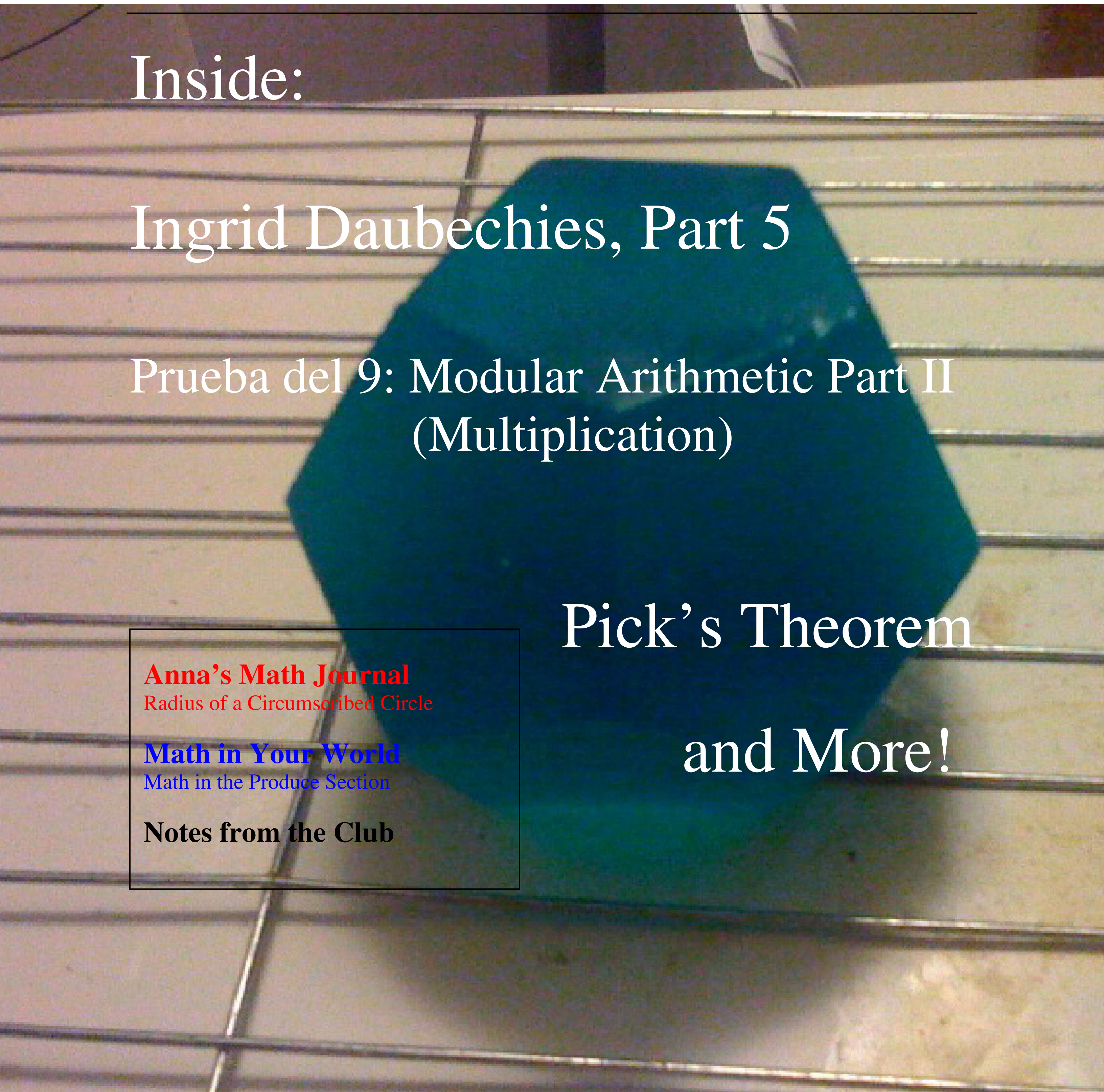
Girls' Angle Bulletin

April 2009 • Volume 2 • Number 4

To Foster and Nurture Girls' Interest in Mathematics



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From the Director

Girls' Angle has been given another generous donation, this time from Science House. This donation is specifically earmarked to jumpstart the video wing of Girls' Angle.

One goal of Girls' Angle is to produce quality math educational materials. As part of this goal, Girls' Angle will be producing educational videos that will appear on our website. Our members have already witnessed one aspect of this video development when Vanessa Gould filmed Sara Seager's visit back in February, and today, as Vanessa came again to film Gigliola Staffilani's visit.

The Science House donation will be used to fund two other video projects. One, conceived of and led by Girls' Angle director Eli Grigsby, will feature women in math explaining some piece of mathematics that excited them when they were the age of our club members. The other will be a series of short math educational clips. Expect to see some of these videos appearing on our website by year's end!

All my best,
Ken Fan
Founder and Director

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

Individuals

Charles Burlingham Jr.
Anda Degeratu
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Girls' Angle Bulletin

The official magazine of
Girls' Angle: A Math Club for girls

girlsangle@gmail.com

This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: **Hadassah** and **Grace**, together with their family, helped Girls' Angle produce its first geometric soap bar: a dodecahedron!

Ingrid Daubechies, Part 5

This is the final part of a multi-part interview with Princeton mathematics professor Ingrid Daubechies.

Ken: So, I'm going to ask you a completely different question. I've been wanting interviewees to address this but many are understandably reluctant to. Do you think there is gender bias in the field of mathematics today?

Prof. Daubechies: [pause] There certainly are still instances that I hear about. If you ask any young woman, she has stories. Among the graduate students I think they've encountered it more among their peers than the professors. Some of the professors are not necessarily very friendly towards graduate students…it's not that they are not friendly, it's just that they are oblivious in a kind of gender neutral way…but I think it might be resented more by women. But definitely there are people who think yes, who kind of assume that a woman will not be as good and who may even articulate that. Usually, they are more perceived as jerks, but it exists. And then, definitely, if you look at statistics, or, well…the statistics are complicated by many, many issues. Because women also unfortunately drop out more from the field at transition points than men and so we want to do something partly because the numbers are low, I think. So, I think…I haven't seen many cases of blatant discrimination, but I do think there still is some. I think it's at the level where it's just a consequence of the fact that there are much fewer women than men in the profession. I mean, people make implicit assumptions based on what they see.

Ken: There is an explanation put forth by some psychologists that the low representation of women in the upper levels of mathematics is not due to gender bias but is due instead to differences in preferences between what girls like and what boys like. What do you think of that argument? Do you think that it is valid?

Prof. Daubechies: I think that it does not explain why more women than men leave the field. I mean, these women already liked mathematics to begin with and then concluded it is harder for them to stay in and combine a family and career, to make their mark…whatever…so I mean, I think I have no a priori reason to assume that… I don't believe in it because I've seen plenty of women very interested in mathematics. And I've seen women talented in mathematics and who did leave the field usually because they don't like a job where the tenure clock is racing at the same time as raising a family and things like that. Another thing is that many mathematicians seem to believe that, well, there are two things. First of all, many mathematicians who are considered to have left the field, for instance by the AMS, are still doing mathematics in a broader sense…in the sense that they are applying very much their analytical skills and their precise reasoning skills in their new jobs and they find that mathematics trained them very well for that, and they enjoy it. So, we haven't really "lost them". The second thing is that so many people in academia seem to feel that it's almost a failure not to go to academia. For your girls, that's not an issue, they are still young, but I think that is absolutely so ridiculous. Even people who agree that it is ridiculous still have this implicit assumption. If they have a very good student who then decides to do something else, they say, "oh, such a pity, he could have been!" But what does it mean? It still means you have an implicit value system…I mean, [the thought that] "only the people that couldn't have been do something else"…which is ridiculous. Mathematics is a skill that is very fundamental that many people have and we need many people who have it in spades, and we need it everywhere, and we should be happy to spread it around.

But, we do lose women more than men. I think it is partly because, well, the track itself is kind of hard for women if they want to raise a family.

Ken: One of the goals of Girls' Angle is to become a community of support for all women engaged in the study, use or creation of mathematics. One of the issues I think about is this issue of raising a family. I've often wondered, you know, every person has two parents…why does it seem that the burden is on the mother to do the child rearing…why is it that when a male mathematician has a child there is no stigma associated with whether he is still going to be able to do the math?

Prof. Daubechies: Well, yeah, well, I don't know. Well, part of it is that when the baby is really little, the mother is nursing and that is something only the mother can do. But, yeah, I agree. So actually, when graduate students and postdocs ask me how do you combine a family and a career, I tell them that the first thing to do is to choose their husband well. It may seem a bit silly. But it is very important that you discuss ahead of time how you see things going in a family and in some detail so you don't have vague expectations. Well, I'm happily married now but earlier I had a relation with a man who was a very nice man but I think he expected something different from his fiancé than what he expected from his wife, which can be very frustrating for a young woman to meet a man who really appreciates you because you are intellectual, because you have all these interesting pursuits in life and so on, but then expects that after marriage that you will take care of the house and cooking and…which wouldn't leave you any time anymore to be the person that they liked before. I had implicitly assumed that since he liked these things, we would work together, that we both would have the time left to be the persons we had been. And it didn't work that way. But, we hadn't discussed it. And it took me a while to figure out that was what was wrong. So I tell them, that's important.

The support of a husband who views it in the same way is absolutely important. But after that, there's still the rest of the world and people may find it strange that this couple works in a different way from most couples they've seen. So, I think having a support group like that would be great.

Ken: Do you have any suggestions for how this can be implemented?

Prof. Daubechies: Ummm, I don't know…I mean I know that I've had support groups with women and we'd hang out, and we'd get together and chat…so, definitely have a corner, I don't know how big your facility is…but having a corner where people can just chat, or even a separate room, where they can chat and not be overheard by everybody else working on math would be good.

Ken: Someday we hope to expand the age range to include tots and have a daycare center overseen by people who love math. So mothers could…

Prof. Daubechies: Oh, that would be very good.

The End

Thank you, Professor Daubechies, for sharing your thoughts and time so generously. I learned so much from you and I feel very fortunate to have had the opportunity to interview you. -Ken

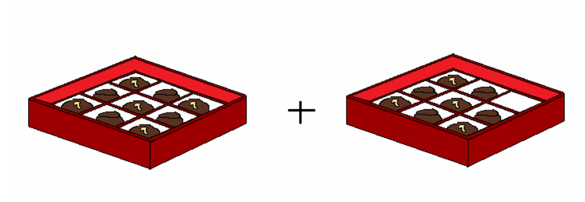
Prueba del 9: Modular Arithmetic Part II (Multiplication)

By Hana Kitasei • Illustrated by Yume Kitasei • Edited by Kay Kirkpatrick

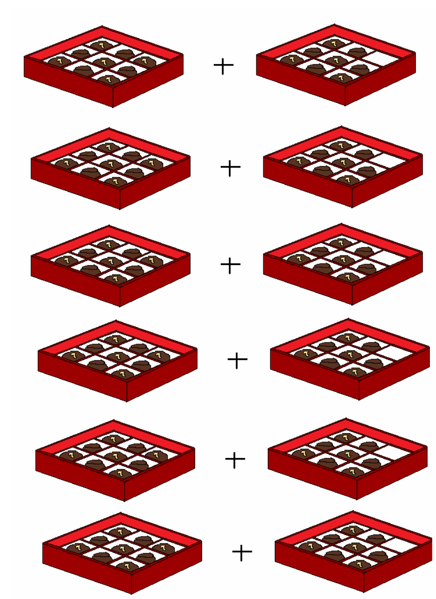
Last time we introduced modular arithmetic. In arithmetic modulo 9, we are concerned with remainders of integers when divided by 9. The integers then fall into nine classes: {0, 1, 2, 3, 4, 5, 6, 7, 8}. For example, 9 has a remainder of 0 when divided by 9, while 16 has a remainder of 7. We introduced the notation "mod" (short for "modulo"), so that "$n \equiv m \bmod 9$" is read as "n is congruent to m modulo 9" and means $n = 9a + m$ for some integer a, or, that $n - m$ is divisible by 9. And we investigated how addition respects these classes.

Now that we've learned how to add two numbers in modular arithmetic, it is natural to wonder how multiplication works in this system. Do you see? As it turns out, multiplication works similarly to addition. We can prove that the product of two integers is congruent to the product of their remainders.

Let's start off with some visual intuition for modular multiplication, in terms of our chocolate boxes with spaces for 9 chocolates in each. Say we have 17 chocolates, which can fill one box and partly fill one box with only 8 chocolates.



Now we wonder, if we had six times this many chocolates, then how many chocolate boxes would be completely full, and how many would be left over? That is, if I multiply 17 chocolates by 6 and attempt to fill as many boxes as possible, will there be an incomplete box? And if so, how many chocolates will be in it?



To multiply 17 chocolates by 6, we can take the original arrangement of one full box and one box of 8 chocolates, and copy that arrangement five more times. Copying the full box gives 6 full boxes; and copying the box with 8 chocolates gives 6 boxes of 8 each. The 6 full boxes have no left-over chocolates; so we concentrate on the $8 \bullet 6$ chocolates from multiplying the box of 8. This shows that $17 \bullet 6 \equiv 8 \bullet 6 \bmod 9$.

And in particular, we find that when we take 6 times 17 chocolates, we can fill a bunch of boxes and have 3 chocolates in the last box. We can express this in symbols as $17 \bullet 6 \equiv 3 \bmod 9$.

Now to generalize this example, let's write two integers m and n as $m = 9a + b$ and $n = 9c + d$ for some integers a and b and remainders b and d (integers between 0 and 8).

We multiply m and n together using the distributive law:

$$\begin{aligned} mn &= (9a + b)(9c + d) \\ &= 9a \bullet 9c + 9ad + b \bullet 9c + bd \end{aligned}$$

The first three terms are multiples of 9, so they will not contribute to the remainder when mn is divided by 9, as we know from previous issues. This means that we only need to look at the last term, bd, the product of the remainders of m and n. In other words, mn has the same remainder as bd when both are divided by 9.

This proves that, if we wish to know the remainder of mn when divided by 9, we can simply look at the remainders of m and n (which will generally be much smaller numbers), and find the remainder of *their* product when divided by 9.

What have we learned so far in these four installments of *Prueba del 9*? In the first two installments, we learned that the reduction procedure of "the trick" has a nifty property – a number and its reduced form will have the same remainder when divided by 9. In the last installment, we learned that *the sum of two numbers is congruent to the sum of their remainders.* And here we have learned that *the product of two numbers is congruent to the product of their remainders.* In other words: If m has remainder b, and n has remainder d, then $mn \equiv bd$ mod 9.

Can you see how this is heading towards a proof of the trick? Tune in next time for the final explanation!

More questions

What set of remainder classes do we get when we replace nine with 2? 5? 10? 100? What numbers can you multiply by 17, and get only full boxes of 9? What if you had 11 chocolates? 13 chocolates? Can you complete the table below?

Multiplication Modulo 9

$m \backslash n$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0								0						
1	0		2					7							
2							3								
3							0								
4												8			
5															
6															
7															
8				6											
9															
10															
11															
12						6									

Table of Addition Modulo Nine

Addition Modulo 9

m \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5
1	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6
2	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7
3	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
4	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0
5	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1
6	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2
7	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3
8	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4
9	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5
10	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6
11	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7
12	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
13	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0
14	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1

Here's the addition table from the last installment of *Prueba del 9* filled out. If you haven't tried to fill out the table yourself, please try to do that first before looking at this table. Figuring things out for yourself does take longer and is harder, but the benefits are well worth the effort.

What patterns do you see in this table?

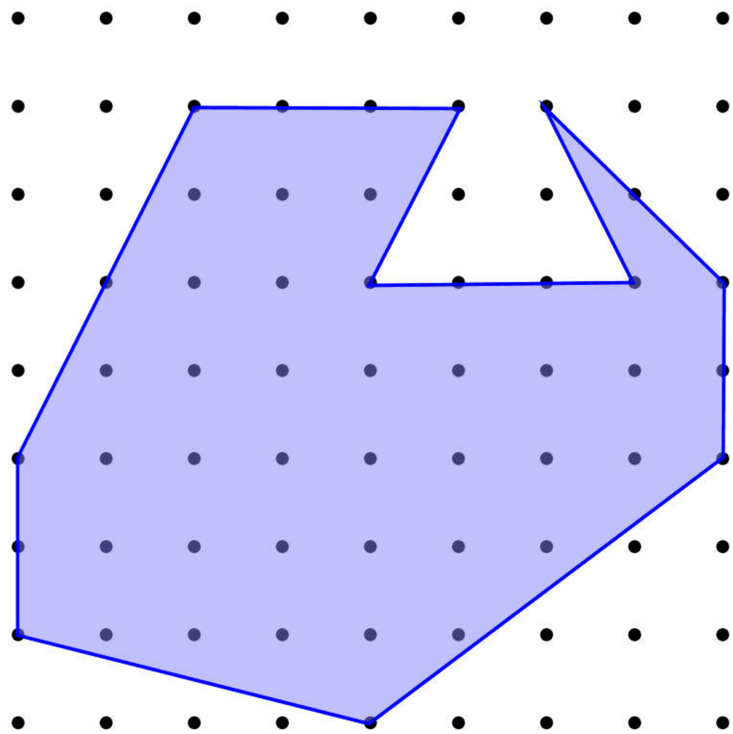
What does the first row within the table represent (the row with a 0 against a blue background as its leftmost entry)?

Can you describe how this table would look if you extended the table to all positive integers? What happens of you include all integers, both positive, negative and zero?

What do analogous tables look like for addition modulo 2, 3, 4 or 5?

If you make a table of addition modulo 10 and compare it to a regular addition table, how do they relate to each other? Why?

Pick's Theorem



Several girls have studied Pick's theorem at the club. Pick's theorem is concerned with the area of **lattice polygons**. A lattice polygon is a polygon whose vertices are part of a square lattice, as in the above figure. The distance between consecutive horizontal or vertical lattice points is taken as the unit distance.

Let I be the number of lattice points strictly *inside* the polygon.

Let E be the number of lattice points on the *edge* or boundary of the polygon.

Pick's theorem states that the area of this polygon is equal to $I + \frac{E}{2} - 1$ square units.

In the above figure, $I = 30$ and $E = 18$. Pick's theorem tells us that the area of the polygon is 38 square units. Try to compute the area of the polygon without using Pick's theorem. Which method do you find easier? Try out Pick's formula for other lattice polygons and see if you can convince yourself that the formula is valid.

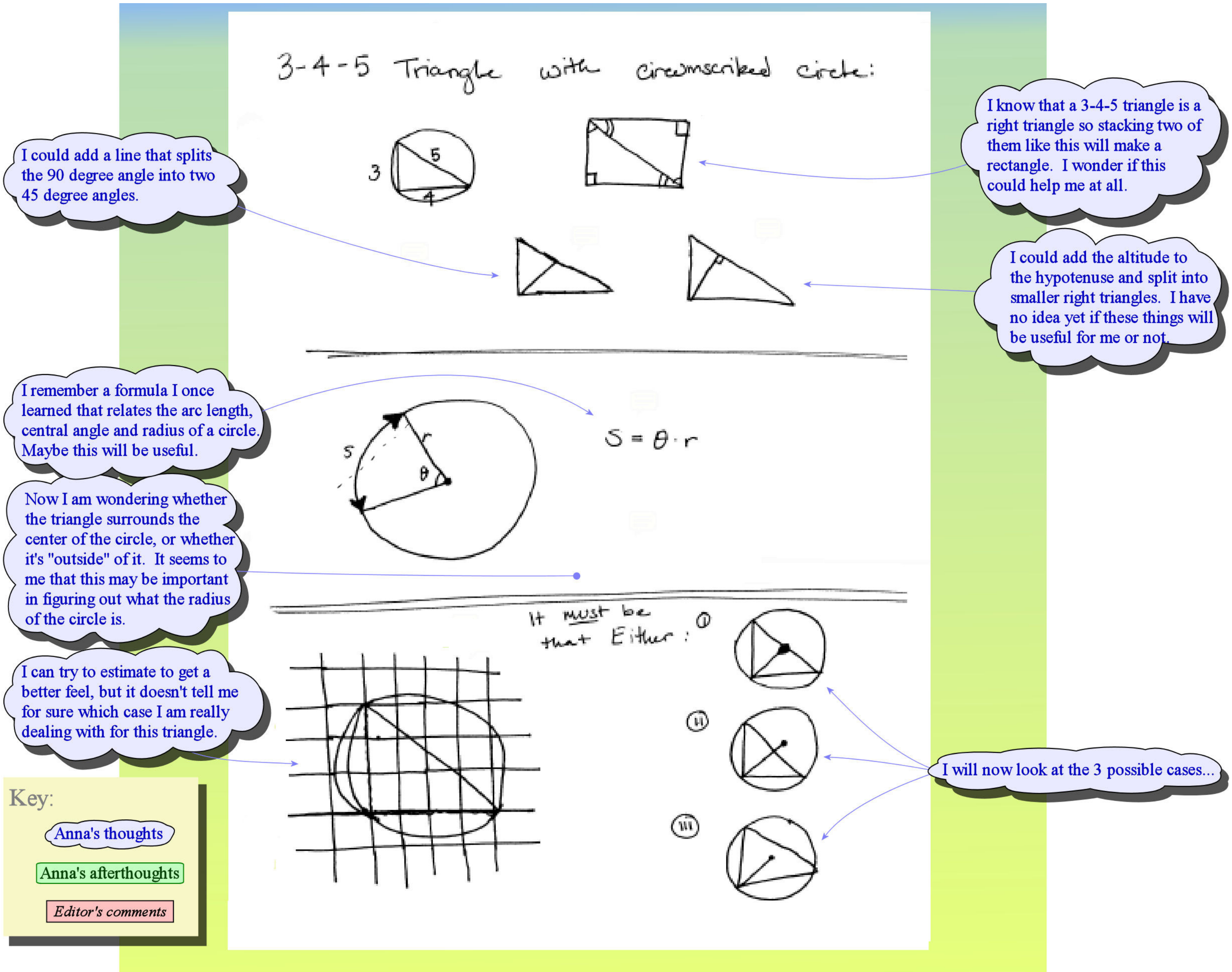
Here's a question for you: Suppose you have a polygon in the coordinate plane whose vertices all have *rational* horizontal and vertical coordinates. Can you prove that the area of such a polygon must be a rational number?

Anna's Math Journal

By Anna Boatwright

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Here, Anna finds **the radius of a circle** circumscribed about a 3-4-5 triangle.



Suppose ① is true,

then

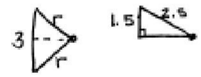
If the center of the circle lies on the hypotenuse of the triangle, then it would have to be exactly at the midpoint, since each side is the radius length. Now I will try to see if I can figure out a reason why this case either *must* be true or why it *cannot* be true.

which implies $r + r = 5$

$2r = 5$

$r = 2.5$

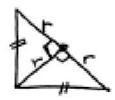
and therefore



So what ?

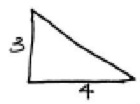
I feel like I am getting nowhere.

① cannot be true because,



Now I see something!

but

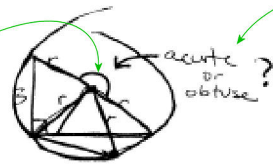


Implies these sides are equal

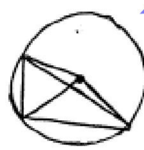
by definition, these sides have length 3 and 4, and of course $3 \neq 4$!

I mislabelled this. "Acute" and "obtuse" apply to angles less than or greater than 90 degress, not 180 degrees.

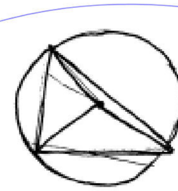
After taking a break from this problem I came back to it, to see if anything new and fresh might come to me. Since I ruled out Case I, I am re-phrasing the question as, "is this angle greater than 180-degrees or less than 180-degrees?"



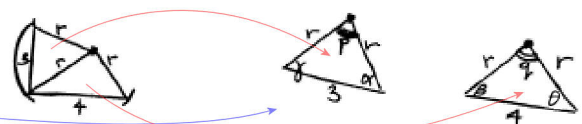
If the angle in question is greater than 180-degrees, then it'll look like this.



If the angle in question is less than 180-degrees, then it'll look like this.



I have split apart the triangle into two smaller triangles: one with side lengths 3-r-r and one with side lengths 4-r-r.



I know that $\alpha + \beta$ must add up to 90 because the 3-4-5 triangle is a right triangle. Also, I know that $\alpha = \gamma$ and $\beta = \theta$ because each of these smaller triangles is isosceles.

$\alpha + \beta = 90$

$\left.\begin{array}{l}\alpha = \gamma \\ \beta = \theta\end{array}\right\}$ b/c isos. triangle

$p = 180 - 2\alpha$ $\qquad$ $q = 180 - 2\beta$

$$\begin{aligned} p + q &= 180 - 2\alpha + 180 - 2\beta \\ &= 360 - 2(\alpha + \beta) \\ &= 360 - 2(90) \\ &= 180 \end{aligned}$$

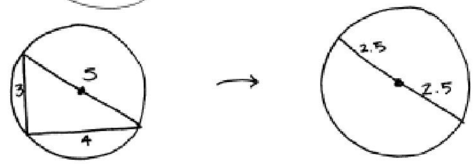
Using substitution, I discover that p and q must add up to 180!!! But that means that Case 1, which I already ruled out, must actually be true! Did I make a mistake before? Am I making a mistake now? I have to go back and take a look...

Which means that case ① must be true !?!

No.

Yes.

Looking back, I see that I **did** make a mistake before. I assumed that the angles p and q each must be exactly 90 degrees. And that is certainly not true. However, it *is* true that $p + q = 180$.



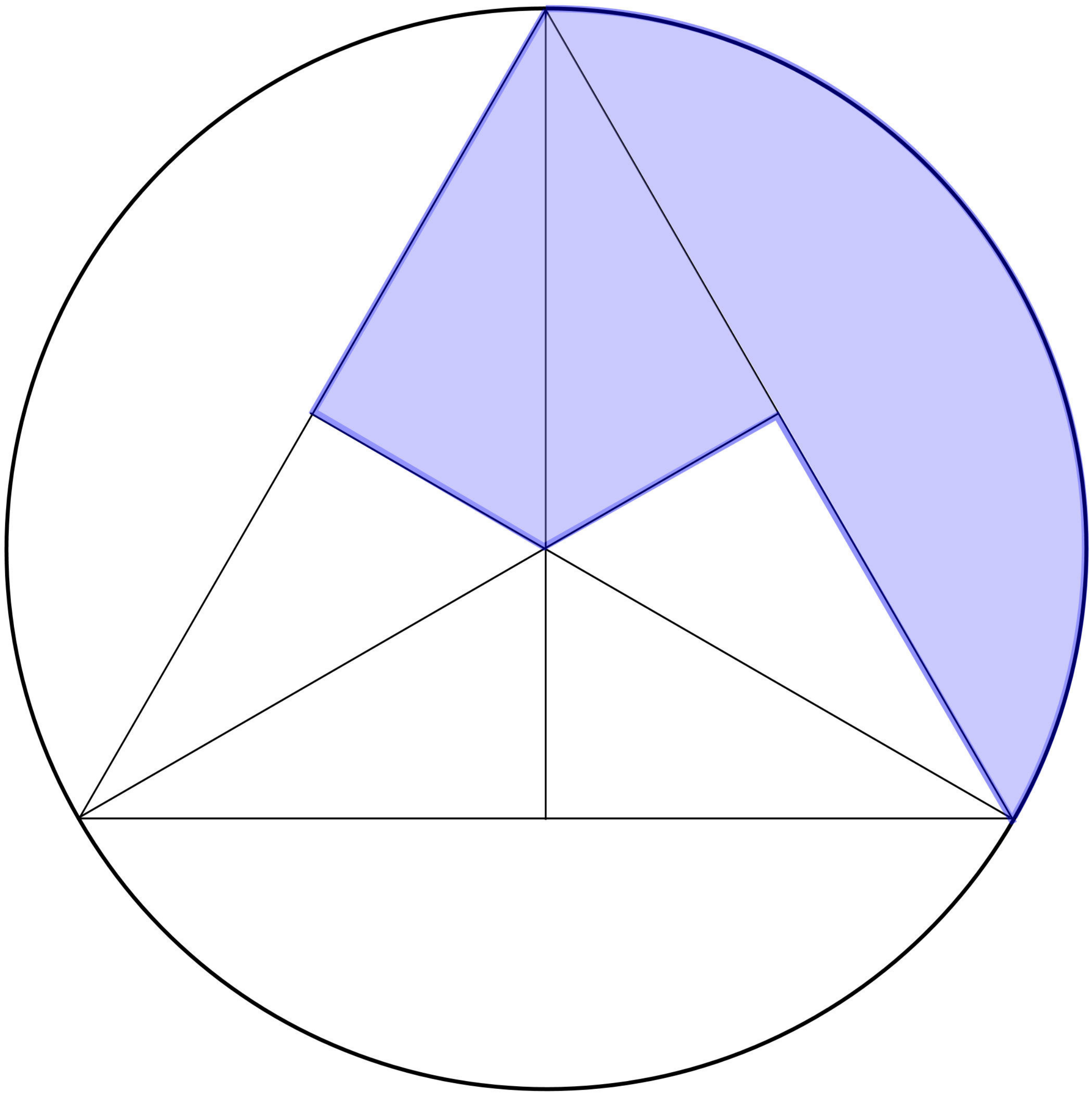
radius = 2.5

ABB 4.23.07

In fact, it does not matter that the sides lengths were 3 and 4. My reasoning only relies on the fact that the triangle has one 90-degree angle. So, this result is true for *any* right triangle with hypotenuse of length 5. I have proved more than I set out to do!

Key:
- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

A Halving Problem



The patterns of stars members have been making for flag designs have led to many discussions of symmetry. Here is a problem that further explores the concept of symmetry.

The figure shows a circle circumscribed about an equilateral triangle. The three lines inside the triangle connect vertices to midpoints of opposite sides. From these lines, a curious looking blue shape is selected.

Here's the problem: Figure out how to divide the blue shape into two *identical* halves.

Feel free to send your answer to girlsangle@gmail.com.

Math in the Produce Section

By Katy Bold

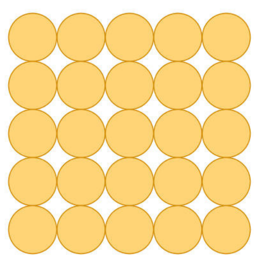
MATH IN YOUR WORLD

Logo Design by Hana Kitasei

A few weeks ago I moved from Virginia to Texas, and I brought along most of my books (about 170 pounds!). As I packed the books into boxes, I thought about how to make them fit into as few boxes as possible (without any one box being too heavy to carry, of course). Within each box, I could lay the books flat or stand them up length-wise or height-wise. Since I sent the books through the mail, I wanted the books packed tightly so they would not slide around during shipping.

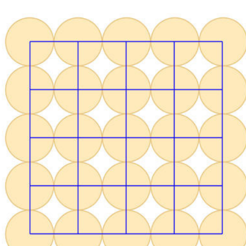
This is an example of a **packing problem**. Generally, a packing problem requires finding the best way to place objects (such as books) into a container (such as a box). You may be familiar with a packing problem at your local grocery store. Have you ever noticed how the oranges are displayed?

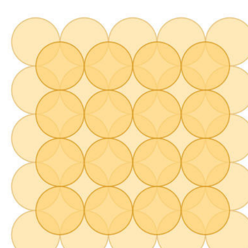
Let's say a grocer has room for 5 rows of oranges such that each row contains 5 oranges. The bottom layer of the oranges usually looks like this:



This is a top view.

The arrangement of the oranges is called a **square lattice**. Connecting the centers of neighboring circles creates lots of squares. In the next layer, oranges sit in "holes" between the oranges in the first layer. See the pictures below.





In the bottom layer, there are $5^2 = 25$ oranges. The next layer has $4^2 = 16$ oranges, so there are 41 total oranges in the bottom two layers.

How many oranges are in the next layer? And how many layers will there be total? Figure out how many oranges will be in each layer and fill in the chart to determine the total number of oranges.

	Number of rows and columns	**Oranges in this layer**	**Total number of oranges**
Bottom Layer	5	25	25
	4	16	41
	3		
	2		
Top Layer	1		

What is the pattern? The total number of oranges is a sum of squares:

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \_\_\_\_\_\_\_\_\_.$$

If the grocer has a lot of space and makes a pile of oranges with base 20 oranges by 20 oranges, how many oranges can she stack in total?

You could add up all of the squares from 1^2 to 20^2, but that would take a while. There is a shortcut formula for the sum of the first N squares. Do you know it? How many oranges will be in a stack with base N by N?

If a grocer has 500 oranges to stack, what dimension should she make the base layer? How many empty spaces will there be for oranges at the top of the stack?

For circular and spherical objects, we know the best way to pack the objects into any shape. But for ellipses of different dimensions (different lengths and thicknesses), it is not easy to find the best packing method. Computer algorithms find the best (or nearly the best) packing by pretending to shake the ellipses in a box. That is my favorite aspect of packing problems – if you throw everything in a box and shake the box, the resulting configuration will be close to the optimal packing. If only it were so easy to pack books into boxes…

Take it to your world

Take out 25 pennies. What is the tightest way to pack them on a tabletop? (Hint: The answer is not a square lattice!) By connecting the centers of neighboring pennies, what shape do you see in this lattice?

Try stacking your favorite candies in a cubic lattice or another lattice. Which can be packed more tightly – Mike & Ike candy or M&M's?

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete and, to nonmembers, they may not even be coherent!

Session 4 – Meet 4 – February 26, 2009

Mentors: Lauren McGough, Jennifer Melot, Mia Minnes, Maria Monks

Special Visitor: Dr. Tanja Bosak, Earth and Planetary Sciences, MIT

For the first half, we played another round of the Define This Game game. **Trisscar** got a particularly difficult word to define: dimension. But, she pulled it off quite spectacularly by systematically discussing a progress from length to area to volume as one goes from a line segment to a square to a cube.

Sylvie had to define a negative number. She described it as being a number "below zero" and "next to a dash". This was a valiant attempt, but **The Cat** cleverly stole the point by observing that the "1" in the fraction $\frac{0}{1}$ is both "below zero" and "next to a dash"!

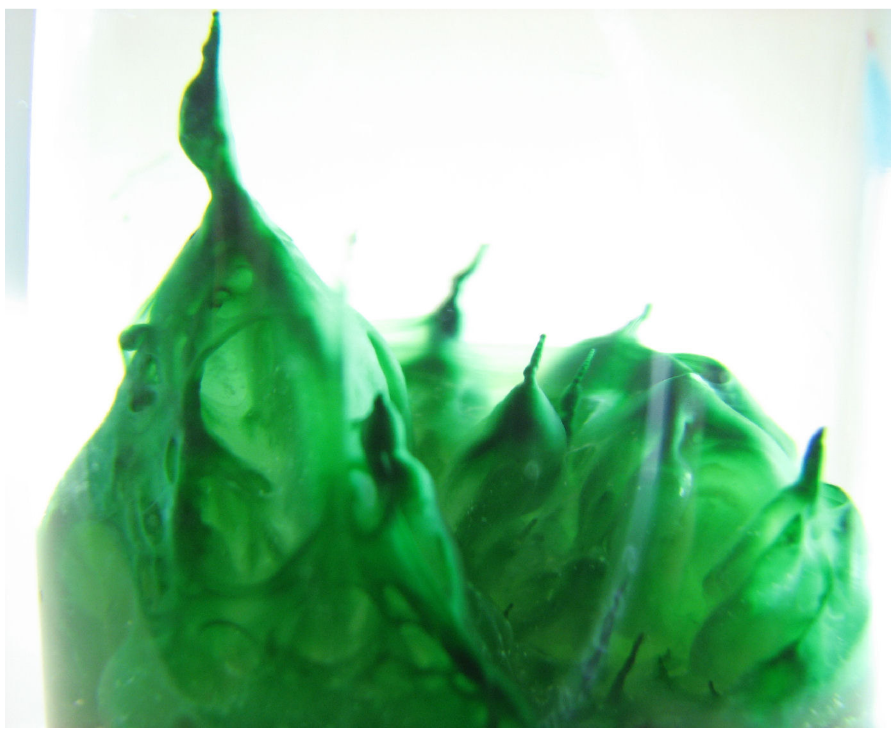


Photo courtesy of Dr. Tanja Bosak

Is this structure an artifact of life or is it formed by a geological process?

Tanja Bosak talked about how she uses math to help her decide whether a structure found in a rock is the result of a geological process or if it is an artifact of a life form. Amazingly, she literally carried in what she described as the oldest rock thought to contain preserved life forms. One hint can be obtained by determining known characteristics of life forms that are alive today and that seem to make similar structures. By modeling these characteristics mathematically, one can more objectively decide if such structures are the possible result of a life process.

Session 4 – Meet 5 – March 5, 2009

Mentors: Grace Lyo (Head), Lauren McGough, Mia Minnes, Maria Monks, Leia Stirling

Special Visitor: Dr. Leia Stirling, Boston Children's Hospital

The first half was spent in small groups.

In Mia's group, the discussion began by considering vertices, edges and faces of a cube. But a debate arose about whether an edge contains infinitely many vertices. The debate was the launch point for a journey that ended up in a discussion about rational and irrational numbers and algorithms for computing the digits of π.

Leia led a new game which aims to increase computational proficiency.

Lauren and Maria worked with girls on the combinatorics of hypercubes. **Caitlin** managed to find a formula for the number of vertices and edges in an n dimensional hypercube.

Jo wondered how computers add. So, we introduced the girls to binary numbers and talked about the addition algorithm for binary numbers. Once an algorithm is determined, the question becomes, how can this algorithm be implemented in a machine?

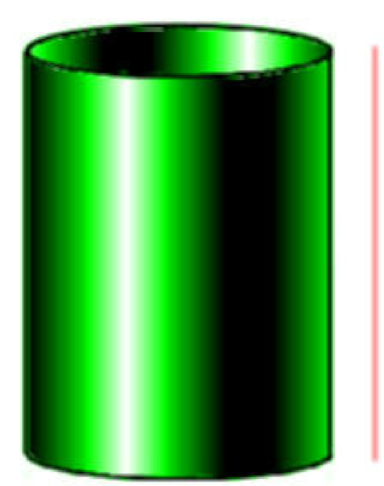
After the break, Leia Stirling led a robot activity where girls had to design a robotic door that automatically opened and closed for use in a miniature golf course.

Session 4 – Meet 6 – March 12, 2009

Mentors: Grace Lyo (Head), Cammie Smith Barnes, Jennifer Melot, Mia Minnes

To follow up on binary numbers, the mentors began the sixth meet by performing the Binary and Ternary Line dances. In these dances, mentors line up in a row and each mentor represents a different digit of, say, a ternary number. The digit zero is represented by sitting down, one is represented by standing up with arms at the sides, and two is represented by standing up with arms raised in the air. As quickly as possible, the mentors begin counting in ternary. Many thanks to Jennifer for taking on the responsibility of being the units digit! She got a good workout.

The girls are challenged to form a team and challenge the mentors to do a Ternary Line dance race. This is an open challenge…bring it on!



Will the red string wrap all the way around the mouth of this cylinder?

To celebrate National Pi day, many of the activities were related to π. For example, Grace brought in a number of cylinders and the girls were asked whether a string cut to the height of the cylinder would wrap around the cylinder's mouth. Try this at home. If you'd like to see for which cylinder the string will just make it around the mouth, roll a piece of origami paper into a cylinder by taping opposite edges together (with no gap or overlap).

Even the break snacks were related to π. All of them were circular! We had chocolate mousse cake, apple tart, apple pie, chocolate chip cookies and waffles.

After the break, we broke into small groups.

In one group, the study of factors of numbers by arranging dots into rectangular arrangements evolved into the study of patterns of dots with high symmetry. This led to the idea of designing a flag for the United States if a 51st state happened to be annexed. The new flag would then have to have 51 stars. How could these stars be arranged into an aesthetically pleasing pattern?

In Jennifer's group, different methods for computing the area of composite shapes were explored. **Tree** invented a neat approximation method which amounted to overlaying a grid and counting squares. The finer the grid, the better will be the approximation. This is an excellent idea and is the germ of the idea of integral calculus. She later refined her technique by including triangles. The inclusion of triangles is a refinement just like what is done to create the so-called "trapezoid" approximations that you'll see when you study calculus.

Mouse said that she thought numbers with infinite decimal expansions are strange. So, let's think about them some more. One broad classification of such numbers is into the repeating and non-repeating decimals. Recall that a repeating decimal is one where after a certain point, the digits cycle through the same pattern of digits. The decimal expansion of one third is a good example of a repeating decimal. In fact, we could even consider decimals with a finite expansion as repeating if we allow ourselves to include endlessly repeating zeros. That is, think of 1.5, for instance, as 1.5000000… It turns out that if we include numbers with an endless string of zeros as repeating decimals, then the repeating decimals correspond exactly to the set of rational numbers, which are ratios of integers. The non-repeating decimals correspond to irrational numbers such as $\sqrt{2}$ and π. There were early mathematicians who didn't even get to the stage of regarding irrational numbers as strange because to them, such numbers *didn't even exist*!

By the way, notice that $1.5\overline{0} = 1.4\overline{9}$.

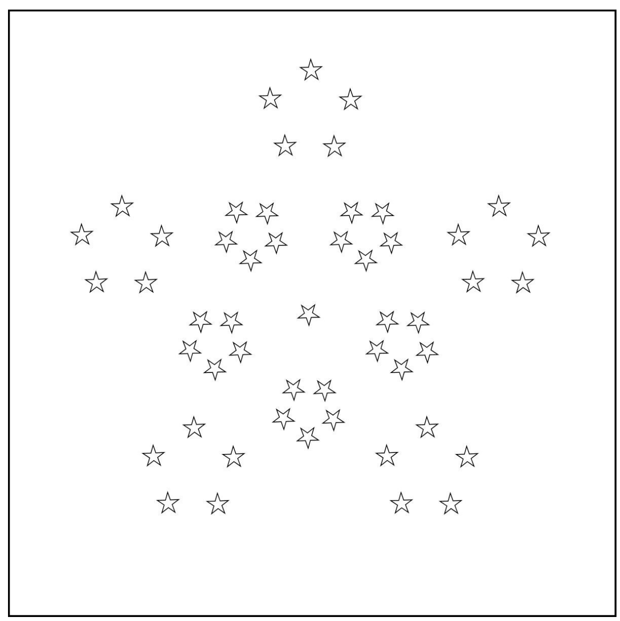
Session 4 – Meet 7 – March 19, 2009

Mentors: Annie Huang, Kay Kirkpatrick, Lauren McGough, Jennifer Melot, Maria Monks

Special Visitor: Taylor Walker, Architect, Dimella Shaffer

Following up on the flag design question from last meet, several girls created patterns that involved 51 stars. In each pattern, symmetries were counted.

Ilana became curious about the analog of the Four Color theorem on the surface of a torus. Kay led a



How many symmetries are there?

group on this topic, showing how the problem can be phrased in terms of graph theory.

Annie led a group activity concerning Pick's theorem. Pick's theorem gives the area of the kinds of figures that **Tree** was making as she approximated areas during the previous meet, namely, polygonal shapes whose vertices all belong to a fixed square lattice. One consequence of Pick's theorem is that such polygons can only have areas of the form $\frac{n}{2}$, where n is an integer. Annie exploited this fact to show that there is no way that 3 points in a square lattice can form the vertices of an equilateral triangle. Can you show this? Annie challenges the girls to figure out which regular polygons can be realized as such "lattice polygons"?

Photo courtesy of en.wikipedia.org/wiki/File:Gateway_Arch.jpg

The Gateway Arch in St. Louis, MO.
The shape is a catenary curve.

Taylor Walker started by talking about the golden mean and its appearance in the Parthenon. She then talked about optical illusions and discussed various ways that architects compensate for the way we perceive things in order to make us believe that we are looking at something else. For instance, in certain contexts, true parallel lines do not look parallel, so the architect will actually design non-parallel lines in order to fool us into thinking that we are looking at parallel lines. She also talked about various mathematical curves that appear in architecture, such as the catenary curve. Then she had the girls compute the actual dimensions of her home given a scale drawing and some rulers.

Session 4 – Meet 8 – April 2, 2009

Mentors: Cammie Smith Barnes, Annie Huang, Kay Kirkpatrick,
Lauren McGough, Jennifer Melot, Maria Monks

The eighth meet was spent entirely in small groups.

Some girls designed geometric soap shapes. Others designed symmetric flag star patterns. One group analyzed the combinatorics of the card game Set, while another studied applications of the pigeon hole principle.

Aba-ka-dabra, **Mouse** and **Grace** computed the total number of "sets" in the card game Set. They also began looking for a maximal subset of cards that are "set-free".

Thanks to 4 Sisters Soap, Girls' Angle hopes to create a line of geometric soap bars to raise funds for the program. The cover shows our very first soap bar. We made the mold and 4 Sisters Soap made the bar. We hope that our members will design interesting geometric shapes that are suitable for soap, concentrating on the theoretical side of the equation. Girls' Angle staff will then take their ideas and create a soap mold which will be turned over to 4 Sisters Soap who will use the mold to produce real soap bars! For example, already **Lucky** has designed a highly symmetric truncated tetrahedron which we intend to turn into a soap bar soon. A number of girls designed excellent soap bar shapes that we hope to use, such as **Trisscar's** rhomboid-hedron.

During this meet, **Ilana** asked, "What is the radius of a 4-D ball whose (hyper)volume is equal to its (hyper)surface area?" She conjectured that it should be 4. Why? Because the radius of the circle whose area is equal to its circumference is 2 and the radius of the ball whose volume is equal to its surface area is 3, so 4 would continue a simple pattern. In fact, **Ilana**'s conjecture is true and it is worth thinking about why. If you have an explanation, please send it along to us at girlsangle@gmail.com.

Session 4 – Meet 9 – April 16, 2009

Mentors: Cammie Smith Barnes, Lauren McGough, Jennifer Melot, Mia Minnes, Nike Sun

Special Visitor: Prof. Eleanor Duckworth, Harvard Graduate School of Education

Meet nine began in small groups.

These days, a lot of math goes on during a typical Girls' Angle meet. Design of soap solids led to discussions about measuring interior angles of polygons and Euler's formula. Study of the card game Set led to combinatorial problems and probability questions. Symmetric flag designs led to discussions about symmetry in functions. Probability questions led to discussions about independent events.

After the break, Eleanor Duckworth led her activity on mirrors. She had two girls stand in two different places in the room. She then asked, "Where should a mirror be placed flat against the wall so that the girls can see each other in the mirror?" From this direct and straight-forward question, there followed a lot of experimentation, thought and conjecture.

The Hubble Telescope uses a mirror to direct gathered light along engineered pathways enabling us to see astronomically distant objects.

Some girls proposed precise descriptions of how to locate the position for the mirror. Does it fascinate you that one can control light so precisely?

By the way, the behavior of light is rather sophisticated and subtle. It is so subtle that it is dangerous to extrapolate one's experiential knowledge of light behavior to the way light will behave under extreme conditions, such as when the light is very, very dim or when you are moving very, very fast.

Calendar

Session 4: (all dates in 2009)

January	29	Start of fourth session!
February	5	Sara Seager, Earth and Planetary Science, MIT
	12	
	19	Winter break - No meet
	26	Tanja Bosak, Earth and Planetery Sciences, MIT
March	5	Leia Stirling, Boston Children's Hospital
	12	
	19	Taylor Walker, DiMella Shaffer Architecture
	26	Spring recess - No meet
April	2	
	9	No meet - Rescheduled for April 23
	16	Eleanor Duckworth, Harvard Graduate School of Education
	23	
	30	Gigliola Staffilani, Mathematics, MIT
May	7	

Session 5: (all dates in 2009)

September	10	Start of fifth session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	No meet
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	
	10	

Girls’ Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls’ Angle? Girls’ Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls’ interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls’ Angle mentors, the Girls’ Angle Support Network, the Girls’ Angle Bulletin and Community Outreach.

Who are the Girls’ Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls’ Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls’ Angle Bulletin? The Girls’ Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls’ Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members’ efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 2 ways: **membership** and **active subscription** to the Girls’ Angle Bulletin. **Membership** is granted per session and includes access to the club and extends the member’s subscription to the Girls’ Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. **Active subscriptions** to the Girls’ Angle Bulletin allow the subscriber to ask and receive answers to math questions through email. Please note that we will not answer email questions if we think that we are doing the asker’s homework! We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of “catching up with the group” doesn’t apply. Note that you can receive the Girls’ Angle Bulletin free of charge. Just send us email with your request.

Where is Girls’ Angle located? Girls’ Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________ (first) ______________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: ______________ Cell Phone: ______________ Email: ________________

Emergency contact name and number: ____________________________________

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ______________________________

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ____________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a 12 session membership
  - ☐ $50 for a one year active subscription
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ______________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: __________________________________

Girls' Angle

A Math Club for Girls