Girls' Angle Bulletin

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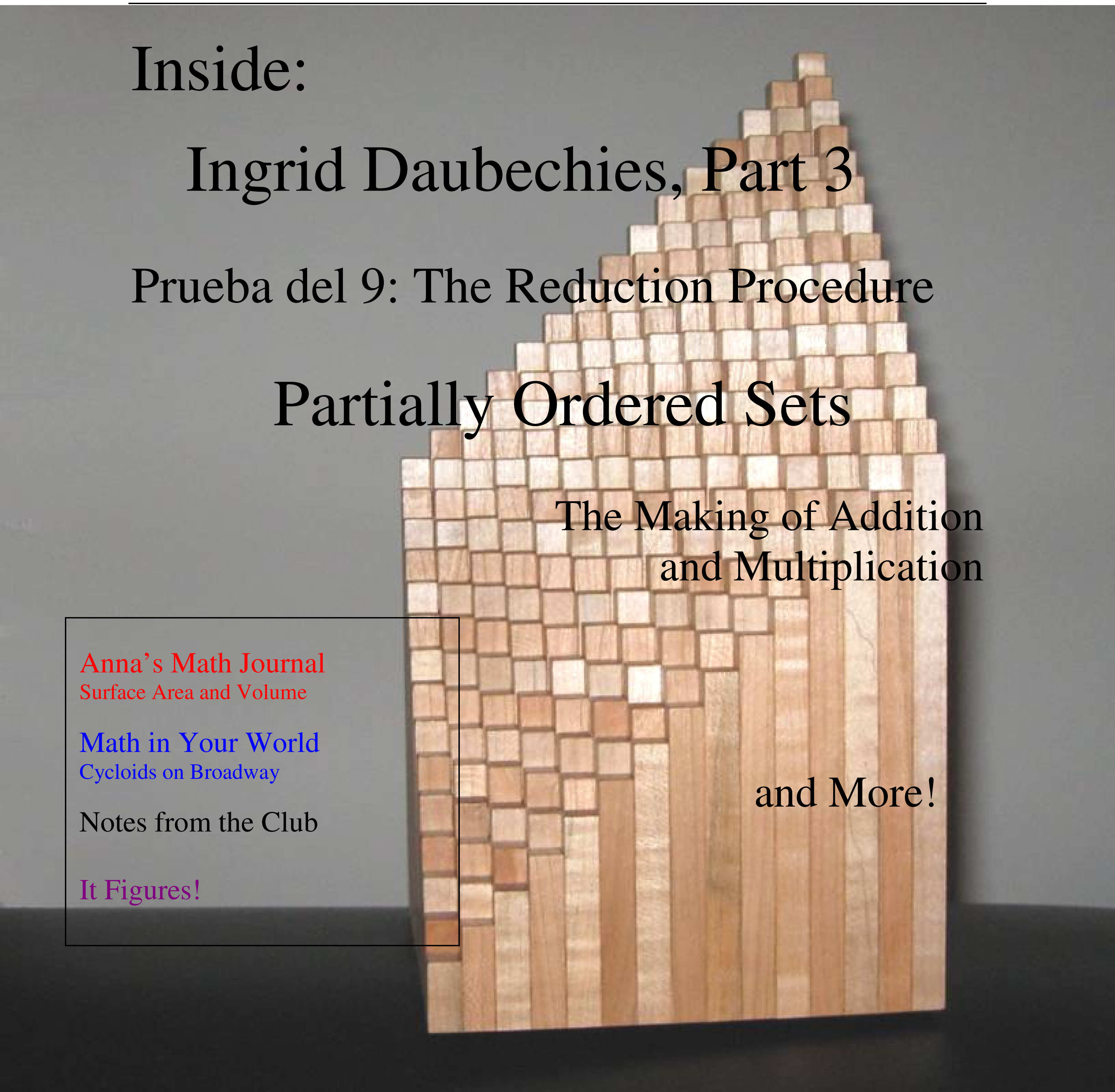
To Foster and Nurture Girls' Interest in Mathematics



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From the Director

I'm thrilled to announce that Girls' Angle has its first corporate benefactor: Big George Ventures, an eco-friendly real estate development company based in Carson Valley, Nevada.

I'm also excited to announce a new Girls' Angle advisor: Mia Minnes, a CLE Moore Instructor at MIT. Mia received her Ph.D. in mathematics from Cornell.

Their contributions to Girls' Angle are critical to helping us provide the highest quality math education for girls that is humanly possible.

For the girls: Remember, even though we're on break, you are *all* welcome to keep in touch through email to share and discuss any math thoughts you have. We hope to see all of you in the fourth session which begins January 29.

Happy Holidays!
Ken Fan
Founder and Director

p.s. A correction: Anna Boatwright, the author of *Anna's Math Journal*, is a Mt. Holyoke alumnus who also earned a post-baccalaureate certificate in mathematics from Smith College.

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

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Girls' Angle Bulletin
The official magazine of
Girls' Angle: A Math Club for girls

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: *Addition* built by Jane Kostick and designed by Ken Fan and Jane Kostick.

Ingrid Daubechies, Part 3

This is part 3 of a multi-part interview with Princeton mathematics professor Ingrid Daubechies.

Ken: Do you have a confidence that if you keep working, you will find new results?

Prof. Daubechies: No.

Ken: No?

Prof. Daubechies: (laughing) No.

> Ken: *…mathematicians make a lot of mistakes.*
>
> Prof. Daubechies: *Oh yeah!*

You never know!

Well, we don't really know where ideas come from. It's a very strange thing. I think they come from trying to remain wide open to many things. You have to really know your stuff You have to really dig into understanding strands of the different things you're looking at. You should not live in a very narrow crack between boards. And it's good to go to seminars and it's good to talk to other people because you have no idea where ideas just fall from.

When my daughter was diagnosed with ADD [Attention Deficit Disorder], and they told me on what basis they diagnosed this, I said, well, I have all those things too. So they asked me some questions and they decided I have ADD too. So I have a mild form of ADD…I didn't know…and when I learned this, I told my daughter, "well, we can stop worrying" because what it means is that I have tons of things that flip through my head. First of all, it means you see a whole lot of humor in many situations, which is fun. I think any good cartoonist must have ADD of some sort. But, on the other hand, I think it also makes me more creative, to see connections where there might not be any. And most of these connections don't really exist, but sometimes, it shows something…some patterns. But you don't know where they come from, so when you start working on something new, since you don't control it, how will you know that it's going to happen again?

After some time you start saying, well, it's happened a couple of times before, so many times before, maybe it will happen again and you say, well, they gave me a job, so they believe it will happen again. But I still, for a long, long time, I felt like a complete fake. I felt if people only knew how inside I was insecure they would never ever…I mean there was a complete gap between the person they thought they had in front of them and the person who I knew inside.

Ken: How did you cope with the insecurity then? How did you manage to keep working and trying?

Prof. Daubechies: Well, first, you want to keep up appearances [laughter]. You don't want to lose that job! But then, I think that it must be a very rare person who does not feel insecure like that on the inside. After all, you have a very asymmetric way of looking at the world. I mean, you are the only person that you see from the inside. Everybody else you see from the outside. And so from everybody else you don't see the insecurities unless they have some kind of neurosis and really expose them very badly. I think Orwell said in an essay on Salvador Dali…he said that Dali always struck him as completely insincere in his interviews because he says no person feels on the inside the way Dali pretends he feels. On the inside, you always feel like a whole assembly of failures with the occasional good thing in between. But, first of all, realizing that everybody feels that way helps. And then second, by wanting to do research…that is to say, finding things that nobody else has found before, you're bound to be on a bad track at times. So when you find something, it's great! You feel, absolutely… you feel very, very elated. So I tell my students, when you find something new you should enjoy [it] for half an hour and then you check the details because it could be a mistake, but at least you've had that half hour of pure joy!

Salvador Dali was a Spanish painter who lived in the 20th century. He also wrote books, such as *50 Secrets of Magic Craftsmanship.*

Ken: For half an hour??? Not even an hour?

Prof. Daubechies: Well, ok…but you should always check for mistakes because there could be mistakes. And if there are no mistakes, you feel even better. But it lasts very little. Even if you have no mistakes, it lasts a couple of days. After that, you have understood it even better and you begin to feel very stupid for having looked so long before you found it! And you kind of make it part of the tissue of mathematics that you know and at some point it becomes completely absorbed just like we absorbed decimal notation in the mathematics we teach our children. And at that point, it's no longer that big joy. It might still be fun to explain it to others who don't know it, but that pure thing of "Wow!!!"…that's over. So you choose a profession where you're frustrated a great deal of the time, you don't know when you'll find something, and when you find something, you feel, "woah!!!" and then that high is over a couple of days later. So it's a frustrating thing…it can be frustrating…but it's also a lot of fun. And I like teaching also…I like talking about mathematics to people and teaching students. So it's not just these wows and dips in between.

Ken: I often feel, I don't know if you agree, that a large part of doing mathematics is psychological, but it doesn't seem that there are any courses in grad school that try to help students deal with these issues.

Prof. Daubechies: Yeah, well… I tell our incoming students always that they have to really work on building a social group…a social network around themselves…that they will learn half their mathematics from their peers anyway…not from their professors. Just like mathematics, doing mathematics is a very human pursuit and it's very good to do it with others. I like collaborating with people. I also like having my time to figure it out because sometimes you have to figure out things in the privacy of your own office before explaining it to someone else. So in collaborations, we typically explain things to each other then we work alone then we get together again and so on. I try to foster a lot of getting together of the students. I like to give them reading courses where I explain for each chapter of the book what it is about and then they have to work through the details and they have to assign problems and discuss problems and so on. And then, after they have digested that chapter, explaining it to each other, worked through the problems, done everything, we go to the next chapter. They find that it teaches them a lot. But it's also the doing it together that teaches them. So that is very important. So, I try to make sure we have nobody falling through the cracks who is too isolated.

If you're down and you see other people who are not…that helps you. Also, you talk about more things; talking about more things, being exposed to more things, leads to more ways out of a problem.

Ken: We have a girl at Girls' Angle who I personally think is quite gifted. But she has kind of a disease where whenever she solves a problem she immediately thinks, it must have been trivial because she feels that if she can solve it, it must be something very simple. So, she tends to think of herself as very unintelligent. She's even said, "I'm so stupid".

Prof. Daubechies: Yeah, but, uh, well…I don't know her, so I don't know whether it is…it could be the result of something at work that is something in the way girls are brought up…girls are not brought up to feel they are stupid, but girls are brought up in a way to try to find common ground among each other, to find ways of sharing. And it might be that by saying, "I am smart" she feels that she is counteracting…that she is not doing the acceptable thing. Or she might even feel that by being smart she is making herself too different from the others…I don't know. I really don't know. I know that played a role for me at some point. At the last mentoring program we had for women, we had a discussion about professional interactions between women and men in the math department or, rather, not professional, but how collegial interactions can be different. And one of the senior women said that junior women should be aware of the fact, she said, you know when you meet other women…one way in which women bond is that one of them will say something in a funny way but something about a personal weakness…something disparaging about herself. And the way other women will counter will be by doing the same thing about herself. So you've shared a weakness and that creates a bond so you're both not perfect. Well, this is something that you shouldn't do with most men in the department. If you do this with a man, most likely, he could start feeling superior or he could start explaining to you how to solve your problem. [laughter] But this is part of a way of interacting…it's not a ploy…it's a tactic people use, not consciously, but as a way of making a bridge.

So, I don't know if it would help, instead of feeling, "I'm so stupid" to think well, "I can do this, but so and so is better at that," so as to feel she is not singling herself out by being able to do the problem. I don't know. That's one thing I can think of, but there are others.

But there's also the fact that always what you can do yourself doesn't seem as miraculous as what other people can do because you've seen all the mistakes that you did on the way towards it while the others just come out with the answer. How old is she?

Ken: She's twelve.

Prof. Daubechies: I hope she doesn't discourage herself from doing mathematics.

Ken: Also, she has an issue with making mistakes. She hates to make mistakes, to the point where I think it is an obstacle. This is a problem we're having at the club: trying to convince the girls that mathematicians make a lot of mistakes.

Prof. Daubechies: Oh yeah!

Ken: Actually, maybe you could address this?

To be continued…

"…talking about more things, being exposed to more things, leads to more ways out of a problem."

Prueba del 9: The Reduction Procedure

By Hana Kitasei

In the last issue we mentioned a math trick **littleMeme** brought to Girls' Angle. (See *Prueba del 9*: *The Trick*.) The trick involved a "reduction" procedure. Do you remember it? See the box for a little refresher.

<u>The Reduction Procedure</u>

Start with any positive integer, a. If a has multiple digits, add its digits together to get a new integer, b. If b has multiple digits, add its digits up to get c. Repeat the process of adding digits until you come to a number that is a single digit. The result of the reduction procedure is this single digit.

By the way, have you thought about why, when you perform the reduction procedure, you will always eventually arrive at a single digit number?

We promised an explanation of how the math trick works, and in the next couple of issues we'll gradually gather the tools we need to understand it.

In this issue, we'll focus on the reduction procedure.

Choose any positive integer, a. Apply the reduction procedure to obtain the single digit A. Divide a by 9 and A by 9. A key fact about the reduction procedure that we'll need to explain the trick is this: **No matter what *a* you chose, both divisions will leave the same remainder!**

Example: Start with $a = 17$. Perform the reduction procedure, $1 + 7 = 8$. This is a single digit so $A = 8$. Dividing a by 9, we see that 17/9 is 1 remainder 8. Dividing A by 9, we see that 8/9 is 0 remainder 8. So both a and A leave a remainder of 8 if you divide them by the number 9.

Now try it yourself a few times!

Why does this work? If you try it yourself a few times, you might get an idea. For my explanation, read on.

The reduction procedure is rather complex. It involves summing the digits of numbers possibly several times. In fact, we have no idea how many times one would have to sum digits of numbers until one would finally arrive at a single digit number. So, how can we prove the equality of the remainders upon division by 9 of the *starting* number and the *end* result of the reduction procedure?

If you compute the remainders upon division by 9 at each step in the reduction procedure, you may begin to suspect that the remainder upon division by 9 is preserved at *each* step of the procedure, not just in the final single digit result. That is, given any number a, the sum of its digits, let's call it b, seems to have the same remainder as a, if you divide both by 9. If each step of the reduction preserves the remainder, then the single digit result of the reduction procedure will also have the same remainder as a.

So, instead of trying to relate something between the starting number and the end result, we just try to relate something between a number and the sum of its digits. Because the starting number and the end result are linked by a chain of digit sums, if we are successful, we will have established our desired result!

Example: If $a = 17$, we only need to perform the reduction one time to get a single digit result. Let's make sure my claim works for a number when we have to perform the reduction multiple times to get a single digit result.

Let $a = 157$. Perform the reduction: $1 + 5 + 7 = 13$ which is not a single digit, so we sum again, $1 + 3 = 4$. Divide 157, 13, and 4 by 9, and check yourself that they each have remainders of 4.

Why do 157 and $1 + 5 + 7$ have the same remainder if you divide by 9?

Before we go any further, let us define precisely what we mean by "remainder." After all, without a clear idea of what a remainder is, how would we be able to prove anything about it?

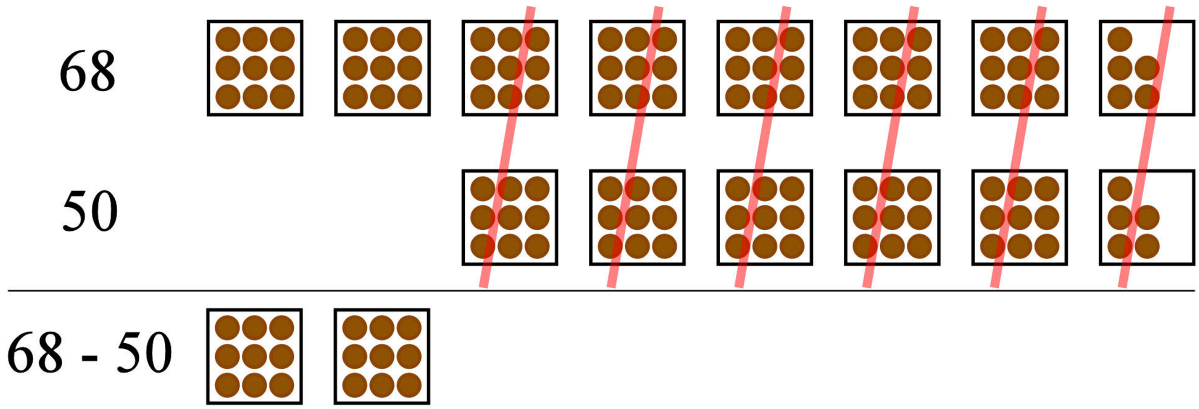
Since this issue's trick requires division by 9, let's use division by 9 as an example. Dividing a number is like sorting chocolates into chocolate boxes. Suppose you have a number of chocolates and boxes that can fit 9 chocolates each. You keep filling up boxes and when they are filled you set them aside. Sometimes the number of chocolates is divisible by 9, in which case you will end up with a pile of boxes that are all filled. This is like having no remainder. When a number is not divisible by 9, you will have a box that isn't full—and you can't sell a box that's not full! The chocolates in this box are the scraps, and their number corresponds to the remainder.

Definition of Remainder

In more precise mathematical terms, if you divide a number a by a number d, the remainder is the smallest nonnegative integer r such that d divides evenly into $a - r$. Another way of putting it is that if x is the largest nonnegative integer (x corresponds to the number of filled chocolate boxes) such that xd is less than a, then the remainder $r = a - xd$, or, equivalently, we can write $a/d = x + r/d$.

Now that we agree what a remainder is, let's turn our attention to showing that a positive number and the sum of its digits will both leave the same remainder upon division by 9.

We want to show that two positive numbers, say a and b, leave the same remainder upon division by 9. Notice that this is the same as showing that 9 divides evenly into $a - b$. To see this, imagine arranging a chocolates into boxes that can hold 9 chocolates each. You'll have a bunch of filled boxes plus, possibly, a single unfilled box containing the remaining chocolates. If you now subtract b of those chocolates by taking them away and the result is divisible by 9, that is, there are a bunch of filled boxes of chocolates, then it must be that b chocolates can also be fit into some number of filled boxes plus one box with the same number of leftovers that you got with a chocolates, and vice versa. (See illustration on the next page.)



If you have two numbers that leave the same remainder when you divide by 9, like 68 and 50, then their difference is divisible by 9. That's because the unfilled boxes will cancel each other leaving only filled boxes left, some of which will remain, others of which will cancel when you subtract. Conversely, if the difference of two numbers is divisible by 9, then the two numbers must leave the same remainder when you divide by 9 because if they didn't, you wouldn't be able to cancel chocolate boxes in such a way that you would be left with boxes that are all completely filled.

In our case, the two numbers we want to show leave the same remainder upon division by 9 are a number and the sum of its digits. Let's revisit our example of $a = 157$. Let's show that 9 divides 157 minus the sum of its digits:

$$\begin{aligned}
157 - (1 + 5 + 7) &= 1 \times 100 + 5 \times 10 + 7 \times 1 - (1 + 5 + 7) && \text{(write 157 in expanded form)}\\
&= 1 \times 100 + 5 \times 10 + 7 \times 1 - 1 - 5 - 7 && \text{(use the distributive law)}\\
&= 1 \times 100 - 1 + 5 \times 10 - 5 + 7 \times 1 - 7 && \text{(rearrange terms)}\\
&= 1 \times (100 - 1) + 5 \times (10 - 1) + 7 \times (1 - 1) && \text{(the distributive law again!)}\\
&= 1 \times (99) + 5 \times (9) + 7 \times (0) && \text{(simplify inside parentheses)}\\
&= 9 \times (1 \times (11) + 5 \times (1) + 7 \times (0)) && \text{(distribute out the factor of 9)}
\end{aligned}$$

This shows that the difference is a multiple of 9, and we're done!

The same proof will work for any positive integer: after you subtract the sum of its digits, rewrite the number in expanded form, group terms that correspond to corresponding digits together, use the distributive law, and see that the result is a sum of terms that are each a multiple of 9.

Now we can see that if we take a number a and use the reduction procedure to obtain from it a single digit number A, then both a and A will leave the same remainder upon division by 9.

> Notice that:
> $10 - 1 = 9$,
> $100 - 1 = 99$,
> $1000 - 1 = 999$,
> and so on.
>
> In general, $10^n - 1 = 999\ldots9$ (the number whose digits consist of exactly n nines).

By the way, the single digit number A must be one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, or 9, and each of these digits leaves a *different* remainder upon division by 9. *That means that if a and b leave the same remainder when you divide them by 9, then if you apply the reduction procedure to both a and b to obtain A and B, respectively, then A = B.* That's because a, b, A, and B must all leave the same remainder upon division by 9, but both A and B are single digit numbers and so if A and B leave the same remainders upon division by 9, they must be equal.

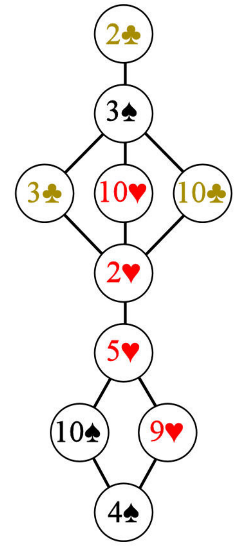
Partially Ordered Sets

When it comes to preferences, we may like one thing more than another, but we may also have no preference at all between two things. So if we order things according to our preferences, the order may not be “complete” in the sense that there may be two things that are not comparable.

That’s a different situation from the natural order on (real) numbers. If I have two numbers a and b, then exactly one of the following three situations *must* hold:

$$a < b \quad \text{or} \quad a > b \quad \text{or} \quad a = b.$$

The natural order on numbers is called a *total* order.



Z

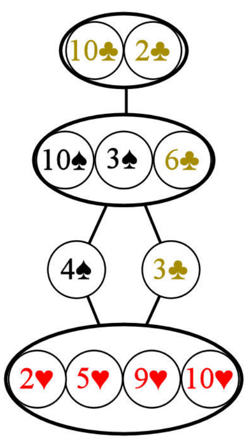
In both partially ordered and totally ordered sets, the following are always true:

1. We always have that $a \geq a$.
2. If $a \geq b$ and $b \geq a$, then $a = b$.
3. If $a \geq b$ and $b \geq c$, then $a \geq c$.

(By the way, notice that “$a > b$” is the same thing as saying that “$a \geq b$ and $a \neq b$”. Also, “$a \leq b$” is the same thing as saying that “$b \geq a$”.) The only difference from a totally ordered set is that in a partially ordered set we do not require that every pair of elements be comparable whereas in a totally ordered set, we demand it!

At the first meet, we did an activity that involved subsets of the set of girls at Girls’ Angle. The collection of subsets of girls at Girls’ Angle is an example of a partially ordered set if we say that one set is less than or equal to another when the first set is included (as a subset) in the other. Can you see why this is an example of a partially ordered set but not an example of a totally ordered set? Can you draw a diagram representing the partial order of subsets of a set with 4 elements?

To explore partially ordered sets further, we had a mini-chocolate tasting at Girls’ Angle. We used playing card symbols to identify different chocolates. The girls tasted the chocolates and put the chocolates into a partial order according to their preference.



cat in the hat

Above right is an illustration of **Z**’s partial order on chocolates. The way this diagram works is that each circle represents a different chocolate. If chocolate a is greater than chocolate b, and there is no chocolate that is in between the two (i.e. there is no chocolate c such that $a > c > b$), then chocolate a is drawn above chocolate b and a line is drawn to connect them. From this diagram, you can see that **Z** liked 2♣ best of all. For **Z**, chocolates 3♣, 10♥ and 10♣ were incomparable. But, all three were better than 2♥.

At left is **cat in the hat**’s partial order. Chocolates grouped into ovals were deemed to be indistinguishable from each other. That is, with respect to their taste, they were considered equals. We used hearts for milk chocolates. Evidently, **cat in the hat** likes milk chocolates the least.

Honda

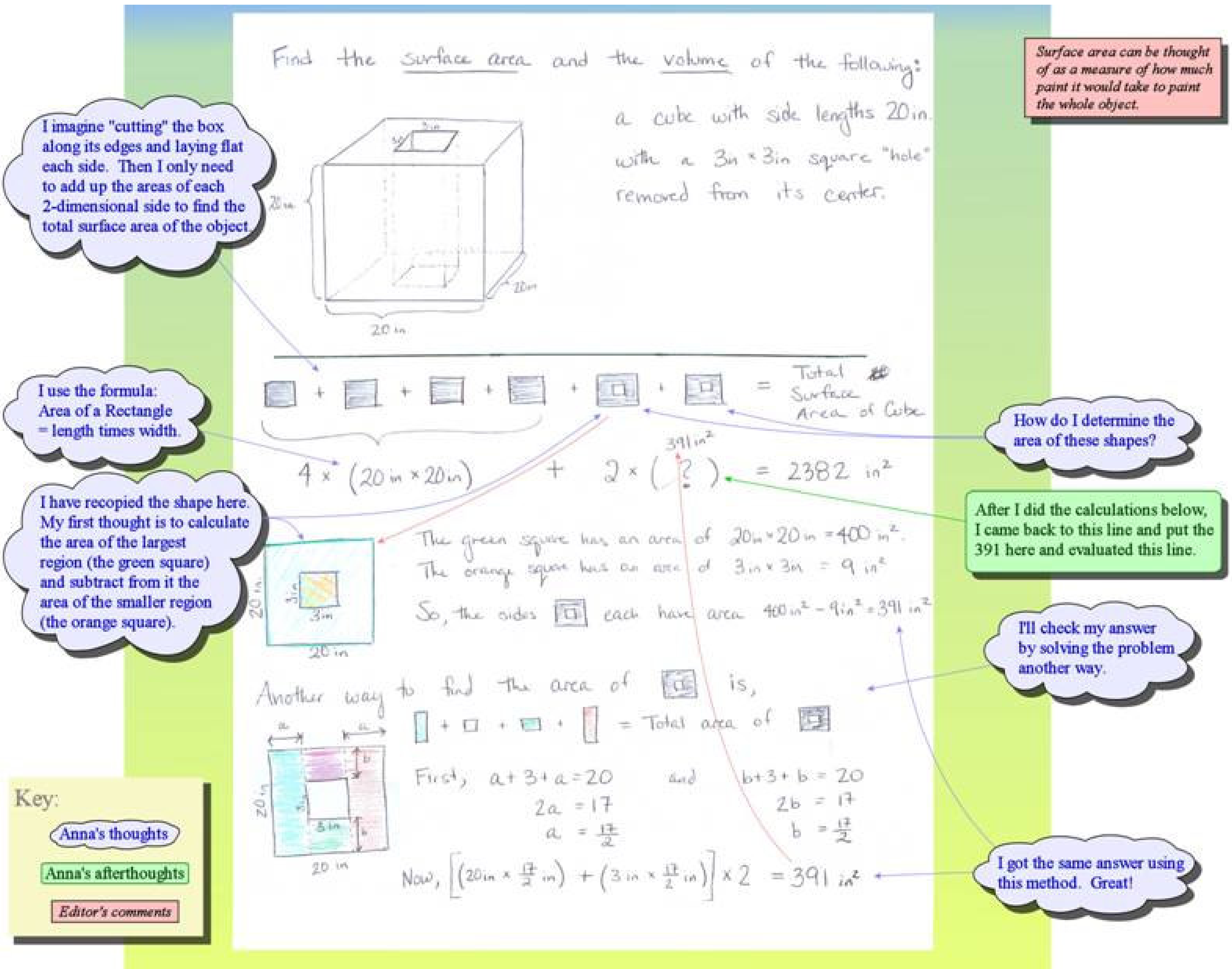
At right we see that **Honda**’s partial order on chocolates is, in fact, a total order. **Honda**, unlike **cat in the hat**, seems to prefer milk chocolates.

Anna's Math Journal

By Anna Boatwright

> *Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.*

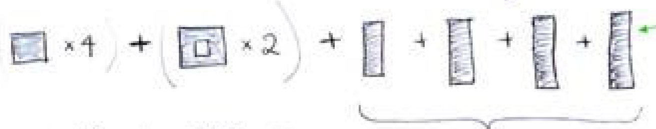
Here, Anna finds the **surface area and volume of a cube with a hole.**



Wait! I forgot about the inside part!

> The object also has 4 "inner" walls as part of its surface.

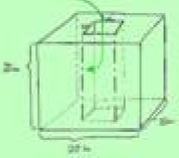
Total Surface Area of Cube =



$= 1600 \text{ in}^2 + 782 \text{ in}^2 + 4 \times (3\text{in} \times 20\text{in})$

$= 2622 \text{ in}^2$

> This rectangular face corresponds to one of these inner faces in the drawing I made from the previous page.



> One thing I experienced here is that checking my answer by solving the problem in two different ways is a good way to catch errors in my calculations, but there are some types of error made in setting up the problem that it will not catch.

Total Volume = Volume of Cube − Volume of "hole"

> I can use a similar approach as before...I'll try to express the volume in terms of pieces whose volumes are easy to see.

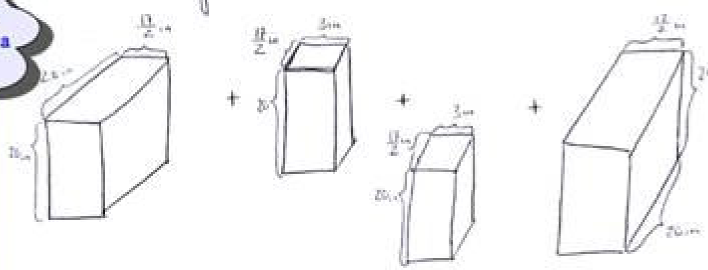
$= (20\text{in} \times 20\text{in} \times 20\text{in}) - (3\text{in} \times 3\text{in} \times 20\text{in})$

$= 8000 \text{ in}^3 - 180 \text{ in}^3$

$= 7820 \text{ in}^3$

Another Way to find the total volume is

> Now I will check my answer by breaking up the object in a different way.



= Total Volume

$2\left(20 \times 20 \times \frac{17}{2}\right) + 2\left(20 \times \frac{17}{2} \times 3\right) = \text{Total Volume}$

$6800 \text{ in}^3 + 1020 \text{ in}^3 = \text{Total Volume}$

$7820 \text{ in}^3 = \text{Total Volume}$

> I get the same answer. Great!

If you'd like to try your hand at solving a similar problem, compute the surface area and volume of the following: Take a cube, 30" on a side. Now, drill three square holes, 10" by 10", right through the centers of its faces. These three holes are made so that they intersect in a smaller cube, 10" on a side, in the middle of the larger cube.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

ABB 12.23.08

Addition and Multiplication Sculptures

By Ken Fan

I recently commissioned Jane Kostick to make two wooden sculptures to represent addition and multiplication. These sculptures were brought to the club at the sixth meet of the session and to the Girls' Angle social event. Here, I'll explain exactly what these sculptures are and in the next article, Jane will explain how she built them. Addition appears on this issue's cover. For more of Multiplication, see *It Figures!*

We've all made addition and multiplication tables. In fact, if you've got a spare moment, how about making some right now? Fifteen by fifteen. We'll do the addition one below. You do the multiplication table!

Addition

m \ n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
4	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
5	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**

Let's focus in on the red section in the table. Those numbers are the various sums of pairs of numbers between 1 and 15, inclusive. Imagine placing a column of wood over each red entry. If you make the height of the column of wood correspond to the entry itself, you will obtain some three dimensional wood sculpture. That is exactly what the Addition sculpture is! Multiplication is similar, only one uses a multiplication table.

Using 15 by 15 tables and a unit length of 3/8", Addition will have a square base 5 5/8" on a side and a height of 11 1/4". Multiplication will have the same square base, but will soar over 7 feet!

Certain patterns in addition and multiplication are visually more apparent in the sculptures than in the tables. The reason is that the tables are filled with symbolic forms of numbers that are rather sophisticated. If you look at the symbol "5" and the symbol "6", unless you have learned the meanings assigned to these symbols by society, there would be no reason for you to regard one as being "one more" than the other. In order to get this sense of "one more", you would have to have trained your mind to recognize those symbols and associate with them their quantitative meanings. In sculptural form, however, the heights represent the numbers and you can see "one more" in a visual way.

Whenever possible, as you learn more and more mathematical concepts, try to create multiple representations of the concepts. For example, these sculptures are visual representations of addition and multiplication. Doing so will help you avoid manipulating symbols mechanically and help you to remember the meaning of things.

At the Girls' Angle social event, many advanced mathematicians were present. We presented the two sculptures, but did not explain what they represented. We asked the guests to figure out their mathematical significance. Amusingly, I can honestly write that there are Ph.D.s in mathematics who do not recognize multiplication when they see it! The most common error was to be thrown off by the sheer height of Multiplication. Visually, it grows so fast that many immediately began thinking that they were observing some kind of exponential growth.

Multiplication does grow fast! In fact, it grows so fast, if the wooden columns over each entry were placed end to end, how long would the result be, assuming that each unit is represented by 3/8"? This would be a pretty important question to answer before making the sculpture because it would give an idea of how much wood one would need and that could affect the cost of the sculpture. To figure this out, we have to compute the value of

$$\begin{array}{l}
1 \times 1 + \ 1 \times 2 + \ 1 \times 3 + \ldots + \ 1 \times 13 + \ 1 \times 14 + \ 1 \times 15 + \\
2 \times 1 + \ 2 \times 2 + \ 2 \times 4 + \ldots + \ 2 \times 13 + \ 2 \times 14 + \ 2 \times 15 + \\
3 \times 1 + \ 3 \times 2 + \ 3 \times 4 + \ldots + \ 3 \times 13 + \ 3 \times 14 + \ 3 \times 15 + \\
\qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad \vdots \\
13 \times 1 + 13 \times 2 + 13 \times 3 + \ldots + 13 \times 13 + 13 \times 14 + 13 \times 15 + \\
14 \times 1 + 14 \times 2 + 14 \times 4 + \ldots + 14 \times 13 + 14 \times 14 + 14 \times 15 + \\
15 \times 1 + 15 \times 2 + 15 \times 4 + \ldots + 15 \times 13 + 15 \times 14 + 15 \times 15 +
\end{array}$$

and then multiply by 3/8". Can you see that the above sum of products is equal to the square of the sum of the first 15 positive integers, that is, $(1 + 2 + 3 + \ldots + 13 + 14 + 15)^2$? Can you compute that $1 + 2 + 3 + \ldots + 13 + 14 + 15 = 120$ and $120^2 = 14{,}400$? So if one unit is 3/8", the length of wood needed would be 5,400 inches which is 450 feet of wood! That's a long piece of wood! How much wood would be needed for Addition? If you figure it out, feel free to email us what you got.

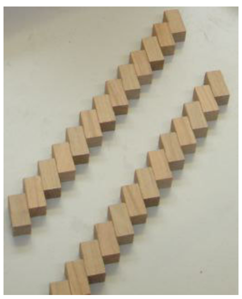
Fortunately, Jane figured out how to make the sculptures without using so much wood! Instead of making them solid, she…

The Making of Addition and Multiplication

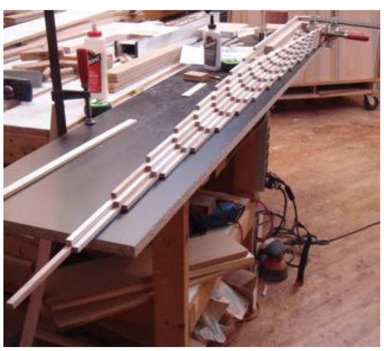
By Jane Kostick

…was able to save a lot of wood by making the sculptures hollow.

Building Addition involved a lot of repetition of making identical parts. I began with large boards of lumber that I sawed into many sticks roughly 1/2” thick by 1/2” wide. Passing each stick over a jointer, I got two of the faces nice and flat, with a perfect 90 degree angle between them. Then with a thickness planer I milled the opposite faces of the wood in order to make sticks with a cross section of 3/8” by 3/8”. These sticks then got diced up into the various lengths necessary to build the outside perimeter of the sculpture. I used a small router to round over the edges of the sticks prior to assembling the sculpture. For the 169 sticks that make up the inside of the sculpture, I was able to save a lot of wood by making each stick not much longer that its visible portion. The following pictures show the two sculptures in progress.



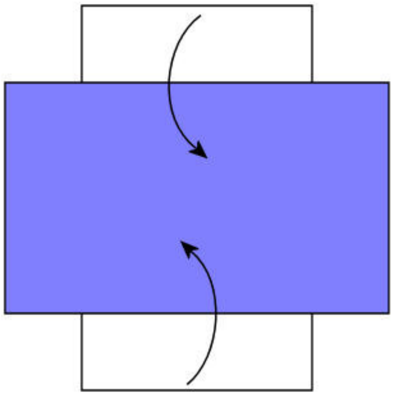




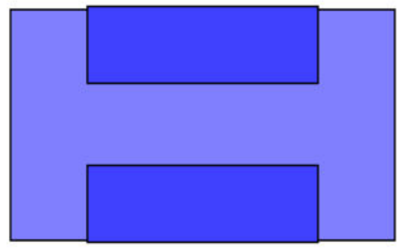
Jane Kostick in her studio.

Index Card Cube

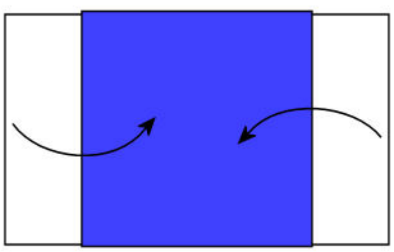
Materials: 6 rectangles. Index cards and business cards work well.



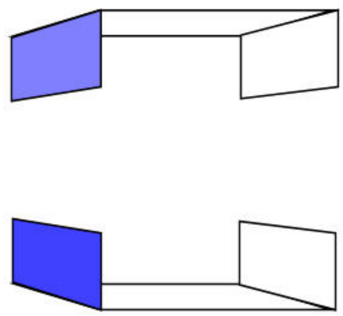
1. Take two index cards and form a “Red Cross” shape. Fold the ends of one index card over the other.



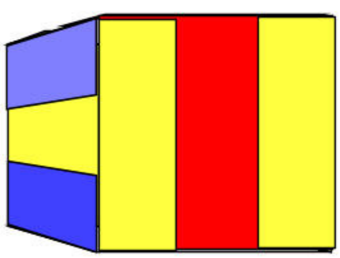
2. Step 1 finished.



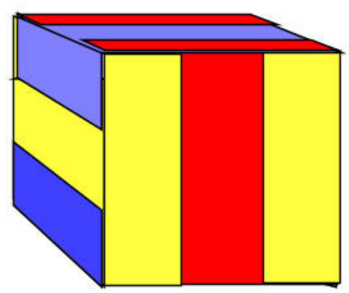
3. Flip and repeat, then separate.



4. You should have two staple-like objects. Repeat steps 1-3 two more times with the remaining 4 index cards.



5. Assemble the cards into a cube.



6. Here’s another view of the cube.

Cycloids on Broadway

By Katy Bold

Logo Design by Hana Kitasei

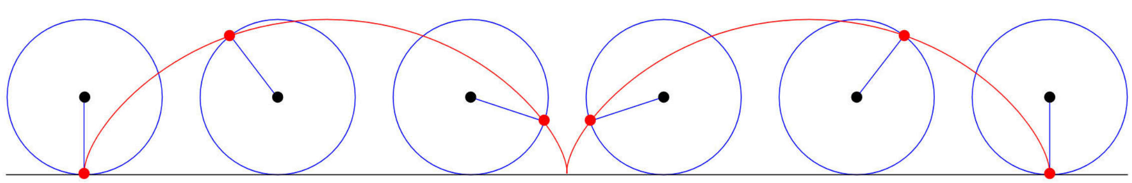
The Lion King is a Disney story of a young lion cub, Simba, who is wrongly exiled from his African homeland by a jealous uncle. Simba grows up in the jungle with new friends Timon (a meerkat) and Pumbaa (a warthog) before returning home and reclaiming his rightful place as king.

Originally a cartoon, it was a challenge to bring to life the animals of the African plains in a live theater on Broadway. Simba and his family are portrayed by actors wearing makeup and large head pieces. Simba's friends Timon and Pumbaa are life-size puppets. Giraffes walk across the stage – brave actors wear two pairs of stilts, a shorter pair for the legs and a longer pair for the arms. Several dancers move across the stage, each carrying a large gazelle prop in each hand, making the gazelles appear to leap through the air.

At one point during the show, a man pushes a small cart across the stage, with lots of small gazelles attached. The small gazelles are held up by sticks connected to the rims of the cart's wheels. As the wheels turn, the gazelles climb higher into the sky and then as the wheel turns further, they come back down.

The path of the gazelles has a special name: a **cycloid**. One way to think about a cycloid is to imagine a bicycle wheel with a dot of red paint on the face of the tire closest to you. As the bicycle wheel turns, the red dot is moving, and the curve created by the red dot is a cycloid.

Maybe a mathematician was among the set and costume designers who designed and built the cart that made the gazelles leap!



The cycloid is a very interesting curve that is famous for being the answer to the *brachistochrone* and *tautochrone* problems. The brachistochrone problem requires finding the shortest-time path of a bead falling between two points when the only force acting on the bead is gravity (and assuming no friction). This problem will be the subject of a future Math in Your World column.

The tautochrone problem requires finding the curve with this special property: starting from rest anywhere along the curve, a bead will reach the bottom of the curve in the same amount of time. Like the brachistochrone problem, the bead moves under the force of gravity, and friction is ignored.

Properties of the Cycloid

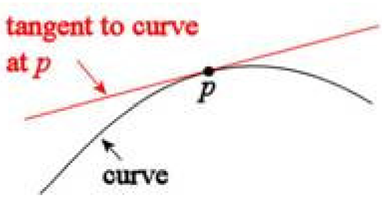
The cycloid has many interesting properties, and here we will touch on a few of them.

If a cycloid is traced out by a circle of radius 2 feet, what are the **maximum** and **minimum** heights of the cycloid?

If the stick holding the gazelle is 4 feet long, how high is the gazelle at the peak of its leap?

Does the cycloid ever cross over itself?

Recall that the **slope** of a straight line is the ratio of the change in the y-values to the corresponding change in the x-values ("rise over run"). This idea can be generalized to curves by taking the slope at a point *p* on the curve to be the slope of the **tangent** line to the curve at *p*. The figure below shows the tangent to a curve at a point *p*.



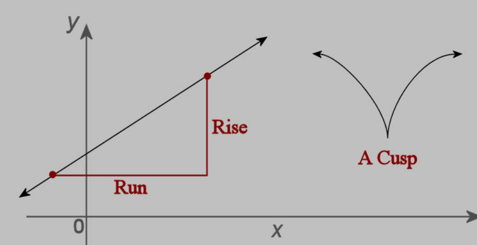
Where does the cycloid have slope equal to zero? Hint: Think about a line with slope zero – what does it look like?

The slope of a vertical line is undefined. On a curve, if the slopes are undefined on both sides at a particular point, the point is called a **cusp**. At which points on the cycloid is there a cusp?

"Mactionary"

The largest value a function yields is its **maximum**. The smallest value a function yields is its **minimum**.

The **slope** of a line is the ratio of the change in y-values to the corresponding change in x-values. Informally, the slope is the "rise over run." (See the Slope Problem Sets from the Summer Fun section of volume 1, number 4 of this Bulletin.)



A point on a graph at which the curve has undefined slope to the right and left of the point is called a **cusp**.

A **tangent** to a curve at a point *p* on the curve is the line that best approximates the curve in the neighborhood of *p*.

A function is **periodic** if it repeats itself. Formally, this is written $f(x + T) = f(x)$, where T is the **period**.

Why is the cycloid **periodic**?
If we think of the cycloid curve as a function $f(x)$, where x represents distance along the horizontal, what is the period of the curve (assume a circle of radius r)?

Take it to your world

Use a piece of chalk to mark an "x" on a bicycle wheel. Ask a friend to ride the bicycle slowly so that you can watch the shape of a cycloid.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete and, to nonmembers, they may not even be coherent!

<u>Session 3 – Meet 7 – October 30, 2008</u>

Mentors: Lauren McGough, Jennifer Melot

The first half of the first meet of the second half of the third session began with Algebra Target Practice. In this game, girls are presented with a function and a target number. The girls split into three teams. Each team had to figure out what number to feed the function so that the output of the function would be as close as possible to the target. The absolute value of the difference between the value of the output and the target was added to a cumulative score for each team. The winning team was the team with the lowest cumulative score. Here are the results:

The Function	**Target**	**One Team**			**Another Team**			**The Third Team**		
		x	$f(x)$	**Score**	x	$f(x)$	**Score**	x	$f(x)$	**Score**
$f(x) = 4x + 2$	**54**	13	54	0	13	54	0	13	54	0
$f(x) = 3(x + 2)$	**0**	-2	0	0	-2	0	0	-2	0	0
$f(x) = -2x$	**3**	-1.5	3	0	-1.5	3	0	-1.5	3	0
$f(x) = x^2$	**25**	5	25	0	5	25	0	5	25	0
$f(x) = x^2$	**10**	$3\frac{1}{3}$	$11\frac{1}{9}$	$1\frac{1}{9}$	$3\frac{1}{3}$	$11\frac{1}{9}$	$1\frac{1}{9}$	3.33	11.0889	1.0889
$f(x) = x^2 - 1$	**8**	3	8	$1\frac{1}{9}$	3	8	$1\frac{1}{9}$	3	8	1.0889
$f(x) = x^2 + x$	**12**	3	12	$1\frac{1}{9}$	3	12	$1\frac{1}{9}$	3	12	1.0889
$f(x) = x^2 + x$	**-1**	-0.8	-0.16	$1.95\overline{1}$	-0.5	-0.25	$1.86\overline{1}$	0.5	0.75	2.8389
$f(x) = (x - 5)(x + 6)$	**0**	-6	0	$1.95\overline{1}$	-6	0	$1.86\overline{1}$	-5	-10	12.8389
$f(x) = x^2 + x + 1$	**7**	2	7	$1.95\overline{1}$	2	7	$1.86\overline{1}$	2	7	12.8389

The eighth function, $f(x) = x^2 + x$, is an example of a *quadratic* function. The target was -1, and, as it turns out, there is no real number x for which $f(x) = -1$. So, in this case, teams just had to try to come as close to the target as possible.

Notice that $x^2 + x$ can be rewritten as $(x + 0.5)^2 - 0.25$. (Check this! If you are having trouble, please read the article on the distributive law in the last issue.) When written in this form, we can see that the function is equal to a constant (-0.25) plus the *square* of a number: $(x + 0.5)^2$. When you multiply a real number by itself, the result is always greater than or equal to zero. This means that the function will always be greater than or equal to -0.25. So the closest one can get to the target of -1 is -0.25 and this value is attained when the squared number is zero, that is, when $x + 0.5 = 0$, in other words, when $x = -0.5$.

A bonus question was given with $f(x) = x^3 + x^2$ and a target of 1,000,000. Trying to solve the equation $f(x) = 1{,}000{,}000$ is not so easy, but how can one get close to the target? The function consists of two terms, x^3 and x^2. The first of these terms is x times bigger than the second. As x gets bigger and bigger, the first term gets much, much bigger than the second term. When x is 100, the first term is 100 times bigger than the second term. Another way of putting it is that when x is bigger than 100, the second term is less than 1 percent of the first term. This means that to get a good *approximation* of $f(x)$, when x is big, we can just *ignore the second term*. If we do that, we are then looking for the cube root of 1,000,000, which is 100. So we know that $f(100)$ will be within a percentage point of 1,000,000. Any other integer will be farther away. Here's a table of values showing various values of the function near 100:

x	98	99	100	101	102
$x^3 + x^2$	950,796	980,100	1,010,000	1,040,502	1,071,612
Difference from 1,000,000	49,204	19,900	10,000	40,502	71,612

To hit the target exactly, the value of x required sits somewhere between 99 and 100.

For the second half of the meet, we had a mini-chocolate tasting event to introduce the concept of the partially ordered set. See page 9. **Grace** and **Hadassah** also rated teas. Both produced tea orders that turned out to be total orders.

Session 3 – Meet 8 – November 6, 2008

Mentors: Lauren McGough, Jennifer Melot, Doris Dobi

Special Visitor: Catherine Havasi, Computation and Linguistics, Brandeis

The first half of meet 8 involved more target practice, albeit with more exotic functions. For example, let $f(n)$ be the number of divisors of n where n is a positive integer. For example, $f(6) = 4$ because 6 has four divisors: 1, 2, 3 and 6. Here are some questions for you about this function:

1. What is another name for the numbers n for which $f(n) = 2$?
2. What numbers n have $f(n)$ odd?
3. If I pick a positive integer m, can you always find n so that $f(n) = m$?

Ilana figured out that the smallest positive integer n for which $f(n) = 6$ is $n = 12$. Indeed, the divisors of 12 are 1, 2, 3, 4, 6 and 12.

4. What is the smallest positive integer n for which $f(n) = 2$? 4? 8? 16? 32?

Catherine talked about language and communication. She pointed out that we use our common sense to fill in a lot of blanks and this enables us to interact more effectively. This kind of common sense knowledge can also help to improve our interaction with computers. For example, to help us type faster, a computer might try to offer word completions. If you start typing a word and it begins with the letter "d", a computer equipped with some database of common sense knowledge would be programmed to regard it more likely that the intended word is going to be "dog" rather than the word "defenestrate".

Catherine also discussed correlated knowledge. Sometimes the kinds of things that belong to one category of objects is highly related or identical to the things that belong to another category of objects. Programming a computer with this kind of knowledge also helps the computer to reduce possibilities. For example, there is a strong correlation between the "things that we want" and the "things that make us happy". And, as a point of fact, understanding of mathematics is indeed both something that we want and something that makes us happy!

<u>Session 3 – Meet 9 – November 13, 2008</u>

Mentors: Lauren McGough, Jennifer Melot, Doris Dobi, Clara Chan, Nike Sun

Meet 9 was spent entirely in small groups attacking a variety of different math problems, including this one that Doris brought in:

> A family came upon a bridge. The family consisted of mom, dad, a grandfather and a daughter. The bridge was only strong enough to support at most two people at a time. It was very dark, so whoever was crossing had to use a flashlight. The family only had one flashlight. Crossing alone, the mom could make it in 1 minute, the dad in 2 minutes, the grandfather in 10 minutes, and the daughter in 5 minutes.
>
> What is the minimum time required for all the members of the family to make it across the bridge?

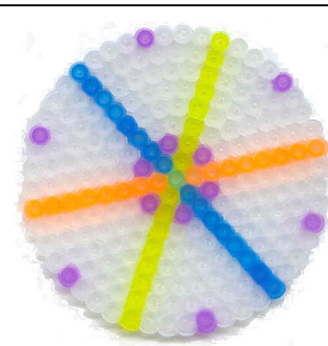
Some girls worked with Clara on the monkey/banana transport problem (see the Notes from the Club from the last issue of this Bulletin). The complexity of the monkey/banana transport problem is high enough that whenever one succeeds in transporting some number of bananas, without proof, one is always left wondering if that was really the maximal number of bananas that could be transported. With Clara, some girls tried to produce a proof that an optimal strategy was found.

Some girls worked on some of the problems from an advanced algebra target practice.

And, a number of girls played the Cliffhanger card game. The girls by and large did the best they could with the hands they were dealt, but that day, **Tree** managed to stay furthest from the cliff.

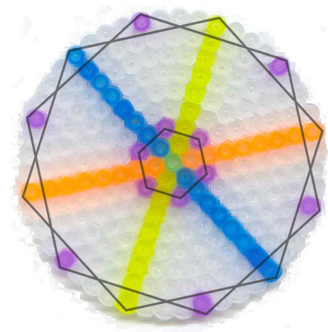
<u>Session 3 – Meet 10 – November 20, 2008</u>

Mentors: Jennifer Melot, Lauren McGough, Doris Dobi
Anda Degeratu, Grace Lyo


littleMeme's design

On a number of occasions, members would make drawings of geometric patterns and designs. So, we began meet 10 with a brief talk on regular polygons and tessellations. For example, earlier, **littleMeme** brought in a circular piece of art she made. Geometrically it involves three diameters of a circle. There are many implied regular hexagons within her design.

By the way, how many pieces is this design made out of? Notice that each sector (between colored diameters) has 1 + 2 + 3 + 4 + 5 + 6 + 7 pieces. These sums of consecutive integers have been appearing a lot!

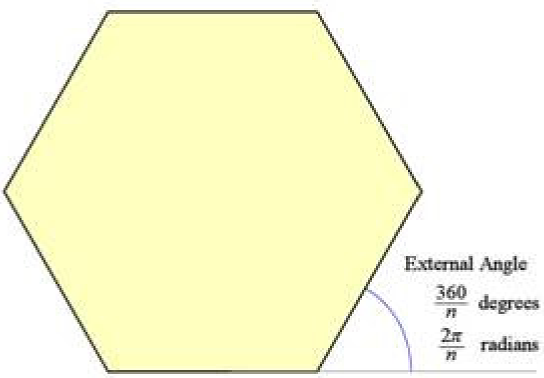


littleMeme's design with some of the implied regular hexagons drawn in.

Regular polygons have a lot of symmetry and much can be said about them. We spent most of the meet in small groups exploring different aspects of polygons, regular polygons, regular polyhedra and polyhedra.

For example, if you have a regular polygon with n sides, all the *external* angles are equal. As we discussed at the meet, the measure of these external angles are all equal to $360/n$ degrees.

We also saw that the sum of the interior angles of a regular polygon with n sides is $180(n - 2)$ degrees. Because all the interior angles are equal in a regular polygon, you can divide this number by n to determine the measure of each interior angle.



You can also find the interior angle by observing that it, together with an external angle, make up a single straight angle. Because a straight angle measures 180 degrees, you can find the measure of the internal angle by subtracting the measure of the external angle from 180 degrees.

If the polygon is not regular, then the various internal and external angles might differ from each other. (Though they can still all be equal…can you think of an example?) If you have a non-regular polygon with n sides, you could even have n interior angles all different from each other. However, all is not lost! It turns out that the *average* of these measures will be the same for all n-sided polygons, regular or not! Or, what's the same, the *sum* of the interior angles of any polygon with n sides only depends on n and will equal $180(n - 2)$ degrees. Try to prove this fact!

What can you say about the average of the measures of the exterior angles of a general polygon?

Some girls worked on figuring out ways to construct regular polygons. For example, **cat in the hat** used folding techniques to construct a square inscribed in a circle. Others studied properties of polygons. Others made tessellations. (For more on tessellations, see *A Puzzling Problem for Penrose* by Allison Henrich and Sarah Wright in the fifth issue of this Bulletin.)

One question that came up was whether or not a tessellation could be made with a heart shape. That is, could one invent a heart shape and, using only copies of that shape, tile the entire plane? **Aba-ka-dabra** came up with a heart shape that comes very close to tessellating the plane. If you can come up with a tiling heart shape, send it in and we'll put it in the Bulletin!

Some girls looked at the 3-dimensional analog of regular polygons: *regular polyhedra*. These 3-dimensional objects have a lot of regularity. First, all of their faces are the same regular polygon. Second, the arrangement of faces around any vertex is the same. **Anonymous**, **Jo** and **Grace** explored equidistant configurations and related them to triangles and tetrahedra. We'll talk more about these highly symmetric objects in the future, but you might enjoy making models of the two with the fewest number of faces: the regular tetrahedron (with 4 faces) and the cube (with 6 faces). If you have a square piece of paper, you can fold it into a regular tetrahedron using techniques of origami. See volume 1, number 6 of this Bulletin. Can you figure out a way to make an origami cube? See page 15 for a way to make a cube with six index cards.

Session 3 – Meet 11 – December 4, 2008

Mentors: Jennifer Melot, Lauren McGough, Nike Sun, Doris Dobi

Special Visitor: Amanda Cather, Waltham Community Organic Farms

Amanda explained the myriad ways she uses basic mathematics in organic farming. So much of farming can be quantified. There's the size of the farm, the number of rows for vegetables in the farm, the number of rows to allot to each vegetable, the cost of the seeds, the number of vegetables that must be produced, the amount of fertilizer to use, how much to pay workers and how much to charge customers, and more! Sorting all this out to make a farm work efficiently involves a lot of computation. Each computation may be a simple addition, subtraction, multiplication or division, but taken together, there are a whole lot of computations making the running of a farm quite a complex task indeed! Amanda gave us an idea of how this complex task is organized.

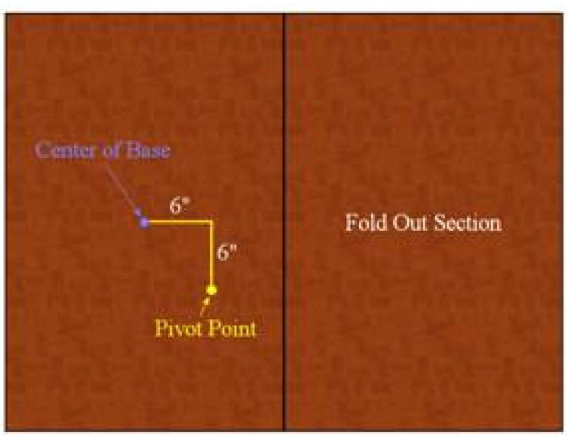
For the remainder of the meet, the girls worked on a number of different problems, including our first Community Outreach commission from Jane Kostick (see box at right).

Community Outreach

Community Outreach is a component of Girls' Angle where we accept commissions from people or organizations in the community to solve math problems. In exchange for solving the problem, the commissioning agent agrees to credit the girls of Girls' Angle for the solution.

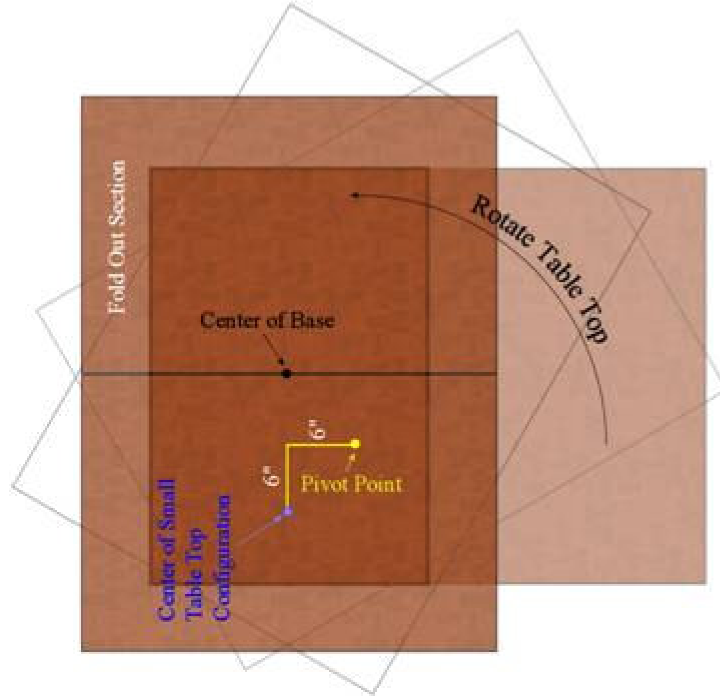
Our first Community Outreach problem comes from Jane Kostick who asked the girls to solve a problem concerning a special table that could be set up in two different configurations. In one configuration, the table is 24" by 36" and sits on a rectangular stand with the same dimensions. However, the table top is actually two layers both 24" by 36" connected by hinges along one of the long sides. The top can be unfolded to produce a new, larger table top that is 48" by 36". If you just unfold this top part out, the table will become unstable because half of the table top will hang out unsupported. To fix this, Jane asked the girls to figure out where a single pivot could be placed so that the 48" by 36" folded out table top could be rotated around this point to a position centered on and square with the original 24" by 36" table base.

Trisscar, **The Cat**, **Mouse**, and **littleMeme** tackled the Community Outreach problem. By making a careful scale model of the table top, they were able to find multiple locations where the pivot could be placed. **Trisscar** took the further step of making a scale model to the scale of 1 centimeter = 1 inch and verified that the pivot point would work for a table of that size. The picture shows one possibility, as they determined. Shown is the folded out configuration of the table top prior to being rotated into place. The base sits directly under the left half.



This is a practical solution, in the sense that it will indeed work: if a table is so constructed with the pivot point located as indicated, one would be able to rotate the larger table around the pivot and center it over the original base.

However, because the solution was found using a scale model, there remains a theoretical question. If the table is rotated about this pivot point, will it, in fact, *exactly* position itself centered and square over the original base? Or, could this pivot be just a little bit off producing a small error that is negligible for a table of the desired size? Can we, for instance, say that if the units were not inches but miles, this pivot point would do the job for the gigantic 48 mile by 36 mile table top? Or would the center of the large table top miss the center of the base by a few inches?



Can you demonstrate that the pivot point is in *exactly the right spot*?

Speaking of the right spot, many times this session, the girls had to search for all points that satisfied some given condition. Such a set of points is called a **locus**. For example, **cat in the hat** and **Trisscar** found the locus of points with a 90 degree viewing angle of a painting.

Session 3 – Meet 12 – December 11, 2008

Mentors: Jennifer Melot, Lauren McGough, Doris Dobi, Mia Minnes, Grace Lyo, Beth Schaffer

For the last meet, we packaged a bunch of gifts for the girls in a box, wrapped it, and wound it round and round with ribbon. A closer inspection revealed two combination locks holding the ribbons together. To unlock the locks, the girls had to solve a number of math problems. They split into several groups and tackled the problems in parallel. By break time, half the answers were found, but the remaining problems seemed rather daunting. Would the girls be able to unlock the treasure in time, or would the treasure remain hidden for another year?

Some Problems from the Locked Treasure Puzzle

An author wrote three interesting math books. She is going to a book signing at a local bookstore. If she sells three copies of the first book, five copies of the second book and seven copies of the third book, she will make \$141. If instead, she sells five copies of the first book, seven of the second and three of the third, she will make \$161. But if she manages to sell seven copies of the first book, three of the second and five of the third, she will make \$163. One of the numbers in one of the combinations is the cost, in dollars, of buying one copy of each of the three books.

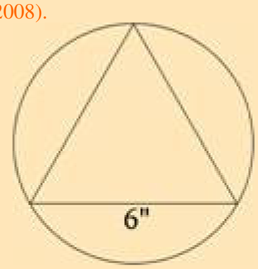
A medieval princess adored squares of perfect squares. She called such numbers beloved numbers. She loved the number 16, for instance, because it is the first nontrivial beloved number. Whenever she came upon a new number, the first thing she would do is determine how many beloved factors the number had. When she turned 15, she was given a gift of $15! = 15 \times 14 \times 13 \times \ldots \times 3 \times 2 \times 1$ gold coins. One of the numbers in one of the combinations is the number of beloved factors in 15!.

The function f takes a number and returns a number. If you plug in 8, it returns 0. That is, $f(8) = 0$. If m and n are any numbers, it so happens that

$$f(m + n) = f(m) + f(n) + 0.1.$$

One of the numbers in one of the combinations is $f(2008)$.

The figure shows a circle *circumscribed* about a triangle. The triangle is an equilateral triangle with sides of length 6 inches. The area of the circle, in square inches, can be expressed as a multiple of π. The number of multiples of π is a number in one of the combinations.



With 10 minutes left, all 6 numbers for the two combination locks were found. But which numbers belonged to which lock?

In a rapid brainstorming session, using the last remaining clue, the numbers were sorted out leaving each lock with just six possible combinations. With just 4 minutes remaining, suddenly, the second lock went *click!*

Because it was so important for the girls to work in concert to solve this in the nick of time, we'll refrain from singling out any girl here, though there definitely were many, many clever ideas that we could discuss.

By the way, I recently gave the fourth problem in the box to a number of adults (outside the field of mathematics). None of them were able to solve it.

To the girls, Great job and congratulations!

Calendar

Session 3: (all dates in 2008)

September	11	Start of third session!
	18	
	25	Sarit Smolikov, Harvard Medical School
October	2	Leia Stirling, Boston Children's Hospital
	9	Yom Kippur - No meet
	16	Jane Kostick, Carpenter
	23	
	30	
November	6	Catherine Havasi, Computer Science, Brandeis[1]
	13	
	20	
	27	Thanksgiving - No meet
December	4	Amanda Cather, Waltham Community Organic Farms
	11	

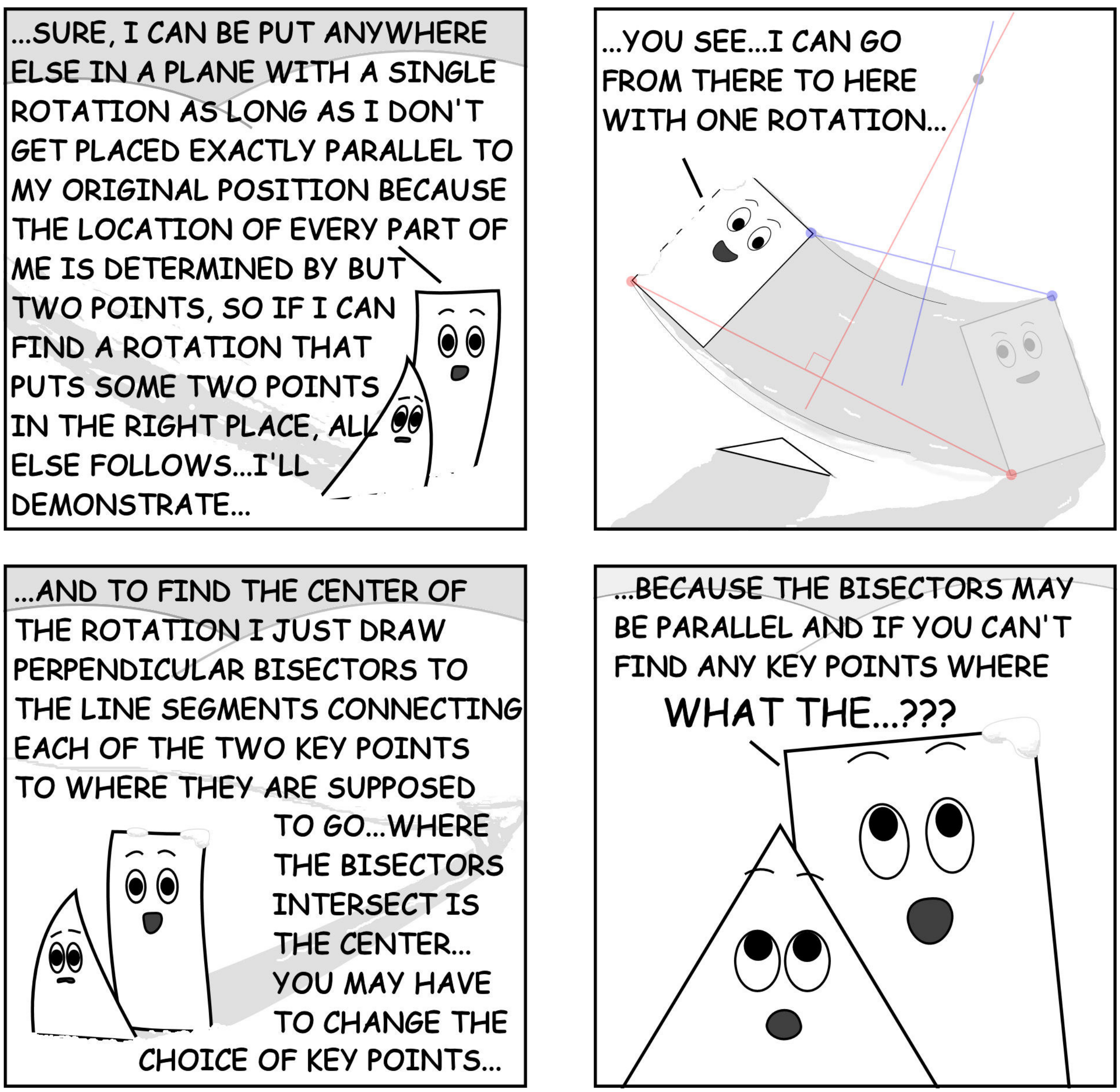
[1]Catherine's visit has been rescheduled to come one week earlier.

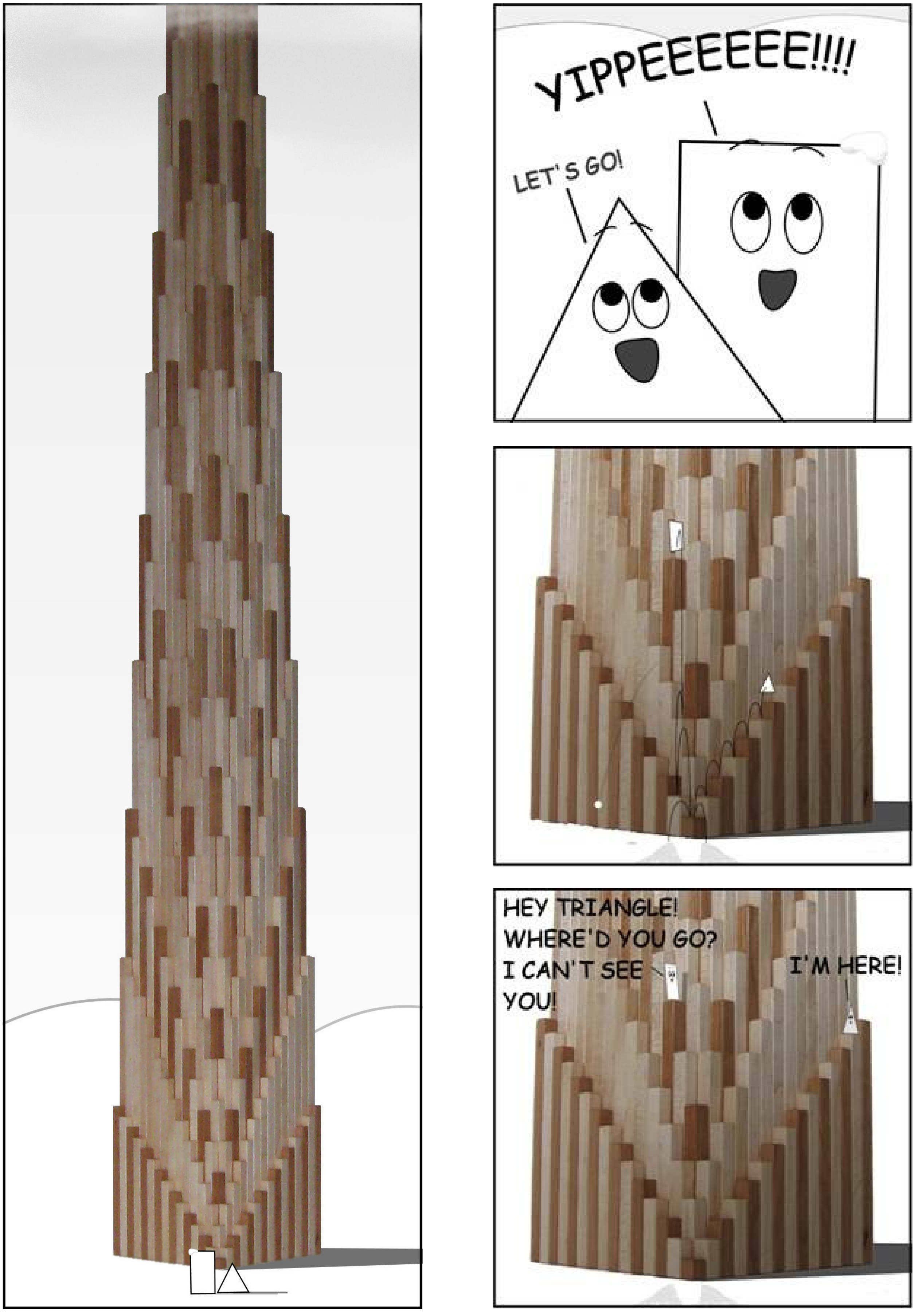
Session 4: (all dates in 2009)

January	29	Start of third session!
February	5	Sara Seager, Earth and Planetary Science, MIT
	12	
	19	Winter break - No meet
	26	Tanja Bosak, Earth and Planetery Sciences, MIT
March	5	Leia Stirling, Boston Children's Hospital
	12	
	19	Taylor Walker, DiMella Shaffer Architecture
	26	Spring recess - No meet
April	2	
	9	
	16	Eleanor Duckworth, Harvard Graduate School of Education
	23	Spring break - No meet
	30	Gigliola Staffilani, Mathematics, MIT
May	7	

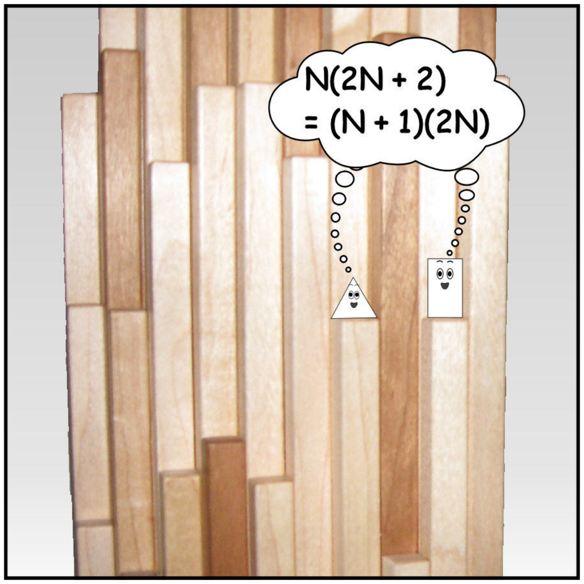
It Figures!

by CKFan




YIPPEEEEEE!!!!
LET'S GO!
HEY TRIANGLE! WHERE'D YOU GO? I CAN'T SEE YOU!
I'M HERE!

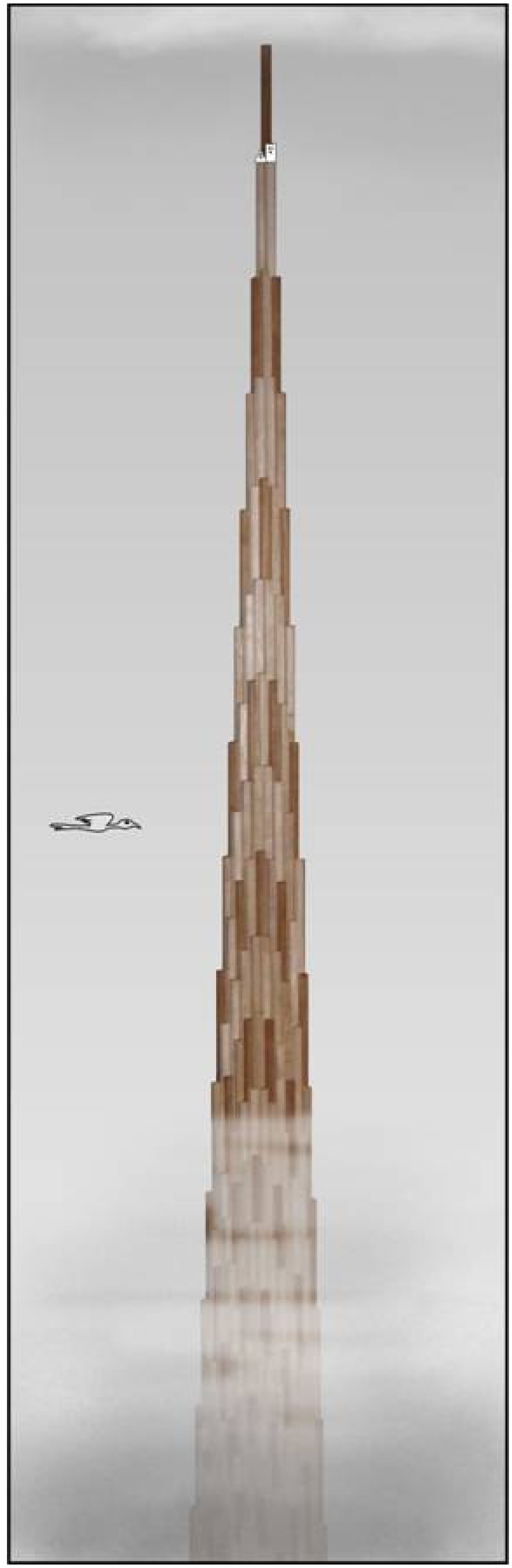

WEEEEE!



N(2N + 2)
= (N + 1)(2N)

YOU COMING,
RECTANGLE?

ON MY
WAY!



Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 3 ways: **membership**, **subscription** and **premium subscription.** **Membership** is granted per session and includes access to the club and extends the member's premium subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session, you will get a subscription to the Bulletin, but the premium subscription will start when total payments reach the premium subscription rate. **Subscriptions** are one-year subscriptions to the Girls' Angle Bulletin. **Premium subscriptions** are subscriptions to the Girls' Angle Bulletin that allow the subscriber to ask and receive answers to math questions through email. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) _____________________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________
(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ \$216 for a 12 session membership
  - ☐ \$100 for a one year premium subscription
  - ☐ \$20 for a one year subscription
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per session basis at \$20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ______________________________________Date: ________________

Print name of applicant/parent: ______________________________________

Print name(s) of child(ren) in program: ________________________________

Girls' Angle

A Math Club for Girls