking one point to be the origin, drawing a horizontal and vertical line through the origin, and specifying a point by talking about its Cartesian coordinates with respect to these axes (see page 8)? Again, let's take the center of the circle to be the origin, just for simplicity. So, as we saw in the previous example, we want a way to say that "the distance from the origin to the points whose coordinates satisfy this equation is equal to a specific number," where the "specific number" is just the radius of the circle. So, our question becomes, how can we represent the distance of any point from the origin using the Cartesian coordinate system? Note that all we have now are the Cartesian coordinates of any point, which are, up to sign, the distances of the point to each axis. We need to figure out a way of expressing the distance of any point from the origin given its Cartesian coordinates.

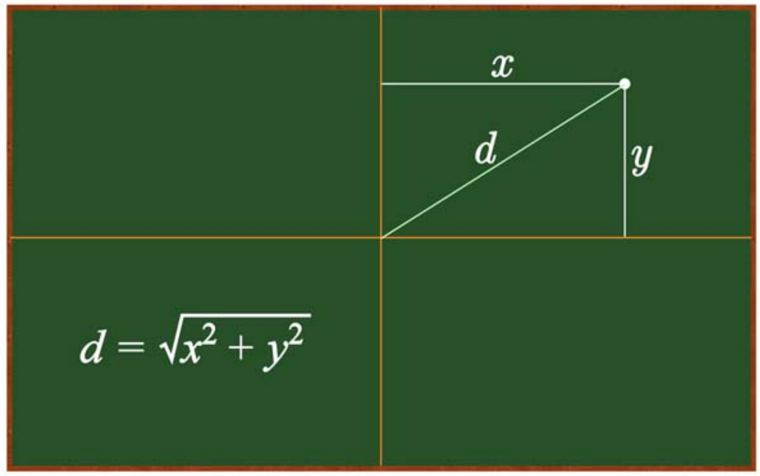


To do this, we can use another aspect of geometry that we discussed at Girls' Angle – the Pythagorean Theorem. As a reminder, the Pythagorean Theorem states that for any right triangle with leg lengths (the lengths of the two shorter sides) a and b and hypotenuse (the length of the longest side) c, we have $a^2 + b^2 = c^2$.

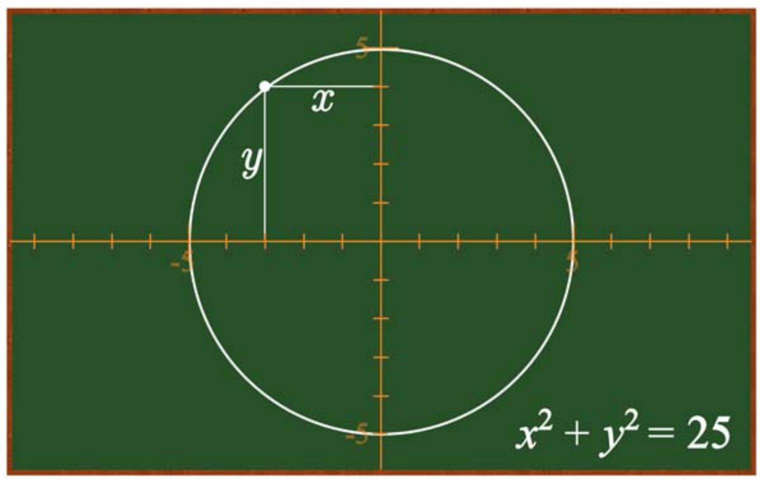
Let's think about how we can apply this to finding the distance of a point in the Cartesian coordinate system from the origin given its Cartesian coordinates. Let's label these coordinates x and y, where x, up to sign, is the distance from the vertical axis and y, also up to sign, is the distance from the horizontal axis. Draw a horizontal line segment from the point to the vertical axis and a vertical line segment from the point to the horizontal axis. The lengths of these line segments are $|x|$ and $|y|$, respectively. (Recall that $|a|$ is the absolute value of a.) These line segments form two sides of a rectangle with our point at one corner. Now, consider drawing a line from our point to the origin, so that we have cut our rectangle into two right triangles by drawing a diagonal. The distance of the point to the origin will just be the length of this line from our point to the origin. We can find this length using the Pythagorean Theorem – can you see how? (Warning: answer given on the next page!)

We can use the right triangles we have formed by cutting the rectangle in half: each right triangle has leg lengths of $|x|$ and $|y|$ (it doesn't matter which triangle we use because they are *congruent*).

Let's call the point's distance from the origin *d*. Because *d* is just the hypotenuse of a right triangle, we have the following: $x^2 + y^2 = d^2$. (Why can we get away without the absolute value symbols in this formula?) So, for a point whose Cartesian coordinates are *x* and *y*, if we want to say that its distance from the origin is *d,* this is the same as saying that $x^2 + y^2 = d^2$! We can use this to write an equation for a circle: for any radius *r* we want, we know that the points that are a distance of *r* units from the origin satisfy $x^2 + y^2 = r^2$, and any point whose Cartesian coordinates satisfy this equation represents a point on the circle, so in fact, this is an equation of the circle! As an example, let's "draw" the circle of radius 5 centered at the origin in Cartesian coordinates. This is just the same as saying $x^2 + y^2 = 5^2$ or $x^2 + y^2 = 25$. With that, we have an equation for the circle! We're done!



So, even if you are uncoordinated like me, you, too, can use equations to represent perfect circles of any size. In fact, these mathematical representations of circles are ideal and no artist, no matter how talented could produce a circle so perfect! So far, we have two equations to represent just a circle of radius 5: in polar coordinates, *r* = 5; in Cartesian coordinates, $x^2 + y^2 = 25$.



In this article, we've discussed how to "draw" circles in polar coordinates and Cartesian coordinates, as long as our circle is centered about the origin. As a problem for further exploration, can you figure out how to "draw" a circle centered about *any* point in the coordinate system (not necessarily the origin)? Or can you figure out how to write equations for circles in other coordinate systems you might be interested in exploring, like the one you used for the treasure hunt? In the end, there are lots of ways of using mathematical equations and coordinate systems to connect equations with geometry – so that even if you can't draw (like me), you can have a way of expressing geometrical figures using equations, instead of your hands.

The Number Guessing Game

In this variant of number guessing games, a mentor thought of two or more numbers. Girls were allowed to ask any question that could be answered by a single number but whose statement did not require the word "number" or any synonym of "number" in singular form. Each time the game is played, none of the questions from prior rounds may be used again.

For example, a girl could ask, "What is the larger of the two numbers?" and "What is the smaller of the two numbers?" For some situations, this would resolve the game pretty quickly! However, once asked, these questions cannot be used again. (The girls never actually used these questions.)

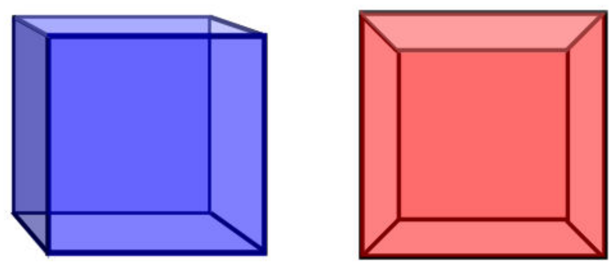
The first day we played this game, the mentors always used two numbers.

In one round, two questions were asked: "What is the sum of the two numbers?" and "What is the positive difference of the two numbers?"

In another round, the numbers were guessed after these two questions were asked: "What is the product of the two numbers?" and "What is the larger of the two quotients of the numbers?"

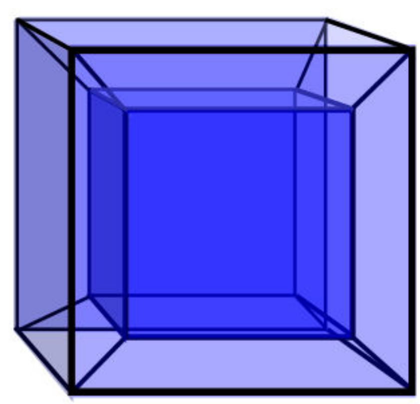
At the last meet of the first session, we went to three numbers, and the numbers were guessed correctly!

Drawing Hypercubes



Cubes. Or, more precisely, projections of cubes on a plane.

First, look at drawings of a cube. The drawings are not actually cubes. For one thing, they're flat, but a cube is three dimensional. Notice, that in some drawings, some line segments that represent non-intersecting line segments of the cube intersect in the drawing. This often happens when you draw a two-dimensional representation of a three-dimensional object. Also, notice that many angles which are right angles in the actual cube become something else in the drawing.



Projection of a hypercube on the plane.

Now, consider the hypercube. Although, unlike the cube, we cannot build a real one because we don't have actual physical access to a fourth spatial dimension, we can make an actual *drawing* of one. Just as we can project a cube onto a piece of paper, we can project a hypercube. You can also make a real model that represents the projection of a hypercube onto our three dimensional world. And, just as with cubes, the drawing of the hypercube shown here is just one of infinitely many different perspectives on it. Can you draw others?

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete.

<u>Fourth Meeting – October 11, 2007</u>

Mentors: Cammie Smith Barnes, Alison Miller, Beth O'Sullivan, Ken Fan, Lauren McGough

This meet began with individual instruction. Topics worked on included the geometry of cubes, equations with one variable, and the Introduction problem described in the first issue of this Bulletin.

Just to mention one of these topics, **Trisscar** and Cammie explored the various cross sections that can be obtained from a cube. **Trisscar** saw that you could get a square and a triangle. What else is possible?

For the second half, each girl wrote down the largest number they could think of in 30 seconds! The girls got interested in large numbers after Sarit's visit in meet three. Sarit talked about exponentials and how quickly they grow. Back in January, at MIT, there was a contest between two philosophers, MIT associate professor Agustin Rayo and Princeton associate professor Adam N. Elga, to see who could write the largest number on a chalkboard. We were going to do something similar, but sometimes things don't go as planned: The first numbers that the girls produced were already quite a challenge to order! In fact, the order of these numbers is yet to be determined! Here are the numbers that **The Cat**, **Resday**, **Trisscar**, **August**, and **Sylvia** created:

$$999{,}999^{99{,}999^{9{,}999^{999^{99^{9}}}}}$$

$$10{,}000{,}000{,}000^{2^{2^{2^{2^{2}}}}} \times 2{,}000^{2}$$

$$99^{99^{99^{99^{99^{99^{99^{99^{99^{99^{99^{99^{99^{99^{99}}}}}}}}}}}}}}$$

$$9^{9^{9^{9^{9^{9^{9^{9^{9^{9^{9^{9^{9^{9}}}}}}}}}}}}}$$

$$999{,}999{,}999{,}999{,}999$$

Some Things to Ponder

1. What regular polygons can be found as cross sections of a cube? What are the various cross sections of a cylinder? a sphere? a pyramid?

2. What is the correct order of the five big numbers shown above?

Fifth Meeting – October 18, 2007

Mentors: Beth O'Sullivan, Ken Fan, Lauren McGough

Special Guest: Kimberly Pearson, Harvard School of Public Health

We talked about the Pythagorean theorem. We proved the theorem and explored a way to construct right triangles with all integer side lengths using origami.

Kim Pearson discussed the problem of trying to decide whether a certain characteristic of a population can be considered normal, or whether it points to something unusual and would be worth investigating. For example, a certain community may have a certain incidence of chicken pox. How can one decide if the incidence of chicken pox is typical for communities of that size, or if there is something unusual about the community that is causing them to have more or fewer cases of chicken pox than normal? To that end, Kim introduced the statistical notions of mean, median, mode, range, and standard deviation. She explained that it is often difficult to determine the mean value of some characteristic of the population because it becomes too costly to sample the entire population. So instead, researchers sample a subset of the population and compute the mean of the subset. Under certain assumptions, one can then construct a *confidence interval* around the computed mean which tells us that the actual mean of the entire population has a certain high probability of being inside this confidence interval. To illustrate, the girls sampled bags of M&Ms and counted the percentage of yellow ones in each bag. From this, they could deduce a confidence interval that informed us that the actual percentage of yellow M&Ms in the world sat between two bounds with a 90% probability.

The assumptions made to draw such statistical inferences are rather subtle. Never forget that these inferences require certain assumptions about the way things are distributed! When statistics is applied to the real world, these assumptions, in the end, cannot, in fact, be confirmed. However, if the assumptions are accepted the logical deductions that one can make are often quite startling!

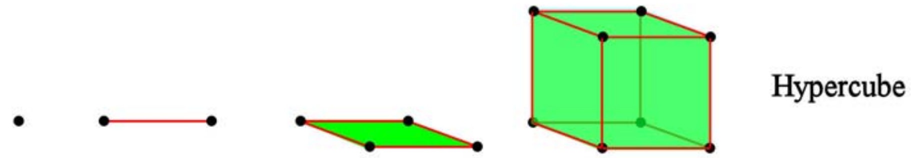
Sixth Meeting – October 25, 2007

Mentors: Beth O'Sullivan, Alison Miller, Hana Kitasei, Ken Fan, Lauren McGough

Special Visitor: Tamara Awerbuch, Harvard School of Public Health

We began looking at coordinate systems, which are systematic ways of labeling the points (see page 8) of some geometric object.

At the end of the meet, we had our first and only Dream Time of the first session. Dream Times are wanderings through some mysterious corner of the mathematical universe, led by a mentor. For the first Dream Time, we followed **Sylvia**'s vision of seeing a cube as a thickened quadrilateral from the second meet into the fourth dimension.



Dream Time. A point moving in a straight line covers a line segment. A line segment moving along a straight line perpendicular to itself covers a square (drawn in perspective). A square moving along a straight line perpendicular to itself covers a cube (**Sylvia**'s vision). A cube moving along a straight line perpendicular to itself covers a *hypercube*.

We imagined what we would see if a hypercube passed through our world. If a cube passed through a plane parallel to one of its faces, at the moment of contact, a square would appear in the plane, and this square would persist as the cube passed through. Once the cube passed all the way through, poof!, the square in the plane would disappear. By analogy, one scenario of a hypercube passing through our world would be the sudden appearance of a cube, which would persist as the hypercube passed through, and then, suddenly, the cube would disappear!

Now, imagine what we would see in a plane if a cube passed through but in such a way that its major diagonal was perpendicular to the plane. Figuring out what you would see has everything to do with the cross-sections of a cube that **Trisscar** and Cammie explored during the fourth meet. **The Cat** wondered about five-dimensional hypercubes and beyond…and she also wondered how such objects could be depicted. See page 16.

Questions

Suppose our world is perpendicular to the major diagonal of a hypercube. What would we see as that hypercube passed through our world?

How many vertices does an n-dimensional hypercube have? How many edges? How many faces?

Tamara Awerbuch introduced the idea of a *difference equation* using mathematical models of population growth as a premise. Here's the idea: The size of a population changes with time, in other words, the size of a population is a function of time. In some applications, it would be useful to know a formula that gave the population as a function of time. However, it may not be so easy to see what this formula is. So instead, try to understand *how the population changes from one moment to the next*. This might be an easier problem because it involves studying how the population changes over short periods of time instead of trying to grasp how the population changes over very long periods of time. By providing a precise description of how the population changes, say, over a one day period, you can then draw out the behavior over several days by applying this precise description over and over again. Tamara made graphs of population size versus time with the aide of a computer to illustrate. All kinds of different graphs result from different difference equations.

Tamara's talk is the tip of an iceberg! A huge and extremely important unsolved problem in mathematics and science today is the question of how to understand the large scale behavior from knowledge of the small scale behavior. In Tamara's talk, the small scale behavior corresponds to understanding how a population changes from one day to the next. The large scale behavior is to understand how the population will behave over centuries or millennia.

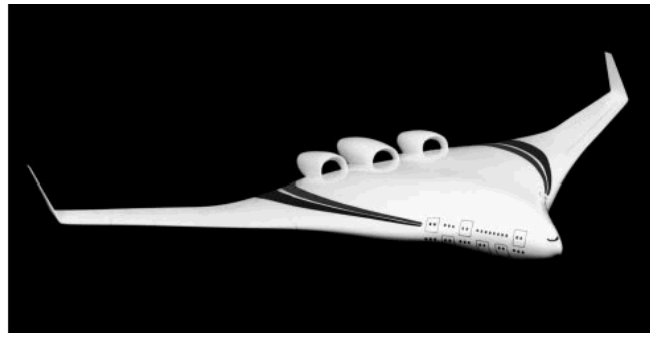
Here are more examples of large scale behavior that would seem to be explainable in terms of small scale behavior but where the connection remains elusive: ant colonies and ants, brains and neurons, bird swarms and birds. Instead of the words “small” and “large”, the words “local” and “global” are often used.

Seventh Meeting – November 1, 2007

Mentors: Cammie Smith Barnes, Beth O’Sullivan, Hana Kitasei, Ken Fan, Lauren McGough

Special Visitor: Karen Willcox, Department of Aeronautics and Astronautics, MIT

We continued exploring coordinate systems (see page 8).



Dr. Willcox designed aircraft like this flying wing for Boeing.

Karen Willcox explained how to use math to design a better airplane. First, she explained that with so many parameters, it is impossible to test all possible airplane designs, even with the aide of computers. On the spot, the girls came up with a number of parameters including the weight of the plane, the distance it can travel, its speed, and its wingspan. So one idea is to isolate a single parameter and make a few planes that are identical except with respect to this one parameter. Test these planes for their flight characteristics and try to use this data to understand how the flight characteristics depend on the isolated parameter. If successful, one would then be able to determine the value of the parameter which maximizes desirable flight characteristics and/or minimizes undesirable ones. To that end, the girls all folded the same paper airplane adding a paper clip at various positions along the fuselage to isolate the parameter of weight distribution. These planes were flown and the distance they flew was measured. A graph of the data was interpolated to produce a curve that had a maximum somewhere between the second and third paper clip positions.

After the meet was over, Karen gave the mentors an interesting piece of advice: bring along Vic’s Vaporub if you ever fly on the airplane used by NASA’s Reduced Gravity Research Program.

Eighth Meeting – November 8, 2007

Mentors: Cammie Smith Barnes, Beth O’Sullivan, Alison Miller, Ken Fan, Lauren McGough

We had a massive treasure hunt. See page 11.

Ninth Meeting – November 15, 2007

Mentors: Beth O’Sullivan, Hana Kitasei, Ken Fan, Lauren McGough

Special Visitor: Elissa Ozanne, Harvard School Medical School

We began by discussing general strategies for solving equations, using the equations from the Treasure Hunt as examples. One basic principle that is exploited over and over is that if you do the same thing to both sides of a true equation, you will get a new equation that is also true. By carefully selecting what you do to both sides, you may be able to simplify the equation and bring it closer to solution. Another principle which we didn't really discuss, although **The Cat** used it instinctively in her hunt solution, is the principle of *substitution*. We'll discuss substitution later.

During this discussion, somehow the question of the parity of the number 9^9 came up. **Trisscar** explained how parity behaves under multiplication. **The Cat** then made a keen observation. Parity and multiplication behave the same way as the logical operator AND!

Parity Rules		
×	even	odd
even	even	even
odd	even	odd

Truth Table		
AND	false	true
false	false	false
true	false	true

Observations that recognize similarities in different structures often form a bridge between seemingly disparate subfields of mathematics and lead to deeper understanding. Connecting logic with algebra is a very fruitful idea!

We digressed briefly to discuss the logical operators NOT, OR, AND, NAND, NOR, and XOR.

We also played a number guessing game (see page 16).

Elissa Ozanne analyzes the benefits and risks of various treatments and diagnostics related to breast cancer, with the aim of helping women make decisions that are in their health's best interest. The bewildering array of options and factors that affect such decisions rapidly become overwhelming without her work. The main theme of Elissa's workshop with Girls' Angle was that mathematics is a tool that can be used to help make the most rational decisions. She presented girls with various games that involved random elements. Each game had a payout for winning, and this gave the game a certain value. Girls had to decide how much they would be willing to pay for a chance to play. In effect, this means that the girls had to decide exactly what the value of the game was. The games became more complicated and some involved choices with unknown elements. Elissa discussed some of the mathematical tools used to handle such situations.

One of the scenarios revealed a kind of irrational behavior in humans. She presented the girls with two jar choices. In one, a jar contains 50 red marbles and 50 blue marbles. In the other, the jar contains an unknown number of red and blue marbles, but the total is still 100 marbles. You choose which jar to pick a marble from, and then if you pick a red marble, you make \$10, but if you pick a blue marble, you make nothing. The question is: Which jar would you choose to pick from? Most people select the first jar with 50 red and 50 blue marbles, but mathematically, it doesn't matter which jar is chosen; both give a 50/50 chance of making \$10.

I think it would be an interesting project to change the mix in the first jar to, say 49 red marbles and 51 blue marbles and ask people again. Would most people still choose the jar with a definite number of red and blue marbles? If so, this aspect of irrational behavior can actually be proven to be detrimental.

Tenth Meeting – November 29, 2007

Mentors: Cammie Smith Barnes, Beth O'Sullivan, Alison Miller, Hana Kitasei, Ken Fan, Lauren McGough

Special Visitor: Dina Sonenshein, Upstairs on the Square restaurant

The last meet of the first session had a festive atmosphere full of professionally baked chocolate chip cookies, thanks to Dina and the assistant chef at Upstairs on the Square!

Our last challenge for the girls was to play the number guessing game with three numbers (see page 16) instead of two.

The Cat, **August**, and **Trisscar** did so well with the three numbers that we gave them additional systems of equations in three unknowns to solve. One was this system of linear equations:

$$\begin{aligned} a + b + \mathrm{c} &= 6 \\ a + b - c &= 0 \\ a - b - c &= -4 \end{aligned}$$

The other was this non-linear system:

$$\begin{aligned} a + b + \mathrm{c} &= 9 \\ ab + ac + bc &= 27 \\ abc &= 27 \end{aligned}$$

This last system is equivalent to solving the *cubic* equation $x^3 - 9x^2 + 27x - 27 = 0$.

Dina Sonenshein explained how math is used to bake a better chocolate chip cookie. She discussed the use of fractions in recipes and how to convert between different units of measurement. She also explained how to alter a recipe for different numbers of people or for different sizes of equipment. The girls mixed their own cookie dough and were able to take dough home with them to bake at home. Dina also brought two different types of chocolate chip cookie made from very similar recipes and she talked about how the subtle differences in the recipes led to such different cookies. One was thin and chewy; the other was thick and crumbly.

Dina was the professional baker for Upstairs on the Square, but she recently switched to the role of financial manager. One never knows how knowing math will affect one's career choices!

Special Announcements

Congratulations to **The Cat** for all the hard work she put into the introduction problem. She found the formula!

Calendar

Session I: (all dates in 2007)

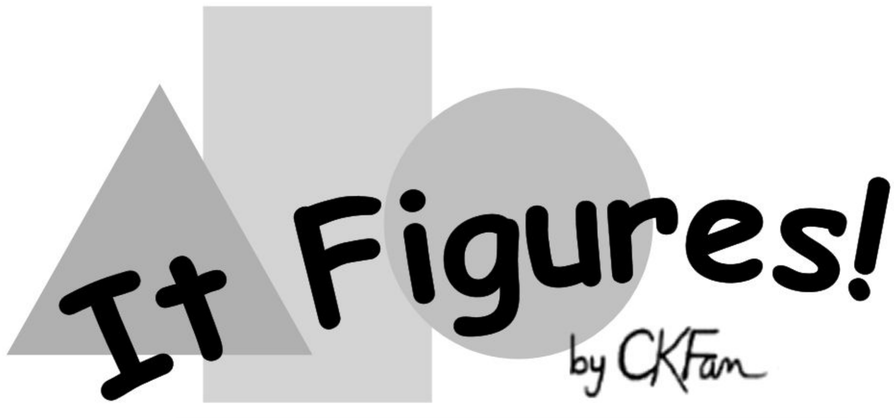
September	20	Grand opening!
	27	Guest: Elissa Ozanne
October	4	Visitor: Sarit Smolikov, Harvard Medical School
	11	
	18	Visitor: Kimberly Pearson, Harvard School of Public Health
	25	Visitor: Tamara Awerbuch, Harvard School of Public Health
November	1	Visitor: Karen Willcox, MIT Aeronautics Department
	8	
	15	Visitor: Elissa Ozanne, Harvard Medical School
	22	No meet – Thanksgiving.
	29	Visitor: Dina Sonenshein, Upstairs on the Square

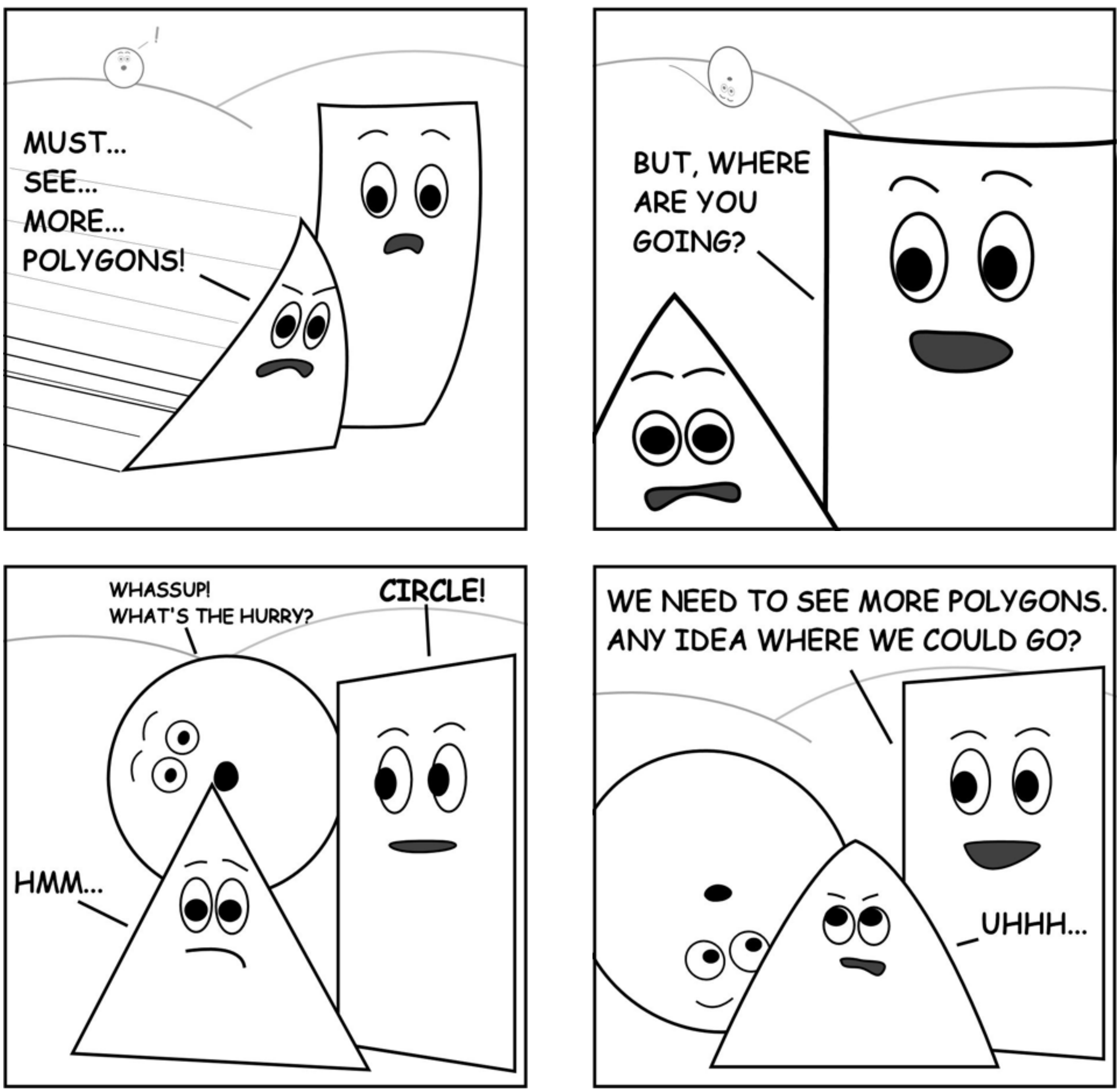
Session II: (all dates in 2008)

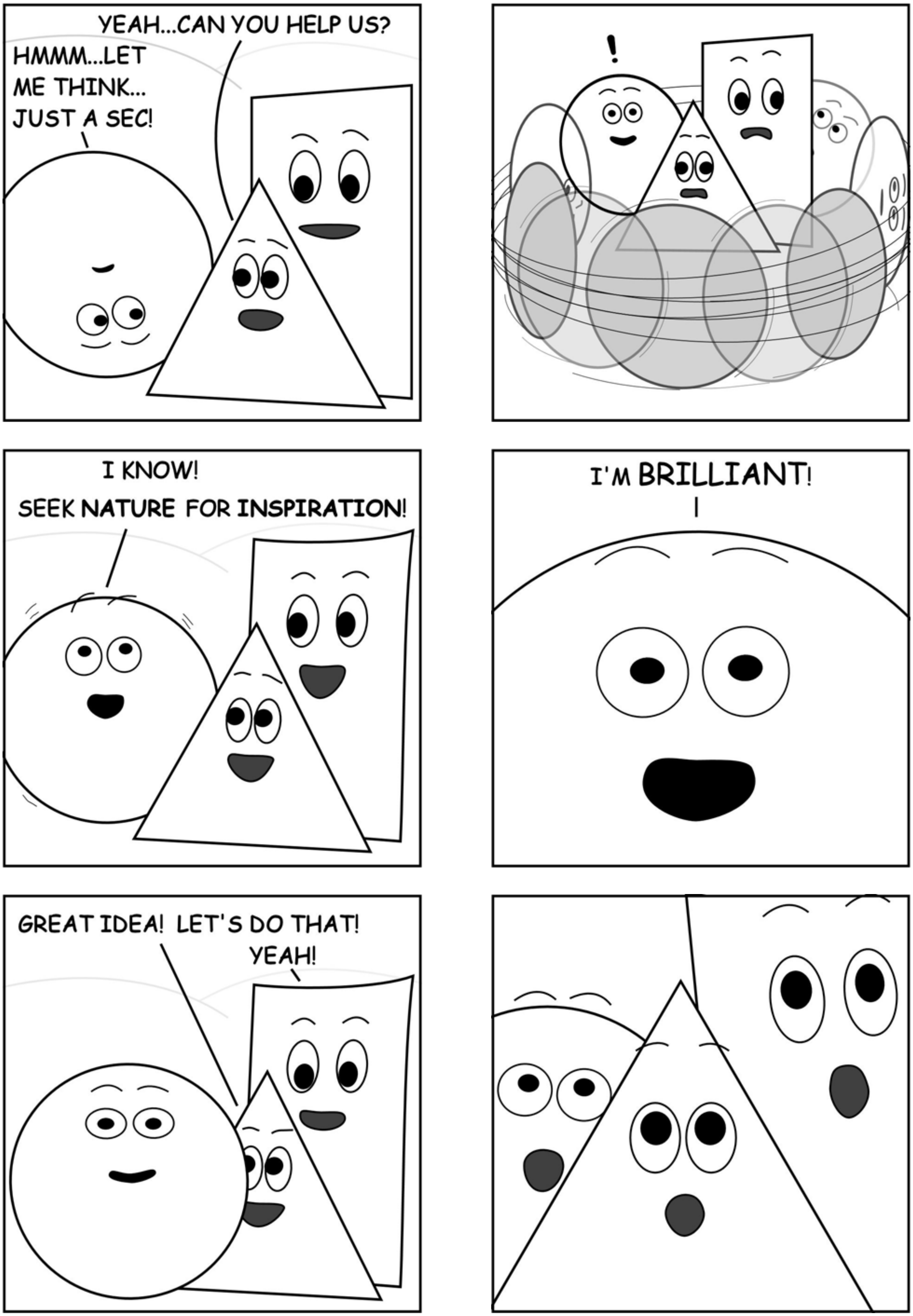
January	31	Start of second session!
February	7	
	14	
	21	No meet
	28	
March	6	
	13	
	20	
	27	No meet
April	3	
	10	
	17	
	24	No meet
May	1	
	8	

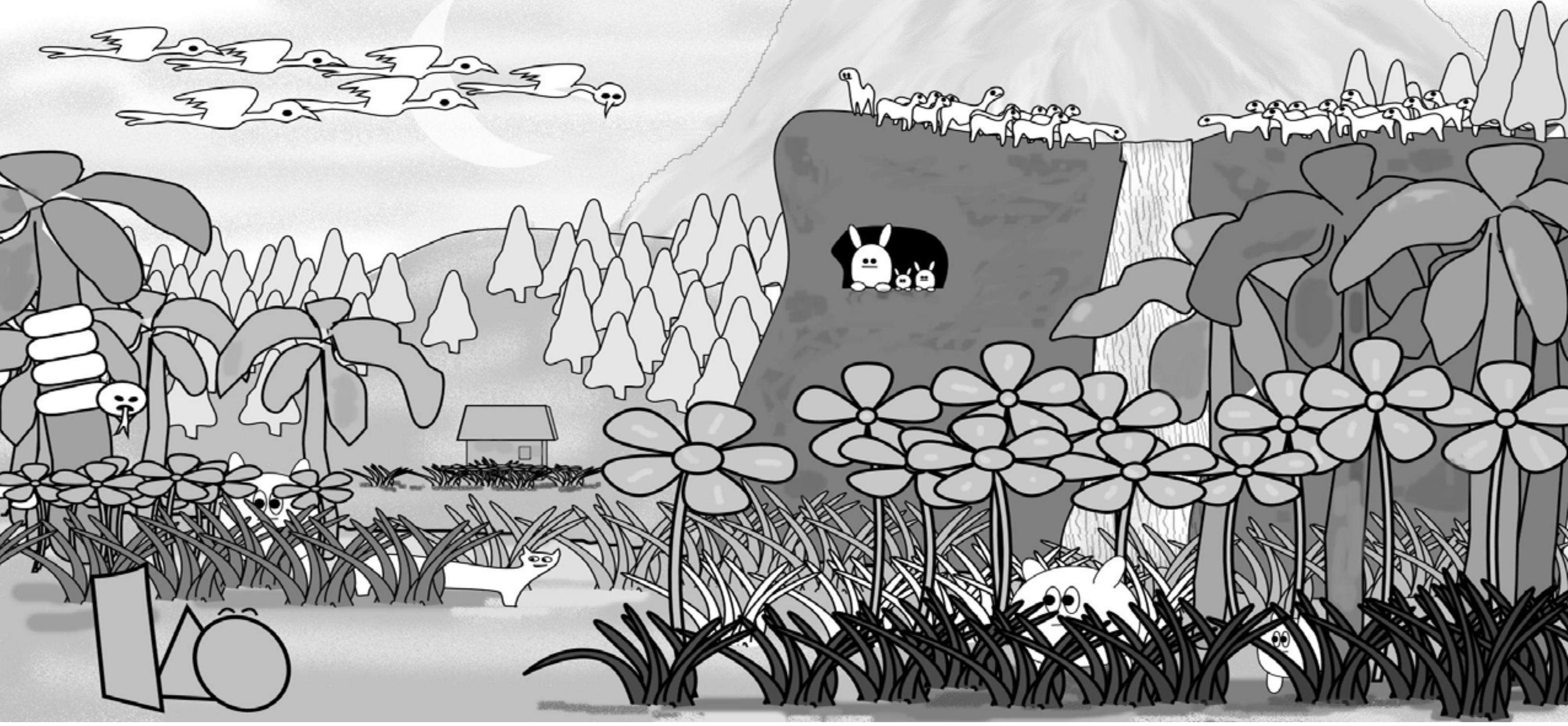
Feedback

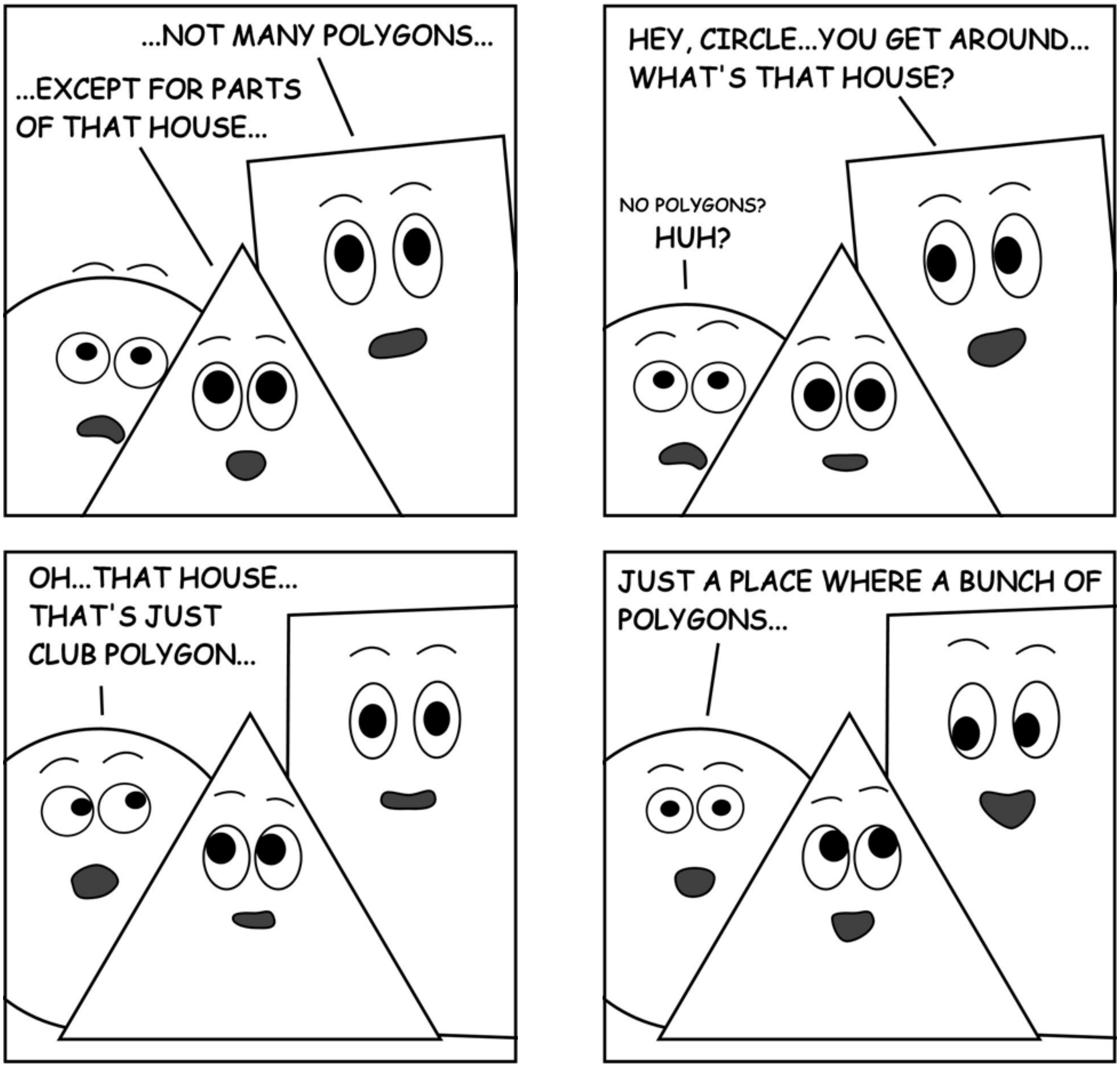
Please send feedback to girlsangle@gmail.com. We'd love to hear from you!


It Figures!
by CKFan

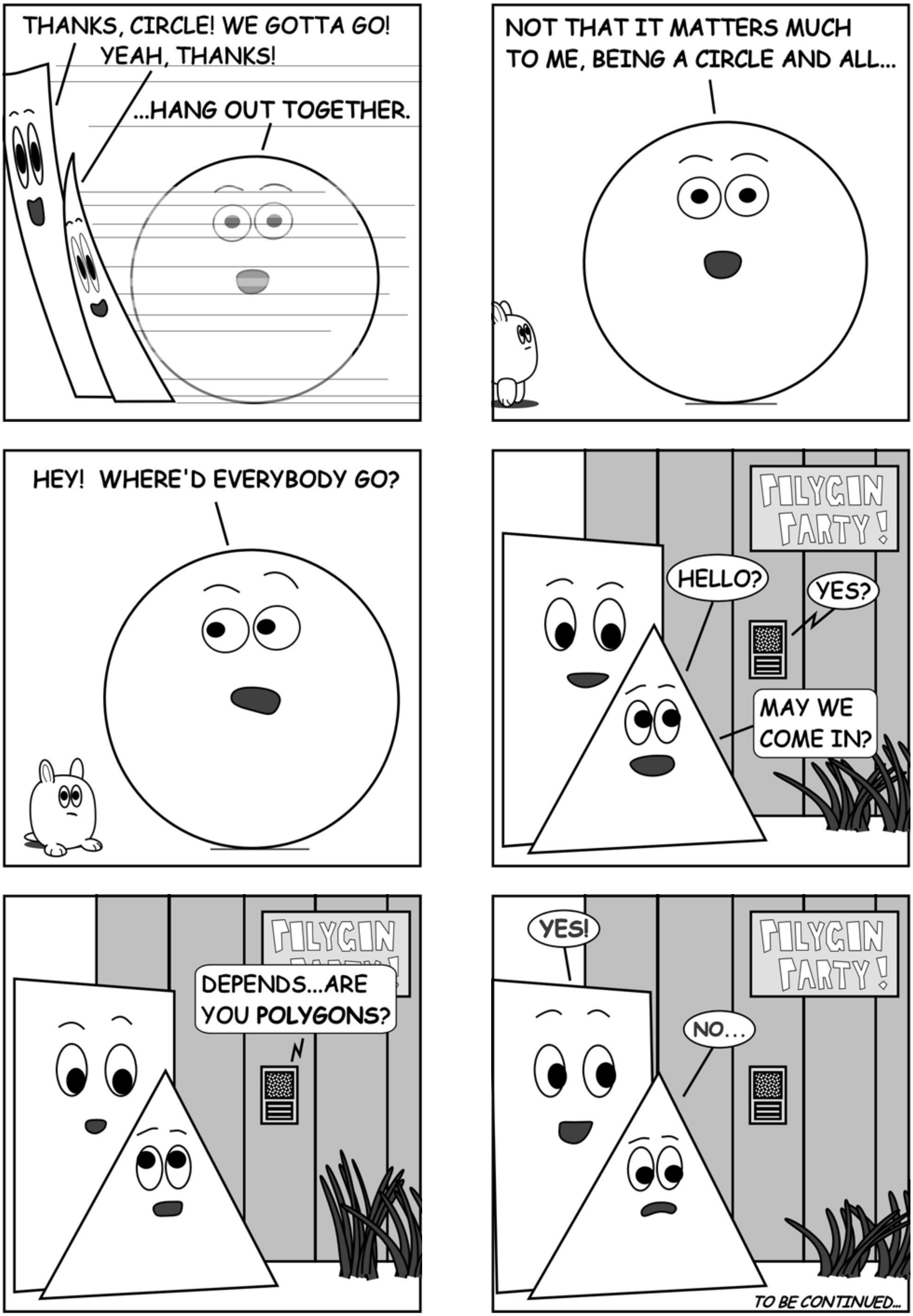

MUST...
SEE...
MORE...
POLYGONS!
BUT, WHERE ARE YOU GOING?
WHASSUP! WHAT'S THE HURRY?
CIRCLE!
HMM...
WE NEED TO SEE MORE POLYGONS. ANY IDEA WHERE WE COULD GO?
UHHH...


YEAH...CAN YOU HELP US?
HMMM...LET ME THINK... JUST A SEC!
!
I KNOW!
SEEK NATURE FOR INSPIRATION!
I'M BRILLIANT!
GREAT IDEA! LET'S DO THAT!
YEAH!





...NOT MANY POLYGONS...
...EXCEPT FOR PARTS OF THAT HOUSE...
HEY, CIRCLE...YOU GET AROUND... WHAT'S THAT HOUSE?
NO POLYGONS?
HUH?
OH...THAT HOUSE... THAT'S JUST CLUB POLYGON...
JUST A PLACE WHERE A BUNCH OF POLYGONS...


THANKS, CIRCLE! WE GOTTA GO!
YEAH, THANKS!
...HANG OUT TOGETHER.
NOT THAT IT MATTERS MUCH TO ME, BEING A CIRCLE AND ALL...
HEY! WHERE'D EVERYBODY GO?
POLYGON PARTY!
HELLO?
YES?
MAY WE COME IN?
POLYGON PARTY!
DEPENDS...ARE YOU POLYGONS?
YES!
POLYGON PARTY!
NO...
TO BE CONTINUED...

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls that aims to foster and nurture girls' interest and ability in mathematics. Instead of modeling after the traditional classroom experience, Girls' Angle is inspired by the lively activity in math department common rooms. Our philosophy is that mathematical ability is best developed through interaction with people who have both a deep understanding of mathematics and a genuine interest in helping others learn. Rather than 'teach math' at the club, we'll have helpers who work on motivation, motivation, motivation! The helpers, who will mostly be women, will introduce the girls to all kinds of activities, objects, and ideas that are math related trying to hook their interest. Once hooked, we will encourage them to explore, to think, and to ask and seek the answers to questions. We will show them all kinds of techniques that help one find answers, and we will show them how to craft questions that promote progress. The goal is to empower girls to be able to tackle any level of mathematics in the future so that no field, no matter how technical, will be off limits. We aim to overcome math anxiety and build solid foundations, so we will be welcoming all girls, not just those deemed gifted in mathematics.

Who can join? Ultimately, we hope to open membership to all women. Initially, we will be opening the doors primarily to girls in grades 5-8. We welcome *all girls* regardless of perceived mathematical ability.

In what ways can a girl participate? There are 3 ways: **membership**, **premium subscriber**, and **basic subscriber. Membership** ($216 or $20 per session) is granted for 12 weeks and includes access to the club, the math question email service, and a subscription to the Girls' Angle Bulletin. **Premium subscriptions** ($100) also last 12 weeks and include the math question email service and subscription to the Girls' Angle Bulletin. **Basic subscriptions** ($20) are one-year subscriptions to the Girls' Angle Bulletin. We operate in 12-meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin will be a bimonthly (6 issues per year) electronic publication that will feature articles and information of mathematical interest as well as a comic strip that teaches mathematics.

What is the math question email service? The math question email service allows a subscriber to email math questions that will be answered by staff or addressed during club meetings. Please note that we will not do math problems that *appear to us* to be for homework.

What do members get? Members get a one-year subscription to the Bulletin and 10 weeks of access to the club and the math question email service. The club will be a friendly place staffed mainly by women who have been selected for their deep understanding of mathematics and their desire to truly help others learn math. Helpers will take a personal interest in each member, assessing her mathematical abilities and working with her to motivate an interest in mathematics and mathematical topics by encouraging questions and explaining strategies and techniques for finding answers. Helpers will also organize fun activities that serve to introduce, explain, and clarify mathematical topics.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. Please inquire about the calendar. It is very important that you pick up your child promptly at 5:45.

Can you describe what the activities at the club will be like? Girls' Angle activities will be tailored to each girl's specific needs. We will assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. If you believe in our approach and goals and want to help support us, we appreciate any contribution you can make. Currently, Science Club for Girls, a 501(c)3 corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, 27 Jefferson St., Cambridge, MA 02141.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken believes that mathematics education in this country can be improved significantly. Also, through the years, he has witnessed instances of gender bias in mathematics and in math education. The last two summers Ken volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were hung at Boston Children's Museum. The girls of Science Club for Girls showed a lot of creativity and ingenuity and were able to realize their ideas in the final project, something that may not have happened in a co-ed environment. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Lauren Williams, assistant professor of mathematics, Harvard
Kathy Paur, graduate student in mathematics, Harvard
Yaim Cooper, graduate student in mathematics, UC Berkeley
Lauren McGough, advanced high school student who founded her school's math club
Connie Chow, executive director of Science Club for Girls
Beth O'Sullivan, co-founder of Science Club for Girls.

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to go on and tackle *any* field regardless of the level of mathematical competence required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle helpers can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

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Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Member (Access to club, Math question email service, Receive Bulletin)
- ☐ Premium Subscriber (Math question email service, Receive Bulletin)
- ☐ Basic Subscriber (Receive Bulletin)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in the grade 5-8 range are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________
(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a 12 session membership
  - ☐ $100 for a 12 week premium subscription
  - ☐ $20 for a one year basic subscription
  - ☐ I am making a tax free charitable donation.

- ☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, 27 Jefferson St., Cambridge, MA 02141. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

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Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

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