Girls' Angle Bulletin

October 2008 • Volume 2 • Number 1

To Foster and Nurture Girls' Interest in Mathematics

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$1 + 2 + 3 + 4 + 5$

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From the Director

Year over year, the number of members at Girls' Angle has more than doubled. It's a really fabulous group of girls and we all look forward to working with them every week.

This issue is the Bulletin's anniversary issue. To mark this special occasion, we are commencing with new regular columns.

Katy Bold launches her column "Math in Your World" with an article on fairly sharing a cookie among three people. Her column will explore applications of mathematics to real world problems. She holds a Ph.D. from Princeton.

Anna Boatwright initiates a new genre of mathematical exposition with "Anna's Math Journal". Here, she gives us a glimpse into her process of mathematical discovery. The vast majority of papers written in mathematics are highly polished gems with all errors and false leads removed. However, as Professor Daubechies points out in her interview, this gives a very misleading impression of how math is actually done. Anna's column fixes this by showing the value of not fearing mistakes. She is a graduate of Smith College.

Ken Fan
Founder and Director

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Girls' Angle Bulletin
The official magazine of
Girls' Angle: A Math Club for girls

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: **Mouse**'s summary of her results about sums of even and odd numbers.

More with Ingrid Daubechies

This is part 2 of a multi-part interview with Princeton mathematics professor Ingrid Daubechies.

Ken: How did you get interested in mathematics?

Prof. Daubechies: Well, I've always been interested in figuring out things. I remember when I was little I was always asking questions…and, actually, resenting it when my father would come with an half hour explanation instead of two sentences, like all kids. But I would want to figure out things. For instance, I remember…I must have been about the age of the girls at Girls' Angle…I remember once being asleep in a car and waking up and I could see the neon street lights through the green strip at the top of the windshield. I knew that this green thing discolors everything, but the neon looked just as orange as always. It wasn't discolored. I mean, I moved my head and looked at it through the clear glass and the green glass, and it looked exactly the same. And I was really puzzling about that. And then later [I realized that], well it's because it is almost monochromatic. It actually has a double line [see box at right] but it's almost one monochromatic line. And, of course, it was *dimmer* but what got through had to be the same color. A green thing doesn't shift [the frequency of the light]…it may absorb something…but it doesn't shift. It absorbs some things more than others, so when you use a composite light, you have a different distribution so it may look different. But when you have a monochromatic light, either it doesn't come through at all or what comes through has to be that color. That really brought home to me very precisely what monochromatic light is. I mean, that's not mathematics…that was a physics thing, but still…I noticed things and I wondered, how is that possible? How can that be? And so on. And I've always had that. But I also had dolls, like many kids and like many girls and I would have a period where I wanted to make clothes for them. And then you ask yourself, how can you, from a flat piece of clothing make something that follows all these curves? And so, that's when you discover, that you have the shaping and so on and so you discover how to make a non-flat thing out of something flat.

Spectral Lines

In this interview, Prof. Daubechies refers to double lines and monochromatic lines. These lines refer to spectral lines. If you shine light through a prism, the various frequencies of light (which we perceive as different colors) bend by different amounts so that you can see what frequencies the light is made out of. White light produces the whole rainbow spectrum:

However, a laser produces just one frequency of light and it shows up as a single slice of color from the rainbow. The spectral lines produced by a neon lamp sit mainly inside the orange part of the spectrum.

Full spectrum courtesy of en.wikipedia.org/wiki/Image:Spectral_lines_continous.png

So I always wanted to see how things worked. I always like to put things together too. I still like working with my hands a lot and like seeing things that fit nicely together. So, in that sense, I've always been interested in things. But also, I found it really easy because it was so…I mean, you followed rules and you would get there.

"I find it impossible to read a book cover to cover in mathematics."

My father came from a blue collar family. He was the first person in the family who ever went to college. And his parents came from a coal mining region. So when in high school, the teacher said you should let this boy go to college, it never occurred to his parents that there were many things you could do in college. They knew the coal engineers and they knew doctors and those were the people who went to college. And he couldn't stand blood so he became a coal mining engineer. It was very, very logical. Later, I think my father would have preferred to be a scientist. He had great regard for science and he was trying to learn more science on his own. So he tried to teach me math and especially physics. That's probably why I went into physics and not mathematics at first. So, he would teach me the math so I could do the physics. He taught me calculus early on. I thought it was really easy…and it is easy…there are just a couple of rules…if you are willing to take it like that…as a game with rules…then it is really easy. It's doing things with it that makes it somewhat harder. I mean, when you understand what you are doing, it's actually not hard. I'm not an advocate of teaching children calculus at that early age, but I thought there were other things in life that were much harder than that.

Ken: How old were you when you did this calculus?

Prof. Daubechies: Twelve. But to me, by and large, it was manipulating symbols. I don't think I could have solved a serious multivariable problem. That came later. But one variable calculus…there's not much to one variable calculus really. And the idea of continuity and convergence- it's a really intuitive idea. If you start doing the epsilon-delta stuff- that's another thing- that's to make it precise and articulate, but if you start by saying 3, 3.1, 3.14, and so on, it's obvious! I mean, that's another example I use to show that math is constructed…it's obvious to us now and it's obvious to kids in elementary school, that you have a number line and that you can pinpoint things like that and the more decimals you have, the more precise you are. It wouldn't have been obvious to the Greeks or Romans; they didn't have that notation. The notation already tells you. It has become such a part of what we know that we know it at a very early age. Eudoxus had all these problems with this notion. He had a glimmering of it- in fact, I think he really had a feeling for it. But if you see all the contortions you have to go through in order to define it really precisely- it is quite a bit of construction. But we don't have that problem anymore because we have notation for it. I mean, continuity…you have to have it…it just says we can make sense of the world…that if we didn't measure things with absolute precision…if we are almost there…well, then, the result will be almost there as well. That happens in most cases…not always…but it happens in most cases. We're all used to that. So, all these concepts I feel are very natural.

Eudoxus was a Greek mathematician and astronomer who lived about 2,400 years ago.

Ken: There are some girls who do get the curiosity about some phenomena they notice and they ask themselves, "why did that happen?" or "how does that work?" but then, another thought interjects itself, and they think, "oh, it's probably too hard for me to understand" and before they try to make progress, they already decide that it is too difficult. Did this ever happen to you? And if it didn't, what is it about your attitude that prevents that?

Prof. Daubechies: Well, it is, uh, I tell all my graduate students that you have to be a bit ornery to work in science. You have to…I find it impossible to read a book cover to cover in mathematics. Because you start, and you hit a difficult proof. And you say, "why do they make it so hard? Come on! There has to be an easier way." And so, you try. And you work, and you work, and you work…and you start understanding why all the different bits are in it. Well, sometimes, you do find an easier proof, which is nice. But this gives you an understanding for why…also you have to understand how math books are written. Math books are not written by saying, look, we would like to get there, but if you want to do this you would do this and this and this but look, there's this stumbling block, so let's see how we get around it. We almost find no math books like that. What they do is they figure all that out and then the write it down so that the stumbling block is sort of swept aside. All these logical lines follow through and then at the end you get this polished gem and you say, my, how did they find *that*? Well, they didn't! They did all the rest, and then they polished it. So, that's some beef I have with how math books are written. But that's another thing. But still, you have to be ornery, you have to ask questions, you have to say, "but, come on, wait a minute!" and when you work together with somebody it's a little bit like that too. If someone says to you, "I don't believe it" you should be able to say, "ah, but this" and so on, and that's how the whole conversation goes. But thinking, "this is too hard for me," well, we all have that to some extent. I mean, I don't work on the Riemann hypothesis[1]. I'm just not the kind of person who likes to tackle a big thing like that. I do work on things that nobody else has done or that are problems, and there are problems that I haven't been able to solve and that I set aside and I often work on several projects at the same time. So I think it's ok to think that something is too hard and you'll have to set it aside and come back at a later time. It's not ok to think that *everything* is too hard. I mean, you have to be prepared to tackle some things. And if it is too hard, say, "how can I make it simpler so that I do understand it?"

Professor Daubechies enjoys making pottery.

Ken: Do you have a confidence that if you keep working, you will find new results?

To be continued…

"…if it is too hard, say,
'how can I make it simpler so that I do understand it?'"

[1] The Riemann hypothesis is a famous conjecture made by the mathematician Bernhard Riemann in 1859 and remains unsolved to this day.

Prueba del 9: The Trick

by Ken Fan

At the second meet, **littleMeme** brought in a math trick. The math trick involves a “reduction” procedure that is performed several times. So before getting to the trick, first I’ll explain this reduction procedure.

The reduction procedure starts with any positive integer and produces a single, positive, digit. To perform it, you add up the digits of the number you start with. If the result is a single digit, then you are done. Otherwise, you add up the digits of the result. You keep going, adding the digits of the results until you end up with a single digit.

Let’s try this procedure starting with the number 95,782.

First, we add up its digits: $9 + 5 + 7 + 8 + 2 = 31$.

The result, 31, is not a single digit, so we add up its digits: $3 + 1 = 4$.

The number 4 is a single digit number so we stop; the result of the reduction procedure is 4.

Here’s another example, starting with the number 123,456,789: 123,456,789 $\rightarrow$ 45 $\rightarrow$ 9.
And here’s another example: 1,999,999,999,999 $\rightarrow$ 109 $\rightarrow$ 10 $\rightarrow$ 1.

Try this procedure with the number 7,777,777. Did you get 4?

Now, let’s get back to the trick.

Pick two positive integers, let’s call them m and n.

Now, compute their product mn. Reduce this product to get a single digit number A.

Now let’s try a different computation. First, reduce m and n separately to get two single digit numbers X and Y. Now, compute XY and reduce this product to get a single digit number B.

Miraculously, you will always find that $A = B$!

WHY?

Try it and see for yourself.

In the next few issues of the Bulletin, we’ll explain why this works. See if you can figure it out for yourself before we explain it!

Example: I start with 18 and 22. Their product is 396. When I reduce this, I get 9. If instead, I reduce 18 and 22 separately, I get 9 and 4, respectively. The product of 9 and 4 is 36, and when I reduce 36 I get 9. So, both ways, I end up with 9.

Filtering the Natural Numbers: The Sieve of Eratosthenes

by Lauren McGough

Sift the Twos and sift the Threes
The Sieve of Eratosthenes.
When the multiples sublime,
The numbers that remain are Prime.[1]

What if I asked you to list all of the primes that are less than 5? You might first consider all of the positive whole numbers less than 5: 1, 2, 3, and 4. Then, all you would have to do to answer the question is check if each of these numbers is prime:

1 is, by definition, not prime
2 has only the factors 2 and 1, so it is prime
3 has only the factors 3 and 1, so it is prime
4 has factors 1, 2, and 4, so it is not prime.

We can then see that the primes less than 5 are just 2 and 3.

But what if I asked you to list the primes that are less than 10? 30? 100? 10,000? You could, in principle, make a list of all of the numbers, then, for each number, check to see if it is prime, and if it is, add it to your list. But checking each number is slow – you have to see if a number has any divisors, which involves dividing each number by other numbers. This might take a long time if your number is very large! Even for a computer, such a large number of divisions, especially for large numbers, might be inefficient.

So I ask, can you think of a way to decrease the number of arithmetic operations you have to do in order to list all of the primes less than some number *n*? (You might be interested in thinking about this question before reading the rest of this article, since I'm going to give away an answer!)

The Sieve of Eratosthenes is a method that does just this: it is a clever method of generating a list of the primes less than a number *n* without going through each number and listing its divisors to see if it is prime. This is how it works:

First, we start with a list of the natural (positive, whole) numbers from 1 to *n*, just as we did in the example where we looked at which primes are less than 5. For example, if we're looking for the primes that are less than 10, we start with a list like this:

1, 2, 3, 4, 5, 6, 7, 8, 9

We cross off 1 right away because we know 1 is not prime, just by definition. We are left with the list:

~~1~~, 2, 3, 4, 5, 6, 7, 8, 9

[1] William S. Clocksin and Christopher H. Mellish. *Programming in PROLOG.* Springer-Verlag.

One might think, how can we get the primes in this list without directly seeing if each number is prime? Well, saying that we want all of the primes in the list is exactly the same as saying that we want the numbers that are neither 1 nor composite numbers; we want the numbers that are not multiples of any smaller numbers, besides 1.

How can we cross off numbers that are not multiples of any smaller numbers? What if we go to the first number that has not been crossed out, which happens to be 2, and cross off all of the numbers greater than 2 that are multiples of 2, and then, for the next smallest number that has not been crossed out yet, cross out all of its nontrivial multiples as well and continue? Eventually, we will cross off all numbers that are multiples of smaller numbers – that is, all composites, and, what will be left is a complete list of prime numbers! (Do you see why?)

So, to continue the Sieve, we cross off the multiples of 2 that are greater than 2, since we know they can not be prime. This leaves the following:

$$\cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}, 9$$

The next number that has not been crossed off is 3. This 3 must be prime because we have already crossed off the multiples of the numbers below it (besides 1 – every number is a multiple of 1!) and hence 3 can't be a multiple of smaller numbers. We then cross off numbers greater than 3 that are multiples of 3, and we are left with the following:

$$\cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}, \cancel{9}$$

Now, we see that the next uncrossed number is 5. This 5 must be prime by the same reasoning that we used to show that 3 must have been prime. (Notice, however, that when performing the Sieve, we don't have to check that the uncrossed numbers are prime; the Sieve is designed so that it works out so.) At this point, we would next cross out all multiples of 5 that are greater than 5, but there are no such numbers in our list. In fact, we could conclude that we are done, since if there are no multiples of 5 greater than 5 on our list, for any number p that has not been crossed out yet, there won't be numbers greater than p that are multiples of p on our list. (Do you see why?) And since we will not be able to cross out any more numbers, there must be no more composites on our list. Thus we are left with the list of primes less than 10:

$$\cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}, \cancel{9}$$

Notice that we were able to generate this list in just a few steps! This is much faster than before, when we had to go through all n - 1 numbers less than n and determine if it was prime by doing multiple divisions.

The Sieve of Eratosthenes

To find the prime numbers from 1 to n:

1. List the numbers from 1 to n and cross out the 1.
2. Find the next smallest uncrossed number.
3. Cross out all multiples of that number greater than the number itself.
4. Repeat steps 2 and 3 until the list is exhausted.

The Sieve of Eratosthenes is extremely useful if we want to determine all of the primes less than a given number, or if we want to count the primes less than a given number. *With the key realization that in order to get a list of primes, we can instead cross out composites (and 1), we can find all of the primes without ever directly determining if any number is prime!*

Now, you might ask, how fast is this method, exactly? And when do we know that we're done? Can you determine how many times we will have to go through the list crossing out multiples of primes p before we have finished generating the whole list? One final question: Is there a way to do this even faster?

Anna's Math Journal

By Anna Boatwright

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Here, Anna answers: **Which prime numbers are even? Which are odd?**

Parity and Prime Numbers

I know that 1 is not prime so I start by crossing it out.

By convention, 1 is not considered prime. Ask the mentors about this!

I'll start listing prime numbers using the sieve of Eratosthenes.

See page 7 for more on the sieve of Eratosthenes.

What are the prime numbers?

~~1~~ 2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ 15 ~~16~~ 17 ~~18~~ 19 ~~20~~ 21 …

2 is the first prime. So I cross out all multiples of two, other than two itself...

1 2 3 5 7 ~~9~~ 11 13 15 17 19 ~~21~~ …

3 is the next prime after 2, so I cross out multiples of three, other than three itself...

1 2 3 5 7 11 13 15 17 19 …

There. This is a list of the first few primes.

Oh...I guess I forgot to leave off the ones! Those ones shouldn't be there because 1 isn't prime.

What are the even numbers?

0 2 4 6 8 10 12 14 16 …

Hey, wait a minute! Is 0 a prime number? What about negative numbers? Are they prime? I'll have to come back to this later. But if negative numbers can be prime, I suppose -2 would be another even prime.

What are the odd numbers?

I should put ellipses on both sides actually!

… 1 3 5 7 9 11 13 15 …

Until I figure out about 0 and the negative even numbers, I can already see that all of the other evens were crossed out of my list except for 2. So that means 2 might be the only even prime.

Claim

Two is the only even prime.

All other prime numbers are odd.

It occurred to me that I can answer this question quite directly. So I added this later.

Remark

[illegible] Whenever 2 divides n, then n is even.

"2 divides n" implies that "2 is a factor of n".

So, the only time that a number could be both even and a prime is if n is 2 itself.

ABB 10.31.08

Next question I'm wondering: are all odd numbers prime numbers?

No...I crossed off 9 from my list of primes but it is still on my odds list. Can I figure out anything more?

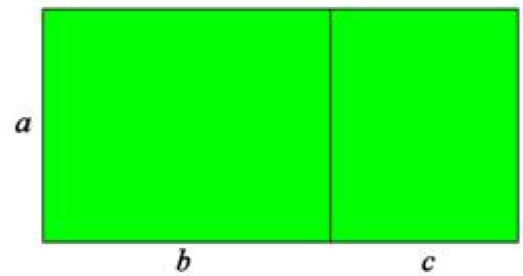
Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

The Distributive Law



If you have a rectangle with width a and length $b + c$, what is its area?

The area of a rectangle is given by its width times its length, so the area of this rectangle must be $a(b + c)$ square units.

However, the area is also the sum of the areas of two rectangles, one a by b and the other a by c. So the area must also be given by $ab + ac$.

In other words, $a(b + c) = ab + ac$.

This identity is called the **distributive law of multiplication over addition**.

In algebra, the distributive law is often used to simplify or rearrange expressions. When applying the law, make sure to handle signs with care. For example, what is an equivalent way of writing $-(x + 3)$? Is it $-x + 3$ or $-x - 3$?

Let's find out. The negative sign before the first parenthesis is equivalent to multiplication by negative one. So $-(x + 3)$ is the same as $(-1)(x + 3)$. Now let's carefully apply the distributive law. If we want to recognize $(-1)(x + 3)$ as an expression of the form $a(b + c)$, we could take $a = -1$, $b = x$, and $c = 3$. The distributive law tells us that $a(b + c) = ab + ac$. So, we carefully replace a, b, and c with -1, x and 3, respectively:

$$(-1)(x + 3) = (-1)(x) + (-1)(3) = -x + (-3) = -x - 3.$$

Here's another example. How can $-3(a - b)$ be written without the use of parentheses?

There are many ways to proceed, but no matter how you proceed, if you proceed without erring, you will find that $-3(a - b) = -3a + 3b$. If you got $-3a - 3b$, you didn't distribute the -3 properly.

Here's one way to get it:

$$-3(a - b) = (-3)(a + (-b)) = (-3)a + (-3)(-b) = -3a + 3b.$$

Sometimes people explicitly state a "distributive law over subtraction": $a(b - c) = ab - ac$. But this law really isn't anything new. It follows from the distributive law over addition:

$$a(b - c) = a(b + (-c)) = ab + a(-c) = ab + (-ac) = ab - ac.$$

Sometimes the distributive law can be very helpful to do quick mental computations. For example, what is $23 \times 17 - 23 \times 7$? You could multiply 23 by 7 and subtract the result from the product 23 times 17. Alternatively, however, you can use the distributive law and recognize that this expression is equal to $23(17 - 7) = 23 \times 10 = 230$.

1 + 2 + 3 + 4 + 5

Math is all about finding patterns and deducing implications of these patterns.

Mouse recently used a pattern to figure out the sum of the first N odd numbers. Her formula, on the cover is

$$\frac{(n+1)^2}{4},$$

where n is the last odd number in the list.

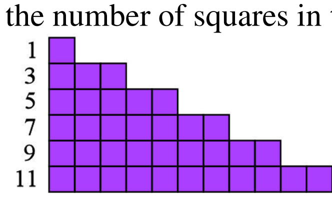
For example, to find the sum 1 + 3 + 5 + 7 + 9 + 11, we plug $n = 11$ into the formula and get 36.

When you have an assortment of numbers and have to add them all up, there usually isn't a pattern to the numbers and, usually, one has little choice but to add them up one by one.

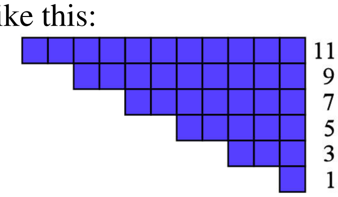
However, with the sum of the first odd numbers, the numbers being added aren't just *any* numbers! They are very special numbers indeed! And they have a lot of regularity. They are evenly spaced from the first to the last.

The more structure there is to something, the more that can be said about it. With an arbitrary sum of a jumble of numbers, not much can be said. But if we try to add up numbers that are evenly spaced like the odd numbers are, we should be able to say something more. And, as **Mouse** has shown, one can say a lot more: one can provide a formula!

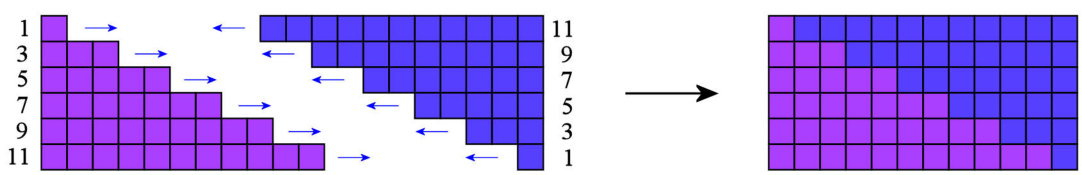
You can see one way to take advantage of the evenly spaced pattern to deduce the formula on the cover. Here's a geometric interpretation of the same idea. The figure below shows rows of unit squares. The area of each row (i.e. the number of squares in that row) is written to its left.



Writing the sum in reverse looks like this:



If we join these two pieces, we get a rectangle:



The area of a rectangle is the base times the height. In this case, we get 6 × 12 = 72. This is twice the sum because we had to use two copies of the sum to get the rectangle. So the sum is half of 72 or 36. Try to repeat this derivation using the variable n to stand for the last odd number instead of using the specific number 11 and see if you can recover **Mouse**'s formula!

How to get your fair share!

By Katy Bold

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Have you ever shared a cookie with a friend? How did you do it?

When my brother Ethan and I were younger, we used the rule “one cuts; the other takes” to share a cookie. If he cut, then theoretically he would make the two pieces equal and would be happy with either half of the cookie. Since I got to choose first, I would be happy because I could choose the bigger piece.

This is a common way for two people to share. The sharing **algorithm** (see the Mactionary, a math dictionary) has nice mathematical properties: it is **fair** and **envy-free**.

“Mactionary”

An **algorithm** is a set of instructions.

The algorithms are **fair** because each girl gets what she believes to be 1/2 (or 1/3 with three people) of the cookie.

The algorithms are **envy-free** because each girl is happy with her piece and does not want to swap with anyone else.

Since Ethan cut the cookie in half, he could not say that it was “unfair” because he did the cutting. No matter which piece I took, Ethan should not have been jealous because he made two equal pieces.

The same ideas are true for me, too. Since I got to choose, I could choose the bigger piece and cannot claim that my brother was unfair to me. I could not be jealous of my brother because I rejected his piece of the cookie.

Life is not always so easy though. Sometimes my cousin Jessica visited and we had to share a cookie three ways. We never had a good way to share a cookie, and someone ended up feeling envious of the others and thinking that the cookie was shared unfairly.

Fortunately, there is a way to share a cookie three ways so that everyone is happy with the outcome -- this is another fair and envy-free algorithm.

Take it to your world

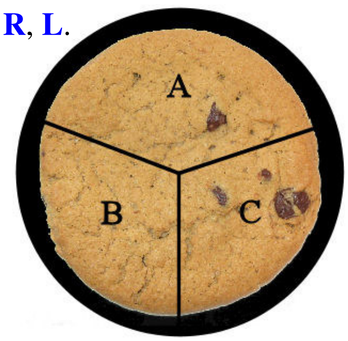
Gather two friends (or siblings or cousins or neighbors) to share something you all like, such as a cookie, a banana, or bread with peanut butter on it. Follow the algorithm on the following page to share the item in a fair and envy-free way. Did you get a fair piece? Are you jealous of anyone else?

Sharing a cookie with two other people

Though the pictures below are for a cookie, you can share an object of any shape. Think about it – what would change if you share a banana?

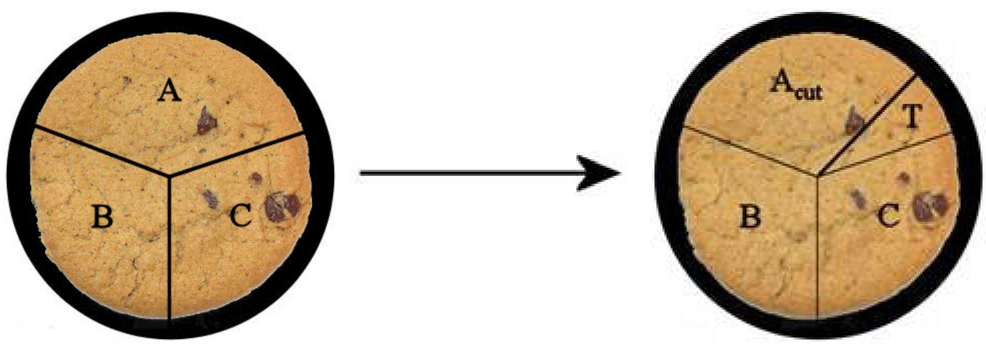
Before using a knife, always ask an adult for permission.

Before you get started, each person should be assigned a letter: **G**, **R**, **L**.



1. **G** tries to cut the treat into three equal pieces: A, B, C.

2. **R** examines the three pieces that **G** made. If **R** thinks one piece is too big, **R** cuts off a small bit (called the trim) and sets the trim aside. Suppose **R** thinks that piece A is too big; the trim is labeled with a "T" and the leftover piece is labeled A_{cut}.



3. **L** looks at the three pieces (A_{cut}, B, C) and takes the piece that she wants.

4. **R** takes the next piece.
   If **L** took piece A_{cut}, then **R** can pick between B and C.
   If, however, **L** did not choose A_{cut}, then **R** must choose A_{cut}.

5. **G** takes the remaining piece. In our example, this will be either piece B or C.
   Think about it – Why is it impossible for girl **G** to take piece A_{cut}?

6. Now it is time to divide the trim T.
   First, we have to figure out who gets to cut the trim.
   Either **R** or **L** took trimmed piece (A_{cut}). Whoever of **R** and **L** did NOT take this piece gets to cut the trim. This person cuts the trim into three equal pieces.

7. Whoever took the trimmed piece (either **R** or **L**) gets to choose the first piece of the trim.

8. **G** chooses the next piece of trim.

9. Whoever cut the trim receives the remaining piece.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete.

<u>Session 3 – Meet 1 – September 11, 2008</u>

Mentors: Hana Kitasei, Lauren McGough, Alison Miller, Jennifer Melot,
Amanda Redlich, Nike Sun

The first meet of the third session began with an ice-breaker activity: the set game. Girls defined subsets of girls. Those girls in the subset raised a hand and the girl who defined the set had to name all the girls who raised a hand. Here are some examples of subsets that were defined:

The set of girls who have an older brother.
The set of girls who like horses.
The set of girls who have a pet.
The set of girls who are 11 years old.
The set of girls who wear glasses.

During the break each girl was given a secret number. After the break, the girls had to collectively figure out what number each girl was given. The way they could get clues was by defining subsets of numbers. The only condition was that when a girl defined a subset, that girl's number could not be a member of the subset.

At first, girls defined subsets of numbers by specifying which numbers were in the subset. For example, one set was those numbers between 1 and 10. **Jo** wondered aloud, "are all the secret numbers whole numbers?" The mentors asked her, "can you define a subset that would answer your own question?" After a digression about whole numbers and integers, **Jo** decided that she'd start by finding out if all the secret numbers were integers and came up with the following set definition:

The set of numbers that are *not* integers.

This was the first example of specifying a set of numbers by describing what was *not* in the set. This eventually led to the girls figuring out that what you could do is reveal your own number simply by defining the set to be the complement of one's number!

In the course of playing this game, the question arose as to how to define the set of integers. What is an integer? Here's a definition: An integer is any number you can get by adding and subtracting any number of ones starting at zero. Can you think of another definition?

During the meet, the notion of a **partition** of a set also came up. A partition of a set is a collection of subsets which are mutually disjoint and whose union is the whole set. **The Cat** and **Ilana** found a nice way to partition the set of integers into n sets for any n. They did this by defining for each k from 0 to n - 1, inclusive, the subset of integers that leave a remainder of k when you divide by n. This partition is very important in mathematics, so keep it in mind!

Session 3 – Meet 2 – September 18, 2008

Mentors: Hana Kitasei, Lauren McGough, Grace Lyo, Jennifer Melot,
Amanda Redlich, Hilary Finucane

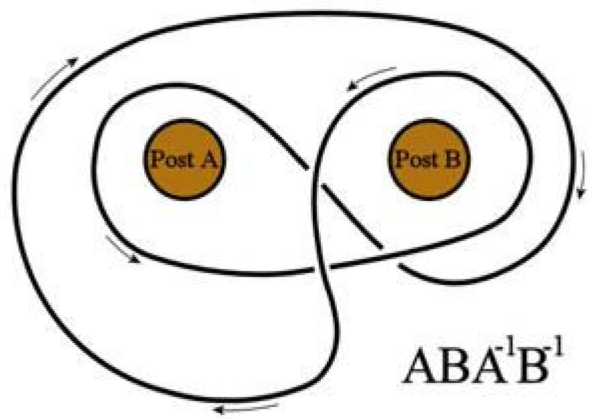
The second meet was spent entirely in small groups.

One group studied the combinatorics of bead arrangements. This arose out of the ice-breaker activity from the first meet. All the girls were arranged around a table and the question arose: How many ways can the girls arrange themselves around the table?

Hilary reports: "The problem we tried to solve was, 'how many different bracelets can you make with n beads?' We started by saying two bracelets are the same if you can rotate the beads of one to get a bracelet identical to the other, but that flipping it over makes it a new bracelet. We then designed an algorithm to determine whether two bracelets are the same: put the bracelets on top of each other, then line up the orange beads from the two bracelets, then try to line up the rest of the beads. We made one bracelet with two beads, and quickly realized there was only one." They gradually increased the number of beads. She continues, "While trying to figure out the case with 5 beads, we started to notice a pattern…" Eventually, the girls figured out that "the number of bracelets you can make with $n + 1$ beads is n times the number of bracelets you can make with n beads!"

Hana also worked with girls on bead arrangements. She reports, "We began by talking about beads on a bracelet in a larger group but we soon ran into trouble in our discussion of what we meant when two bracelets were the same. I decided to pose them the question of lines of beads instead and then work up to bracelets. **Tree**, **littleMeme**, and I also talked about other things like dance and how that might be related to combinatorics (with combinations of a set of steps)."

One of the keys to variety is combinatorics.



Remove either post and the loop pulls free.

Another group, led by Lauren, Jennifer and Amanda, discussed things related to infinity, including the Hotel Infinity, infinite geometric series, and real numbers.

Just in case you are wondering what real numbers have to do with infinity, notice that many (in fact, in a certain sense, just about all) real numbers, such as π, must be specified with an infinite number of digits after the decimal point. Also, consider the number $0.\bar{1}$, that is, the real number whose decimal expansion consists of an infinite string of ones after the decimal point. What number does this stand for? It stands for the number

$$\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\ \dots$$

This is an infinite geometric series.

A third group, led by Grace, worked on a topology problem. The question is, how can one loop a dog leash around two posts in such a way that if either post is removed, the dog is set free? Some notation was introduced to describe looping schemes. For example, A^2 was used to denote looping around post A two times in the clockwise direction and BC^{-1} was used to denote looping around post B once clockwise and then around post C once counter-clockwise. **Anonymous** made the keen observation that any solution would have to have the property that for each post, the corresponding exponents must sum to zero.

Session 3 – Meet 3 – September 25, 2008

Mentors: Amanda Redlich, Lauren McGough, Lauren Williams,
Jennifer Melot, Josephine Yu, Katy Bold

Special Visitor: Sarit Smolikov, Harvard Medical School

Lauren Williams and Josephine continued to work with the girls on bead arrangements. Lauren writes, "we worked on figuring out how many linear arrangements of black and white beads there are, if we use x black beads and y white beads. We started by computing small examples and I got them to explain the idea when there is 1 black bead and y white beads. We also talked about how the number for x black beads and y white beads is the same as the number for x white beads and y black beads. We made a chart (rows and columns indexed by black and white beads). And they also figured out how to compute the answer when there are 2 black beads and some number of white beads, by choosing the positions for the black beads in $(n + 2)(n + 1)$ ways and then realizing they'd double-counted. After looking at the chart we observed some patterns." Josephine worked with **The Cat** and writes, "she figured out the multinomial coefficient formula, the general solution to: how many (linear) necklaces can you make with C_1 beads of the first color, C_2 beads of the second color, ... , C_n beads of the nth color. We briefly talked about how these numbers are related to the expansions of $(a + b)^3$, $(a + b + c)^2$, etc."

Amanda and Katy continued working with the girls on loops and began talking about knots too. Amanda writes, "they gradually figured out the relationship between the missing post [see meet 2] and cancellation. We started talking about knots. I introduced them to the idea of a loop being the un-knot versus a 'real' knot. They had good intuition about this. Then Katy started drawing some knots and asking if they thought they were un-knots or not. Each girl got a piece of string and tried to come up with their own knots and un-knots. After exploring for a while, I started asking which 'un-knotting' moves are fair and which should be cheating. They decided that cutting the knot is cheating, but pulling loops isn't cheating. Then we looked at the drawings again to see how to do those moves on the picture instead of on the physical string. They came up with two maneuvers which they named Kate and Theo." The "Kate" and "Theo" maneuvers correspond to what is known by mathematicians as the type 1 and type 2 Reidemeister moves, named after the mathematician Kurt Reidemeister.

Lauren McGough and Jennifer worked with the girls to understand a math trick that **littleMeme** brought in at the second meet (see page 6). This led to a discussion on modular arithmetic. **Hadassah** and **Cat in the Hat** explored the coefficients that result by taking power of the binomial $a + b$. (See Eli's Summer Fun problem set in volume 1, number 4 of this Bulletin.) Both made a number of interesting observations.

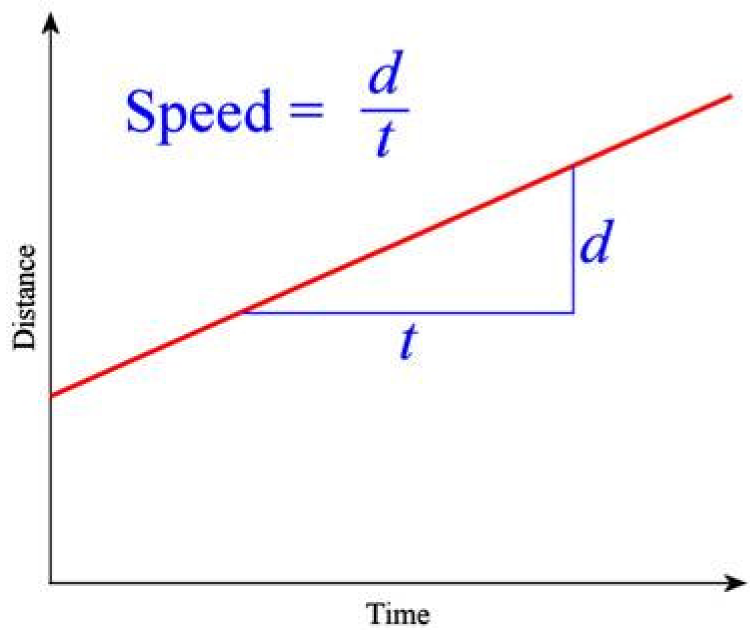
Sarit talked about the use of statistics in her research. She introduced the concepts of average, standard deviation and normal distribution. Often, in her work, she needs to determine whether two populations of worms (or collection of worm eggs) are truly different from another with respect to some characteristic. *On the assumption that the distribution within a given characteristic follows a normal distribution*, she can use statistics to find the probability that two populations are different from each other. The normal distribution is also known as the Gaussian distribution and corresponds to the bell curve.

Session 3 – Meet 4 – October 2, 2008

Mentors: Jennifer Melot, Lauren McGough, Nike Sun

Special Visitor: Leia Stirling, Boston Children's Hospital

The fourth meet began with a return visit by Leia Stirling, now at Boston Children's Hospital. She had the girls program Lego Mindstorm robots to travel in a straight line for a certain distance. Girls made plots of the distance traveled for various times. The slope of the resulting line gave the speed of the robot. Using this information, the girls were able to make their robots travel a precise distance by computing the length of time to activate the motor.



After Leia's visit, the girls split into two groups. One group studied patterns in Pascal's triangle and the other worked through an algebra exploration worksheet. It is important to develop the ability to manipulate expressions that involve variables. For more on Pascal's triangle, see Eli's Summer Fun problem set (volume 1, number 4 of this Bulletin).

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Ilana conjectured that every entry of the *n*th row of Pascal's triangle (except for the first and last entries) are divisible by *n* whenever *n* is a prime number. (Here, the numbering of the rows starts at zero.) For example, the seventh row of Pascal's triangle is:

1 7 21 35 35 21 7 1,

and you can see that other than the ones, each entry is a multiple of 7.

Prime numbers often get singled out in counting problems. When they do, the reason for it can always somehow be traced back to the fact that prime numbers are the only numbers with exactly two factors.

Session 3 – Meet 5 – October 16, 2008

Mentors: Jennifer Melot, Lauren McGough, Beth Schaffer, Cammie Smith Barnes, Yulan Qing

Special Visitor: Jane Kostick, woodworker

Two sculptures built by Jane Kostick were on display. Both sculptures have mathematical significance and some girls studied patterns in the sculptures. **Trisscar** made a makeshift ruler and observed an odd number progression in one of the sculptures. **Ilana** saw that the same sculpture involves arithmetic progressions with common differences increasing by one. We'll discuss these sculptures in detail in a future issue. Other girls were given a stack of index cards bearing algebraic expressions. They had to sort the cards into stacks of equivalent expressions.

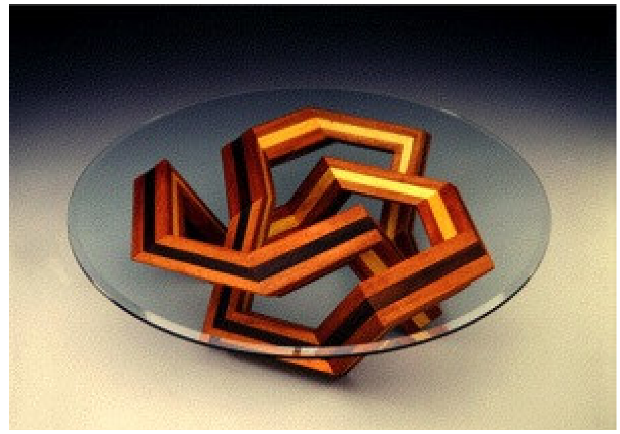
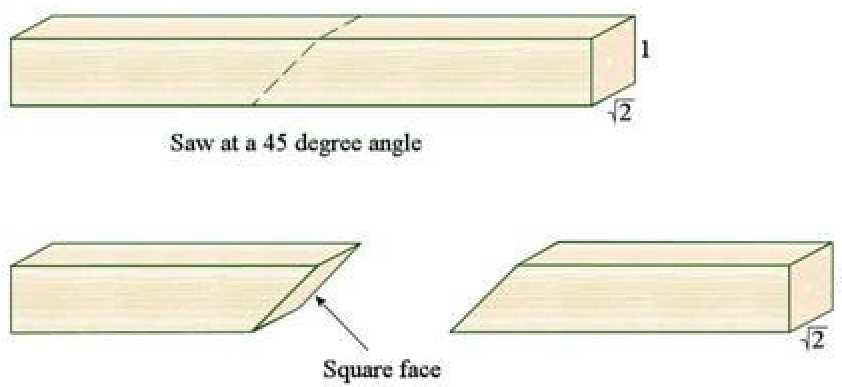


Photo courtesy of Jane Kostick

Trefoil Knot Table
Designed by Koos Verhoeff and built by Jane Kostick

Jane illustrated the strong connection between geometry and her woodwork through numerous examples of wood objects she built. For example, she brought in a model of a coffee table base that she built using a design of Koos Verhoeff. The base can be abstractly regarded as a loop. It is loop with a special name: the trefoil knot.

The base of the Trefoil Knot Table is constructed out of a number of straight beams that join together in a specific way. To understand the way they join, imagine taking a straight beam of wood with a $\sqrt{2} \times 1$ rectangular cross section. Now, saw this beam along a cut that passes through the beam at a 45 degree angle. (See figure at right.) The resulting cross-section will be a square. If you rotate one beam around an axis perpendicular to this square face, every 90 degrees, the square faces will match up. Rotating in this manner through 180 degrees results in a join that produces the right angled corner of a window frame. The beams in the Trefoil Knot Table are created by rotating in this manner through 90 degrees.



This photo shows one of the sculptures that was on display being built in Jane's studio.

Another table base she built is based on the pentagon. The way the posts are linked illustrates an instance of the Introduction game from the first meet of the first session (see volume 1, number 1 of this Bulletin). In particular, the table base illustrates the case of 5 people and skipping by 2. Using **The Cat**'s formula, we know that in this case, everyone gets named. What this implies is that the table base should result in a single loop, which, as you can see, it does!

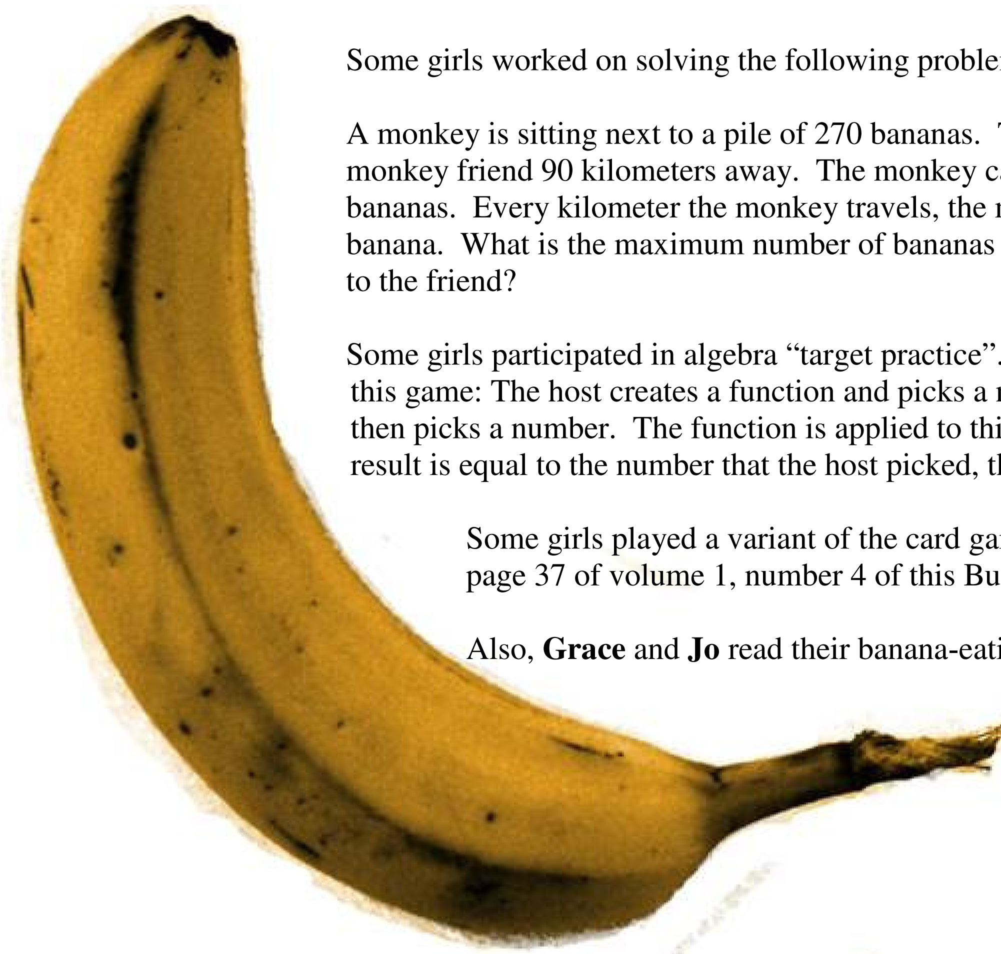
Photo courtesy of Jane Kostick

Pentafoil Knot Table built by Jane Kostick
Designed by Jane Kostick and Glen Whitney

Session 3 – Meet 6 – October 23, 2008

Mentors: Jennifer Melot,
Lauren McGough, Doris Dobi

The girls tackled *many* problems during the sixth meet!



Some girls worked on solving the following problem:

A monkey is sitting next to a pile of 270 bananas. The monkey has a monkey friend 90 kilometers away. The monkey can carry at most 90 bananas. Every kilometer the monkey travels, the monkey consumes a banana. What is the maximum number of bananas the monkey can carry to the friend?

Some girls participated in algebra "target practice". Here's how to play this game: The host creates a function and picks a number. The shooter then picks a number. The function is applied to this number and if the result is equal to the number that the host picked, that's a hit!

Some girls played a variant of the card game Cliffhanger (see page 37 of volume 1, number 4 of this Bulletin).

Also, **Grace** and **Jo** read their banana-eating algorithms which were performed by a quasi-intelligent ape. Notably, the algorithms worked quite well and 1.5 bananas were eaten without having to swallow any skin! Good job!

Special Announcements

The first Girls' Angle Social was held on October 7 to thank the members of the Support Network and all the people who have been helping Girls' Angle. Special thanks to Grace Lyo for organizing the event. We also benefited from the advice of Suzanne Oakley, owner of Experience Chocolate, who made a generous donation to help us acquire all the chocolate.

Calendar

Session 3: (all dates in 2008)

September	11	Start of third session!
	18	
	25	Sarit Smolikov, Harvard Medical School
October	2	Leia Stirling, Boston Children's Hospital
	9	Yom Kippur - No meet
	16	Jane Kostick, Carpenter
	23	
	30	
November	6	Catherine Havasi, Computer Science, Brandeis[1]
	13	
	20	
	27	Thanksgiving - No meet
December	4	Amanda Cather, Waltham Community Organic Farms
	11	

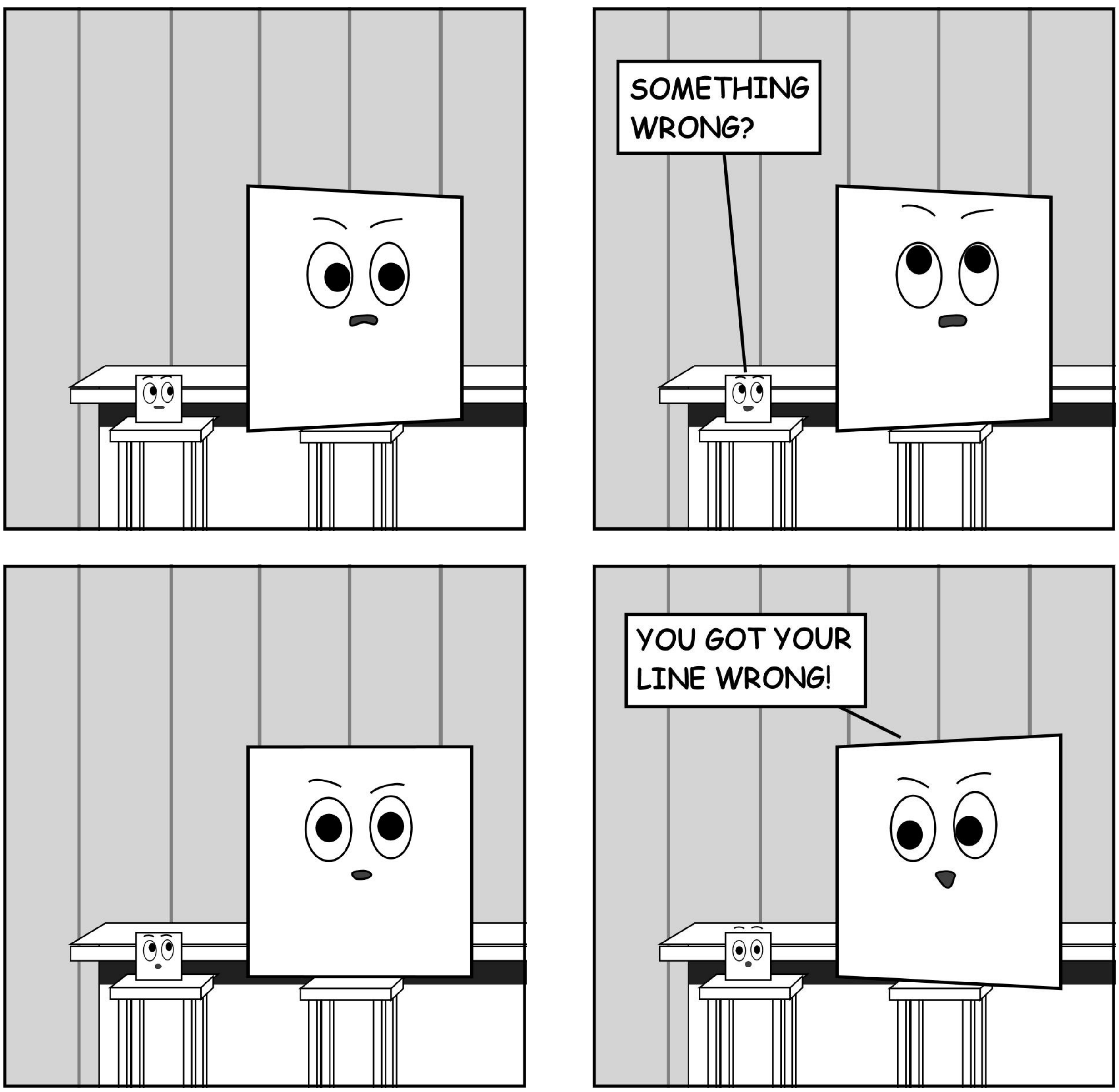
[1]Catherine's visit has been rescheduled to come one week earlier.

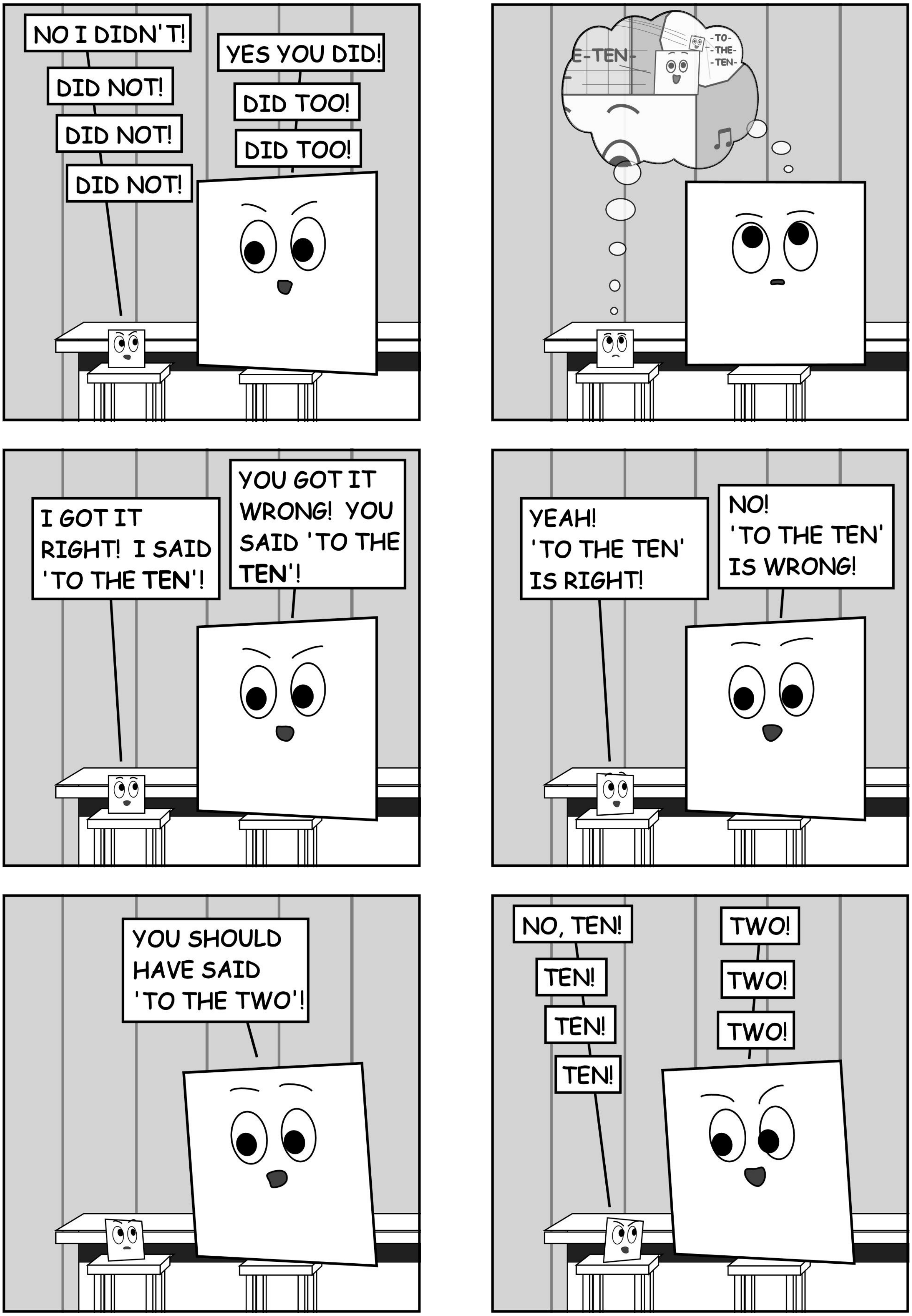
Session 4: (all dates in 2009)

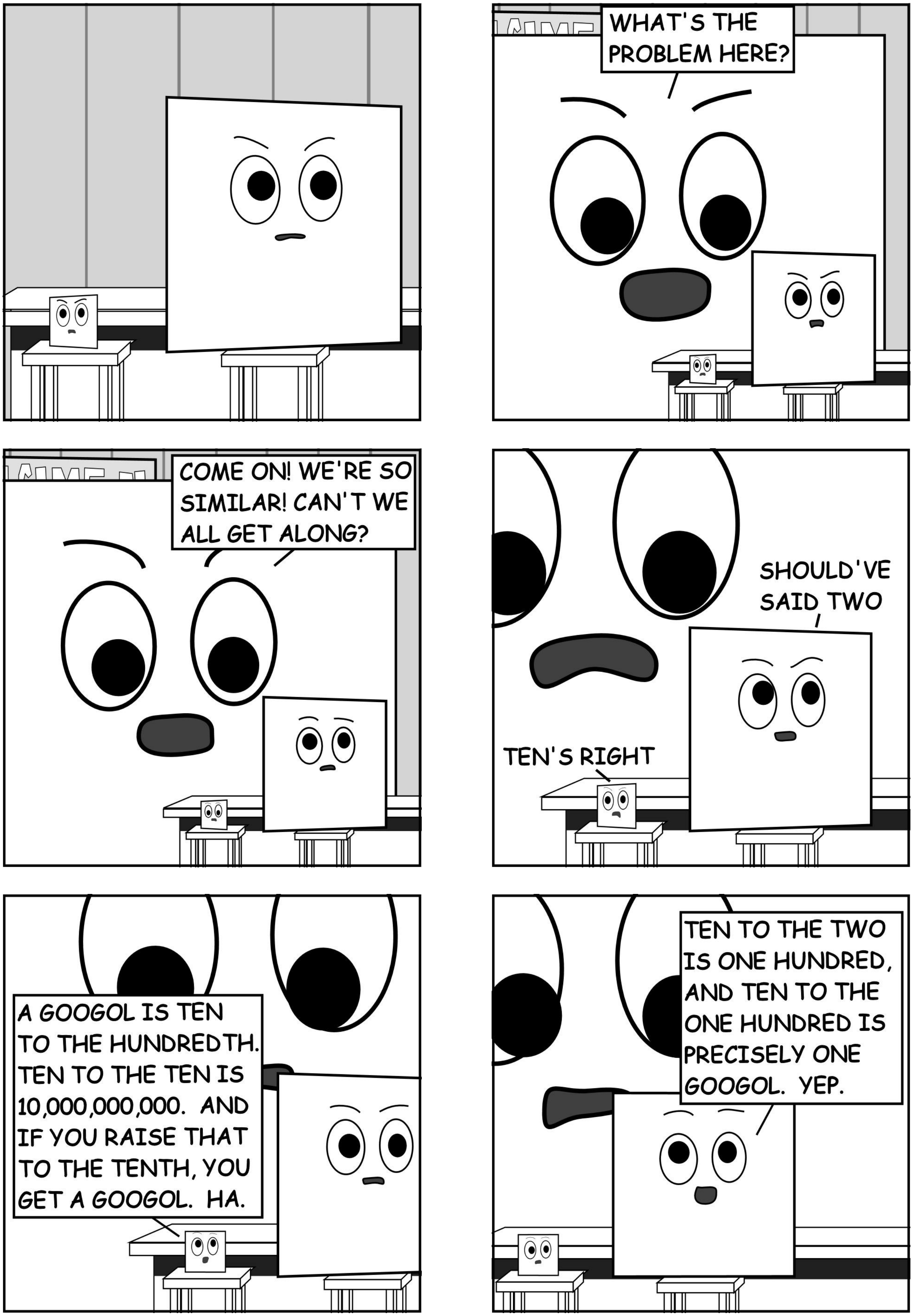
January	29	Start of third session!
February	5	Sarah Seager, Earth and Planetary Science, MIT
	12	
	19	Winter break - No meet
	26	Tanja Bosak, Earth and Planetery Sciences, MIT
March	5	Leia Stirling, Boston Children's Hospital
	12	
	19	Taylor Walker, DiMella Shaffer Architecture
	26	Spring recess - No meet
April	2	
	9	
	16	Eleanor Duckworth, Harvard Graduate School of Education
	23	Spring break - No meet
	30	Gigliola Staffilani, Mathematics, MIT
May	7	

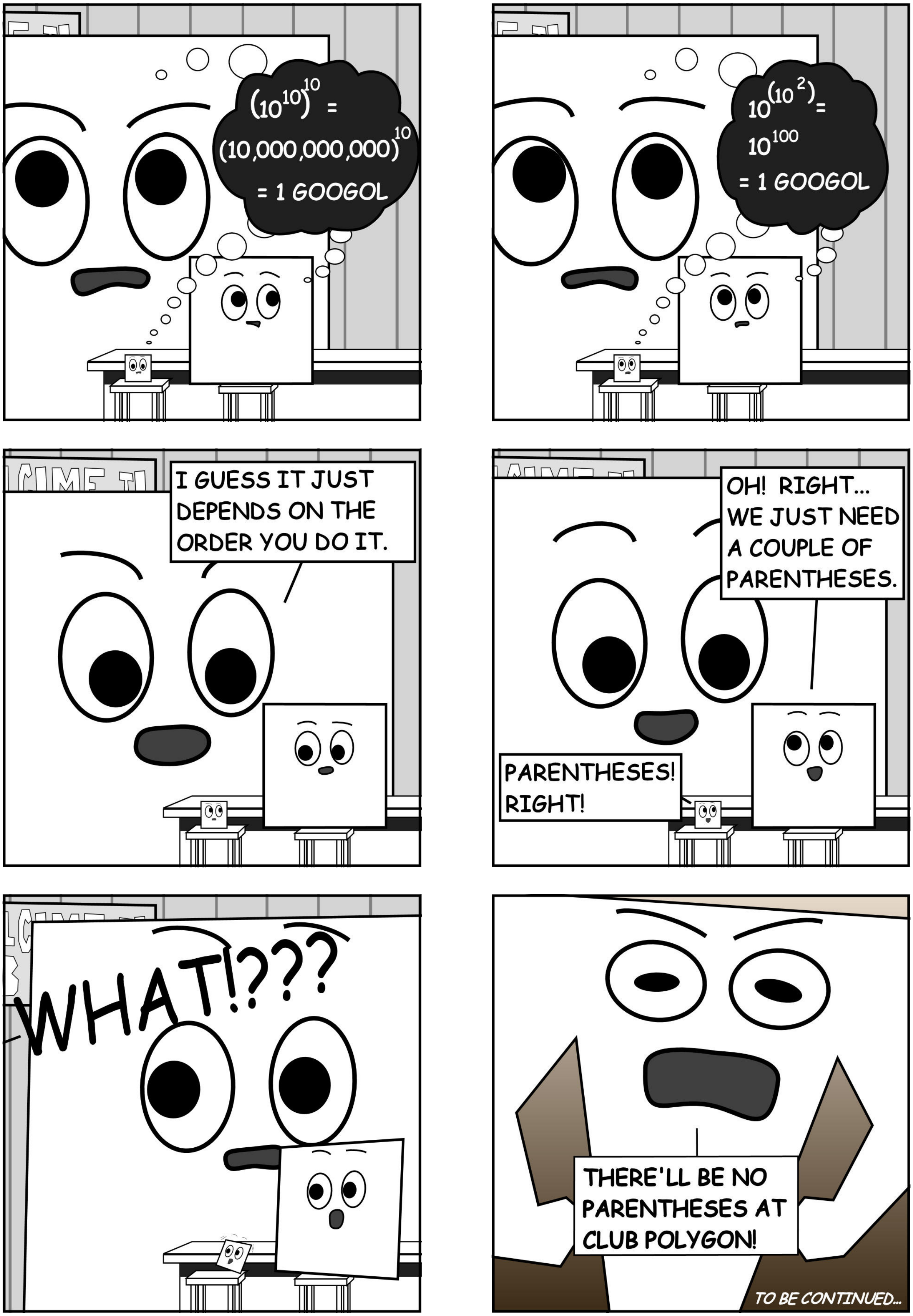
It Figures!

by CKFan




NO I DIDN'T!
YES YOU DID!
DID NOT!
DID TOO!
DID NOT!
DID TOO!
DID NOT!
E-TEN-
-TO-
-THE-
-TEN-
I GOT IT RIGHT! I SAID 'TO THE TEN'!
YOU GOT IT WRONG! YOU SAID 'TO THE TEN'!
YEAH! 'TO THE TEN' IS RIGHT!
NO! 'TO THE TEN' IS WRONG!
YOU SHOULD HAVE SAID 'TO THE TWO'!
NO, TEN!
TWO!
TEN!
TWO!
TEN!
TWO!
TEN!


WHAT'S THE PROBLEM HERE?
COME ON! WE'RE SO SIMILAR! CAN'T WE ALL GET ALONG?
SHOULD'VE SAID TWO
TEN'S RIGHT
A GOOGOL IS TEN TO THE HUNDREDTH. TEN TO THE TEN IS 10,000,000,000. AND IF YOU RAISE THAT TO THE TENTH, YOU GET A GOOGOL. HA.
TEN TO THE TWO IS ONE HUNDRED, AND TEN TO THE ONE HUNDRED IS PRECISELY ONE GOOGOL. YEP.



$(10^{10})^{10}$ =
$(10,000,000,000)^{10}$
= 1 GOOGOL
$10^{(10^2)}$=
10^{100}
= 1 GOOGOL
I GUESS IT JUST DEPENDS ON THE ORDER YOU DO IT.
OH! RIGHT... WE JUST NEED A COUPLE OF PARENTHESES.
PARENTHESES! RIGHT!
WHAT!???
THERE'LL BE NO PARENTHESES AT CLUB POLYGON!
TO BE CONTINUED...

Girls’ Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls’ Angle? Girls’ Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls’ interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls’ Angle mentors, the Girls’ Angle Support Network, the Girls’ Angle Bulletin and Community Outreach.

Who are the Girls’ Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls’ Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls’ Angle Bulletin? The Girls’ Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls’ Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members’ efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 3 ways: **membership**, **subscription** and **premium subscription.** **Membership** is granted per session and includes access to the club and extends the member’s premium subscription to the Girls’ Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session, you will get a subscription to the Bulletin, but the premium subscription will start when total payments reach the premium subscription rate. **Subscriptions** are one-year subscriptions to the Girls’ Angle Bulletin. **Premium subscriptions** are subscriptions to the Girls’ Angle Bulletin that allow the subscriber to ask and receive answers to math questions through email. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of “catching up with the group” doesn’t apply.

Where is Girls’ Angle located? Girls’ Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls’ Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, UC Berkeley
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, assistant professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a 12 session membership
  - ☐ $100 for a one year premium subscription
  - ☐ $20 for a one year subscription
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

A Math Club for Girls