Girls' Angle Bulletin

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To Foster and Nurture Girls' Interest in Mathematics

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From the Director

A few weeks ago, I interviewed Ingrid Daubechies, the first woman ever tenured in the Princeton mathematics department. While I am honored to have had that opportunity, I do find it odd that today, in the twenty first century, the first woman tenured there is still active. Shouldn't the *first* woman tenured there have been tenured a long, long time ago?

Then again, Harvard has yet to tenure a woman at its math department, and Harvard is even older than Princeton.

The next issue will be the Girls' Angle one-year anniversary issue! To celebrate, we are going to commence with some exciting, new regular columns. One of them, by Katy Bold, will address applied mathematics. Another, by Anna Boatwright, will personalize the process of doing mathematics. You'll get to see her problem solving attempts as she gives us an honest (and, I might add, very brave) glimpse into her process of *doing* mathematics, errors and all.

We also have an exciting line up of visitors for the 3rd session. We hope you've all had a wonderful summer, and to the members, we hope to see you again when the next session begins on Thursday, September 11!

Ken Fan
Founder and Director

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle: A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

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On the cover: A collage of some of the things members came up with during the first year of Girls' Angle. Excellent work, all!

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An Interview with Ingrid Daubechies, Part I

en.wikipedia.org/wiki/Image:Fine-hall-princeton.jpeg

Professor Daubechies works in Fine Hall, home of the mathematics department at Princeton University in New Jersey.

Ingrid Daubechies is the first female full professor of mathematics at Princeton University. She received tenure there in 1993 after working at Bell Labs. She was born and raised in Belgium. For this interview, I actually traveled to Princeton and met with her in her office at Fine Hall. The full transcript of the interview will be revealed in parts.

Ken: Thank you so much for agreeing to sit down with me for this interview! I think that the nature of the mathematics profession remains a bit elusive to our members so the opportunity to speak with one of today's premier mathematicians is a very fortunate event for us. I'd like to start by asking you about mathematics generally…what is it to you?

Prof. Daubechies: I've done work that is pure and work that is much more applied. I did this work on wavelets- it doesn't really matter what they are for this purpose- but the thing is, it is something that is motivated by concrete applications in signal analysis, in particular, for images and it's useful for those and it involves some special functions.

Now, when people talk about those they always feel, I feel, everybody feels, they have been constructed. Now, coming from a math background, you probably know that many people feel in mathematics that mathematics is discovered and it is an outside thing and they discover it and they get inside and so on. But about these things [the special functions] people feel that they are constructed. But the feeling, I can tell you, from working on them and figuring it out and so on is exactly the same whether you do pure math or that. So, I try to find out why is that?

I think it's partly because most of mathematics was motivated by physics. And in physics, you try to describe things that exist while when you do it for technology, it is, obviously, that you try to find ways, patterns, and so on, that help you construct things yourself. But the math part of it feels the same and even people think of it differently. So, in the math, where's the boundary [between the discovered and the constructed]? Where do you feel that you go over the boundary? And if you start looking, you don't find it. I believe there is no boundary. I believe that all mathematics is something that we construct. I think that mathematics is something that does not exist "out there".

I think that mathematics is, in a sense, a very human thing. I mean, it's our way…it's the name we give to when we try to really precisely and articulately describe structures that help us in describing nature and studying nature and constructing things and so on. But that whole pattern of precise thinking and making things fit and trying to find patterns, and because we cannot build it all, we have to build smaller building blocks that we then use in building bigger things and

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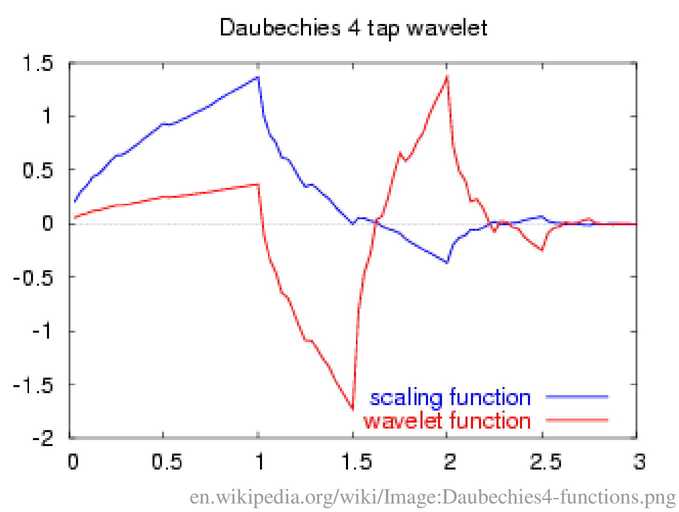
that's why we build all these mathematical concepts and theories. Of course, it is very nice when the thing that you developed here can be used there. To me, that's mathematics.

So, I think there are lots of mathematical types of reasoning where people don't write a single formula if they try to really very precisely and unambiguously reason their way through. I consider that a form of mathematics. Instead of saying mathematics is this abstract thing for which you have to be a kind of weird mutant to be good at it, I say, it's not true. I mean it's something that we all share. It's a very human thing. I could conceive of us meeting an alien race that would do its math *completely* differently and we would have great difficulties in understanding them. I hope we would learn, but I'm not really sure we could, but you hope.

So, I don't believe in this universal thing. I feel it is a very human thing.



This is the graph of a wavelet named after Prof. Daubechies.

So it's also something at which some people are more gifted than others…just like you have Olympic athletes and other people. But, I mean, there's nobody telling all these joggers on the road that they shouldn't run because they will never be an Olympic gold medalist.

There are now all these Sudoku puzzles that are so popular. I thought once, "oh, it's fun…it's not math." Well, that's ridiculous. It's not high level math, but it is definitely math and it is popular because people like doing that kind of figuring out. It's actually much simpler than everyday life because you have such precise rules. I mean, you only have a few rules and that's it. Within that, you have to figure it out.

Now, women and math…I think it is great to have venues where you are especially for girls. Actually, I've been thinking…I'm originally from Belgium and Belgium is a country that is big about cartoons. Now, everybody is about cartoons because people like manga and so on, but Belgium was big on that way before, and they were very different from American cartoons, although they had started from American cartoons. It started after the Second World War when American cartoons were no longer arriving. So, for all the series that were ongoing, they started writing sequels. So a whole industry started.

There are a couple of heroes in Belgium called Suske and Wiske[1], and Wiske is the girl. But mathematics, for a completely different reason is called *wiskunde*. So I was thinking of approaching the estate of that cartoonist to do something like wiskunde with this girl as the emblem. Partly because there is, in many countries in Europe now, a shortage of people taking mathematics- and as a result, a shortage of math-trained high school teachers which is disastrous. I mean, I can see what it does in this country. It didn't happen before in Belgium and in Europe, but it is going to happen. And so, when there is a shortage, all of a sudden, you might not have anymore this thing of, "it is for boys"...if there's nobody. And maybe you can then, and this has happened before, that in things that were no longer attractive to anybody, it was a good niche for

[1] Suske and Wiske are the creation of Willy Vandersteen.

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women. This is a sad thing, but…definitely, I see it in every program where you have women that as soon as you have several women its fun together. Because they are so seldom in the surroundings where there are other bright women also interested in math.

Ken: Yes, that's one thing I love about Girls' Angle. It's a very different feel from the co-ed classed I've taught. We also believe that when the motivation is there, the learning follows naturally. So we spend a lot of time motivating an interest…getting the girls interested in some math problem. And then, once they're hooked…just facilitate.

Prof. Daubechies: One thing I notice when I teach, I try to always relate things. I have classes where I teach for non-math majors and where I try to relate to concrete examples. I put in a lot of examples with my kids and with relational situations and so on and personal anecdotes. And I find that I often relate much better with women…I mean that style of teaching relates much better to the women in the class than with the men because the men consider it not serious. But the women feel, oh yeah, this does make sense.

"I think that mathematics is... a very human thing."

Ken: Have you though about modes of teaching mathematics that might appeal more to girls?

Prof. Daubechies: [pausing] I don't know about that. I've read about it, but I don't really know about it. I think different groups have different feelings about it. I haven't done sufficient experimentation with boys versus girls in the classroom. I could imagine that there might be indeed different modes that might work better. What I do notice is that a certain type of being informal helps me make a bridge to the girls while it sometimes hinders with the boys. Not always, but it sometimes does. With some males I have to do some kind of one-upmanship in order for them to take me seriously, which, with girls, they just clam up. I mean, if people do that with me, I clam up too. I mean, I don't want to work with collaborators who shout. They can be great mathematicians, but they can go shout somewhere else. Now, it takes all kinds of personalities, so none of these are hard and fast rules, I'm sure, but it's definitely the case that with there being so few women that if on average, women like a less authoritative touch and a more collaborative touch and if they don't get it that will discourage them even more…I don't know if that is something that works…it could well be.

Ken: How did you get interested in mathematics?

To be continued…

In 2000, Prof. Daubechies was awarded the National Academy of Sciences Award in Mathematics. At the time of this writing, she is the only woman to have been granted this honor. She was cited, "for fundamental discoveries on wavelets and wavelet expansions and for her role in making wavelet methods a practical basic tool of applied mathematics."

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en.wikipedia.org/wiki/Image:Raffael_058.jpg

Mystery Woman Revealed!

Raphael painted *Philosophy* about 500 years ago. Over 25 feet by 16 feet, he took 2 years to paint it on a wall in the Vatican. Also known as the *School of Athens*, it depicts various scholars. The two central figures are famous philosophers: Plato and Aristotle.

All the figures depicted in the painting are men, except for the highlighted figure. The lone woman in this painting represents the mathematician Hypatia.

Hypatia lived about 1,600 years ago in Alexandria. She wrote works on mathematics, astronomy and philosophy. She made an extensive study of how cones and planes intersect. These **conic sections** play an important role in planetary motion. If you'd like to see some conic sections, take a flashlight and shine it on a wall from various angles and observe the boundary of the lit patch.

For more information on Hypatia, check out *Hypatia of Alexandria* by Maria Dzielska or *Hypatia of Alexandria, Mathematician and Martyr* by Michael Deakin.

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Points, Lines and Circles

Take a plane. A sheet of paper is a handy model.

There are many, many lines in this plane.

Let's mark a point in the plane and ask, what are the lines that pass through this point? If you draw a bunch of lines through the point, you'll see that they fan out like radial lines. You can imagine every line that passes through the point by taking one such line and rotating it around the point like a propeller.

Let's mark another point in the plane. What are the lines that pass through *both* points? Now, there is only one line that passes through both. In other words, two points *define* a unique line.

Let's mark a *third* point in the plane. What lines pass through *all three points*? Now there's trouble. There was only one single line that passed through the first two points, so the third point would have to be on that line for there to even exist a line passing through all three points.

So, in general, there will be no line. Given three points, you cannot expect there to be a single line that passes through all three.

In other words, there's nothing much that is special about finding a line that passes through two points, but it is something special for a line to pass through three or more points. When several points sit on a single line, they are called **collinear**.

The next time you look up at the stars, pick three stars at random. Chances are that they will not line up. If they look like they do, hold up a ruler and you'll probably discover that they didn't really line up. And if they still do, it's an amazing coincidence and is possible only because real stars do not actually model points. By the time the starlight has reached our eyes, it has dispersed a little and the star appears as a tiny smudge with a little bit of thickness.

In mathematics, if you should ever notice three points of interest in some geometric configuration and you observe that the three points are collinear, it is worth pointing out. *It might even be a theorem.*

By the way, what happens if we play this same game in space instead of in a plane? Think about that for a moment. Do two points still define a line in space?

Here's a problem: Suppose you have ten points in a plane. Can you show that there is a line that divides the plane in half so that there are five points on one side and five on the other?

Now let's go back to the plane, and start over, only this time, let's think about circles instead of lines.

Again, there are many, many circles in a plane. Imagine the surface of a pond when it is raining. Each drop will create ripples of expanding concentric circles.

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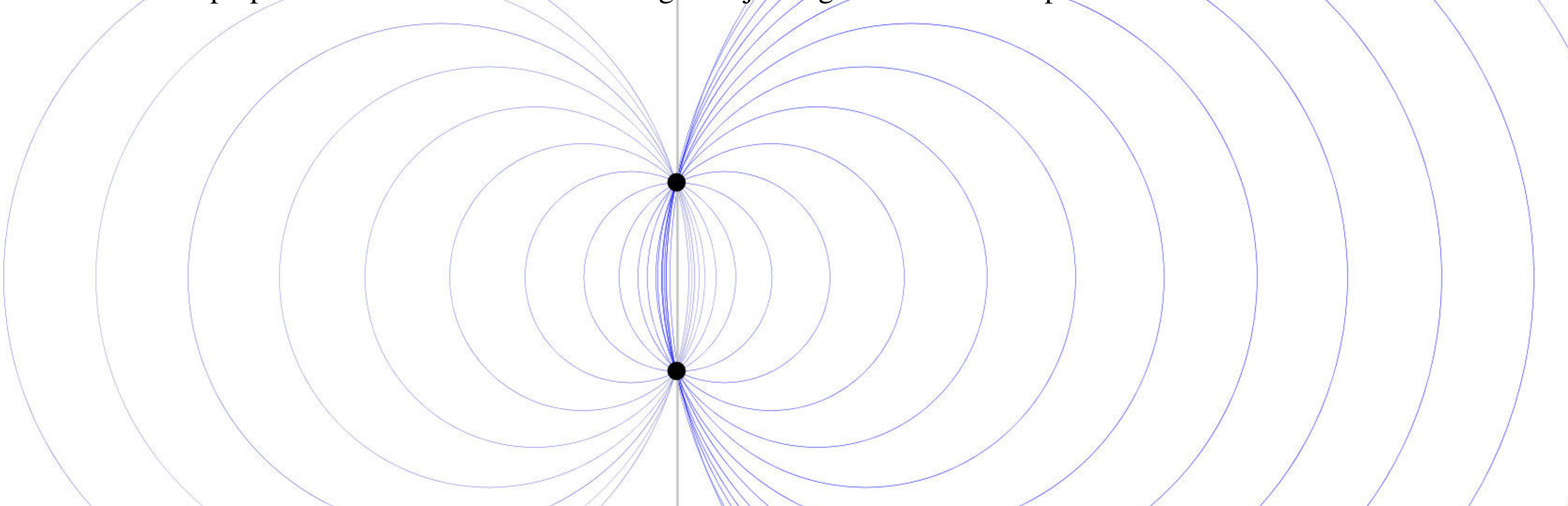
So, just as we did with lines, let's mark a point in the plane and ask, "What circles pass through this point?"

There are still many! Every other point in the plane can serve as the center of a circle that passes through the marked point and each of these circles is unique.

So let's mark a second point. What are all the circles that pass through both marked points? There are *still* many such circles. These circles form a family of circles whose centers all lie on the perpendicular bisector of the line segment joining the two marked points.



Perhaps you can imagine all of these circles by imagining that you are squeezing one of them between the two marked points as if the points were fixed posts. As you push it through, the circle shrinks. Once through, however, the circle now gets caught on the posts and starts expanding again.

Now, let's mark a third point. Except for points on the gray vertical line defined by the first two points, you would be able to find a circle that passes through the three points. If you imagine passing through the family of circles described in the previous paragraph, some of which are depicted in the drawing, you might see that each point in the plane off of the gray line will be touched exactly once by some circle in the family. What this means is that, in general, there is a unique circle through three points. The only time there is no such circle is if the three points are collinear.

In other words, three non-collinear points define a circle. So, while two points define a line, it takes three to define a circle.

If you arbitrarily select four or more points in a plane, it is very unlikely that all will sit on a common circle. If four or more points happen to sit on a common circle, that's a special situation and one should take note!

For example, the four vertices of a rectangle all sit on a common circle and it is, indeed, quite special for four points to be the vertices of a rectangle because a rectangle is a very particular type of polygon.

Let's finish with a question: What happens if you play this game in space using spheres instead of circles?

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Polynomial Primer

Some of you may be starting a school year where you will encounter **polynomials**. Polynomials are very important. Here's why:

There are four concepts that are used a whole lot in many places both in and outside of mathematics: number, addition, multiplication and variable. If you doubt the importance of these concepts, see if you can go for one week without using them. It's not so easy to do!

If you're convinced of the importance of these four basic concepts, then you'll just have to agree that polynomials are important too! Why? Because polynomials *are exactly everything and anything you can get by combining numbers, addition, multiplication and variables*!

Here are some examples:

$$1 \qquad \frac{5}{9}(F-32) \qquad \frac{1}{2}h(b_1+b_2) \qquad \frac{1}{3}Bh \qquad \frac{a+b+c}{3}$$

$$mc^2 \qquad b^2-4ac \qquad 4\pi R^2 \qquad m^2+n^2 \qquad \frac{1}{2}gt^2+vt+h$$

Notice that division by a *number* is the same thing as multiplication by its reciprocal, which is just multiplication by some other number, so it's ok to have things like $\frac{1}{3}Bh$ and $\frac{a+b+c}{3}$. In fact, when it comes to polynomials, we don't really want to include division in the definition because we want to avoid expressions that involve dividing by a *variable*, such as in the expression $\frac{1}{x}$.

Of course, $\frac{1}{x}$ is an important expression too; it's just not a polynomial. It's an example of a **rational** expression. Rational expressions are ratios of polynomials. In fact, you could rewrite this primer by changing its title to "Rational Expression Primer" and talking about *five* basic concepts: the four already listed together with division.

Also, note that even though subtraction is not discussed, you can have subtraction because subtraction is the same as multiplication by negative one followed by addition. For example, we can see that $b^2 - 4ac$ is made using only numbers, addition, multiplication and variables if we write it as $b^2 + (-4)ac$. After all, -4 is just another number.

We also didn't use anything beyond the four basic concepts by including exponents. The exponents just stand for lots of multiplication. However, the exponent has to be a nonnegative integer. Raising something to the zero power is okay because that is just a fancy way of writing one. But a polynomial *cannot* have variables with non-integral exponents nor can it have variables with negative exponents. (So, $\sqrt{x}$ is *not* a polynomial.)

If you have two polynomials, you can get a third by adding them together or multiplying them together. And why not? Doing so continues to use only the four concepts.

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Now I'm going to introduce some terminology associated with polynomials. But first, I'd like to comment on such "terminology" passages, which are common in math books. Without feeling the motivation for the new words, it can be harder to retain them all. If you encounter such a passage and feel frustrated about not being able to remember everything, rest assured that most everybody feels the same way. What I do when I encounter such passages is to just skim through without worrying about memorizing everything. I'll get them later, when I see the need for them and if I never get to them later, I guess I don't really need them! Although, it can be fun, sometimes, to try to guess at why the various terms were invented. You can ask us, too!

When a polynomial expression is written without parentheses, explicit or implied, then the polynomial is said to be in an **expanded form**. Notice that even though $\frac{a+b+c}{3}$ does not have explicit parentheses, there is an implied set of parentheses around the numerator. An expanded form of this polynomial is $\frac{1}{3}a+\frac{1}{3}b+\frac{1}{3}c$. When you write a polynomial in expanded form, its **terms** are the pieces separated by addition and subtraction. For instance, the terms of $b^2 - 4ac$ are b^2 and $4ac$. **Like terms** are terms that can be combined into a single term. For example, $7xy^2$ and $3yxy$ are like terms because they can be combined into the single term $10xy^2$. People use the word "term" in the context of polynomials a little bit loosely. For example, occasionally people would say that the terms of $b^2 - 4ac$ are b^2 and $-4ac$. Doing so suggests that they are thinking of $b^2 - 4ac$ as $b^2 + (-4)ac$. Also, sometimes, people will refer to a term only by its variable factor. For instance, in the polynomial $2x^3 - 5x^2 + x + 2$, people will sometimes speak of "the x^2 term" when they are referring to the $5x^2$ (or $-5x^2$) in that polynomial. The **coefficient** of a term is the product of all of its numeric factors. When computing the coefficient, any negative sign or minus sign *must be* factored in. Indeed, when people ask for the coefficient, they are often mainly interested in whether the coefficient is positive or negative…especially, this is true of the **leading coefficient**, which will be explained in a moment. For instance, the coefficient of 2^5a^3 is 32 and the coefficient of the x^2 term in $2x^3 - 5x^2 + x + 2$ is -5 (not 5!).

An expanded form is **simplified** if its like terms have all been combined. For example, the polynomials $2x^2 + 5x - x^2$ and $x^2 + 5x$ are equal, but only the latter is simplified. In the former, the terms $2x^2$ and $-x^2$ are like terms that can be combined into the single term x^2.

A polynomial that can be expressed as a single term is called a **monomial**. In the table of polynomial examples, there are four monomials. The **degree** of a monomial is the *total* number of its variable factors. For example, the degree of $3x^2yz$ is 4 and the degree of 2^5a^3 is 3 (only the variable factors count toward the degree). Notice how one counts the total number of variable factors and *not* the total number of distinct variables. That is x^2 has *two* variable factors that both happen to be x, so its degree is 2. To find the **degree** of a polynomial, expand it and take the maximum degree of its monomial terms. The **leading coefficient** is the coefficient of the monomial of highest degree, if there is a unique monomial with highest degree. (For example, the polynomial $b^2 - 4ac$ has no leading coefficient because both of its terms have degree 2.)

A simplified polynomial with two terms is a **binomial** and one with three terms is a **trinomial**.

When you study polynomials, you are studying the interplay of addition and multiplication. For more on polynomials, see Eli's Summer Fun problem set on page 16 which is about the polynomials $(x + y)^n$, where n is a nonnegative integer.

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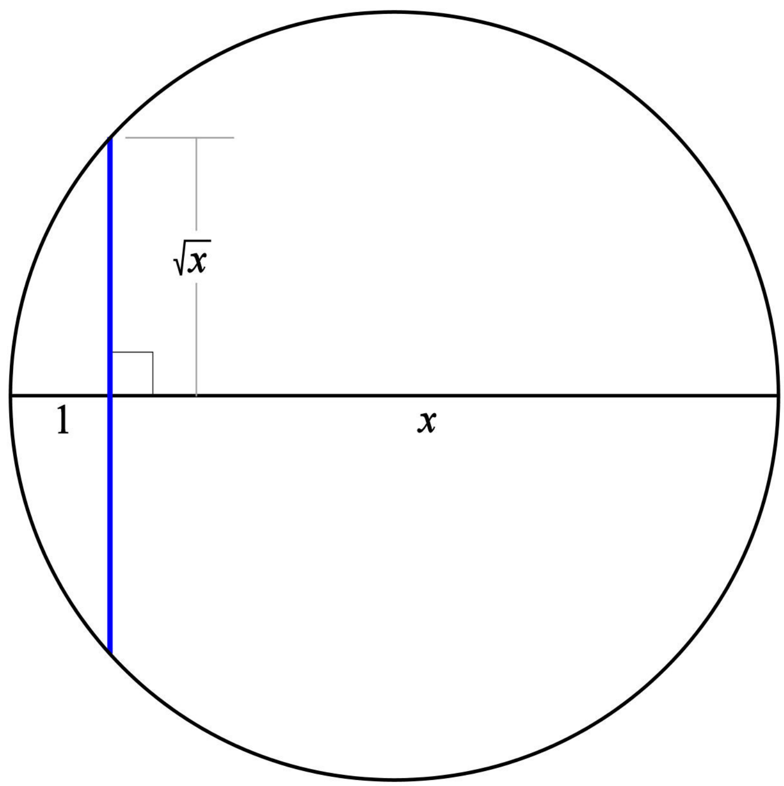
Constructing Square Roots

In the last issue, we briefly discussed square roots.

Here, we are going to indicate, or suggest, how you can construct the length $\sqrt{x}$ if you have a segment of length x using a compass and straightedge (i.e. an unmarked ruler).

The idea is to make a circle whose diameter has length $1 + x$. Then the chord indicated in blue will have length $2\sqrt{x}$ (note that the chord extends all the way from a point on the circle to another point on the circle, passing over the drawn diameter).



Can you prove this?

Even if you can't prove it now, it's fun to try the construction. For example, if $x = 4$ inches, then the blue chord should be *exactly* 4 inches long too. Try it and see!

Notice that part of the construction involves drawing a line segment perpendicular to another. Do you know how to do this with a compass and straightedge? If you don't, can you figure out a way to do it?

Can you think of another way to construct $\sqrt{x}$ with a compass and straightedge?

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Square Roots - True or False?

Assume that $\sqrt{x} = \frac{3}{2}$ and $\sqrt{y} = \frac{4}{3}$.

1. -5 is a square root of 25	T
2. The principal square root of 2.5 is 0.5.	F
3. $3 < \sqrt{10} < 4$	T
4. $\sqrt{123^3} = \sqrt{123}^3$	T
5. If $a = b$, then $a^2 = b^2$.	T
6. If $a^2 = b^2$, then $a = b$.	F
7. The value of $9y$ is 16.	T
8. $\sqrt{xy} = 2$	T
9. $\sqrt{\frac{x}{y}} = \frac{8}{9}$	F
10. If $a > 0$, then $\sqrt{a} < a$.	F
11. The only solution to $\sqrt{a} = a$ is $a = 0$.	F
12. $\sqrt{x+y} = \frac{17}{6}$	F
13. $(\sqrt{2} + 1)(\sqrt{2} - 1) = 1$	T
14. $\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$	T
15. $\sqrt{1800} = 30\sqrt{2}$	T
16. $\frac{1}{\sqrt{2}-1} \neq \sqrt{2}+1$	F

These are the answers for the true/false set from the last issue.

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Summer Fun!

In volume 1, number 4, we invited members and subscribers of the Bulletin to submit solutions to a number of Summer Fun problem sets. In the last issue, we gave solutions to the first problem from each problem set, albeit with deliberately planted errors. Did you find them?

In this issue, solutions to all of the problems are provided. These solutions will sometimes be rather terse and, in some cases, are more of a hint than a solution. We prefer not to give completely detailed solutions before we know that most of the members have spent time thinking about the problems. The reason is that *doing* mathematics is very important if you want to learn mathematics really well. If you haven't tried to solve these problems yourself, the value gained by reading other people's solutions will not be optimized.

Solutions that are especially curt will be indicated in red. Please do not get frustrated if you read a solution and have difficulty understanding it. If you run into difficulties, we are here to help! Just ask!

So, if you haven't thought about the problems, we urge you to do so before reading the solutions. Even if you cannot solve a problem, you will benefit from trying and it will become much easier to read other people's solutions.

With mathematics, don't be passive! Get active!

Move that pencil and move your mind! Your mind may just end up somewhere no one has been before.

Also, the solutions presented are *not* definitive. Try to improve them or find different solutions.

There were eight errors in total in the solutions from the last issue. How many did you find? Those errors are corrected in this issue.

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Parity: Are you Even or Odd?

by Ken Fan

In this problem set, when I say “number”, I mean an integer, that is, a counting number, the negative of a counting number, or zero.

1. We’ve encountered parity many times at the club. Just knowing whether a number is even or odd can often be powerful information! Recall that a number is even if and only if it is divisible by 2. Otherwise, it is odd.

   a. Write down the first 10 (positive) even numbers and the first 10 (positive) odd numbers.
   b. Is zero even or odd?
   c. Which prime numbers are even? Which prime numbers are odd?

2. When you have a concept that pertains to numbers, it is often a good idea to see how that concept relates to basic number operations, such as addition and multiplication. Let N and M be two numbers.

   a. Complete this table:

Table of Parities						
N	M	$N + M$	$N - M$	$N \times M$	N^2	N^3
even	even	even				
even	odd					
odd	even					
odd	odd					

   b. Is it possible for $2N = 2M + 1$?
   c. Can you show that $N^2 + N$ is always even?

3. How does parity relate to the Fibonacci numbers? To the triangular numbers?

4. Here’s another classic problem that involves parity. One hundred lockers are lined up in a row. They are numbered one through one hundred and are all closed. One hundred people pass by this row of lockers. As the first person passes by, she opens any closed locker and closes any open locker. As the second person passes by, she only pays attention to the even numbered lockers, again opening closed lockers and closing open lockers. The third person does the same, only paying attention to those lockers whose number is a multiple of three. In general, the Nth person goes to the lockers numbered by a multiple of N, closing open lockers and opening closed lockers. After all 100 people have passed by the lockers, which lockers end up open?

Summer Fun!

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Solutions

(Ken Fan)

1a. The first 10 positive even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20. The first 10 positive odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17 and 19.

1b. Zero is even.

1c. The only even prime number is 2. (We're taking the prime numbers to all be positive.) To see why, notice that if n is even, then n is a multiple of 2. That means we can write $n = 2m$ where m is an integer. But if $m > 1$, this would exhibit n as a composite number. The only way n could be prime is if $m = 1$, that is, if $n = 2$. We can see that 2 is prime because its only factors are 1 and 2. If the only even prime number is 2, then all the other prime numbers are odd.

2a.

Table of Parities						
N	M	$N + M$	$N - M$	$N \times M$	N^2	N^3
even	even	even	even	even	even	even
even	odd	odd	odd	even	even	even
odd	even	odd	odd	even	odd	odd
odd	odd	even	even	odd	odd	odd

2b. No, it is not possible because an even number can never equal an odd number.

2c. If N is even, then N^2 is even also, and so $N^2 + N$ would be the sum of two even numbers, which is even. On the other hand, if N is odd, then N^2 would also be odd and $N^2 + N$ would be the sum of two odd numbers, which would again be even.

3. Let F_n be the Fibonacci sequence with $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for all $n > 1$. Then the first few terms are: 1, 1, 2, 3, 5, 8, etc. The parities of the first few terms are: odd, odd, even, odd, odd, etc. Because each Fibonacci number (starting with the third one) is the sum of the previous two, once we see the same parity pattern in two consecutive Fibonacci numbers, the pattern must begin to repeat. So, the pattern must repeat odd, odd, even over and over.

The nth triangular number is the sum of the number from 1 to n. This means that you can get the $(n + 1)$st triangular number from the nth triangular number by adding $n + 1$, and $n + 1$ changes parity with n (it has the opposite parity of n). When you add an even number the parity does not change and when you add an odd number, it switches. So the parity of the triangular numbers will alternately stay the same and switch, starting with 1, which is odd. So the pattern is: odd, odd, even, even, odd, odd, even, even, odd, odd, even, even, etc.

4. The lockers that end up open are those numbered 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

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Pascal's Triangle and Binomial Coefficients

by Elisenda Grigsby

Suppose that x and y are variables. We can manipulate the symbols without having to know what numbers they represent. For example, we can write "$x + y$", and it means "add x and y". If x represents 2, and y represents 3, then "$x + y$" represents "5". The point is that the expression "$x + y$" itself doesn't depend on what numbers x and y actually represent, and we can think about what we can say about these expressions in general, without worrying about some particular choice of numbers for x and y.

As an example, let's consider the following question: Let x and y be numbers. What can we say about the expression $(x + y)^n$, when $n = 0, 1, 2, 3, ...$?

So, $(x + y)^0 = 1$; (the result of multiplying a number by itself 0 times is usually defined to be 1)
$(x + y)^1 = x + y$;
$(x + y)^2 = (x + y)(x + y)$;
$(x + y)^3 = (x + y)(x + y)(x + y)$;
etc....

1. Show that $(x + y)^2 = x^2 + 2xy + y^2$ and $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$. What is $(x + y)^4$? (Hint: Use the distributive property of numbers: $(x + y)(z) = xz + yz$, for all numbers x, y, and z. Also, remember that multiplying two numbers doesn't depend on the order in which we multiply them. So, if x and y are two numbers, then $xy = yx$.)

2. Consider the collection of numbers, arranged into rows, shown at right. Can you see a pattern? Can you fill in the next row? Can you state precisely how each row is obtained from the row above it? (Assume that the top row is given.)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

3. This collection of numbers is called *Pascal's triangle* after the French mathematician Blaise Pascal. Do you see a relationship between the rows of Pascal's triangle and the expressions you were writing down in Problem 1? Can you explain why this relationship exists?

4. Why is Pascal's triangle symmetric across the vertical line through the center of the triangle? In other words, notice that the numbers on a row to the right of the vertical line are the same as the numbers to the left. Can you explain why this is the case?

5. (Bonus challenge!) It's standard to call the top row of Pascal's triangle row zero and the leftmost number in each row the zero-eth number (instead of the first). Can you show that the kth entry in the nth row of Pascal's triangle is the number of different k-element subsets that can be formed from a set with n elements? As an example, suppose we want to see how many different 3-element subsets we can form from the set {1, 2, 3, 4}. We see that there are exactly four: {1, 2, 3}, {1, 2, 4}, {1, 3, 4} and {2, 3, 4}. And if we look at the third number in row four, it is indeed four! (Remember to start your count from zero!)

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Solutions

1. (Eli Grigsby) Just multiply!

$$\begin{aligned}(x+y)^2 &= (x+y)(x+y)\\ &= x(x+y)+y(x+y)\\ &= x^2+xy+yx+y^2\\ &= x^2+2xy+y^2\end{aligned}$$

$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y)\\ &= (x^2+2xy+y^2)(x+y)\\ &= (x^2+2xy+y^2)x+(x^2+2xy+y^2)y\\ &= x^3+2x^2y+y^2x+x^2y+2xy^2+y^3\\ &= x^3+3x^2y+3xy^2+y^3\end{aligned}$$

$$\begin{aligned}(x+y)^4 &= (x+y)^3(x+y)\\ &= (x^3+3x^2y+3xy^2+y^3)(x+y)\\ &= x^4+3x^3y+3x^2y^2+xy^3+x^3y+3x^2y^2+3xy^3+y^4\\ &= x^4+4x^3y+6x^2y^2+4xy^3+y^4\end{aligned}$$

2. There are many patterns! One pattern, which can even be taken as the defining pattern, is that each entry is obtained by taking the sum of the two entries above it.

3. The coefficients of the terms (see page 9 for explanation of some language associated with polynomials) are given by the rows in Pascal's triangle. One way to see this is to check that the coefficients in the expansion of $(x + y)^{n+1}$ can be expressed as the sum of the appropriate two coefficients in the expansion of $(x + y)^n$ in the same manner as the numbers in Pascal's triangle are the sum of the two numbers just above it. Keep track of the coefficients in the expansion of $(x + y)^n$ as you multiply by $(x + y)$ to obtain the expansion of $(x + y)^{n+1}$.

4. If you are convinced that each row of Pascal's triangle gives the coefficients in the expansion of $(x + y)^n$, then the symmetry can be seen by observing that $(x + y)^n = (y + x)^n$. Think about this carefully!

5. The key to this problem is to pay very close attention to what happens to the terms in each factor when you multiply everything out. Think, for a moment about expanding $(x + y)^3$. If we carefully multiply out using the distributive law over and over taking care not to disturb the order of the terms (that is, do not commute any of the variables!), we'll see that each term in the expansion is the product of three variables, one from each factor in the original product. Also, if you pick one term from each factor in the original product and then multiply them together, you will obtain one of the terms in the expansion. We can address this in more detail at the club or in a future issue of the Bulletin if you wish. The connection to subsets is to associate the factors in the original product $(x + y)^n$ with the n members of a set and when you expand, interpret choosing, say x, as including that object in a subset and choosing y as not including it. So, in the above expansion, xyy would correspond to a subset with the first object, but not the second or third. Each term in the expanded form that simplifies to $x^k y^{n-k}$ would represent a different subset of k objects. So the coefficient of $x^k y^{n-k}$ in the expansion of $(x + y)^n$ turns out to be *exactly* the number of subsets of size k that you can make from a set with n objects.

$$(x+y)(x+y)(x+y)$$

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

This is an example of using algebra to count things!

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Slope Problem Set I

by Lauren McGough

For this problem set, we're going to be dealing a lot with lines. If you think about a straight line on a coordinate plane where the horizontal and the vertical directions are defined, it has a certain steepness associated with it. Maybe it is completely parallel to the horizontal, in this direction: ––. Or maybe it is parallel to the vertical, in this direction: | . Or maybe it makes some angle with the horizontal, like this: / , or this: \. Each of these lines has a different "steepness" associated with it. There is a number we use to measure this property of steepness: it's called *slope***.** The slope of a line is just the ratio of the amount the line goes up for every unit it goes over. We can measure slope just by taking two points on a line, and calculating the change in the vertical direction divided by the change in the horizontal direction. First, let's make sense of this definition!

1. The first question here is: draw some lines, and calculate their slopes using a few sets of different points. Is the slope always the same no matter what points you use? Why or why not? (We hope it is, because otherwise, the definition of the "slope of a line" doesn't make sense– the line could have a different slope at every point!)

2. Now that we've hopefully found that the definition of slope makes sense, let's see why this quantity actually measures the "steepness" mentioned before:
   a. What does a line of slope 1 look like?
   b. How about a line of slope 0? What does this look like?
   c. What is the slope of a vertical line?
   d. What does a line with negative slope look like?
   e. Draw lines with slopes of 0.5, 1, 5, 0, -0.5, -1, and -5. What does it look like when one line has a more positive slope than another line? How about a line with a more negative slope than another line? What do lines with slopes that are less than 1 and greater than -1 look like?

3. Let's assume that we're working with a specific coordinate plane. For now, let's specify points on this coordinate plane as (a, b), where a is the (signed)* horizontal distance of the point from a specific vertical line, and b is the (signed) vertical distance of the point from a specific horizontal line (these specific lines are called our *axes*). How many lines of a specific slope s, where s is any real number, is it possible to draw on this coordinate plane? Do you notice anything special about all lines of a specific slope?

4. Try drawing pairs of lines that make right angles with each other, and measuring their slopes. Lines that make right angles like this are called "perpendicular." Do you notice anything special about the relationship between the slopes of lines that are perpendicular?

*What we mean by "signed" here is that a is negative when the point is left of the specific vertical line and b is negative when the point is below the specific horizontal line.

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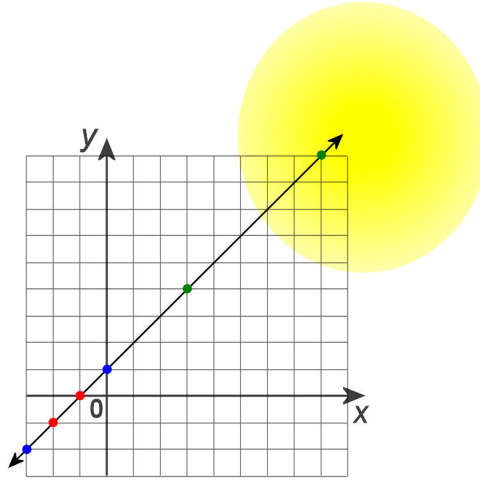
Solutions

1. (Lauren McGough) Let's try the line graphed at right. Some pairs of points on this line are:



(3, 4) and (8, 9)
(0, 1) and (-3, -2)
(-2, -1) and (-1, 0)

Calculating the slope of the line using each of these pairs by calculating the ratio of the vertical change to the horizontal change, we find:

(first pair) $\frac{9-4}{8-3}=\frac{5}{5}=1$

(second pair) $\frac{-2-1}{-3-0}=\frac{-3}{-3}=1$

(third pair) $\frac{0-(-1)}{-1-(-2)}=\frac{1}{1}=1$

We got the same ratio, 1, each time!

Problem 1 is actually quite deep. See page 15 of the last issue. *-Editor*

2a. A line with slope 1 goes up as you move from left to right. It makes a 45º angle with the horizontal.

2b. All lines with slope 0 are horizontal.

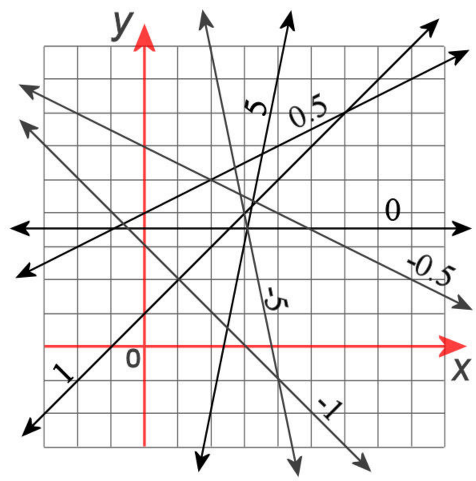
2c. The slope of a vertical line is undefined because you cannot divide by zero.

2d. A line with negative slope goes down as you move from left to right.

2e. Please see the box at right.

3. There are infinitely many lines of a given slope and all such lines are parallel to each other.

4. The product of the slopes of lines that are perpendicular to each other is equal to -1.



2e. This is just a sample. Your solution may differ substantially. What is really crucial is not where you place your line, but that the examples you draw are parallel to the lines with corresponding slope in the graph above. As the slope increases, the line rotates in the counterclockwise direction. The lines of slope 1 and -1 make a 45º angle with the horizontal. Lines with slope in between 1 and -1 make a shallower angle with the horizontal.

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Slope Problem Set II

by Lauren McGough

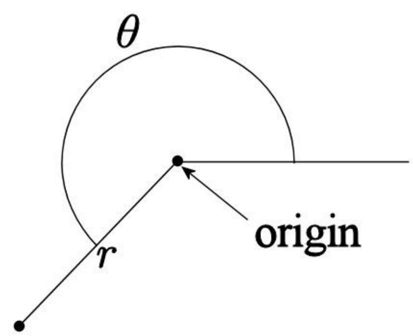
Let's continue using the same set up of Slope Problem Set I.

1. Sometimes, people like to express all of the points on a line using an equation that relates *a* and *b* for all points (*a, b*) on the line. Consider a line of slope 5 that goes through the intersection of the two axes on the plane– that is, through the "origin". Can you think of a relationship that all of the points (*a, b*) satisfy– that is, can you write an equation using *a, b* and the slope of the line such that if *a* and *b* satisfy the equation, then (*a*, *b*) is a point on the line and vice versa?

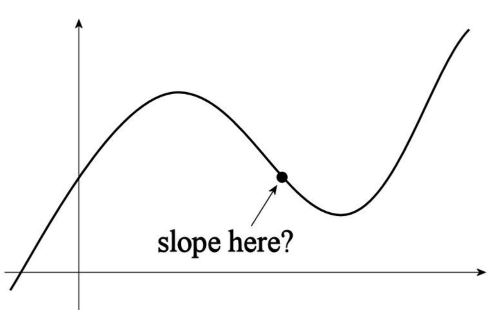
2. Can you generalize the equation you found in problem 1? So, say that we have a line of slope *m* that also passes through the origin. Can you write an equation involving *a*, *b* and *m* such that if *a* and *b* satisfy the equation, then (*a*, *b*) is a point on the line and vice versa?

3. What does an equation for a horizontal line (slope 0) look like, using the generalization you found in problem 2? What happens in the case of a vertical line?

4. So far, we've been using a coordinate system that relies on distances of points from specific horizontal and vertical axes. However, we've talked about other coordinate systems before! Do you remember the coordinate system where points were specified by (r, θ), where r is the point's distance from a special point called the "origin", and θ is the angle the line connecting the point to the origin makes with the horizontal ray pointing from the origin to the right? Consider all of the lines that go through the origin in this type of coordinate system. Say that you have a specific line that goes through the origin in mind. How could you use an equation that uses r and/or θ in order to tell me exactly what line you have in mind?



5. So far, we've been dealing with straight lines, which go off to infinity in the same direction forever. The "slope" of such a straight line defines how "quickly" it rises or falls, in a sense. What about for a curve? Curves rise and fall, too, though they don't necessarily always rise or fall at the same rate as you move along it. How would you express the steepness, or slope, of a curve at a point? Would your definition result in the same slope at all points of the curve? How could you calculate the slope of a curve at a point using your definition?



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Solutions

1. (Ken Fan) The problem asks for an equation that describes the coordinates of a point on a line that passes through the origin and has a slope of 5. The origin has coordinates (0, 0). If (a, b) is any other point on the line, then we can use these two points to compute the slope, and this slope must be equal to 5:

$$\frac{b-0}{a-0}=5$$

This simplifies to:

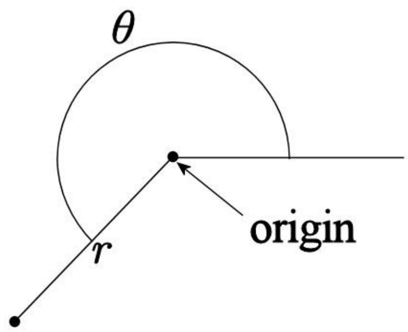
$$\frac{b}{a}=5$$

Notice that a cannot equal zero in this equation because you cannot divide by zero. If we multiply both sides by a we get the equation $b = 5a$. In this last equation, if $a = 0$, then $b = 0$, and this would correspond to the point (0, 0) which we know is on the line. So we conclude that the equation $b = 5a$ describes the relationship between the coordinates of points on the line with slope 5 that passes through the origin.

2. All points (a, b) on a line through the origin and with slope m satisfy the equation $b = ma$.

3. The points (a, b) on a horizontal line satisfy an equation of the form $b = c$, where c is a constant. (In other words, the y-coordinate of all points on the line are equal to some constant.) Similarly, the points (a, b) on a vertical line satisfy an equation of the form $a = c$, where c is again a constant.

4. In order to specify a line that you know goes through the origin, one piece of information you can provide is the angle that the line makes with the horizontal, as measured in the counterclockwise direction. But this is exactly what the coordinate θ specifies in polar coordinates! One technical point of concern is that if we restrict the coordinate r to nonnegative values, we have to make sure that we fully specify a line and not just a ray. So we can specify a line through the origin using polar coordinates with two equations: $\theta = c$ or $\theta = c + 180°$, where c is a constant angle satisfying $0° \leq c < 180°$.



5. One idea is to draw the line that just touches the curve at the point of interest. This line can also be described as the line which best looks like the curve near the point. If you draw a curve and magnify the region around a point, unless there is a kink at that point, the curve will look straighter and straighter as you magnify. At some point, you wouldn't even be able to distinguish what you see from an actual straight line. You could then define the slope of the curve at a point to be the slope of this straight line of best fit. This line of best fit is also known as the **tangent** to the curve at the given point. There are various techniques for computing this slope. As an interesting challenge, try to figure out the slopes at various points along the graph of $y = x^2$ by picking a point on it, such as the point (a, a^2), and figuring out the slope of the line that intersects the graph only in that single point of contact.

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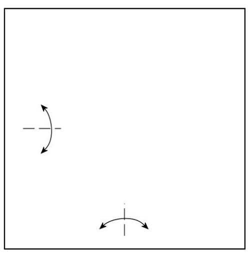
Origami Math

by Ken Fan

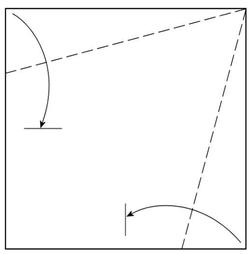
Origami artists start with square pieces of paper and fold them into works of art. There is quite a bit of mathematics related to origami. Christine's article (see the prior issue of this Bulletin) gave an example and this problem set introduces some others.

1. Take an origami square.
   a. If you fold the square in half along a crease parallel to a side, what is the resulting shape?
   b. If you fold the square in half along a diagonal, what is the resulting shape?
   c. If you fold an origami square in half repeatedly a total of n times, how many layers of paper will there be?

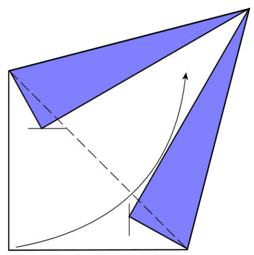
2. Because an origami square starts out as a square, you don't have to do anything to make an origami square! However, an origami equilateral triangle is a little bit harder.



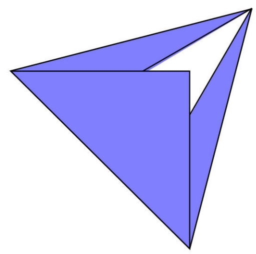
1. Start with the square, white side up. Make two pinches along the horizontal and vertical center lines positioned roughly as shown.



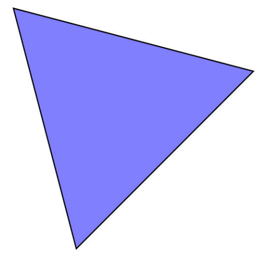
2. Fold indicated corners to the creases made in step 1 along creases that pass through the top right corner.



3. Fold the lower left corner over a crease that passes through the upper left and lower right corners.

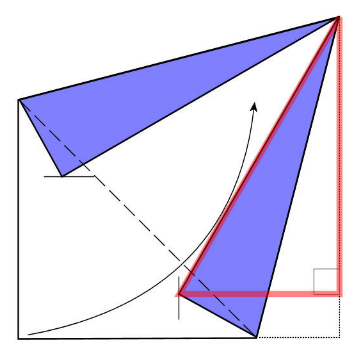


4. Flip over.



5. Finished equilateral triangle.

The diagram to the right is identical to the diagram in step 3 with a few more lines added. Notice the right triangle drawn in red. Let s be the length of the side of the original square.



   a. What is the hypotenuse of this red right triangle?
   b. What is the length of the shorter leg of this red right triangle?
   c. What are the angles of this red right triangle? (Do you recognize the red right triangle as half of a special triangle?)
   d. Can you *prove* that this folding sequence produces an exact equilateral triangle?

3. Can you extend the folding sequence for the equilateral triangle to make a three dimensional regular tetrahedron? A tetrahedron is a 3D object with four triangular faces, six edges, and four vertices. See if you can make an origami regular tetrahedron that holds its shape.

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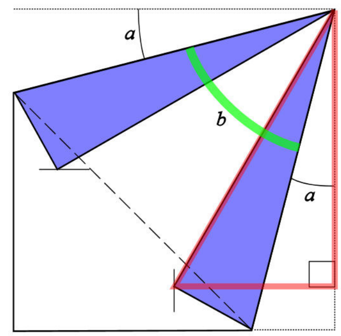
Solutions

(Ken Fan)

1a. A rectangle with proportions 2:1 (or 1:2). 1b. An isosceles right triangle. 1c. There will be 2^n layers.

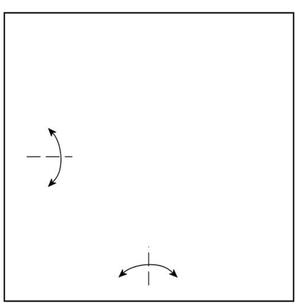
2a. The hypotenuse has length *s*, the side length of the origami square. 2b. By construction, the shorter leg has length half of *s*. 2c. If you reflect the triangle over the longer of its two legs, you will end up with an equilateral triangle. This means that the triangle has angles 30º, 60º and 90º.



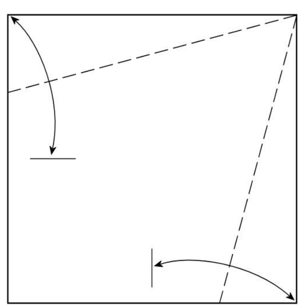
2d. Angles *a* measure half the 30º angle in the red right triangle (why?). So the apex angle *b* of the triangle being folded measures exactly (90 - 15 - 15)º = 60º. Because the triangle being folded is symmetric, the triangle is an isosceles triangle with apex angle 60º. The only such isosceles triangle is the equilateral one. You can use the side-angle-side theorem to conclude this. If you don't know what the side-angle-side theorem is, ask us!

Another way to see that an isosceles triangle with an apex angle of 60º is equilateral is as follows. The total sum of the measures of all angles in a triangle is 180º. If the apex angle is 60º, then the sum of the two other angles must be (180-60)º or 120º. In an isosceles triangle, these two other angles have equal measures. This means that they both measure half of 120º, which is 60º. Thus, all three angles measure 60º and so the triangle is equilateral. (If you're unsatisfied that we didn't actually show that the triangle has *sides* of equal length, ask us about it at the club!)

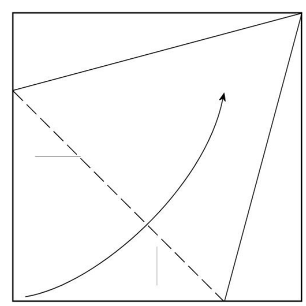
3. Here's a folding sequence for an origami tetrahedron:



1. Start with the square, white side up. Make two pinches along the horizontal and vertical center lines positioned roughly as shown.



2. Fold the upper left and lower right corners to the creases made in step 1 along creases that pass through the upper right corner. Unfold.



3. Fold the lower left corner over a crease that passes through the intersections of the creases made in step 2 with the left and lower sides.

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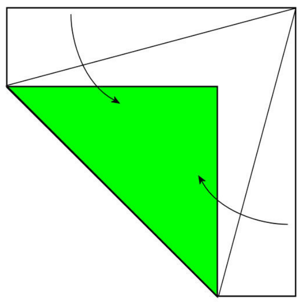
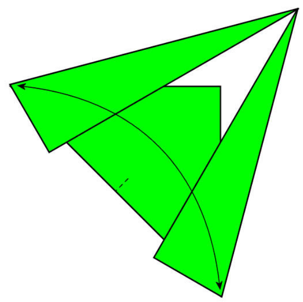
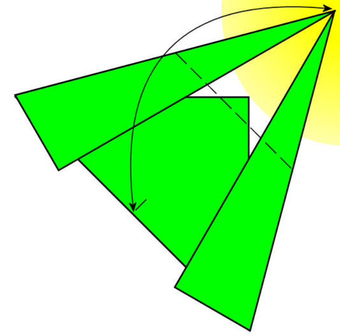


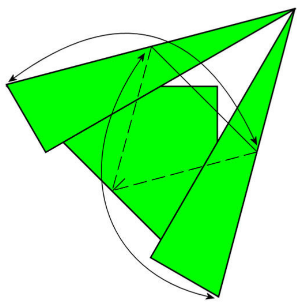
4. Refold the upper left and lower right corners over the creases you made in step 2.



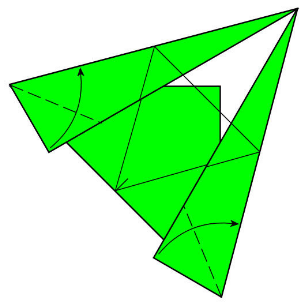
5. Make a small pinch at the midpoint of the lower left edge.



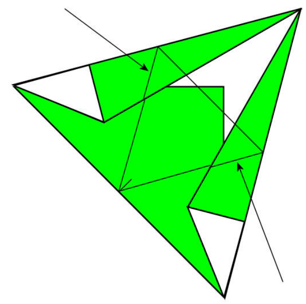
6. Fold the upper right corner to the pinch you just made and unfold.



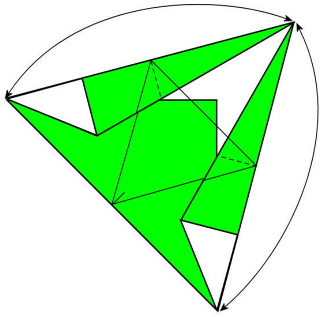
7. Fold the other corners of the major triangle to the midpoints of the opposite sides using the creases formed in step 6 as guides.



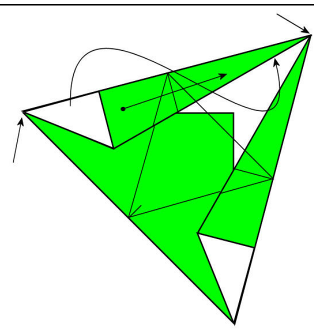
8. Fold the indicated corners of the topmost layer over an angle bisector. These flaps will eventually be used to lock the tetrahedron.



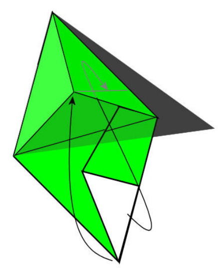
9. Reverse the orientation of the indicated creases. These creases are short segments in the top layer only.



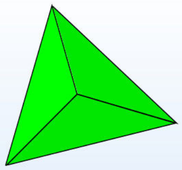
10. Make creases along the medians of the triangle by folding corner to corner. Try to make the crease sharp only in the top layer.



11. Fold the indicated corners of the triangle up over the creases made in steps 6 and 7 so that they meet above the plane. Simultaneously, reform the creases made in steps 9 and 10 so the left part of the top layer becomes flush with its right part. Slip the flap folded in step 8 in between the layers as indicated.



12. Now lift the remaining side into position. Use the flap you folded in step 8 to lock the tetrahedron by sliding it in between the layers on the inside of the upper right side (hidden details are shown in the diagram).



Finished Tetrahedron

Summer Fun!

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Getting a Balanced Diet

by Lauren Williams

You can see this painting by Spanish painter Luis Meléndez at the Museum of Fine Arts in Boston.

Photo courtesy of http://commons.wikimedia.org/wiki/Main_Page

An "average" human being consumes about 1940 calories per day. All food is comprised of carbohydrates, fat, and/or protein. One gram of protein or carbohydrates provides 4 calories, and one gram of fat provides 9 calories. Suppose a person decides to get the 1940 calories by eating 50 grams of protein, 300 grams of carbohydrates and 60 grams of fat. Let's see how this person could do that with different kinds of foods!

1. A pat of butter contains 4 grams of fat (and no significant protein or carbohydrates). A large pear contains 30 grams of carbohydrates (and no significant amount of protein or fat). A can of tuna canned in water contains 40 grams of protein (and no significant quantity of fat or carbohydrates). In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only butter, pears, and canned tuna, how much of each quantity of food should the person eat?

2. A large cantaloupe contains 65 grams of carbohydrates (and no significant protein or carbohydrates). A whole egg contains 5 grams of protein and 5 grams of fat. A chicken pot pie contains 25 grams of fat, 40 grams of carbohydrates, and 15 grams of protein. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only cantaloupe, eggs, and chicken pot pie, how much of each quantity of food should the person eat?

3. One cup of mushrooms contains 2 grams of carbohydrates and 2 grams of protein. One whole avocado contains 20 grams of fat, 10 grams of carbohydrates, and 3 grams of protein. One cup of carrots contains 10 grams of carbohydrates and 1 gram of protein. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only mushrooms, avocados, and carrots, how much of each quantity of food should the person eat?

4. A cookie contains 3 grams of protein, 35 grams of carbohydrates, and 10 grams of fat. A donut contains 6 grams of protein, 41 grams of carbohydrates, and 10 grams of fat. A croissant contains 4 grams of protein, 40 grams of carbohydrate, and 20 grams of fat. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only cookies, donuts, and croissants, how much of each quantity of food should the person eat? What is wrong with your answer? Explain.

Different people have different caloric requirements. Using the same tools that you developed to solve the problems in this problem set, you could consult a nutritionist and design meals for yourself that are best suited to your own needs!

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Solutions

1. (Lauren Williams) The recommended daily amount of protein in a 1940 calorie diet is 50 grams. If we only plan to eat butters, pears and canned tuna, how can we get 50 grams of protein in a day? We're told that butter and pears don't contain protein, but tuna contains 40 grams of protein per can. How many cans of tuna should we consume to get 50 grams of protein? One can is too little and two cans are too much. We can make an educated guess and arrive at 1.25 cans of tuna. Alternatively, we could let T represent the number of cans of tuna that we should eat; in that case we will consume $40T$ grams of protein, which must be equal to 50. Solving $40T = 50$ we get $T = 1.25$.

Similarly, let's see how we can get 300 grams of carbohydrates. The only food among butter, pears, and tuna which contains carbohydrates is pears. Each pear contains 30 grams of carbohydrates. We can again "guess" that the right number of pears to eat is 10. Alternatively, we can let P represent the number of pears we should eat; in that case we will consume $30P$ grams of carbohydrates from the pears. Solving $30P = 300$ we get $P = 10$.

Finally, let's see how we can get 60 grams of fat. The only food among butter, pears and tuna which contains fat is butter, which contains 4 grams of fat. So we can "guess" that the right number of pats of butter is 15. Alternatively, we can let B represent the number of pats of butter we should eat; in that case we will consume $4B$ grams of fat. Solving $4B = 60$ we get $B = 15$.

Therefore we should eat one and one-quarter cans of tuna, ten pears and fifteen pats of butter. Does this menu sound appealing to you?

2. Let C be the number of cantaloupes, E the number of eggs and P the number of chicken pot pies needed to consume 50 grams of protein, 300 grams of carbohydrates and 60 grams of fat. Using the information given, we see that $5E + 50P = 50$, $65C + 40P = 300$ and $5E + 60P = 60$. At this point, there are many ways to solve these equations for C, E and P. One way is to note that $5E$ occurs in the first and third equations. The first says $5E = 50 - 50P$ and the third says that $5E = 60 - 60P$. This means $50 - 50P = 60 - 60P$. Solving for P, we find $P = 1$. But this means $E = 0$ and $65C + 40 = 300$, which tells us that $C = 4$. So the answer is that we would consume 4 cantaloupes and 1 chicken pot pie.

3. Using similar methods as used in the previous solution, we find that we must consume $7\frac{7}{9}$ cups of mushrooms, 3 avocados and $25\frac{4}{9}$ cups of carrots.

4. Using similar methods as used in the previous solution, we find that we must consume -2 cookies, 10 donuts, and -1 croissant. The problem with this solution is that it gives negative numbers for the number of cookies and croissants, and a person cannot eat a negative quantity of anything! What does this mean? It means that it is impossible to achieve the goal of 50 grams of protein, 300 grams of carbohydrates and 60 grams of fat using cookies, donuts and croissants. All of these food items have too many carbohydrates compared to protein (more than six times as many grams of carbohydrates as protein).

Summer Fun!

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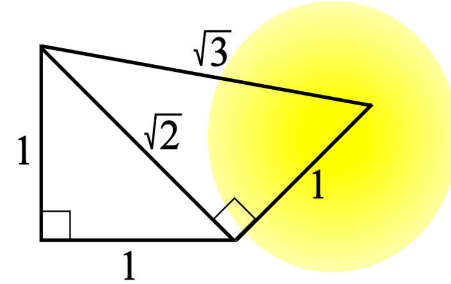
Discovering Square Roots Via the Pythagorean Theorem

by Anda Degeratu



Triangle inequality:

For an arbitrary triangle with sides of length *a*, *b* and *c*, we have

$$a + b > c$$
$$b + c > a$$
$$c + a > b$$

…but if the triangle has a right angle between the sides of length *a* and *b*, more can be said:

$$a < c \text{ and } b < c$$

and we also have the powerful Pythagorean theorem:

$$a^2 + b^2 = c^2$$

Square roots:

Recall that the *principal square root* of a positive number *a* is the positive number *x* that satisfies $x^2 = a$. We denote it by $\sqrt{a}$.

Example: the principal square root of 1.69 is 1.3 because $1.3^2 = 1.69$.

In this problem set we are going to look at square roots using triangles, which we construct using only the following three tools: a ruler which is marked 1, 2, 3 and 4, a right angle, and a compass.

For example, if we construct a triangle with a right angle with short sides of length 1, the Pythagorean theorem tells us the length of the long side: it is $\sqrt{2}$.

Now, using the segment of length $\sqrt{2}$ as a leg we can construct another right triangle with the other leg of length 1. In this new triangle, the long side has length $\sqrt{3}$.

The triangle inequality in this new triangle gives

$$\sqrt{3} < 1 + \sqrt{2}.$$

Moreover, since this is a right triangle, we also get $\sqrt{2} < \sqrt{3}$ (which you already knew since $2 < 3$ implies $\sqrt{2} < \sqrt{3}$).

1. Find **two** ways to construct a segment of length $\sqrt{5}$. (For both ways, you should use only the three tools given to you. Note that the compass can be used to construct segments of a certain length, once you have constructed them somewhere else on your sheet of paper.)

2. Inferring (as much as possible) from the above constructions, try to rearrange the following numbers in increasing order:

$$3,\ \sqrt{2},\ 1,\ \sqrt{5},\ 1 + \sqrt{2},\ \sqrt{2} + \sqrt{3},\ \sqrt{3},\ 1 + \sqrt{4},\ 2.$$

3. Using the three given tools, devise a way of constructing a segment of length $\sqrt{147}$.

4. With the same tools, can you devise a way of constructing a rectangle with area $\sqrt{147}$? Would it be possible to construct a square with area $\sqrt{147}$?

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Solutions

(Ken Fan)

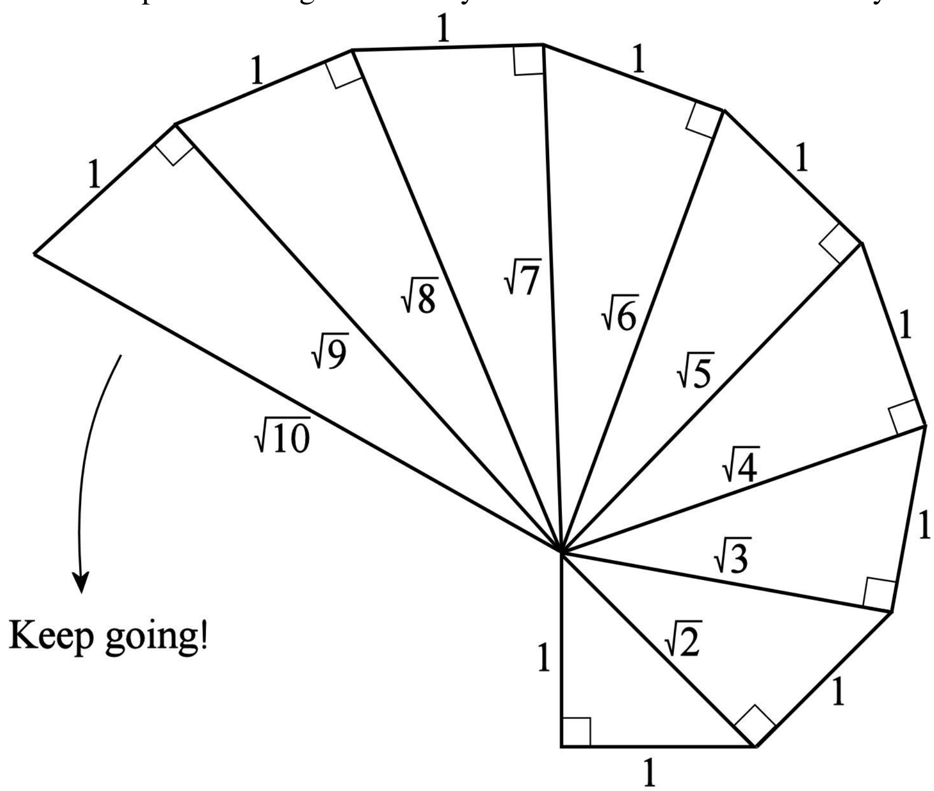
1. Method 1: Make a right triangle with legs of length 1 and 2. The hypotenuse will have length $\sqrt{5}$.

Method 2: Make a right triangle with legs of length $\sqrt{2}$ and $\sqrt{3}$. (Use the method in the statement of the problem to make these lengths.) The hypotenuse will have length $\sqrt{5}$.

2. The numbers 3, $\sqrt{2}$, 1, $\sqrt{5}$, $1 + \sqrt{2}$, $\sqrt{2} + \sqrt{3}$, $\sqrt{3}$, $1 + \sqrt{4}$ and 2 in ascending order are:

$$1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, 1 + \sqrt{2}, 3 = 1 + \sqrt{4}, \sqrt{2} + \sqrt{3}$$

3. A very systematic way of solving this problem is to extend the idea shown in the illustration at the top of the problem set and create a spiral of right triangles. You can use this spiral to get the square root of *n* for all positive integers *n*. Can you think of a more efficient way?



4. Once you have a line segment of length $\sqrt{147}$, you can make a rectangle that is $1 \times \sqrt{147}$. It is also possible to construct a square whose area is $\sqrt{147}$. See page 11.

Summer Fun!

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Special Announcements

The third session of Girls' Angle begins on September 11!

Calendar

Session 3: (all dates in 2008)

September	11	Start of third session!
	18	
	25	Sarit Smolikov, Harvard Medical School
October	2	Leia Stirling, Boston Children's Hospital
	9	Yom Kippur - No meet
	16	Jane Kostick, Carpenter
	23	
	30	
November	6	
	13	Catherine Havasi, Brandeis
	20	
	27	Thanksgiving - No meet
December	4	Amanda Cather, Waltham Community Organic Farms
	11	

This calendar includes Girls' Angle Support Network members who have been scheduled as of this writing..

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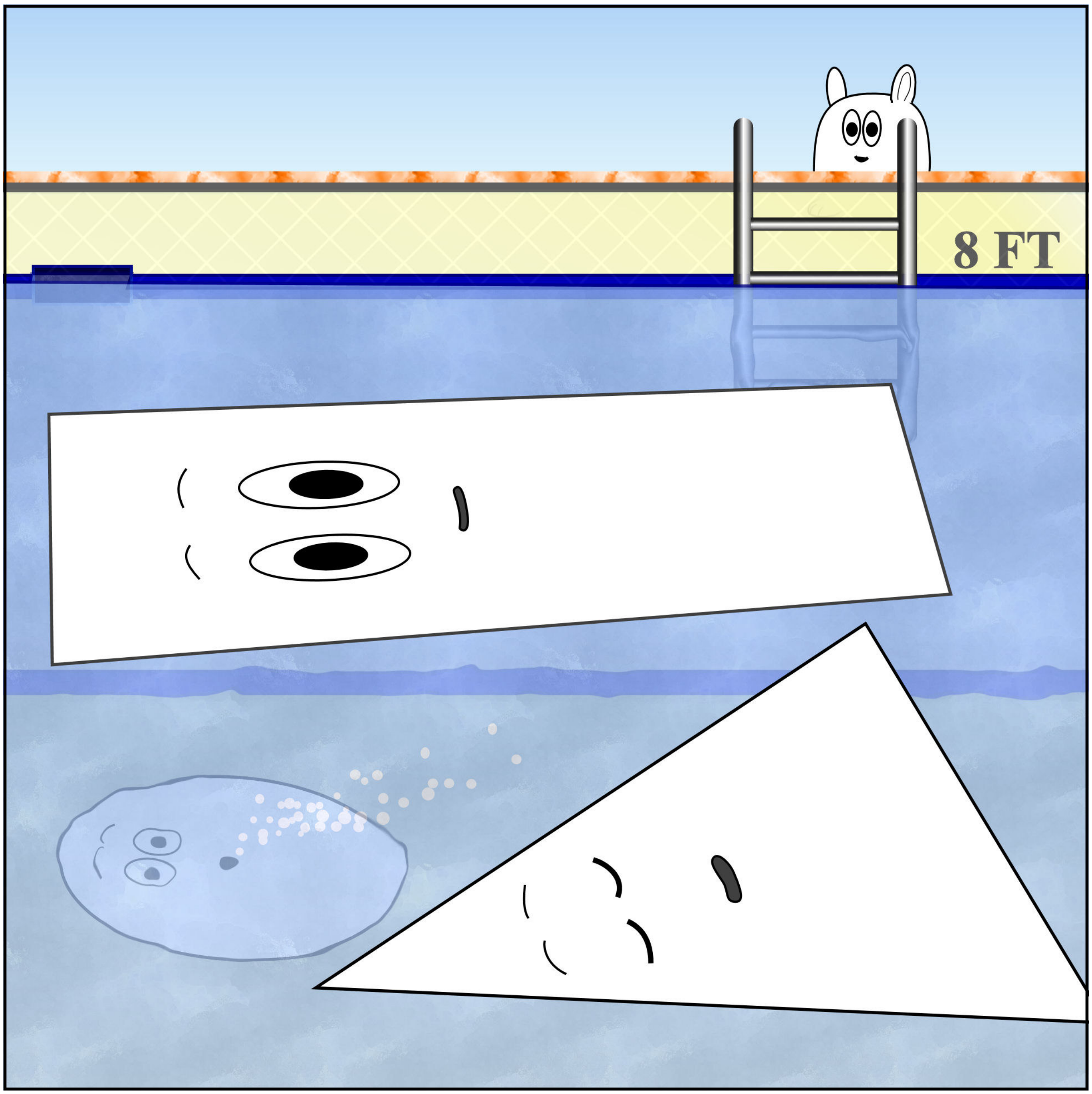
It Figures!

by CKFan



Summertime!

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Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 3 ways: **membership**, **subscription** and **premium subscription. Membership** is granted per session and includes access to the club and extends the member's premium subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session, you will get a subscription to the Bulletin, but the premium subscription will start when total payments reach the premium subscription rate. **Subscriptions** are one-year subscriptions to the Girls' Angle Bulletin. **Premium subscriptions** are subscriptions to the Girls' Angle Bulletin that allow the subscriber to ask and receive answers to math questions through email. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

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Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, UC Berkeley
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, assistant professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

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Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________ (first) ______________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _______________ Cell Phone: _______________ Email: _______________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ____________________
(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a 12 session membership
  - ☐ $100 for a one year premium subscription
  - ☐ $20 for a one year subscription
  - ☐ I am making a tax free charitable donation.

- ☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

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Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ______________________________________Date: ______________

Print name of applicant/parent: ______________________________________

Print name(s) of child(ren) in program: ________________________________

Girls' Angle

A Math Club for Girls

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