Girls' Angle Bulletin

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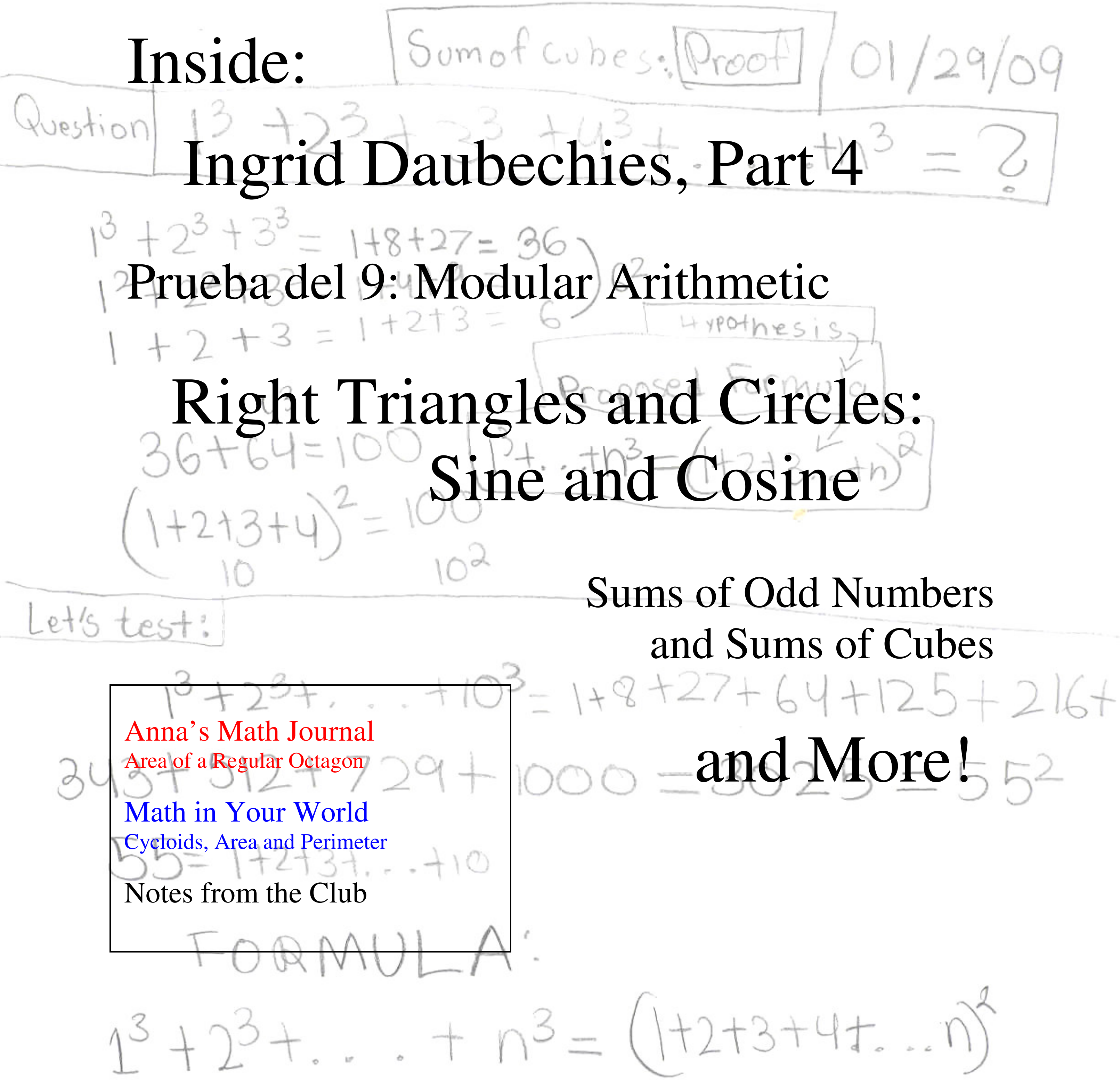
To Foster and Nurture Girls' Interest in Mathematics



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Sums of Odd Numbers and Sums of Cubes

and More!

From the Director

The fourth session is off to a great start.

We have a record number of girls and the group of mentors continues to grow and become stronger.

We filmed a Girls' Angle meet for the first time too. Vanessa Gould, the producer and director of the one-of-a-kind origami documentary *Between the Folds*, came to Boston and filmed Professor Seager's presentation on exoplanets. You can read more about Professor Seager's visit in the Notes from the Club.

To our members: The current installment of the interview with Professor Daubechies contains some very useful advice! If you haven't been looking at these interviews, please start with this one! Also, the most underused benefit that you receive as a member of Girls' Angle is the ability to communicate with mentors through email. All the mentors welcome email from members throughout the session. I encourage all of you to take advantage of the opportunity!

All my best,
Ken Fan
Founder and Director

Girls' Angle Donors

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Girls' Angle Bulletin
The official magazine of Girls' Angle: A Math Club for girls

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: **Henriette** observed a pattern and was able to deduce a formula for the sum of the first n positive cubes.

Ingrid Daubechies, Part 4

This is part 4 of a multi-part interview with Princeton mathematics professor Ingrid Daubechies.

Ken: …This is a problem we're having at the club: trying to convince the girls that mathematicians make a lot of mistakes. Actually, maybe you could address this?

Prof. Daubechies: Oh…everything I start, when I've finished, I joke that I've done it ten times, because I started and I find that I made a mistake, so I start over again, and I find another mistake, so I have start over again, and over again! That's why many of my students, when they write a paper, they write it on the computer, I can't do that. Because even in the writing, I find, it's not right the way I say that…it's not exactly the way I want it and if I do that on a computer, too much of it survives. So I do it in long hand and I erase and so on. I know you can do that on a computer, but it is not the same for me.

But, oh yeah, I make mistakes all the time.

And I have hunches that are wrong. Some hunches are right. But you know, the funny thing is, for one paper that I did, many, many, many years ago, I had kept a notebook with every approach that I did…every attempt…and many of them led nowhere. And, in the end, I solved that problem and it became a paper, and I had the impression that the insight had just come in a kind of flash. I really believe that for many things...and that then, everything had fallen into place. And then, I came upon that notebook some time later, and I realized that it wasn't like that at all. I could look in the notebook and see in all those places where it hadn't worked that there were already some of the germs of the things that did work eventually. So it wasn't true at all that I just found it in a flash, in hindsight. I mean, if I hadn't made all those mistakes before, I wouldn't have had all the building blocks to fit the thing together. So all those early things had been necessary, I mean, had been part of it. I was really surprised, because I had this very different perception of how it happened from what this notebook showed me.

Ken: One thing we're going to try with the Bulletin…there's a woman who just graduated from Smith College [with a post-baccalaureate degree in math], her name is Anna Boatwright. She's going to solve a problem keeping an honest record of her attempts to solve it. Then she's going to write an account of her attempts to solve the problem, errors and all. I think it is very brave of her to do this because I think most mathematicians try to sweep all the errors under the rug before they publish.

Prof. Daubechies: Well, because that's the way you have to publish because publication is expensive. So they try to do it as efficiently as possible. And also because they develop this way of writing with lemma and theorem which made it easier to check whether something is correct or not. In fact what we do when we write a paper is to write it in such a way that somebody who, if all the steps are given, then what they do is give a procedure for checking that the whole thing is correct. It might be ten lemmas and you say, "why would they care whether this obscure fact is true?" Well, because it is a building block later. And of course, they didn't find that lemma first, but they give you a way in which you can see that the whole thing is correct. But that's not the way in which it was found.

Ken: In your opinion, should the literature be a repository of truth or a place to explain?

Prof. Daubechies: I think you need both. That's one thing…I hope you teach the girls this…students who come out of high school and who are good at math…I don't know if they learn this from a teacher or if it is just the culture, but they believe that you have to figure it out in your head. I mean, you give them a problem, and they *stare*. My son would do this, and he would say, "I don't see it". So I would say, "I can't help you if all you tell me is 'I don't see it'!" They don't start writing. I mean, when I start a problem, I start immediately writing. I start writing what I know, I start writing what I want to get. I write how can I get from one to the other. I start from the beginning if I know where to start or from the end if I know where I want to go. And I try to connect…I try to knit it all together. I try to see many ways and hope to build a bridge. Of course, there are people who can do all that in their head, but I think at some point, unless they are Terry Tao, they are going to hit a place where they can't do it all in their heads anymore. And if they are Terry Tao, they don't need me to teach them anyway! [laughter] And when they hit that place, they have to have in place already the tools to handle a situation like that. And the tool to handle a situation like that is to start writing. Putting down the different pieces so that you can look at them and see how it fits together. I teach a freshman multivariable calculus class which we integrate with physics. When I do that class I tell them that we are going to go way faster than what they are used to, that they will have to study material…usually they don't study either…they're used to going so slow that the absorb it by osmosis, and we can't do that anymore. And that they always have to start writing. They have to write what they know, what they want to get, what they know about the kind of problem, and then start seeing whether they can put the pieces together. If you do that, if you don't believe that writing a problem is going here to…and then writing linearly to there without a single mistake and this is the answer and I think it is easier also if you see it as a whole process of having here things, and here things, and here's some knowledge, and now we are kind of embroidering around it until the things start touching, then it might be easier to accept that there are going to be in the end lots of things on the page that have nothing to do with the solution. But that's ok, you just select out the part that does.

When I teach, I also have a course that is a first-proof course based in applications. I teach styles of proof to the students. I do it in two-columns. I say, how would we build this proof? I say, we want to get here and so on…and after we have the proof, we then write the proof on the right hand side which is a very different thing.

Ken: By the way, I'm just curious, you've used a knitting metaphor…do you knit?

Prof. Daubechies: I used to knit. I don't anymore. I used to crochet even more. I would love to if I have time, I would love to have a loom for weaving. But I make ceramics.

Ken: Do you make your ceramics with any connection to mathematics?

Prof. Daubechies: No…I just like making things by hand. I've been thinking maybe after I retire I'd love to make some art works in which you illustrate some mathematics.

Ken: So, I'm going to ask you a completely different question. I've been wanting interviewees to address this but many are understandably reluctant to. Do you think there is gender bias in the field of mathematics today?

To be continued…

Prueba del 9: Modular Arithmetic

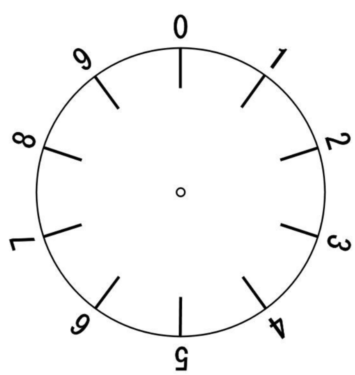
By Hana Kitasei

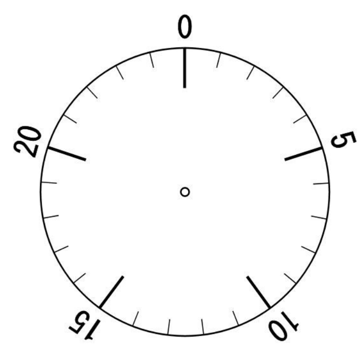
In the fall, **littleMeme** introduced a trick (see *Prueba del 9: The Trick* in Vol. 2, No. 1 of this Bulletin). In the last issue, we explored an aspect of it, the reduction procedure. Let M be an integer and let m be the result of applying the reduction procedure to M. What we saw was that M and m both leave the same remainder upon division by 9. To see why this worked, we learned more about remainders. In this installment, we'll be learning even more about them.

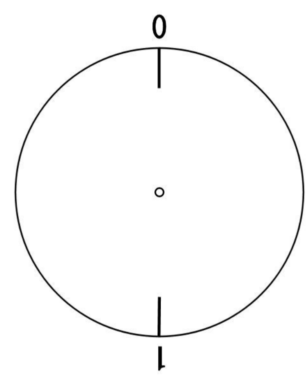
We'll also introduce a cool topic called modular arithmetic. Eventually, it will be clear how modular arithmetic connects to the trick, but we'll leave this for a future issue.

Understanding remainders is so important, it's worth revisiting. In the last issue we visualized filling chocolate boxes with chocolates to understand remainders. Here's a different way of thinking about it: Suppose you want to convert 130 minutes into hours. You know that one hour is 60 minutes. To figure out how many hours 130 minutes is, you could see that $130 = 60 \times 2 + 10$, concluding that 130 minutes is 2 hours with 10 minutes left over. The minutes "left over" are the remainder that results when you divide 130 by 60.

We use clocks that have 60 minutes to an hour. But we can imagine a society that divides the hour up into different units of time. Their "minute" will not be the same amount of time as our minute. Maybe one society has a 10 minute clock, or a twenty-five minute one or even one with just two minutes!



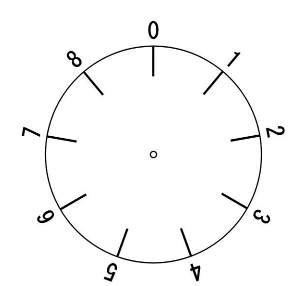




What is the remainder when you divide 51 by 2? by 10? by 25? If you find it helpful, use the clocks above.

Let's return to our example of dividing by 9. Suppose we had a clock with nine "minutes" on it.

In other words, if the minute hand starts from zero, try counting off different numbers around the clock. Where do you end up after each number of minutes? You might notice that certain "families" of numbers will end up on the same place on the clock. For example, multiples of nine will all end up at 0, because they are all divisible by 9. On the other hand, what about the numbers 10, 19 and 28? Where do they end up?



Have you noticed that they all end up at one? What do 10, 19 and 28 all have in common? For one thing, they all have a remainder of one when they are divided by nine.

Let's categorize numbers according to where they would end on the 9 minute clock. There are 9 different choices, and from what we saw above, numbers of the same group will have the same remainder when divided by 9.

Let's introduce some notation to help us communicate these concepts:

Suppose m and n are integers. If $m - n$ is divisible by 9, we say that "m is congruent to n modulo 9" and write "$m \equiv n \bmod 9$".

What numbers are congruent modulo 9? Write True or False next to each congruence relation below.

___ $6 \equiv 7 \bmod 9$
___ $6 \equiv 18 \bmod 9$
___ $3 \equiv 12 \bmod 9$
___ $30 \equiv 12 \bmod 9$

More generally, if d is any positive integer, we say that m is congruent to n modulo d and write "$m \equiv n \bmod d$" if and only if d divides evenly into $m - n$.

Now, let's think about addition. Suppose we have two numbers, say, 12 and 13, and we are asked to find what remainder is produced when their sum is divided by 9.

One way to do that is to add them and go around the 9 minute clock and see where you end up.

But there's another way! It turns out, you can use the clock to find the remainders of 12 and 13 separately, and then add the remainders together going around the clock. That is, if you go around the minutes of the 9 minute clock 12 times, you end up at 3 and if you do the same for 13, you end up at 4. Now, if you start at 3 and go around for 4 further minutes, you end up at 7. That is the same place you'll end up if you just start at 0 and go around 12 + 13 = 25 times. Try it!

Do you see why these two ways produce the same result? Reconsider the picture of chocolate boxes. Suppose we were filling boxes of size 9 with chocolates. With the 12 chocolates, there would be an unfilled box with 3 left over chocolates. With 13 chocolates, we would have a box with just 4 left over chocolates. Neither the box containing the 3 nor the box containing the 4 would be full. However, to get as many *full* boxes as possible, we can combine the remaining chocolates into one box. In this case, we'd end up with a single unfilled box with 7 chocolates, and this also represents the remainder when 12 + 13 = 25 is divided by 9 because packing 25 chocolates into 9-chocolate chocolate boxes would result in the same arrangement.

Compute the remainder of 4 + 7 when you divide by 9 both ways as described above. That is, first compute the sum, then go around the 9 minute clock that many times and see where you end up. Next, compute the remainders of 4 and 7 (divided by 9) separately. Add the remainders and go around the 9 minute clock to see that you end up at the same place. Try this again with the sum 13 + 16.

We can make a table for this kind of "**clock arithmetic**" (see the next page). Each entry should equal the remainder of the sum of the column and row number divided by nine. Try to complete it yourself.

What patterns do you notice? Why do you think these patterns exist? Try making a table for addition modulo a number other than 9 and describe what you see.

Addition Modulo 9

m \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0		1					6								
1					5		7							5	
2		3													
3				6											
4				7											
5															
6															
7						3									
8					3										
9							6								
10					5									5	
11															
12															
13															0
14															

In our discussion so far, we have been paying special attention to the numbers 0 through 8 because these represent all the possible remainders when you divide by 9. But, if we really grasp the idea of modular 9 arithmetic, these 9 numbers aren't really special. Instead, we should think of *all numbers that leave the same remainder upon division by 9 as equivalent*. So, when working modulo 9, try to think of 0 not as the number 0, but instead, as the set of all multiples of 9. Similarly, think of 1 not as the number 1, but instead, as the set of all numbers that leave a remainder of 1 after dividing by 9. What we saw above is a key fact: *addition respects this equivalence*. All the numbers that end up at the same place on the clock are thought of as *equivalent*. The numbers 10 and 1 are not actually equal, of course, but when we work modulo 9, they are considered to be equal because, when divided by 9, their *remainders* are equal.

With this in mind, here are the 9 equivalence classes[1] modulo 9:

$S_0 = \{. . ., -18, -9, 0, 9, 18, 27, 36, . . . \}$
$S_1 = \{. . ., -17, -8, 1, 10, 19, 28, 37, . . . \}$
$S_2 = \{. . ., -16, -7, 2, 11, 20, 29, 38, . . . \}$
$S_3 = \{. . ., -15, -6, 3, 12, 21, 30, 39, . . . \}$
$S_4 = \{. . ., -14, -5, 4, 13, 22, 31, 40, . . . \}$
$S_5 = \{. . ., -13, -4, 5, 14, 23, 32, 41, . . . \}$
$S_6 = \{. . ., -12, -3, 6, 15, 24, 33, 42, . . . \}$
$S_7 = \{. . ., -11, -2, 7, 16, 25, 34, 43, . . . \}$
$S_8 = \{. . ., -10, -1, 8, 17, 26, 35, 44, . . . \}$

Now pick two of these sets. *No matter which two numbers you pick from the two sets, if you add them you will always end up inside the* ***same*** *set*. A number from S_2 and a number from S_3 will always sum to a number in S_5. (Try it!) Can you *prove* that this is the case?

What happens with multiplication? We'll see next time!

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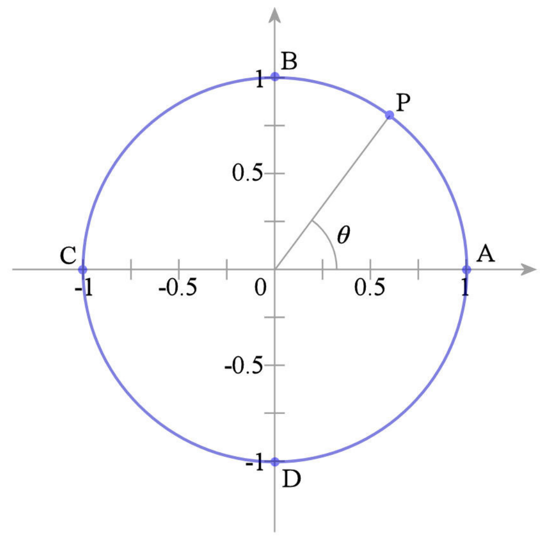
[1] For more on equivalence, see *Equivalence Relations* in Vol. 1, No. 3 of this Bulletin.

Right Triangles and Circles: Sine and Cosine

by Katy Bold and Ken Fan

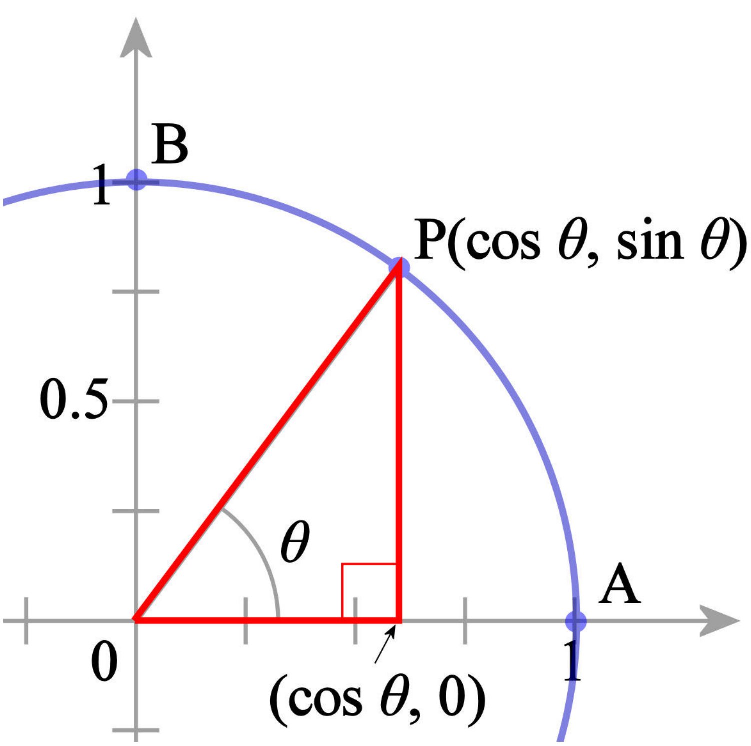


The circle is a fundamental mathematical object, and you probably already know quite a bit about circles. Let's look at a circle in the coordinate plane with radius 1 centered at the origin. At two points, the circle intersects the horizontal axis (A and C), and at two other points, the circle intersects the vertical axis (B and D).

What are the coordinates of points A, B, C, and D?

A: (1, 0)   B: (,   )   C: (,   )   D: (0, -1)

But what about the coordinates of point P? It looks like the horizontal coordinate is something between 0.5 and 0.75 and the vertical coordinate is between 0.75 and 1. You could get out a ruler and try to measure it accurately, but probably your measurement would be a little bit off no matter how careful you are. Well, P has *some* coordinates, and we know what we mean when we talk about them. We know that the coordinates depend on the location of P. Let's specify the location of P by giving the angle θ (the Greek letter "theta") that OP makes with OA. The vertical coordinate of P is some number that depends on θ. When θ is zero degrees, the vertical coordinate is 0. When θ is 90 degrees, the vertical coordinate is 1. In other words, the vertical coordinate of P is a *function* of θ. This function comes in so handy that mathematicians long ago gave it a special name: the **sine** function. Similarly, the horizontal coordinate of P is a function of θ, and mathematicians call it the **cosine** function. As a matter of notation, mathematicians write "sin(θ)" for the sine of θ and "cos(θ)" for the cosine of θ.



Well, giving the horizontal and vertical coordinates of point P in terms of the angle θ special names does not help us understand or find those coordinates. It just gives us some names to call those coordinates. *But circles are beautiful objects with many properties. So, we can expect that the sine and cosine functions will have many beautiful properties.*

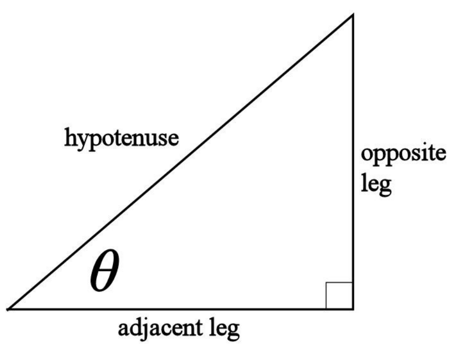
For example, look at the right triangle whose vertices are the origin, the point P, and the point on the horizontal axis with horizontal coordinate cos(θ). If we apply the Pythagorean theorem to this right triangle (why is it a right triangle?), what do we get? The legs have lengths cos(θ) and sin(θ) and the hypotenuse is the radius of the unit circle, which is 1. So, we get $\cos^2(\theta) + \sin^2(\theta) = 1$.

Also, if you go round and round the circle, the point P will pass through the same points on the circle over and over, and the coordinates of any specific point on the circle don't change as the point P is moving round and round. Because there are 360 degrees in a full circle, it must be the case that $\cos(\theta + 360°) = \cos(\theta)$ and $\sin(\theta + 360°) = \sin(\theta)$. These formulas just express the fact that if you go 360 degrees around a circle, you end up at the same place you started.

By the way, I hope you weren't bothered by the fact that I zoomed in to a part of the first figure to create the second figure. Were you? I mean, technically, the lengths in the second figure are actually longer than the corresponding lengths in the first figure…all by the same amount, of course! Hmmm. Well, just so long as you realize that both circles were supposed to represent circles of radius 1, then all's ok.

Actually, I think the radius of the circle in the first figure is 1 inch, or if you like the metric system, it's about 2.5 centimeters. So in the first figure, the horizontal coordinate of P is $\cos(\theta)$ inches. Well, if you like the metric system, it's about $2.5\cos(\theta)$ centimeters. In the second figure, I haven't a clue how many inches the radius of the circle is, but it is definitely more than an inch. Let's say that the radius of the circle in the second figure is X inches. It looks to me like X is maybe a little more than 2. Whatever. Well, whatever X is, then the horizontal coordinate of P in the second figure would have to be $X \cos(\theta)$ inches, right?

Aha! So the point I'm making is that no matter what units of measure you use, the *ratio* of the horizontal coordinate of P to the radius of the circle (which is the same as the length of the hypotenuse of the red right triangle) will always be the same and equal to $\cos(\theta)$.



In other words, if you look at the right triangle to the left, the cosine of θ could be computed by dividing the length of the adjacent leg by the hypotenuse. To see this, just scale the image so that the hypotenuse has length 1, or imagine that you are using a unit of measure that makes the hypotenuse of this triangle have length 1 unit. This is how Anna relates cosine to her computations in her Math Journal in this issue (see page 10). By the way, the terms "adjacent" and "opposite" are interpreted with respect to the location of the angle θ in the right triangle. If θ were placed in the upper right angle of that triangle, then the legs would switch labels.

Similarly, the sine of θ is equal to the length of the opposite leg divided by the length of the hypotenuse.

Can you figure out the values of $\sin(45°)$, $\cos(45°)$, $\sin(60°)$ and $\cos(60°)$?

Hint: use right triangles and the Pythagorean theorem. For a big hint on how to compute $\cos(45°)$, see Anna's Math Journal. To compute $\sin(60°)$ and $\cos(60°)$, notice that the relevant right triangle is half of an equilateral triangle.

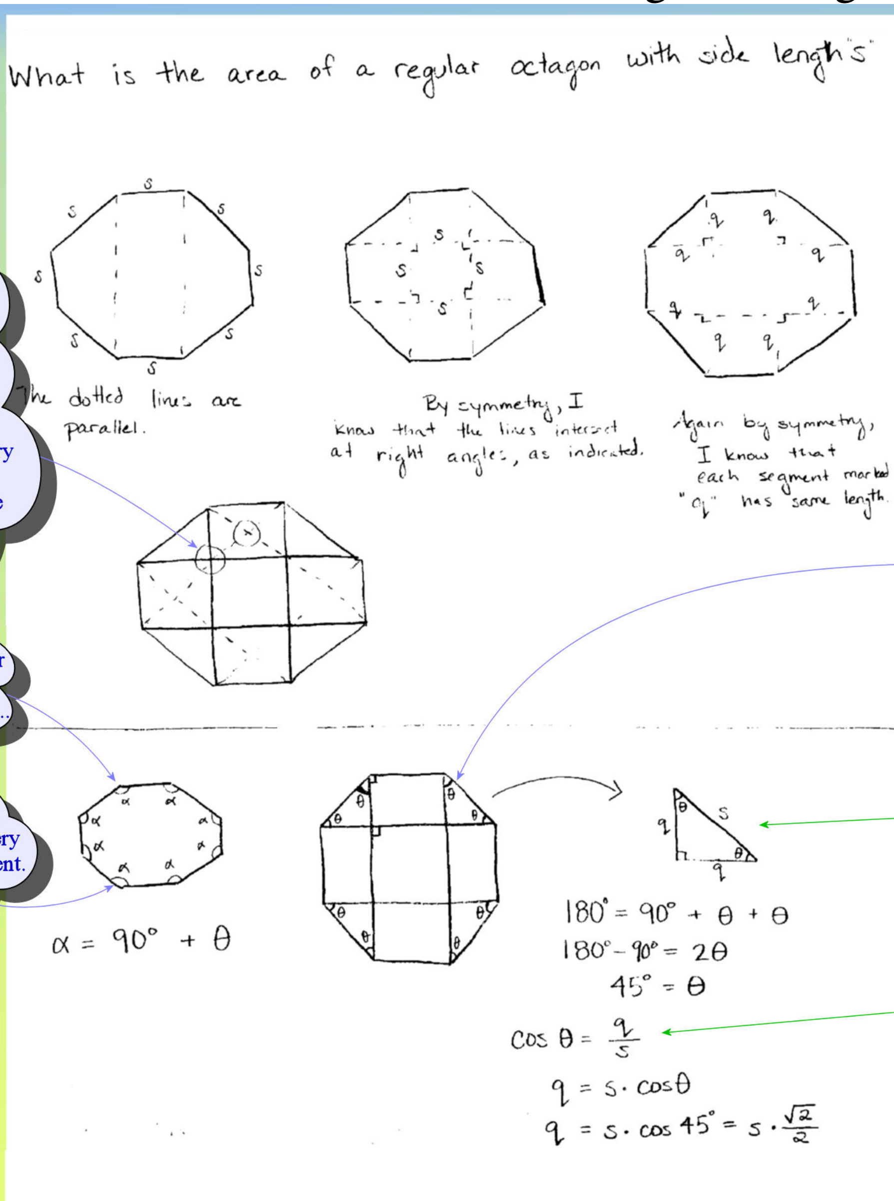
Feel free to send in your answers to girlsangle@gmail.com!

Anna's Math Journal

By Anna Boatwright

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Here, Anna finds **a formula for the area of a regular octagon.**



I have to be careful about conclusions based on how this drawing looks because it may be inaccurate. Some lines that intersect may not actually intersect.
I could get out my ruler and compass and try to draw a very accurate regular octagon, but it is even more accurate to see what I can figure out using logical reasoning.

I've drawn in the interior lines and I see that every angle α gets divided up into a 90-degree angle and another smaller, "left-over" angle. I will label this new angle θ.

Really, what I don't know much about yet are the interior angles of the regular octagon. I think I'll look more into that...

Since this is a regular octagon, every side has length s and every interior angle must be equivalent. I will label this angle α.

Here I looked at one of the corner triangles. I know that the interior angles of any triangle must add up to 180-degrees. And so I figured out that θ is 45-degrees.

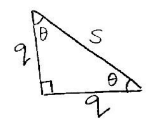
I used trigonometry to write q in terms of s.

See page 8 for an explanation of sine and cosine. Do you see how this can be done using the Pythagorean theorem instead? If not, just turn the page...

Key:

- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

Actually, I don't have to use trigonometry to find q in terms of the side length s, and, in fact, the way I used trigonometry to find q relies on the fact that I have memorized the cosine of 45 degrees. Just in case you don't know about trigonometry, I decided to give an alternative derivation of q in terms of s using the Pythaogrean theorem.



$$q^2 + q^2 = s^2$$
$$2q^2 = s^2$$
$$q = \pm\sqrt{\frac{s^2}{2}} \rightarrow q = \sqrt{\frac{s^2}{2}}$$
$$= \frac{s}{\sqrt{2}}$$
$$= s \cdot \frac{\sqrt{2}}{2}$$

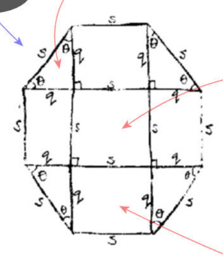
Does $\frac{s}{\sqrt{2}} \stackrel{?}{=} s \cdot \frac{\sqrt{2}}{2}$

Well, $\frac{s}{\sqrt{2}} = \frac{s}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = s \cdot \frac{\sqrt{2}}{2}$

So yes.

Some people prefer not to have radicals in the denominators. It's a matter of personal taste, but I decided to show both forms and show why they are equal.

Let me redraw the regular octagon carefully and label all the sides and angles I know.

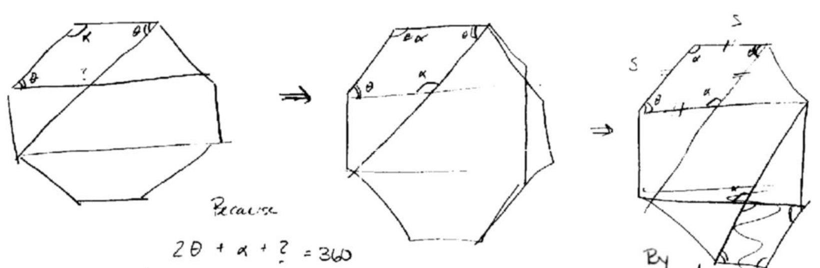


Now I think I'm ready to write the total area of the regular octagon.

$$\text{Total Area} = (\square) + (4 \times \triangle) + 4(\square)$$
$$= s^2 + \left(4 \times \tfrac{1}{2} \times q \times q\right) + (4 \times s \times q)$$
$$= s^2 + \left(2 \times \frac{s\sqrt{2}}{2} \times \frac{s\sqrt{2}}{2}\right) + \left(4 \times s \times \frac{s\sqrt{2}}{2}\right)$$
$$= s^2 + s^2 + s^2 2\sqrt{2}$$
$$= s^2(2 + 2\sqrt{2})$$

Just out of curiosity, I wonder if I can find the area using some of those diagonal lines I drew in my first figure.

Another Track,



Because

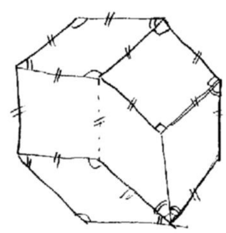
$2\theta + \alpha + ? = 360$

$\Rightarrow$ ~~2(45)+135~~

$? = 360 - 2(45) - 135 = 135$

By symmetry

Furthermore | side = // side



This partition is kind of neat. Can you see that it is mirror symmetric?

Can you help me finish the computation? What are the areas of the squares and parallelograms here? Can you think of a way to partition an octagon into simpler shapes that all look the same?

$$A_{Total} = 2(\square) + 4(\lozenge)$$

ABB 2.23.09

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Sums of Odd Numbers and Sums of Cubes

Sums like the following have begun appearing at Girls' Angle more and more often:

$$1 + 2 + 3 + \ldots + n$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2$$

$$1^3 + 2^3 + 3^3 + \ldots + n^3$$

Thinking about them has led to a lot of mathematics.

In volume 2, number 1 of this Bulletin, **Mouse** looked at a modification of the first formula by summing over just odd numbers or just even numbers instead of summing over positive integers. She found that if you add up the positive odd integers from 1 to the odd number N, the result is $\frac{(N+1)^2}{4}$. On the cover of this issue, **Henriette** gives a formula for the third sum above. She noticed a pattern of perfect squares emerging when she computed the sum for a few values of n and saw that $1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$. In words, the sum of the first n positive cubes is equal to the square of the sum of the first n positive integers.

We've seen before that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$, so we can substitute this result into **Henriette**'s formula to obtain the more compact form:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Both **Mouse**'s and **Henriette**'s formulas simplify the computation of the sum.

What's more, both formulas show that the sum turns out to be a perfect square! That's pretty uncanny, isn't it?

The sum of the first n positive integers isn't always a perfect square. In fact, it hardly ever is a perfect square. Do you feel compelled to ask, for which n, then, is the sum of the first n positive integers a perfect square???

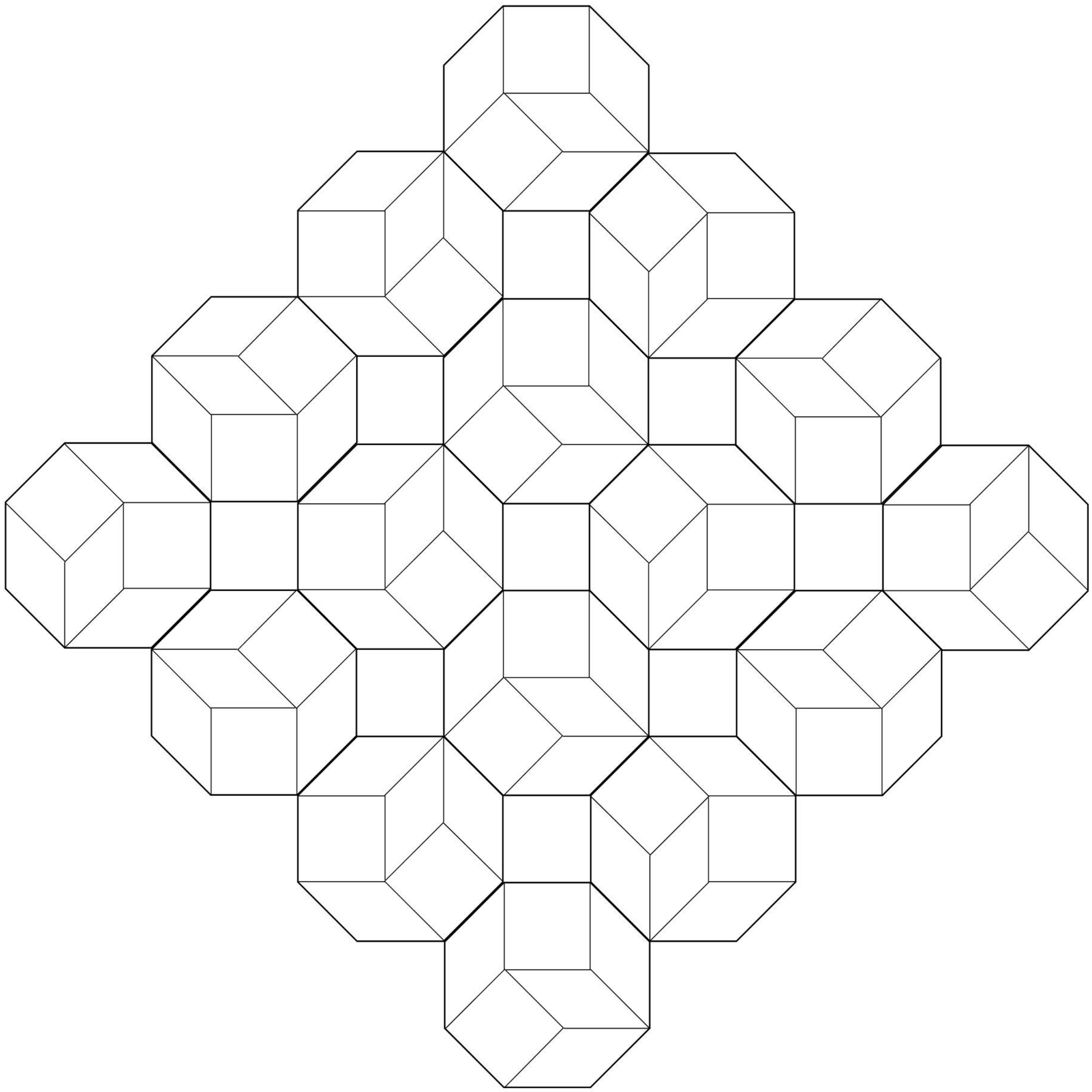
Also, what about the sum of the first n perfect squares? Is there a formula for that sum that is similar to the formulas for the sum of the first n positive odd numbers or the sum of the first n positive cubes? And what about sums of higher powers, like $1^4 + 2^4 + 3^4 + \ldots + n^4$?

Or, what about the sum of the first n positive *odd* perfect cubes?

Or, how about the sum of the first n even perfect squares?

So many questions…send us any answers you find! girlsangle@gmail.com...

An Octagonal Tiling



This tiling pattern is made out of copies of the partition of the regular octagon that Anna Boatwright found in her Math Journal.

Cycloids, Area, and Perimeter

By Katy Bold

Math in Your World

Logo Design by Hana Kitasei

In the last issue of this Bulletin, we saw the cycloid in a Broadway musical. The cycloid has been making appearances for centuries – it is perhaps most famous for its appearance in 1696.

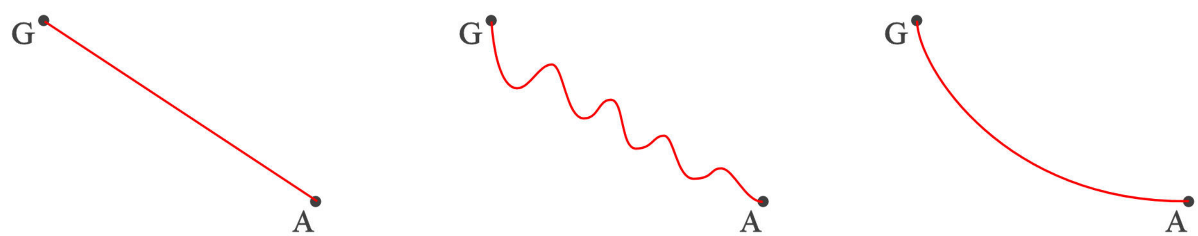
adapted from Wikipedia

Cycloids in Architecture: The Kimball Art Museum.

In 1696, Johann Bernoulli challenged mathematicians to solve the **brachistochrone** problem:

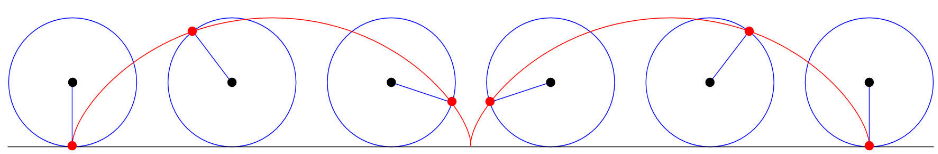
Fix two points in a vertical plane – call them G and A. Place a bead at G, and connect the points G and A with a wire. The wire can take any shape (a straight line, a curve, etc.). What shape should the wire be so that the bead reaches A in the shortest amount of time possible? (The only force to consider is gravity; assume there is no friction. When you let go of the bead, don't push it; just let it be completely still when it starts.)

Of the three curves shown below, which do you think is the solution to the problem? If the bead travels from G to A – which path takes the least amount of time? Which path do you think would take the longest? Why are some paths faster than others?



The answer to this bet is the cycloid, which we saw in the last issue of this Bulletin. Recall that a cycloid is the curve traced out by a blob of paint on the rim of a rolling bicycle wheel.

The brachistochrone problem can be solved using techniques from a branch of Applied Mathematics called the Calculus of Variations. Mathematicians and scientists use calculus of variations techniques to solve a wide range of problems that typically have two properties:



1. Some quantity needs to be **maximized** or **minimized**.
2. There are some **constraints** on the problem.

In the brachistochrone problem, the *time it takes the bead to reach point A* needs to be minimized. The constraint is that *the curve has to go through the points G and A*.

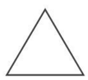
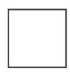
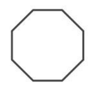
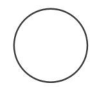
Try it out: Area and Perimeter

Here we'll work through part of a simpler calculus of variations problem:

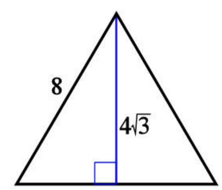
> Of all possible shapes that have perimeter 24, which shape has the largest area?

To solve this problem, we want to *maximize* the *area* of the object, subject to the *constraint* that the *perimeter is 24*.

What shape do you think might work? When a mathematician approaches a new problem, she makes examples to try to understand the problem better. We can gain a lot of insight by just looking at a few examples, so let's look at a few shapes: the equilateral triangle, square, regular octagon, and circle. Can you complete this table? Remember that all shapes have perimeter 24.

				
Side Length	8			-
Area	$16\sqrt{3}$			$\frac{144}{\pi}$

An equilateral triangle has three sides of equal length. If the perimeter of an equilateral triangle is 24, each side has length 8. By the Pythagorean Theorem, the height of the triangle[2] is $4\sqrt{3}$. Since the area of a triangle is half of the product of the base and height, the area of our triangle is $16\sqrt{3}$.



Let's do the same analysis for a square. If a square has perimeter 24, what is the length of each side of the square? Let's call this length s. The area of a square is given by s^2. What is the area of a square with perimeter 24? Fill in your answers in the table above.

Can you do the same analysis for a regular octagon? Check out Anna's Math Journal on page 10 for help with this one. A regular octagon has 8 sides all of the same length. If the octagon has perimeter 24, how long is each side? Let's call this length s. Anna has already helped us out with finding the area of a regular octagon – she shows us that the area is $(2+2\sqrt{2})s^2$. What is the area of a regular octagon with perimeter 24? Fill in your answer in the table above.

Now for the circle, the formula for the perimeter of a circle is $2\pi r$, where r is the radius, so, $2\pi r = 24$. From this, we can solve for r and use the formula πr^2 to get the area. See the table.

Now that you have looked at a few examples – what shape do you think might have the largest area for a fixed perimeter? Why?

[2] The height can also be expressed as the side length times the cosine of 60°. See page 8.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete and, to nonmembers, they may not even be coherent!

Session 4 – Meet 1 – January 29, 2009

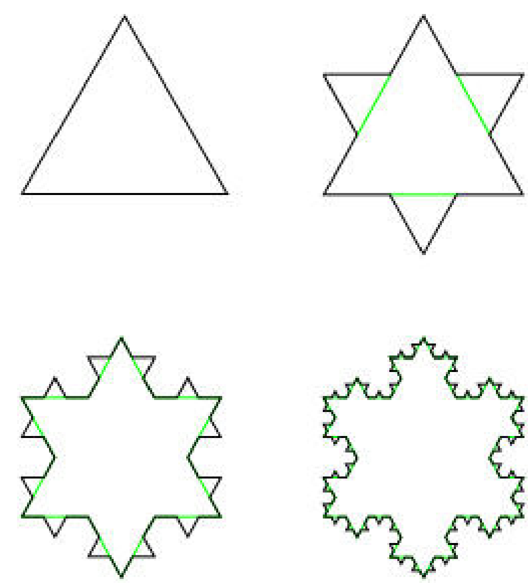
Mentors: Doris Dobi, Lauren McGough, Jennifer Melot, Mia Minnes, Maria Monks

Our ice breaker activity for the first meet of the fourth session was a new game we called "Math Pictionary". Members would draw a figure and another member would try to get her teammates to replicate (to scale) the figure using only verbal cues. The goal of this activity was to develop precision of speech and new vocabulary to describe geometric configurations.

During Math Pictionary, many interesting questions came up. For example, what does it mean to say that two circles are *tangent* to each other? Also, many interesting figures were drawn. For example, Mia illustrated the Koch snowflake.

The Koch snowflake is actually the limit curve that results if you continue the iteration illustrated at right forever.

How many vertices does the nth iteration have? How many edges does the nth iteration have? If the first iteration, which is just an equilateral triangle, has side length 1 unit, what is the length of an edge of the nth iteration? What is the perimeter of the nth iteration? What is its area? Can you figure out the area contained in the ultimate limiting curve? This last computation involves an infinite geometric series.



en.wikipedia.org/wiki/File:KochFlake.svg

The first four iterations of the Koch snowflake.

Session 4 – Meet 2 – February 5, 2009

Mentors: Lauren McGough, Jennifer Melot, Maria Monks, Kate Parrot

Special Visitor: Dr. Sara Seager, Earth and Planetary Sciences, MIT

The first half was spent in small groups working on a number of different math problems.

Dr. Seager talked about extrasolar planets. Thirty years ago, the only planets known to exist in the entire Universe were those orbiting the Sun. Today, dozens of planets are known to exist orbiting other stars. Dr. Seager began by discussing estimates for the number of planets in the Universe, starting with the question, "What is a planet?" Next, she had the girls make models of various planetary systems, including one orbiting a star 390 light years away which was discovered just days before the meet. The newly discovered exoplanet is called CoRoT-Exo-7b.

Thirty years ago, the astronomer Carl Sagan gave a talk to a classroom and predicted that when those students grew up to be his age, we would know of dozens of planetary systems. His prediction came true. As part of her presentation, Professor Seager made the following prediction about what we may know when our current members grow up to be adults:

> We will know the main components of the atmospheric composition of a number of exoplanets.

Think about this prediction. Thirty years ago, we didn't know if there were planets orbiting other stars. Now, we know there are, but we know little about *what those objects are made of*. Professor Seager's prediction anticipates progress on that question in the next few decades. How can we figure out what something is made of when that something is so far away it takes light hundreds of years to reach us?

Session 4 – Meet 3 – February 12, 2009

Mentors: Grace Lyo, Mia Minnes, Maria Monks

The third meet also began in small groups.

At one point, with some of the girls, Maria discussed the algebraic identity

$a^2 - b^2 = (a + b)(a - b)$

To illustrate its power, she showed how you can use it to compute $1{,}000{,}000^2 - 999{,}999^2$ without having to compute $999{,}999^2$. Can you see how to do that?

During the second half, we played the "Define this Game" game, which was first played at the second meet of Girls' Angle.

en.wikipedia.org/wiki/File:Snofru%27s-Red-Pyramid.jpg

Snofru's Red Pyramid in Egypt

In one round, **Rowena** had to define a "4-sided pyramid". She defined it as an object strongly associated with Egypt, having a square base and a pointy top. The girls on the other team cleverly stole the point by thinking of an object that's not a 4-sided pyramid yet satisfies all three of the properties **Rowena** described. Can you think of such an object?

Calendar

Session 4: (all dates in 2009)

January	29	Start of fourth session!
February	5	Sara Seager, Earth and Planetary Science, MIT
	12	
	19	Winter break - No meet
	26	Tanja Bosak, Earth and Planetery Sciences, MIT
March	5	Leia Stirling, Boston Children's Hospital
	12	
	19	Taylor Walker, DiMella Shaffer Architecture
	26	Spring recess - No meet
April	2	
	9	
	16	Eleanor Duckworth, Harvard Graduate School of Education
	23	Spring break - No meet
	30	Gigliola Staffilani, Mathematics, MIT
May	7	

Session 5: (all dates in 2009)

September	10	Start of fifth session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	No meet
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	
	10	

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 3 ways: **membership**, **subscription** and **premium subscription.** **Membership** is granted per session and includes access to the club and extends the member's premium subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session, you will get a subscription to the Bulletin, but the premium subscription will start when total payments reach the premium subscription rate. **Subscriptions** are one-year subscriptions to the Girls' Angle Bulletin. **Premium subscriptions** are subscriptions to the Girls' Angle Bulletin that allow the subscriber to ask and receive answers to math questions through email. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

☐ Enclosed is a check for (indicate one) (prorate as necessary)

- ☐ $216 for a 12 session membership
- ☐ $100 for a one year premium subscription
- ☐ $20 for a one year subscription
- ☐ I am making a tax free charitable donation.

☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

Girls' Angle

A Math Club for Girls