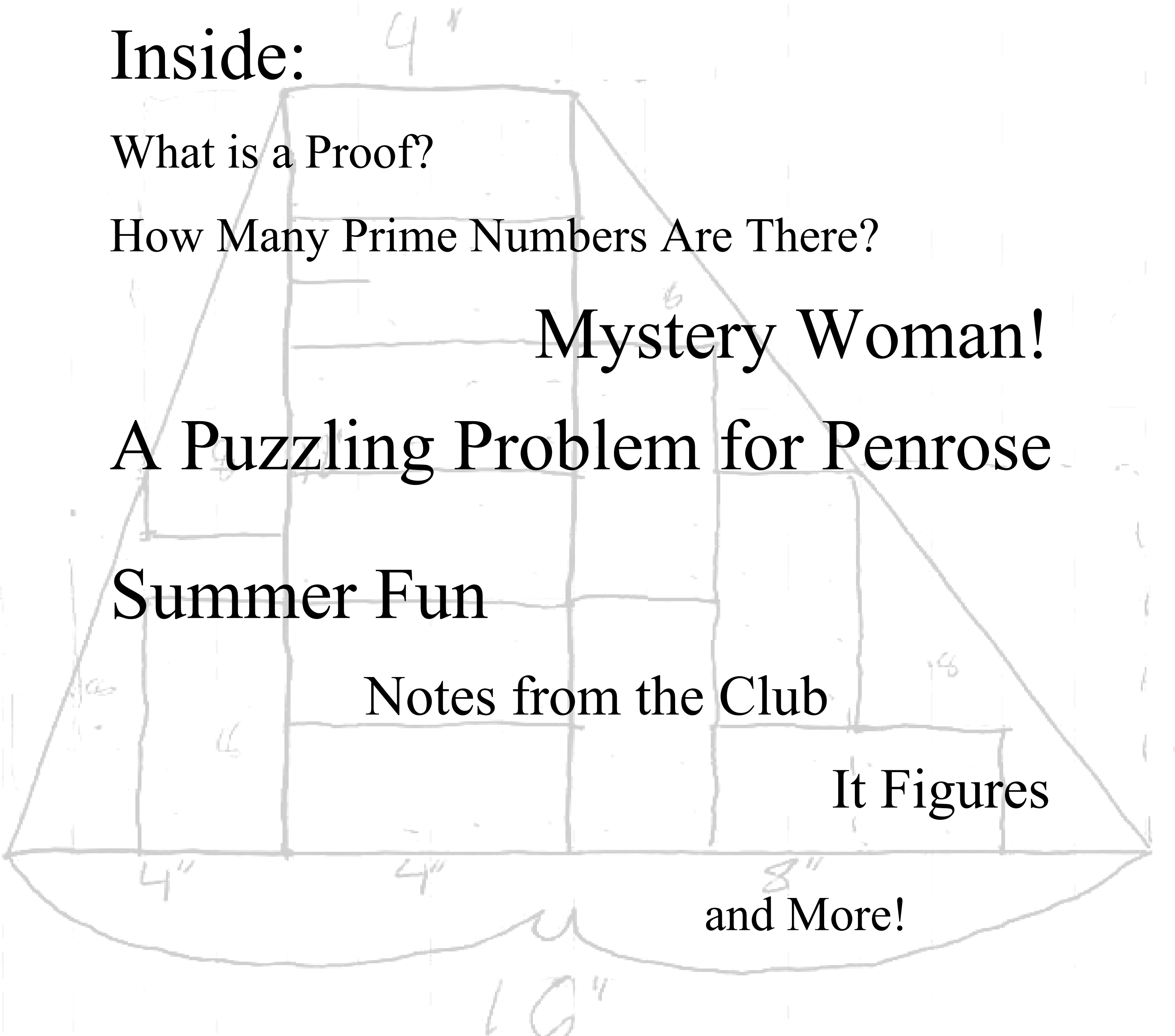
Girls' Angle Bulletin

June 2008 • Volume 1 • Number 5

To Foster and Nurture Girls' Interest in Mathematics



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From the Director

Shortly after our last meet, we had a big Girls' Angle planning meeting. I'd like to share one fact that really stood out to me. One year prior to that meeting, there were exactly six people involved with Girls' Angle. When the meeting occurred, there were 48! And, now, there are well over fifty helping Girls' Angle achieve its mission. Girls' Angle is a product of all this help and all these helpers *are* Girls' Angle.

Girls' Angle welcomes Katrin Wehrheim to the advisory board. Dr. Wehrheim is an assistant professor of mathematics at MIT. She has mentored at Girls' Angle and it was thanks to her that Girls' Angle was able to present at the MIT Women in Mathematics conference.

One consequence of the presentation at MIT is the article on Penrose tilings by Allison Henrich and Sarah Wright. In it, there are templates for two special tiles that you can make copies of and use to explore the world of asymmetric tiling.

Finally, a reminder to our members and subscribers: we encourage you to send us solutions to the Summer Fun problems, email us your mathematical thoughts and feel free to ask us math questions! We'd love to hear from you!

Ken Fan
Founder and Director

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

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Charles Burlingham Jr.
Julee Kim
Beth O'Sullivan
Elissa Ozanne
Patsy Wang-Iverson
Anonymous

Institutions

The Mathematical Sciences Research Institute

Girls' Angle Bulletin
The official magazine of
Girls' Angle: A Math Club for girls

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

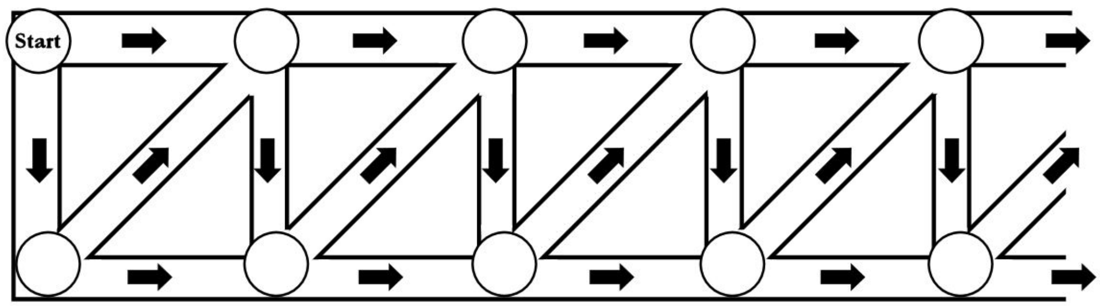
The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: The club voted to use this brownie dissection scheme designed by **Ilana**. See page 26.

What is a Proof?

by Timothy Chow

In the last issue of the Bulletin, we studied the number of paths to the various intersections in the one-way street map below, and we went through a careful *proof* that these numbers were the Fibonacci numbers.



Some of you may have wondered why we went to so much trouble to find a proof. After all, just by calculating the first few numbers, we could see that we were getting the Fibonacci numbers. To be on the safe side, maybe we should have calculated a few more numbers, since we've seen other examples where it's a mistake to jump to conclusions based on just the first three or four numbers in a sequence. But say we worked out the first thousand numbers (maybe with the help of a computer) and they were always Fibonacci numbers. Wouldn't that be convincing proof that we would always get Fibonacci numbers?

This is a very important question. Every professional mathematician will tell you that *proof* is extremely important in mathematics. Some will even say that proofs are the *most* important part of mathematics, and that mathematical proof is what distinguishes mathematics from every other subject. That's a pretty big claim! So it's worth spending some extra time thinking about what a proof is and why mathematicians make such a big fuss about it.

As Ken explained last time, a proof is a very clearly spelled-out argument. This is certainly true, but don't scientists and lawyers and philosophers sometimes give very clearly spelled-out and convincing arguments? What makes mathematical proofs so special?

One of the key features of a mathematical proof is that, once you have carefully checked that the argument is logically correct and that there are no loopholes or mistakes in it, you can be sure that *no future evidence can ever overthrow the conclusion*. For example, let's think about those Fibonacci numbers again. Suppose we had calculated the first thousand numbers and found that they were Fibonacci numbers, but we did *not* have a proof that we would *always* get Fibonacci numbers. Suppose then that some very famous and intelligent person were to come by the next day and say, "I used a much bigger computer than yours and I found that indeed, the first thirteen million numbers are Fibonacci numbers, but then after that, the pattern breaks down and we don't get Fibonacci numbers any more." What would we make of that?

Well, if we were honest about it, we would have to admit that maybe this person is right. We would need to look at the computer program and examine this new evidence to see if it forces us to change our minds. After all, that is how the natural sciences work. For centuries, Newton's laws of physics were believed to hold universally. There was a huge amount of evidence for those laws and very convincing arguments to support them. But then in the twentieth century, physicists found that Newton's laws break down in certain extreme circumstances, and relativity theory and quantum theory had to be invented. Scientists studying the natural world know that we must always keep an open mind, and that new evidence in the future may force us to revise our scientific theories. Therefore, scientific "proof" may be convincing and clear, but it is never *definitive*; it is always *tentative*.

But the situation is different with mathematical proof. If we are armed not only with some calculations, but also a *proof* that the numbers of paths are indeed Fibonacci numbers, then if someone produces a computer program showing that the pattern breaks down after thirteen million, then we can say for sure that the computer program (or the computer itself!) has a bug in it, *without even looking at the computer program*! It does not matter how respected or how intelligent the programmer is. With a mathematical proof in hand, we can *guarantee* that something went wrong with the calculation.

Fibonacci Numbers

To illustrate the ideas presented in this article further, let's consider an aspect of the table of greatest common factors of pairs of Fibonacci numbers on page 8. Notice that just above and below the diagonal there is always a 1. What this means is that consecutive Fibonacci numbers are relatively prime, at least, for those Fibonacci numbers that are in that table. Is this true in general? What would you say if someone said to you, "I have computed the ten billion trillionth and ten billion trillion and first Fibonacci numbers and they are both divisible by 5!"?

Well, I would say, "Liar!" For I have *proven* that consecutive Fibonacci numbers *must be relatively prime*. So I know that any such claim must be false. Maybe "liar" is too strong…maybe the person really did *try* to compute those two Fibonacci numbers but erred. Whether or not the person lied or just made a mistake, I can tell you that those two Fibonacci numbers are definitely relatively prime, and, I don't even know what those two Fibonacci numbers are! Here's my proof:

Proof. Let F_n be the nth Fibonacci number and let d be the greatest common factor of F_n and F_{n+1}. I will show that $d = 1$. For $n > 1$, we know that $F_{n+1} = F_n + F_{n-1}$, by definition We can rearrange like this: $F_{n-1} = F_{n+1} - F_n$. This equation tells us that if a number divides F_n and F_{n+1}, then it must also divide F_{n-1}. From the previous observation, we know that d divides F_{n-1}. Applying the previous observation again using the equation $F_{n-2} = F_n - F_{n-1}$, we see that d divides F_{n-2}. Repeatedly applying the same observation, we see that d divides F_{n-3}, F_{n-4}, F_{n-5}, …, all the way down to F_1. But $F_1 = 1$ and if d divides 1, it must be equal to 1. □

-Editor

Maybe you can see now why mathematicians get so excited about proofs. A proof lasts forever; it will never be undermined in the future, no matter how smart people or computers become. On the other hand, without a proof, you can never be completely sure that what you *think* is true is *really* true, even if there seems to be a lot of convincing evidence for it. That is why mathematicians work so hard to find proofs of mathematical claims, even if the claims seem "obvious".

One last word of caution: A proof only has this magical quality of being everlasting and definitive *if there are no mistakes in the argument*! Everyone, even the world's most brilliant mathematician, makes mistakes sometimes. So, when you are trying to construct a proof, be very careful to make sure that every step is logically correct and has no loopholes in it. At first you may have trouble recognizing when an argument has a loophole in it and when it does not, but that is one thing the mentors are there to help you with. Finding proofs is a tricky business and takes a lot of experience, but it is an art worth mastering. There is nothing like the thrill of finding a proof on your own and getting a taste of eternity!

How Many Prime Numbers Are There?

by Lauren McGough

One of the topics we have talked about often at Girls' Angle is the idea of divisibility: if one integer can be evenly divided by another integer, we say the first number is divisible by the second number. Well, you probably already know that some numbers are special – they have exactly two factors, as they are only divisible by 1 and themselves. Positive integers with this property are called prime numbers. For example, the number 2 is prime – it is only divisible by 1 and itself, 2; the numbers 3, 5 and 23 are other examples of prime numbers.

Prime numbers are tricky things in math because if you start counting from 1 and keep counting into infinity, it is difficult to know just when another prime number will pop up, especially as you get into large numbers. For example, the number 91 may seem prime at first sight, but it is actually the product of 7 and 13, giving it four divisors, 1, 7, 13 and 91. It is often hard to prove that very large numbers are prime, as doing so requires doing many divisions to see if such numbers have any factors other than 1 and themselves.

With all of this difficulty in predicting which numbers are prime without doing many divisions, you might start to wonder, given a prime p, is it always possible to find a prime number larger than p? Maybe there is just some point after which all of the numbers are composite; that is, maybe after some point, all of the numbers have more factors than just 1 and themselves. How can we tell how many primes there are? How do we know, after finding one prime, that there is or isn't some larger prime number out there in existence? After all, we can't just go through all of the integers and test them to see if they are prime – there are infinitely many integers, so that would take infinitely many divisions, and we may not have infinite time!

Luckily for us, there is at least one way to know exactly how many primes there are. If you read the previous article, you might even be able to guess what that way of knowing is. That's right: we have the power of proof. So let's see if we can use that power to figure out, once and for all, how many primes there are.

Let's assume that there are finitely many prime numbers. If we operate under this assumption, perhaps we will succeed in finding the largest prime, or, perhaps we will discover a logical contradiction. If we arrive at a contradiction, then we'll know that this assumption cannot be correct and will have discovered a *proof by contradiction* that shows there are, in fact, infinitely many primes. (See the first issue of this Bulletin for more on proofs by contradiction.) So, if there are finitely many prime numbers, say, n of them, we could in principle list them *all*: p_1, p_2, p_3, …, p_n. This list will have some largest element, the largest prime number, since every finite list of integers has to have a largest element. Ok, well, that's fine. We had considered that maybe there was a largest prime number. But wait, is this fine?

Now that we have this list of prime numbers, we can form a new number N by setting $N = p_1 \times p_2 \times p_3 \times \cdots \times p_n + 1$; that is, N is one more than the product of these prime numbers. This number is larger than any of the primes in our list, since all of our primes are greater than 1, and since the product of several numbers that are greater than 1 will be greater than any of the individual numbers. Since N is greater than any of the primes in our complete list of primes, it must be a composite number and have factors other than 1 or itself. What are its factors?

Is it divisible by p_1? No, it can't be: since N - 1 is divisible by p_1, the remainder when N is divided by p_1 is 1, but if N were divisible by p_1, the remainder would be 0, so N cannot be divisible by p_1. How about by p_2? By the same argument, since N - 1 is divisible by p_2, the remainder when N is divided by p_2 is 1, but the remainder must be 0 if N is to be divisible by p_2, so N cannot be divisible by p_2. Can N be divisible by any prime p in our list? By the same argument, you could go through each prime in our list, no matter how many primes there were, and realize that since N - 1 will always be divisible by p, N divided by p will always leave a remainder of 1, and thus N cannot be divisible by any prime on our list!

Ok, you say, but I didn't say N had to be divisible by a prime – I just said it had to be composite, which means that is has to have factors other than 1 and itself. But in fact, any composite number must have a prime factor. For if you start with a composite number, it must have some factor, say c, other than 1 or itself. If c is not prime, it too would be composite, and we could find a factor of c, say d, that was not 1 or c. If d is not prime, we could find a factor of it, say e, that was not 1 or d, and so on, finding ever smaller factors. Eventually, we'd have to arrive at a prime factor because you cannot keep getting smaller and smaller like that (always smaller *and* always bigger than 1). By the way, I used the fact that if a divides b and b divides c, then a divides c in this argument – do you see how? So N, being composite, *must* be divisible by some prime. But we know that N is not divisible by any of the primes on our list. But our list is supposed to be a list of *all* the prime numbers. Impossible! This is a contradiction!

What went wrong? Our construction of N was perfectly acceptable; our reasoning that N was not divisible by any of the primes on our list came directly out of the construction of N. Since our argument was logical but our conclusion made no sense, we must go back and realize it must have been our initial assumption that was incorrect. If you remember, our initial assumption was just that the number of primes is finite. What does all of this tell us? That in fact, this assumption was wrong: there must be an *infinite number* of primes.

This conclusion is extremely valuable to our knowledge about the integers. First of all, we know that there are infinitely many prime numbers without knowing what they all are! Even without a list of all the primes, we have just proven decisively that there are infinitely many primes. Second of all, we can know that if anyone claims to have found the largest prime number, that person is wrong. We now know that given any prime number, there exist prime numbers that are larger than that prime number. Actually, given any prime number, there exist infinitely many larger prime numbers! The race to determine the largest known prime number will never end.

This power of knowing something infinite without doing infinite amounts of work is one of the many powers given to us by the power of proof. Proofs give us the ability to know answers for sure. The question of how many primes there are is just one of the many questions mathematics can answer. So, now we know there are infinitely many. Of course, we could then ask, how many primes are there less than 100, or 10,000, or 1,000,000, or, more generally, how many primes are there less than x for any x? Well, I don't know, maybe you can tell me? And while you're at it, can you prove it?

en.wikipedia.org/wiki/Image:Raffael_058.jpg

Mystery Woman!

Raphael painted *Philosophy* about 500 years ago. Over 25 feet by 16 feet, he took 2 years to paint it on a wall in the Vatican. Also known as the *School of Athens*, it depicts various scholars. The two central figures are famous philosophers: Plato and Aristotle.

All the figures depicted in the painting are men.

Except for one.

Do you see her?

Who does she represent?

What did this thinker do?

If you figure it out, email us about it at girlsangle@gmail.com.

Table of Greatest Common Factors of Pairs of Fibonacci Numbers

Table of Greatest Common Factors of Pairs of Fibonacci Numbers

F_n \ F_m	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	2
3	1	1	1	3	1	1	1	3	1	1	1	3	1	1	1
5	1	1	1	1	5	1	1	1	1	5	1	1	1	1	5
8	1	1	2	1	1	8	1	1	2	1	1	8	1	1	2
13	1	1	1	1	1	1	13	1	1	1	1	1	1	13	1
21	1	1	1	3	1	1	1	21	1	1	1	3	1	1	1
34	1	1	2	1	1	2	1	1	34	1	1	2	1	1	2
55	1	1	1	1	5	1	1	1	1	55	1	1	1	1	5
89	1	1	1	1	1	1	1	1	1	1	89	1	1	1	1
144	1	1	2	3	1	8	1	3	2	1	1	144	1	1	2
233	1	1	1	1	1	1	1	1	1	1	1	1	233	1	1
377	1	1	1	1	1	1	13	1	1	1	1	1	1	377	1
610	1	1	2	1	5	2	1	1	2	5	1	2	1	1	610

Here is a table of greatest common factors of pairs of Fibonacci numbers for the first 15 Fibonacci numbers. What patterns do you see?

Here are a few patterns:

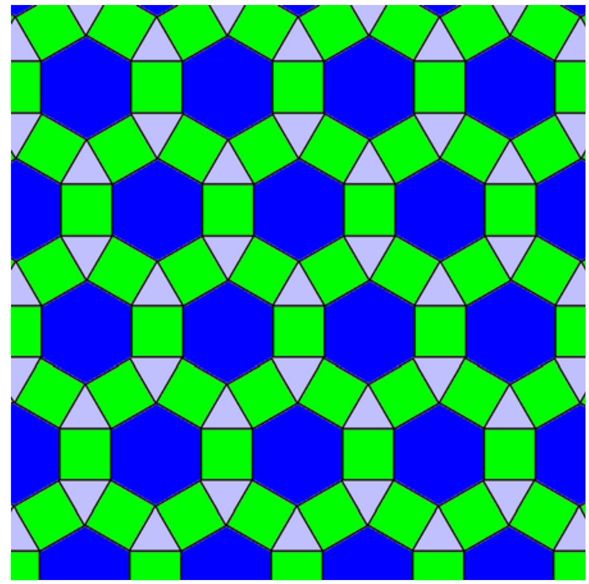
> The first row and column contain only the number one.
>
> The diagonal (places where $n = m$) contains a copy of the Fibonacci sequence.
>
> The table is symmetric about the diagonal.

Here's an unresolved question related to Lauren's article and the Fibonacci sequence: Are there infinitely many prime Fibonacci numbers? It is known that there are infinitely many primes (see page 5), but are there infinitely many primes *in the Fibonacci sequence*? That…nobody knows!

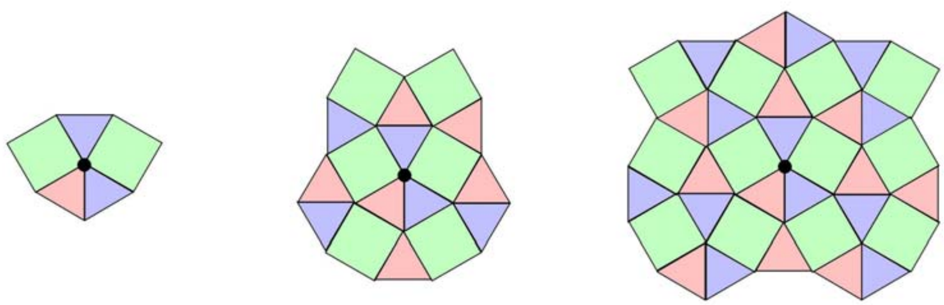
A Puzzling Problem for Penrose

by Allison Henrich and Sarah Wright

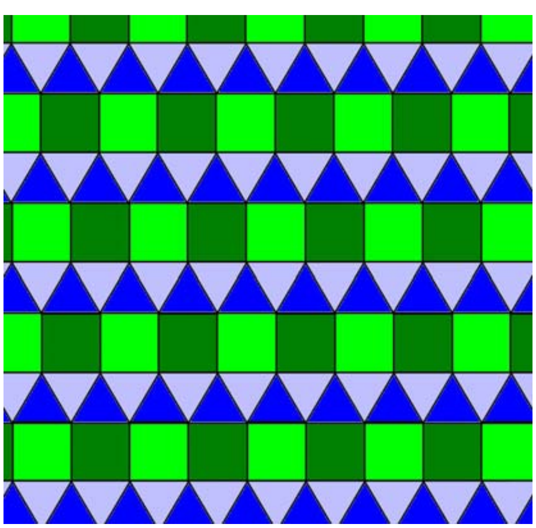
Have you ever played with geometric shapes, trying to get them to fit together or make a pretty picture? If so, then you have something in common with Sir Roger Penrose. Penrose was fascinated by mathematical objects called tilings. Also known as tessellations, tilings are arrangements of geometric objects that fill the plane with no gaps or overlaps:


An example of a tiling.

The geometric shapes that make up a tiling are called tiles. When these tiles are joined so that whenever two tiles share an edge, the edges match up exactly, the tiling is called an **edge-to-edge** tiling. In and edge-to-edge tiling, the points where three or more tiles come together are called **vertex configurations**. If you wanted to build a tiling, you could start with a vertex configuration and add more tiles to expand your picture. Like this:



Tilings of the plane often have very nice properties, called symmetries. There are three types of symmetries we can look for: **translational** symmetry, **reflectional** symmetry and **rotational** symmetry.

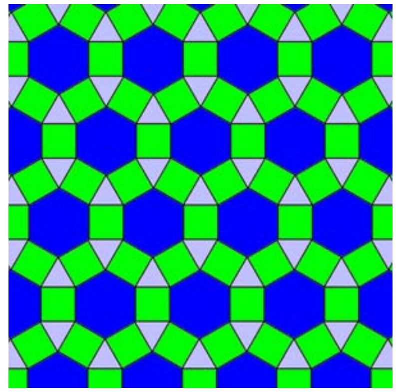


If a tiling of the plane that extends out forever has translational symmetry, this means that you can slide the picture in some direction so that the tiles exactly line up with one another. Can you find translational symmetry in the tiling at left? Imagine that it extends infinitely in all directions.

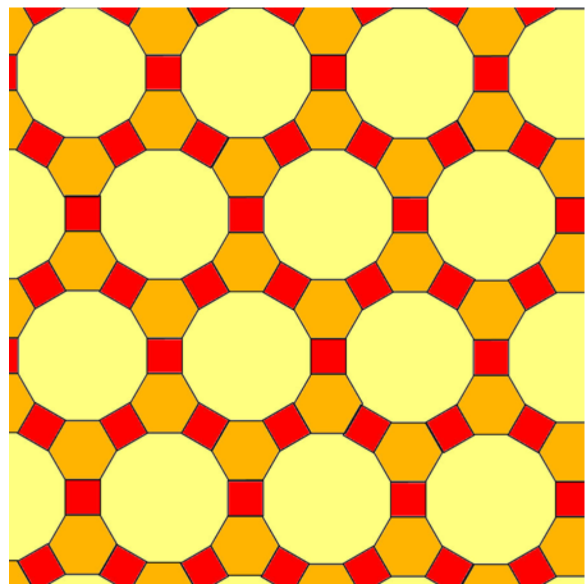
Now what does reflectional symmetry mean? Any guesses? A tiling has reflectional symmetry, also known as **mirror** symmetry, if you can fold it along a straight line and have the tiles match up with one another. Think of a butterfly. If the line of symmetry is the body of the butterfly, you can fold the wings on top of each other so their shapes and patterns match up.

How many lines of symmetry can you find in these pictures?

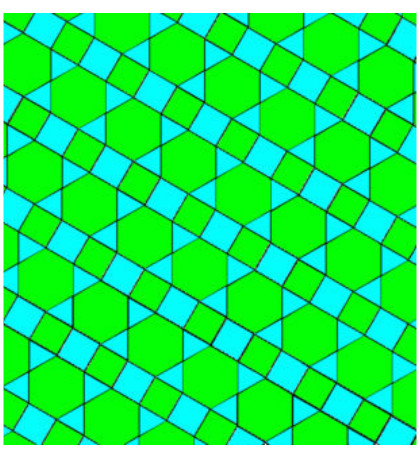
. en.wikipedia.org/wiki/Image:Monarch_In_May.jpg

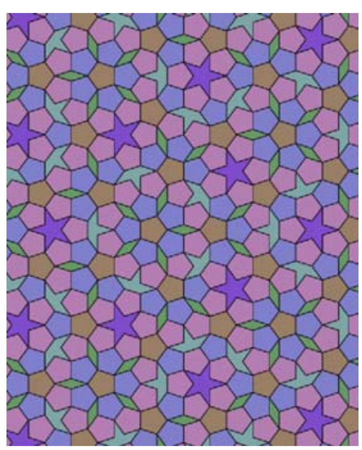


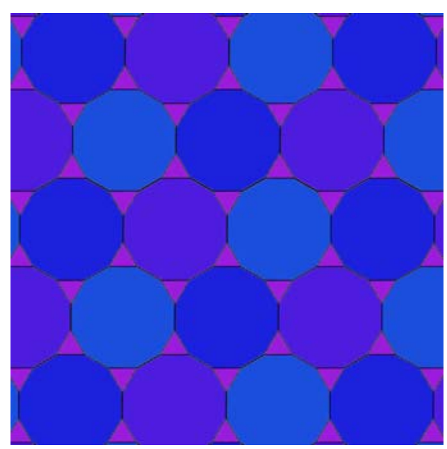
The third type of symmetry is sometimes the most difficult to spot. A tiling has rotational symmetry if you can rotate the whole plane around a point so that the picture looks the same. An object that we've all seen that has rotational symmetry is the wheel of a bike. A wheel with 6 spokes can be rotated by 60 degrees and still look the same as before. In this case, we'd say that the wheel has 6-fold rotational symmetry. In general, a tiling has *n*-fold rotational symmetry if rotating the picture $\frac{360}{n}$ degrees gives the same picture. Try to find 3-fold rotational symmetry in the tiling shown to the right. Are there any other rotational symmetries?



What types of symmetries can you find in these tilings?

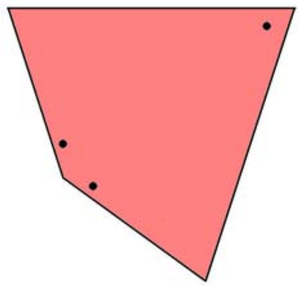




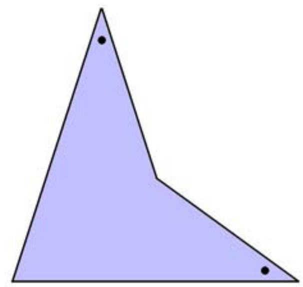


Many of the tilings we've seen so far have a lot of nice properties. Penrose was looking for a tiling that had some definite nice properties, but was missing others. One of the tilings given his name is a tiling with 5-fold rotational symmetry, but no translational symmetry.

The particular Penrose Tiling we'll look at is made up of two types of tiles: kites and darts.



A kite.



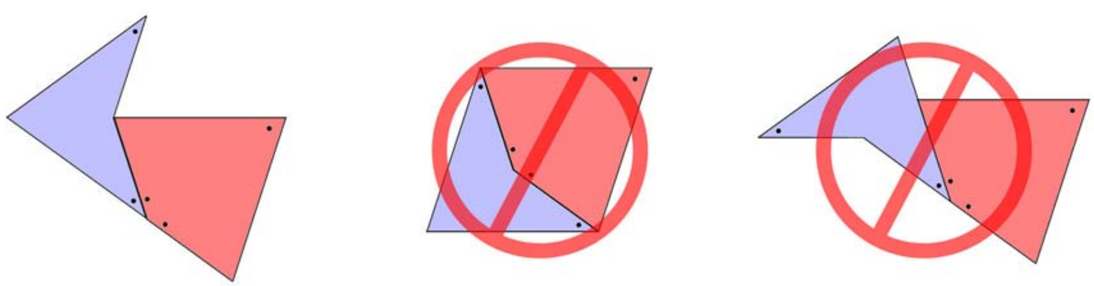
A dart.

Use construction paper to cut out several of each of the tiles above. Since it is important that the angles of the shapes be exact, use the shapes above as a template.

You may notice that the kite and dart fit together nicely to form a diamond, but this configuration is not allowed. There are two rules that you must follow when fitting the tiles together:

1. Make sure that the dots near the edges of the tiles match up with dots in adjacent tiles.

2. There are two lengths of edges, short and long. You are not allowed to put a long edge of one tile beside a short edge of another tile.



Use the tiles you made to see how many different vertex configurations you can find. How many can you find? Do think that is all of them? How could you show you haven't missed any? Is it possible that there are infinitely many? Of the configurations you found, what symmetries can you identify?

Penrose tilings have many interesting properties. For instance, any configuration of Penrose tiles will appear infinitely many times in any Penrose tiling of the plane. Also, there are an infinite number of different tilings that can be made with kites and darts, following the rules above. For more information on Penrose tilings check out the Wikipedia website.

Square Root Information Sheet

Square roots have popped at the club on a number of occasions. This page summarizes properties of square roots. Make sure you understand each property. If you are having trouble with any of them, feel free to write us about it!

Let x be a number, like 0, 1, 1.5 or even π.

It is possible to define square roots for negative numbers, but we'll save that for another day. Today, let's assume that x is not negative, that is, $x \geq 0$.

(If you know about **complex** numbers, you'll see that we have also implicitly assumed that x is a real number.)

Definition. A **square root** of x is any number whose square is equal to x.

Example: -5 and 5 are both square roots of 25.

1. Every number x has exactly *two* square roots with the exception of zero, which has only one square root.

2. If y is a square root of x, then $-y$ is also a square root of x.

This means that if $x > 0$, one of its square roots will be positive and the other will be negative. The *positive* square root of x is denoted $\sqrt{x}$. This is also called the **principal square root** of x.

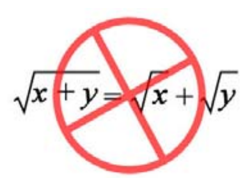
Many, many people forget that there are generally *two* square roots and not just the positive one. They forget that $\sqrt{x}$ stands for the positive square root only and begin to think of $\sqrt{x}$ as *the* square root of x. This is a very common conceptual error! Don't fall into this trap!!!

3. For $x \geq 0$, we have $\sqrt{x}^2 = x$.

4. For any x (including negative values), we have $\sqrt{x^2} = |x|$. (Recall that $|x|$ is the absolute value of x. Why do we have to use the absolute value here?)

5. (Multiplication) $\sqrt{xy} = \sqrt{x}\sqrt{y}$.

Square roots do not behave simply with respect to addition!



6. For $x > 0$, we have $\sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$.

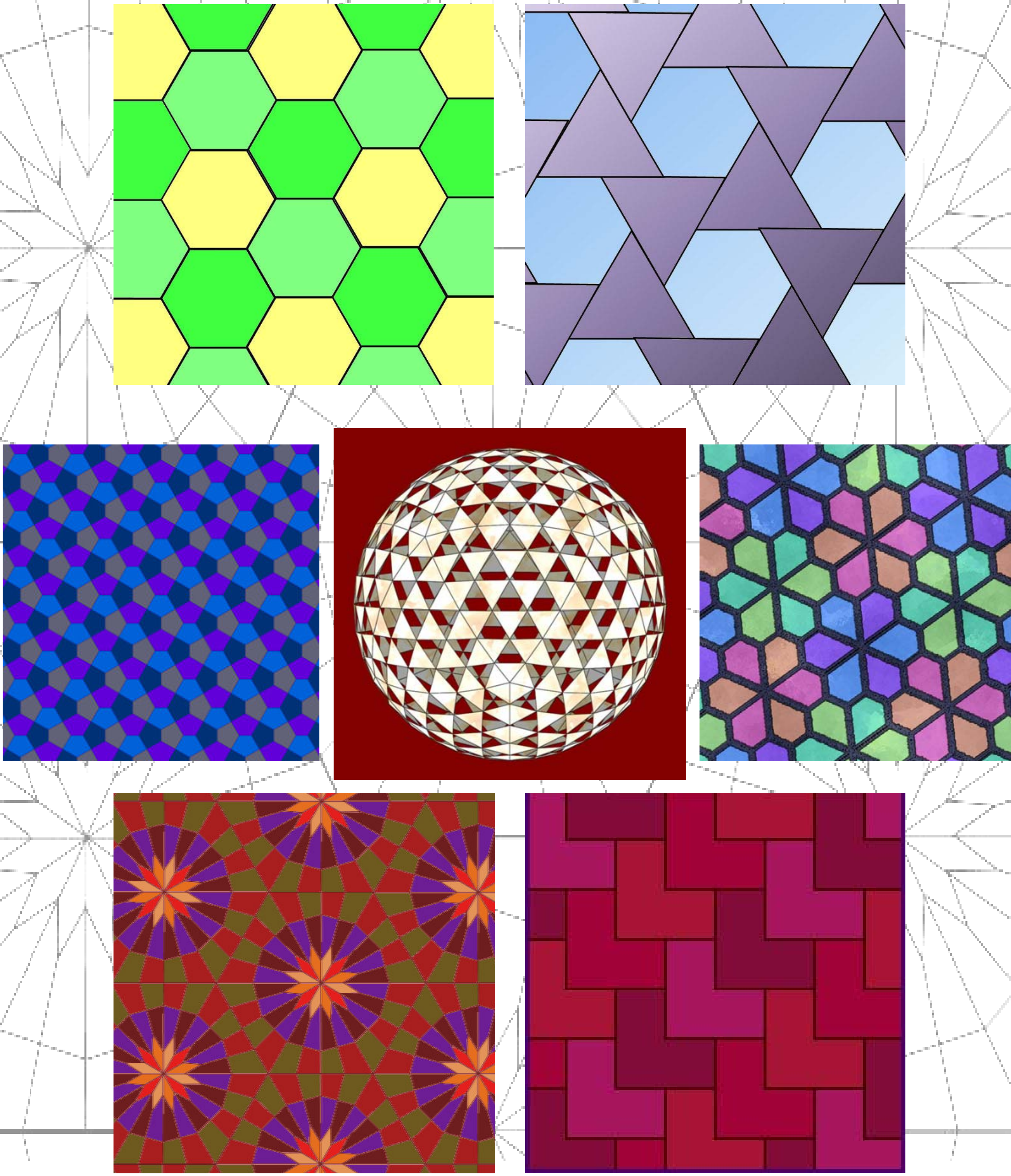
7. If $0 \leq x < y$, then $0 \leq \sqrt{x} < \sqrt{y}$.

Square Roots - True or False?

Assume that $\sqrt{x} = \frac{3}{2}$ and $\sqrt{y} = \frac{4}{3}$.

1. -5 is a square root of 25	T F
2. The principal square root of 2.5 is 0.5.	T F
3. $3 < \sqrt{10} < 4$	T F
4. $\sqrt{123^3} = \sqrt{123}^3$	T F
5. If $a = b$, then $a^2 = b^2$.	T F
6. If $a^2 = b^2$, then $a = b$.	T F
7. The value of $9y$ is 16.	T F
8. $\sqrt{xy} = 2$	T F
9. $\sqrt{\frac{x}{y}} = \frac{8}{9}$	T F
10. If $a > 0$, then $\sqrt{a} < a$.	T F
11. The only solution to $\sqrt{a} = a$ is $a = 0$.	T F
12. $\sqrt{x+y} = \frac{17}{6}$	T F
13. $(\sqrt{2} + 1)(\sqrt{2} - 1) = 1$	T F
14. $\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$	T F
15. $\sqrt{1800} = 30\sqrt{2}$	T F
16. $\frac{1}{\sqrt{2}-1} \neq \sqrt{2}+1$	T F

Answers will appear in the next issue.



More Tessellations

A Comment on Lines

by Ken Fan

In Lauren McGough's first problem from her Slope Problem Set I (see page 19), we're asked to draw some lines and compute the slope using various pairs of points to see that the slope doesn't depend on the choice of points used.

However, no matter how many pairs of points you check this for, it does not suffice to *prove* that the slope will be the same. Also, there are issues concerning the problem of error in measurements made in the real world which can be very distracting but are not actually relevant.

Ultimately, in mathematics, we have to ask, how can we *prove* that the slope will be the same no matter what pair of points are used to compute it?

Proofs require great precision in thought and writing. Each step must carefully be checked to be logically sound, and all the steps that are taken must be *about things that are clearly defined*.

In order to prove *that the slope is the same regardless of which two points on a line are used to compute it, one must clearly* define *just exactly what a line is*.

Without a definition of line, there can be no mathematical proofs involving line.

However, defining a line is quite a tricky business! In fact, there are *many* definitions of line used by mathematicians and each attempts to model our intuitive notion of a line as that "straight" geometric object or that path of "shortest length". Depending on the choice of definition, the *proof* that the slope does not depend on which two points on the line that are used to calculate it can be very straightforward or rather difficult.

Perhaps we'll address some of these definitions in the future.

But for now, keep in mind that Lauren's solution really only *suggests* that the slope is independent of the choice of points used to compute it. (She, by the way, repeated her solution for a few lines, not just the one printed in this issue.) The exercise can also be used to help one develop intuition about lines that could be used to guide the construction of a precise definition.

How would you give a precise definition of line? Send in your ideas to girlsangle@gmail.com!

By the way, let's not forget about the geometric notion of a point! What is a good definition for a point? Believe it or not, for such a seemingly simple concept as the point, it was not until relatively recently in history that a definition of point was given that is mathematically acceptable. In the twentieth century, the mathematical point has become a subtle and sophisticated object. Is it surprising that it took so long for humans to develop a mathematical definition of point? Perhaps when you realize how strong people's intuitive notion of a point is, it is not so very surprising. People tend to overlook things that they take for granted, and it often takes a genius to stop and think about something that everybody else has been taking for granted as far back as people can remember. So, the next time you think something is obvious, catch yourself and give it a closer thought!

Summer Fun!

In the last issue, we invited members and subscribers of the Bulletin to submit solutions to a number of Summer Fun problem sets. In this issue, we are going to provide solutions to the first problem from each problem set.

You can still send any questions and solutions to girlsangle@gmail.com. We'll give you feedback and put your solutions in the Bulletin!

In the August issue, we will give complete solutions. You could wait until the August issue to see the answers, but you will learn a lot more if you try to solve these problems before seeing solutions. If you haven't tried to solve the first problems in each problem set yet, but would like to, please do!

Each of the following pages will reprint the first problems from each problem set followed by a solution.

However, there is a **catch** to this! Some of the solutions have been planted with deliberate errors! See if you can find all the errors. Count how many errors there are and send this number to us at girlsangle@gmail.com. We'll reveal this secret number in the next issue!

So, read carefully!

Parity: Are you Even or Odd?

by Ken Fan

In this problem set, when I say "number", I mean an integer, that is, a counting number, the negative of a counting number, or zero.

1. We've encountered parity many times at the club. Just knowing whether a number is even or odd can often be powerful information! Recall that a number is even if and only if it is divisible by 2. Otherwise, it is odd.

   a. Write down the first 10 (positive) even numbers and the first 10 (positive) odd numbers.
   b. Is zero even or odd?
   c. Which prime numbers are even? Which prime numbers are odd?

<u>Solution</u>

(Ken Fan)

1a. The first 10 positive even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20. The first 10 positive odd numbers are 1, 3, 5, 7, 9, 11, 13, 17 and 19.

1b. Zero is even.

1c. The only even prime number is 2. (We're taking the prime numbers to all be positive.) To see why, notice that if n is even, then n is a multiple of 2. That means we can write $n = 2m$ where m is an integer. But if $m > 1$, this would exhibit n as a composite number. The only way n could be prime is if $m = 1$, that is, if $n = 2$. We can see that 2 is prime because its only factors are 1 and 2. If the only even prime number is 2, then all the other prime numbers are odd.

Number of errors: ___________

Summer Fun!

Pascal's Triangle and Binomial Coefficients

by Elisenda Grigsby

Suppose that x and y are variables. We can manipulate the symbols without having to know what numbers they represent. For example, we can write "$x + y$", and it means "add x and y". If x represents 2, and y represents 3, then "$x + y$" represents "5". The point is that the expression "$x + y$" itself doesn't depend on what numbers x and y actually represent, and we can think about what we can say about these expressions in general, without worrying about some particular choice of numbers for x and y.

As an example, let's consider the following question: Let x and y be numbers. What can we say about the expression $(x + y)^n$, when $n = 0, 1, 2, 3, ...$?

So, $(x + y)^0 = 1$; (the result of multiplying a number by itself 0 times is usually defined to be 1)

$(x + y)^1 = x + y$;

$(x + y)^2 = (x + y)(x + y)$;

$(x + y)^3 = (x + y)(x + y)(x + y)$;

etc....

1. Show that $(x + y)^2 = x^2 + 2xy + y^2$ and $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$. What is $(x + y)^4$? (Hint: Use the distributive property of numbers: $(x + y)(z) = xz + yz$, for all numbers x, y, and z. Also, remember that multiplying two numbers doesn't depend on the order in which we multiply them. So, if x and y are two numbers, then $xy = yx$.)

<u>Solution</u>

1. (Eli Grigsby) Just multiply!

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y)\\ &= x(x + y) + y(x + y)\\ &= x^2 + xy + yx + y^2\\ &= x^2 + 2xy + y^2\end{aligned}$$

$$\begin{aligned}(x + y)^3 &= (x + y)^2(x + y)\\ &= (x^2 + 2xy + y^2)(x + y)\\ &= (x^2 + 2xy + y^2)x + (x^2 + 2xy + y^2)y\\ &= x^3 + 2x^2y + y^2x + x^2y + 2xy^2 + y^3\\ &= x^3 + 3x^2y + 3xy^2 + y^3\end{aligned}$$

$$\begin{aligned}(x + y)^4 &= (x + y)^3(x + y)\\ &= (x^3 + 3x^2y + 3xy^2 + y^3)(x + y)\\ &= x^4 + 3x^3y + 3x^2y^2 + xy^3 + x^3y + 3x^2y^2 + 3xy^3 + y^4\\ &= x^4 + 4x^3y + 8x^2y^2 + 4xy^3 + y^4\end{aligned}$$

Number of errors: ___________

Summer Fun!

Slope Problem Set I

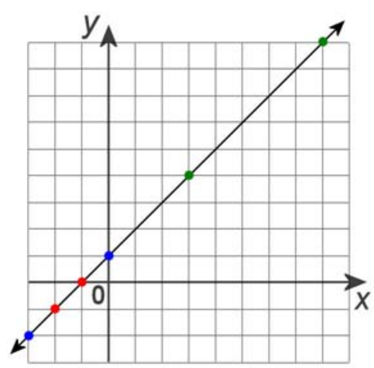
by Lauren McGough

For this problem set, we're going to be dealing a lot with lines. If you think about a straight line on a coordinate plane where the horizontal and the vertical directions are defined, it has a certain steepness associated with it. Maybe it is completely parallel to the horizontal, in this direction: –—. Or maybe it is parallel to the vertical, in this direction: | . Or maybe it makes some angle with the horizontal, like this: / , or this: \. Each of these lines has a different "steepness" associated with it. There is a number we use to measure this property of steepness: it's called *slope***.** The slope of a line is just the ratio of the amount the line goes up for every unit it goes over. We can measure slope just by taking two points on a line, and calculating the change in the vertical direction divided by the change in the horizontal direction. First, let's make sense of this definition!

1. The first question here is: draw some lines, and calculate their slopes using a few sets of different points. Is the slope always the same no matter what points you use? Why or why not? (We hope it is, because otherwise, the definition of the "slope of a line" doesn't make sense– the line could have a different slope at every point!)

Solution

1. (Lauren McGough) Let's try the line graphed at right. Some pairs of points on this line are:



(3, 4) and (8, 9)
(0, 1) and (-3, -2)
(-2, -1) and (-1, 0)

Calculating the slope of the line using each of these pairs by calculating the ratio of the vertical change to the horizontal change, we find:

(first pair) $\frac{9-4}{8-3}=\frac{5}{5}=1$

(second pair) $\frac{-2-1}{-3-0}=\frac{-3}{3}=1$

(third pair) $\frac{0-(-1)}{-1-(-2)}=\frac{1}{1}=1$

We got the same ratio, 1, each time!

Number of errors: ___________

This problem is actually quite deep. See page 15. -*Editor*

Summer Fun!

Slope Problem Set II

by Lauren McGough

Let's continue using the same set up of Slope Problem Set I.

1. Sometimes, people like to express all of the points on a line using an equation that relates *a* and *b* for all points (*a, b*) on the line. Consider a line of slope 5 that goes through the intersection of the two axes on the plane– that is, through the "origin". Can you think of a relationship that all of the points (*a, b*) satisfy– that is, can you write an equation using *a, b* and the slope of the line such that if *a* and *b* satisfy the equation, then (*a*, *b*) is a point on the line and vice versa?

<u>Solution</u>

1. (Ken Fan) The problem asks for an equation that describes the coordinates of a point on a line that passes through the origin and has a slope of 5.

The origin has coordinates (0, 0). If (*a*, *b*) is any other point on the line, then we can use these two points to compute the slope, and this slope must be equal to 5:

$$\frac{b-0}{a-0} = 5$$

This simplifies to:

$$\frac{b}{a} = 5$$

Notice that *a* cannot equal zero in this equation because you cannot divide by zero. If we multiply both sides by *a* we get the equation $a = 5b$. In this last equation, if $a = 0$, then $b = 0$, and this would correspond to the point (0, 0) which we know is on the line. So we conclude that the equation $b = 5a$ describes the relationship between the coordinates of points on the line with slope 5 that passes through the origin.

Number of errors: ___________

Summer Fun!

Origami Math

by Ken Fan

Origami artists start with square pieces of paper and fold them into works of art. There is quite a bit of mathematics related to origami. Christine's article (see the prior issue of this Bulletin) gave an example and this problem set introduces some others.

1. Take an origami square.
   a. If you fold the square in half along a crease parallel to a side, what is the resulting shape?
   b. If you fold the square in half along a diagonal, what is the resulting shape?
   c. If you fold an origami square in half repeatedly a total of n times, how many layers of paper will there be?

<u>Solution</u>

(Ken Fan)

1a. A rectangle with proportions 2:1 (or 1:2).

1b. An isosceles right triangle.

1c. There will be 2^{n-1} layers.

Number of errors: ___________

Summer Fun!

Getting a Balanced Diet

by Lauren Williams

You can see this painting by Spanish painter Luis Meléndez at the Museum of Fine Arts in Boston.

Photo courtesy of http://commons.wikimedia.org/wiki/Main_Page

An "average" human being consumes about 1940 calories per day. All food is comprised of carbohydrates, fat, and/or protein. One gram of protein or carbohydrates provides 4 calories, and one gram of fat provides 9 calories. Suppose a person decides to get the 1940 calories by eating 50 grams of protein, 300 grams of carbohydrates and 60 grams of fat. Let's see how this person could do that with different kinds of foods!

1. A pat of butter contains 4 grams of fat (and no significant protein or carbohydrates). A large pear contains 30 grams of carbohydrates (and no significant amount of protein or fat). A can of tuna canned in water contains 40 grams of protein (and no significant quantity of fat or carbohydrates). In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only butter, pears, and canned tuna, how much of each quantity of food should the person eat?

<u>Solution</u>

1. (Lauren Williams) The recommended daily amount of protein in a 1940 calorie diet is 50 grams. If we only plan to eat butters, pears and canned tuna, how can we get 50 grams of protein in a day? We're told that butter and pears don't contain protein, but tuna contains 40 grams of protein per can. How many cans of tuna should we consume to get 50 grams of protein? One can is too little and two cans are too much. We can make an educated guess and arrive at 1.5 cans of tuna. Alternatively, we could let T represent the number of cans of tuna that we should eat; in that case we will consume $40T$ grams of protein, which must be equal to 50. Solving $40T = 50$ we get $T = 1.5$.

Similarly, let's see how we can get 300 grams of carbohydrates. The only food among butter, pears, and tuna which contains carbohydrates is pears. Each pear contains 30 grams of carbohydrates. We can again "guess" that the right number of pears to eat is 10. Alternatively, we can let P represent the number of pears we should eat; in that case we will consume $30P$ grams of carbohydrates from the pears. Solving $30P = 300$ we get $P = 10$.

Finally, let's see how we can get 60 grams of fat. The only food among butter, pears and tuna which contains fat is butter, which contains 4 grams of fat. So we can "guess" that the right number of pats of butter is 15. Alternatively, we can let B represent the number of pats of butter we should eat; in that case we will consume $4B$ grams of fat. Solving $4B = 60$ we get $B = 15$.

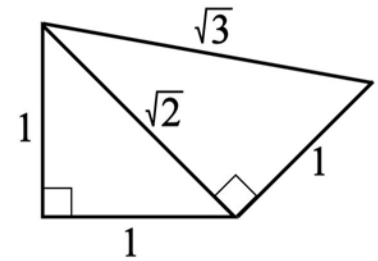
Therefore we should eat one and one-half cans of tuna, ten pears and fifteen pats of butter. Does this menu sound appealing to you?

Summer Fun!

Number of errors: ___________

Discovering Square Roots Via the Pythagorean Theorem

by Anda Degeratu



Triangle inequality:

For an arbitrary triangle with sides of length *a*, *b* and *c*, we have

$$a + b > c$$
$$b + c > a$$
$$c + a > b$$

…but if the triangle has a right angle between the sides of length *a* and *b*, more can be said:

$$a < c \text{ and } b < c$$

and we also have the powerful Pythagorean theorem:

$$a^2 + b^2 = c^2$$

Square roots:

Recall that the *principal square root* of a positive number *a* is the positive number *x* that satisfies $x^2 = a$. We denote it by $\sqrt{a}$.

Example: the principal square root of 1.69 is 1.3 because $1.3^2 = 1.69$.

In this problem set we are going to look at square roots using triangles, which we construct using only the following three tools: a ruler which is marked 1, 2, 3 and 4, a right angle, and a compass.

For example, if we construct a triangle with a right angle with short sides of length 1, the Pythagorean theorem tells us the length of the long side: it is $\sqrt{2}$.

Now, using the segment of length $\sqrt{2}$ as a leg we can construct another right triangle with the other leg of length 1. In this new triangle, the long side has length $\sqrt{3}$.

The triangle inequality in this new triangle gives

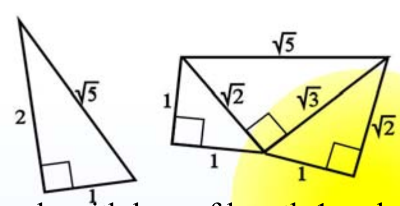
$$\sqrt{3} < 1 + \sqrt{2}.$$

Moreover, since this is a right triangle, we also get $\sqrt{2} < \sqrt{3}$ (which you already knew since $2 < 3$ implies $\sqrt{2} < \sqrt{3}$).

1. Find **two** ways to construct a segment of length $\sqrt{5}$. (For both ways, you should use only the three tools given to you. Note that the compass can be used to construct segments of a certain length, once you have constructed them somewhere else on your sheet of paper.)

<u>Solution</u>

(Ken Fan)



1. Method 1: Make a right triangle with legs of length 1 and 2. The hypotenuse will have length $\sqrt{5}$.

Method 2: Make a right triangle with legs of length $\sqrt{2}$ and $\sqrt{3}$. (Use the method in the statement of the problem to make these lengths.) The hypotenuse will have length $\sqrt{5}$.

Number of errors: ___________

Summer Fun!

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete.

<u>Session 2 – Meet 11 – May 1, 2008</u>

Mentors: Beth Schaffer, Cammie Smith Barnes, Ken Fan, Lauren McGough

Special Visitors: Sarah Ackley and Adele Schwab, MIT '08

We began this meet with some questions that involve periodicity. Periodicity has shown up in subtle ways throughout the entire second session.

1. What is the largest power of 2 that divides 10! = 10 × 9 × 8 × . . . × 3 × 2 × 1? More generally, what is the largest power of 2 that divides $n!$, which is the product of the numbers from 1 to n?

2. Carol has less than $250. She buys several boxes of chocolate. Each box costs $3. If she tries to split the boxes among 7 people, she ends up with 5 boxes left over. If she tries to split the boxes among 12 people, she ends up with 3 boxes left over. How many boxes of chocolate did she buy?

We also slipped in an area question in anticipation of the area activity for the last meet:

What is area of triangle with two sides of lengths s and t units which are separated by an angle of a degrees in terms of the area A of a triangle with two sides of length 1 unit separated by the same angle of a degrees?

Each member also received a Masu box folded by origami artist Christine Edison (see the previous issue of this Bulletin). **Tree**, **The Cat** and **Sylvia** reverse engineered these boxes with Cammie, while the other girls worked on the problems.

Sarah Ackley and Adele Schwab gave a presentation on the mathematics of light. Using laser pointers, the girls explored refraction, diffraction gratings, polarized lenses, and reflections on CDs and peacock feathers. Light can be modeled using periodic functions and diffraction patterns show how complex periodic interactions can be.

A laser pointer emits light with a fixed period. It was Isaac Newton who showed that white light consists of light of all different periods by shining it through a prism and using the fact that the amount that light bends when passing through a prism depends on its period (or color).

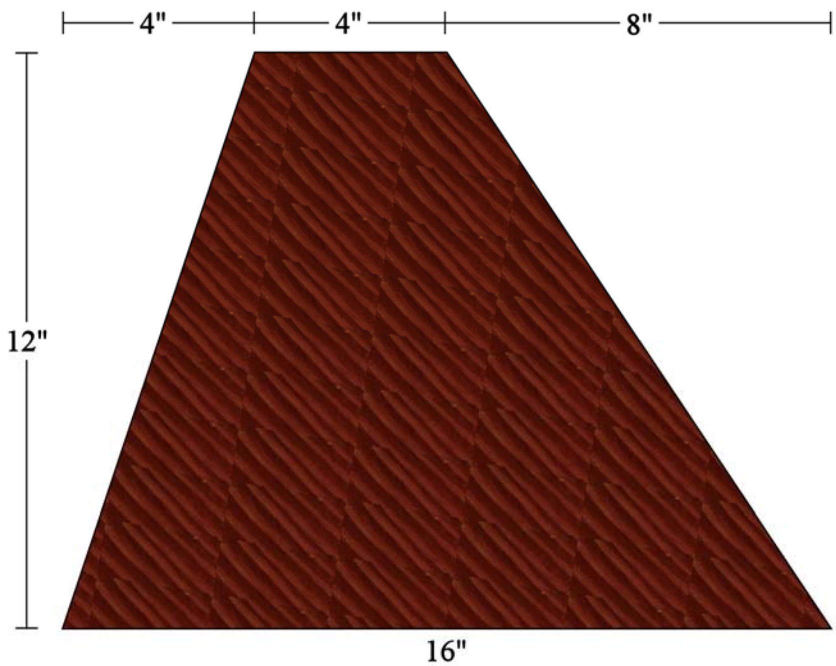
Session 2 – Meet 12 – May 8, 2008

Mentors: Alison Miller, Cammie Smith Barnes, Ken Fan, Lauren McGough, Inna Zakharevich.

We celebrated the last meet with a brownie…but not just any brownie. This brownie was cut into the shape of a large trapezoid. Before any brownie was served, members had to figure out how to cut up the trapezoid into equal pieces.

In order to maximize motivation, we made sure that the brownie tasted great. In fact, we didn't just make sure that the brownie tasted great. We actually were fortunate to have one of the finest, most scrumptious brownies in bean town, courtesy of Rosie's Bakery. They baked a special brownie using their award winning recipe for Girls' Angle. With this mouth-watering brownie, the people of Rosie's Bakery stimulated the study of geometry!

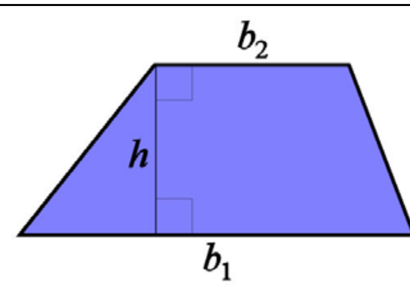
Here is a scale drawing of the brownie we used:



The challenge was to cut this trapezoidal brownie from Rosie's Bakery into 15 equal area pieces.

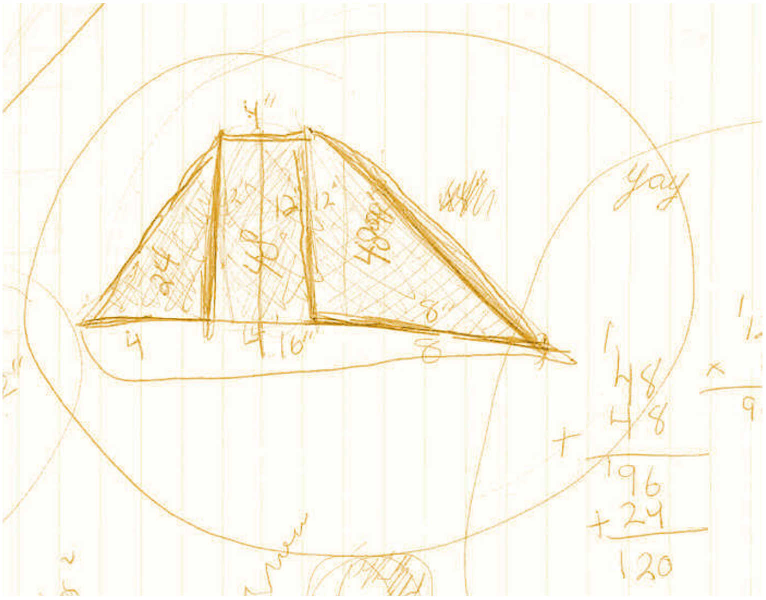
Many began by computing what the area of each individual piece should be. Using various techniques, the girls determined that the area of the trapezoid was 120 square inches. Dividing this by 15 would give 8 square inches of brownie for each person. Yum!

However, knowing the area was not enough. An actual, physical cutting scheme had to be proposed. **August**, **Honda, Ilana**, **PowerPuff, sports car** and **Tree** all used two major cuts that split the brownie into two right triangles and a rectangle as a starting point. But, beyond this first major trisection, their cutting schemes diverged. **Sylvia** approached the problem from an entirely different angle and **The Cat** helped verify that proposed cutting schemes were valid.

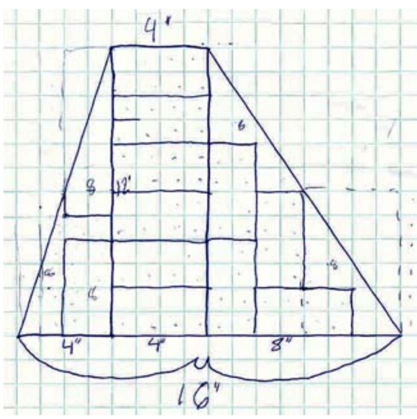


Can you show that the area of this trapezoid is $\frac{1}{2}h(b_1 + b_2)$? Think about it!

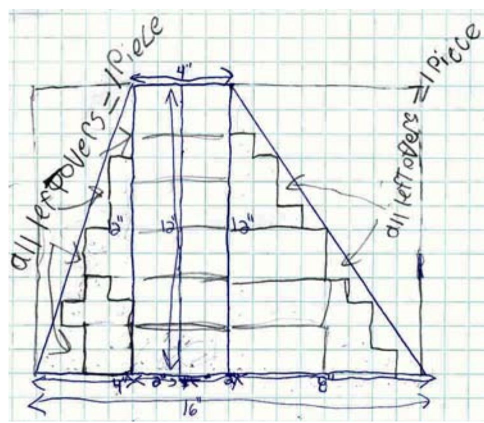
Directly below is **Honda**'s drawing of the trapezoid divided into the two right triangles and a rectangle. Notice that she has also indicated the areas of the three resulting pieces.



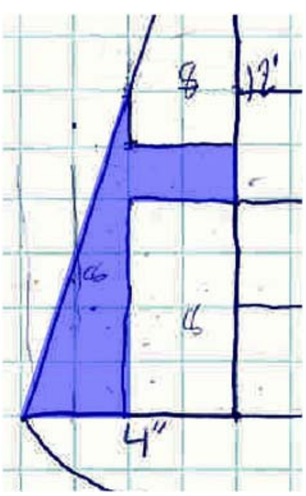
The right triangle on the left is 24 square inches and the right triangle on the right is 48 square inches. The rectangular portion is also 48 square inches. Note the calculation in the lower right that determines the total area of the brownie: 120 square inches. Because all these areas are multiples of 8, we know that each section contains an integral number of pieces— an important consideration if one is to use this major trisection to produce connected pieces. Otherwise, fractional parts would have to be formed into single pieces.



Ilana's cutting scheme

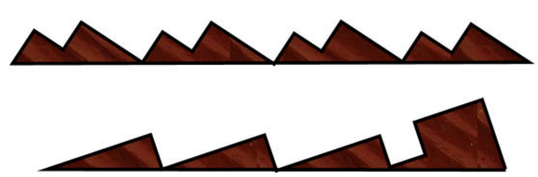


Tree's cutting scheme

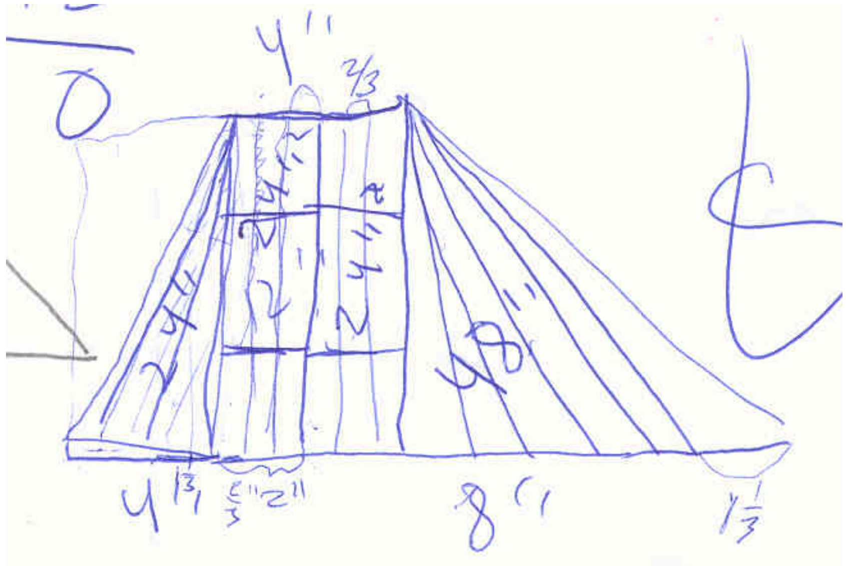


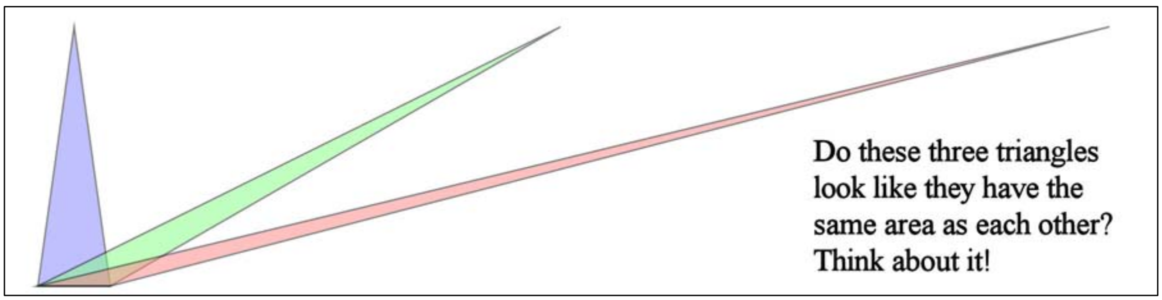
In **Ilana**'s cutting scheme, one of the pieces is an unusually shaped heptagon (at left). How did she know that the area of this heptagon was 8 square inches? The beauty of knowing that the 15 pieces must each have an area of 8 square inches is that because the area of the 14 other pieces in **Ilana**'s cutting scheme were known to be 8 square inches, the area of this heptagonal piece simply *had to be* 8 square inches, no further computation necessary.

Similar reasoning can be used to see that the two jagged pieces in **Tree**'s cutting scheme shown at right are also 8 square inches apiece. Note that each of these pieces really consists of four separate parts.

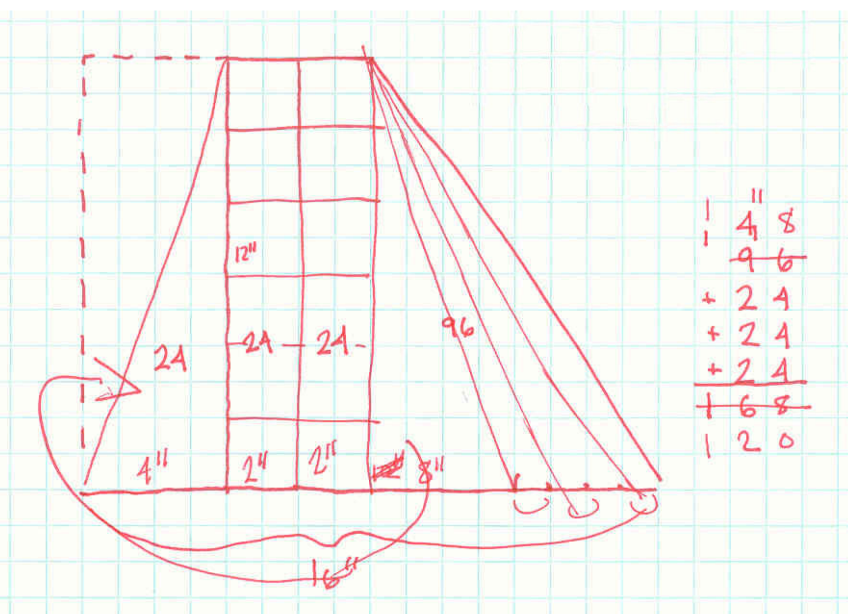


Sports car used a different technique to handle the right triangular sections. As can be seen from her diagram at right, she divided the base of the left triangle into 3 equal parts, and the base of the right one into 6 equal parts. She connected each division point up to the apex. Note however that the units on the values of the areas of different pieces should be square inches, not inches! Inches alone are a unit of measure for length, not area. Try to get into the habit of always using proper units.



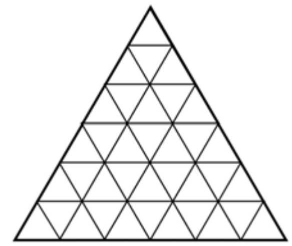


This scheme exploits the fact that triangles with the same base and height have the same area. Visually, this is not always obvious. Often people presented with two triangles with the same base and height do not believe that they have the same area. This can be an example of how mathematics can help you see the truth.

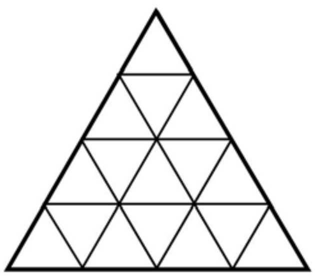


August and **PowderPuff** worked together and came up with a whole cutting *procedure* (see left). They split the larger right triangle into two halves. The right half was used to make 3 pieces by dividing its base into 3 equal parts. Then, they noticed that the other half was the mirror image of the right triangle on the left side of the brownie. By joining these two halves together along their hypotenuses, they could form a rectangle. They could then cut the resulting rectangle into 6 equal pieces using any standard technique for cutting up rectangles into equal parts. So there solution involved cutting, moving and more cutting. Of course, six people would be given two small pieces instead of a single eight square inch piece.

Throughout all this, **Sylvia** took an entirely different approach. Earlier in the session, she had done some work with the division of triangles into a perfect square number of congruent triangles, each similar to the original. Would such a technique work for this trapezoid? Is there a way to dissect the trapezoidal brownie into 15 pieces that are all congruent to each other? If there were such a way, then one would be able to know that everybody was getting an equal share *without having to make any computations involving area at all*. After all, if everybody's piece was identical, then there would be no complaint about someone getting more.



But, is it possible?



Sylvia's first attempt to answer this was to try to *tile* the trapezoid with triangles. Starting with a triangle, she drew multiple copies of it to fill out the trapezoid. There was a snag, however, because as the trapezoid got filled, the triangle would no longer fit snugly into the remaining parts.

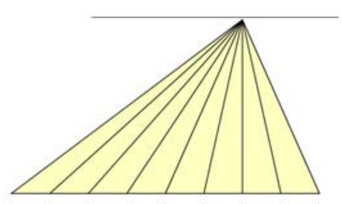
She knew how to divide a triangle into 16 congruent pieces. But she wanted to divide a trapezoid into 15 congruent pieces. What is the difference between a triangle and a trapezoid? What is the difference between 16 and 15?

Sylvia was holding a triangle dissected into 16 congruent pieces, when an idea struck her. She folded the tip of the triangle over…a brilliant idea! The result was a trapezoid…divided into 15 congruent triangles. But, alas, this particular trapezoid wasn't exactly like the brownie shape. The trapezoid she held was an isosceles trapezoid.

One feels that having gotten so tantalizing close to such an elegant solution, there *must* be a way!

Finally, **Trisscar** couldn't attend this last meet of the second session, so we can only wonder what she might have dreamed up in order to divide this brownie. Knowing her, it would have been fascinating!

Comments



Let's underscore two items from this brownie cutting experience.

1. If you divide a triangle into several triangles by drawing lines from a vertex to equally spaced points on the opposite side, all the resulting triangles will have the same area as each other. You probably haven't learned about matrices and determinants yet. But when you do, you will learn that the determinant of a matrix remains unchanged if you add any multiple of one column to another. This fact about determinants boils down to the same principle that underlies this equal area dissection of a triangle.

2. The idea **August** and **PowderPuff** had of joining two non-rectangular pieces to make a rectangle is an idea that can be used to cut any polygon. It turns out that you can take *any polygon*, cut it into some pieces and rearrange those pieces to form *any other polygon with the same area* (including a rectangle). This result is known as the Bolyai-Gerwien theorem.

Special Announcements

We wish all the girls who visited Girls' Angle even just for one meet during this past year a wonderful summer and we hope to see you again in the fall!

Girls' Angle thanks Rosie's Bakery for baking us a delicious brownie for our geometry meet on May 8. Their Cambridge store is located at 243 Hampshire Street.

Calendar

Session 2: (all dates in 2008)

January	31	Start of second session!
February	7	
	14	
	21	No meet
	28	Visitor: Tanya Khovanova, mathematician
March	6	
	13	
	20	Visitor: Elissa Ozanne, Harvard Medical School*
	27	No meet
April	3	
	10	Visitor: Karen Willcox, MIT Aeronautics Department**
	17	Visitor: Leia Sterling, MIT Aeronautics Department
	24	No meet
May	1	Visitors: Sarah Ackley and Adele Schwab, MIT Physics
	8	

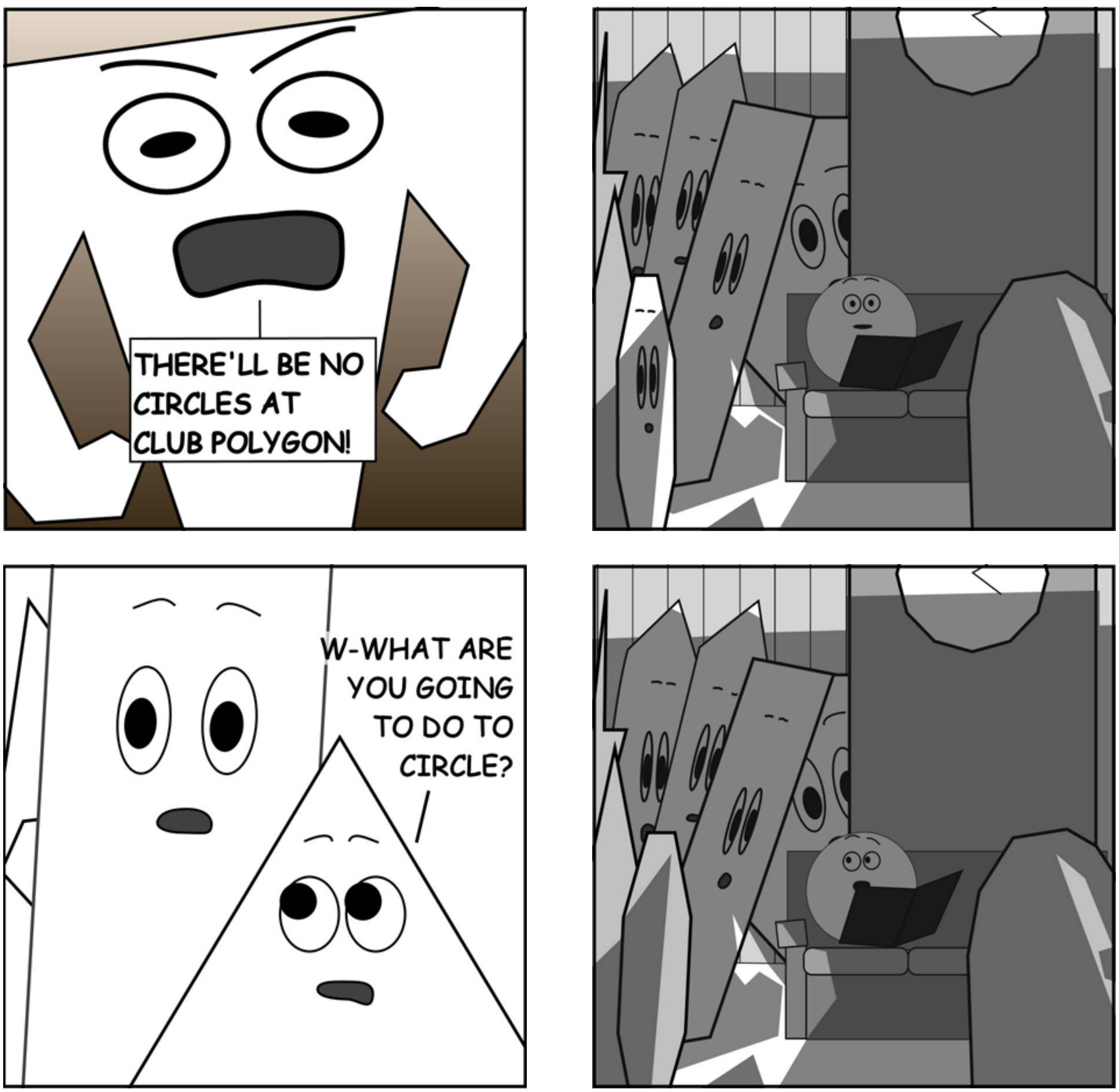
*Dr. Ozanne's visit was postponed one week and took place on March 20.
**Girls' Angle went to MIT for Prof. Willcox's presentation.

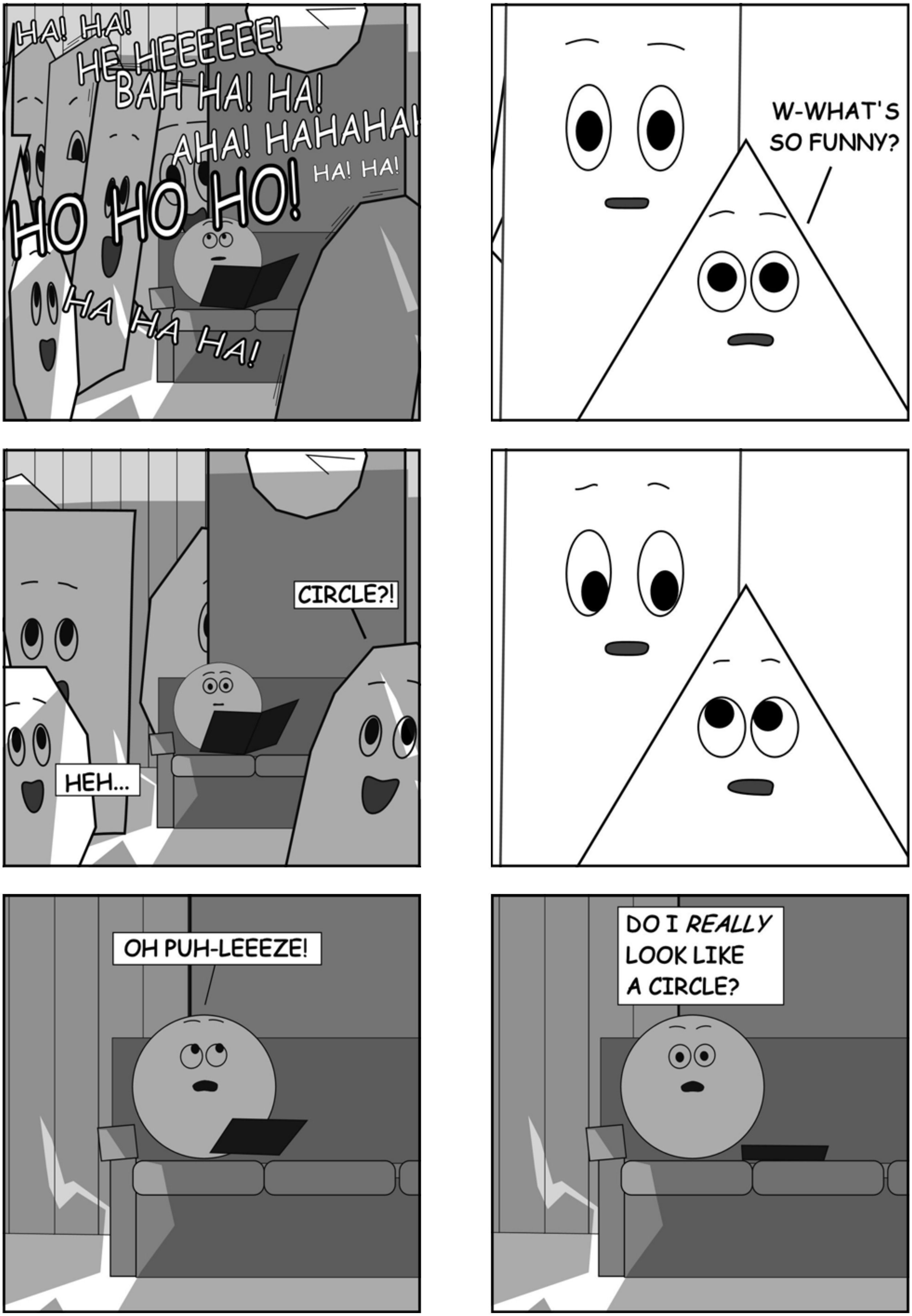
Session 3: (all dates in 2008)

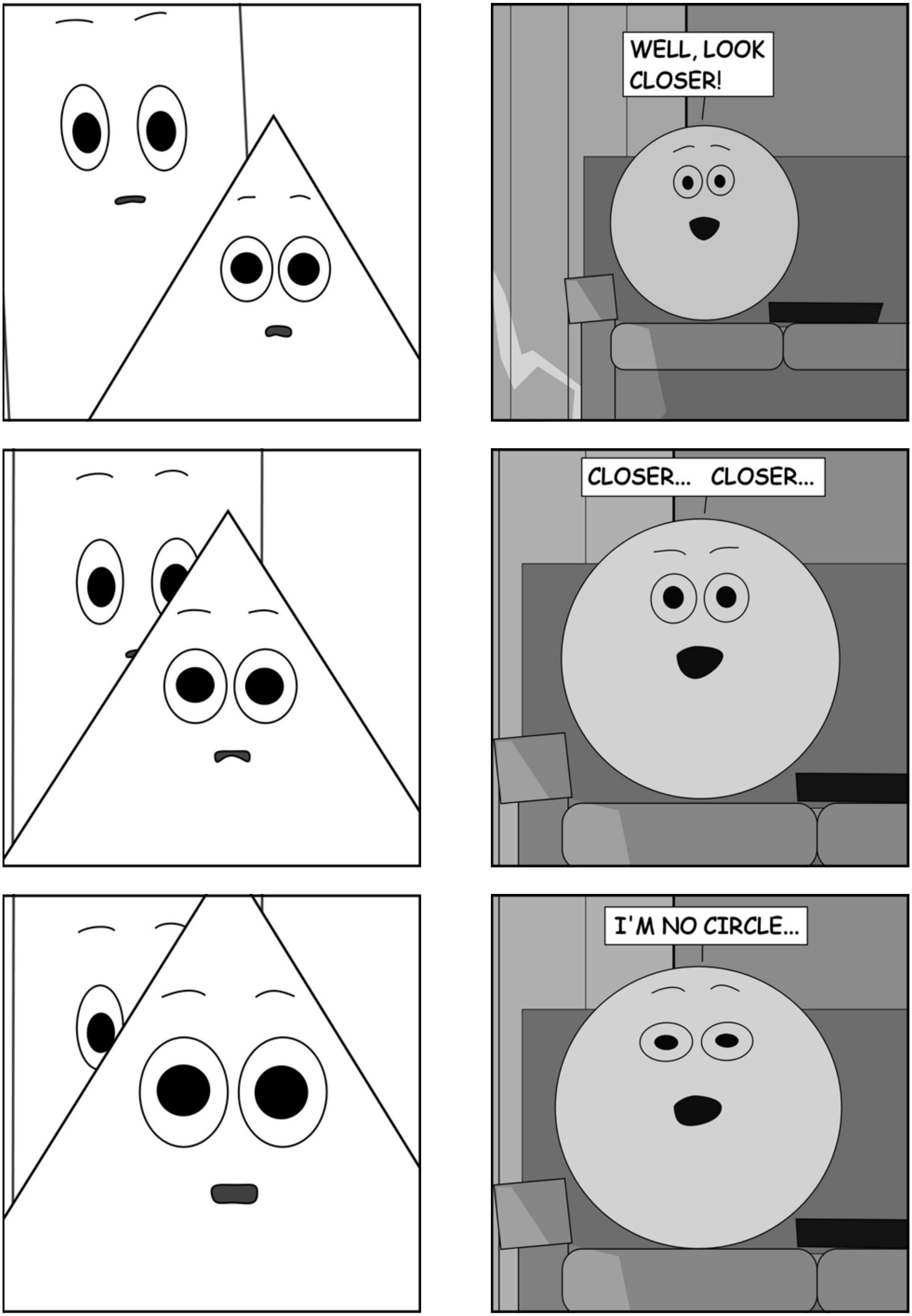
September	11	Start of third session!
	18	
	25	
October	2	
	9	Yom Kippur - No meet
	16	
	23	
	30	
November	6	
	13	
	20	
	27	Thanksgiving - No meet
December	4	
	11	

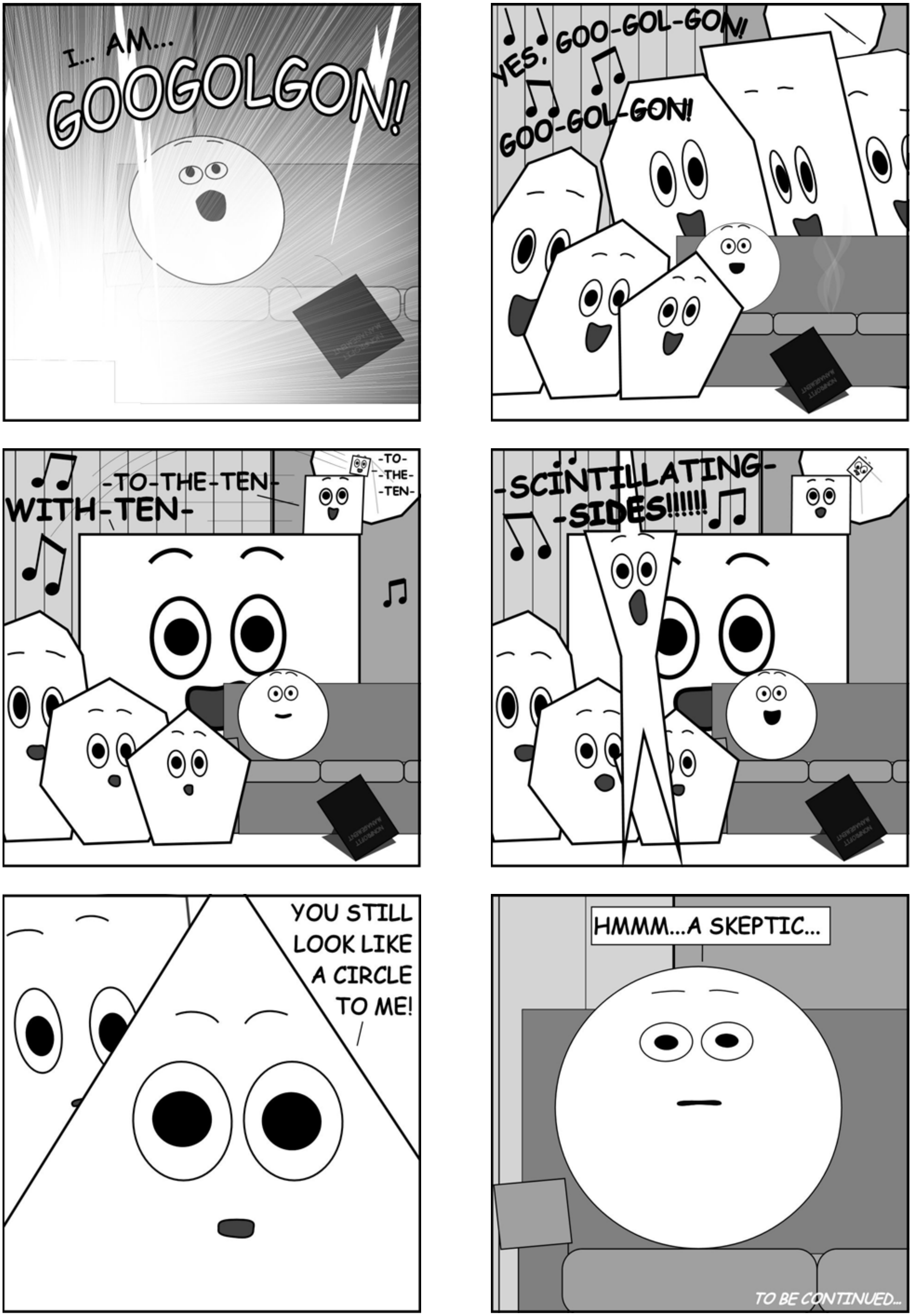
It Figures!

by CKFan




HA! HA!
HE HEEEEEEE!
BAH HA! HA!
AHA! HAHAHA!
HO HO HO!
HA! HA!
HA HA HA!
W-WHAT'S SO FUNNY?
CIRCLE?!
HEH...
OH PUH-LEEEZE!
DO I *REALLY* LOOK LIKE A CIRCLE?


WELL, LOOK CLOSER!
CLOSER... CLOSER...
I'M NO CIRCLE...


I... AM...
GOOGOLGON!
YES, GOO-GOL-GON!
GOO-GOL-GON!
WITH-TEN-
-TO-THE-TEN-
-TO-
-THE-
-TEN-
-SCINTILLATING-
-SIDES!!!!!!
YOU STILL LOOK LIKE A CIRCLE TO ME!
HMMM...A SKEPTIC...
TO BE CONTINUED...

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 3 ways: **membership**, **subscription** and **premium subscription. Membership** is granted per session and includes access to the club and extends the member's premium subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session, you will get a subscription to the Bulletin, but the premium subscription will start when total payments reach the premium subscription rate. **Subscriptions** are one-year subscriptions to the Girls' Angle Bulletin. **Premium subscriptions** are subscriptions to the Girls' Angle Bulletin that allow the subscriber to ask and receive answers to math questions through email. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, UC Berkeley
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, assistant professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

☐ Enclosed is a check for (indicate one) (prorate as necessary)
- ☐ $216 for a 12 session membership
- ☐ $100 for a one year premium subscription
- ☐ $20 for a one year subscription
- ☐ I am making a tax free charitable donation.

☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: ____________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

Girls' Angle

A Math Club for Girls