Girls' Angle Bulletin

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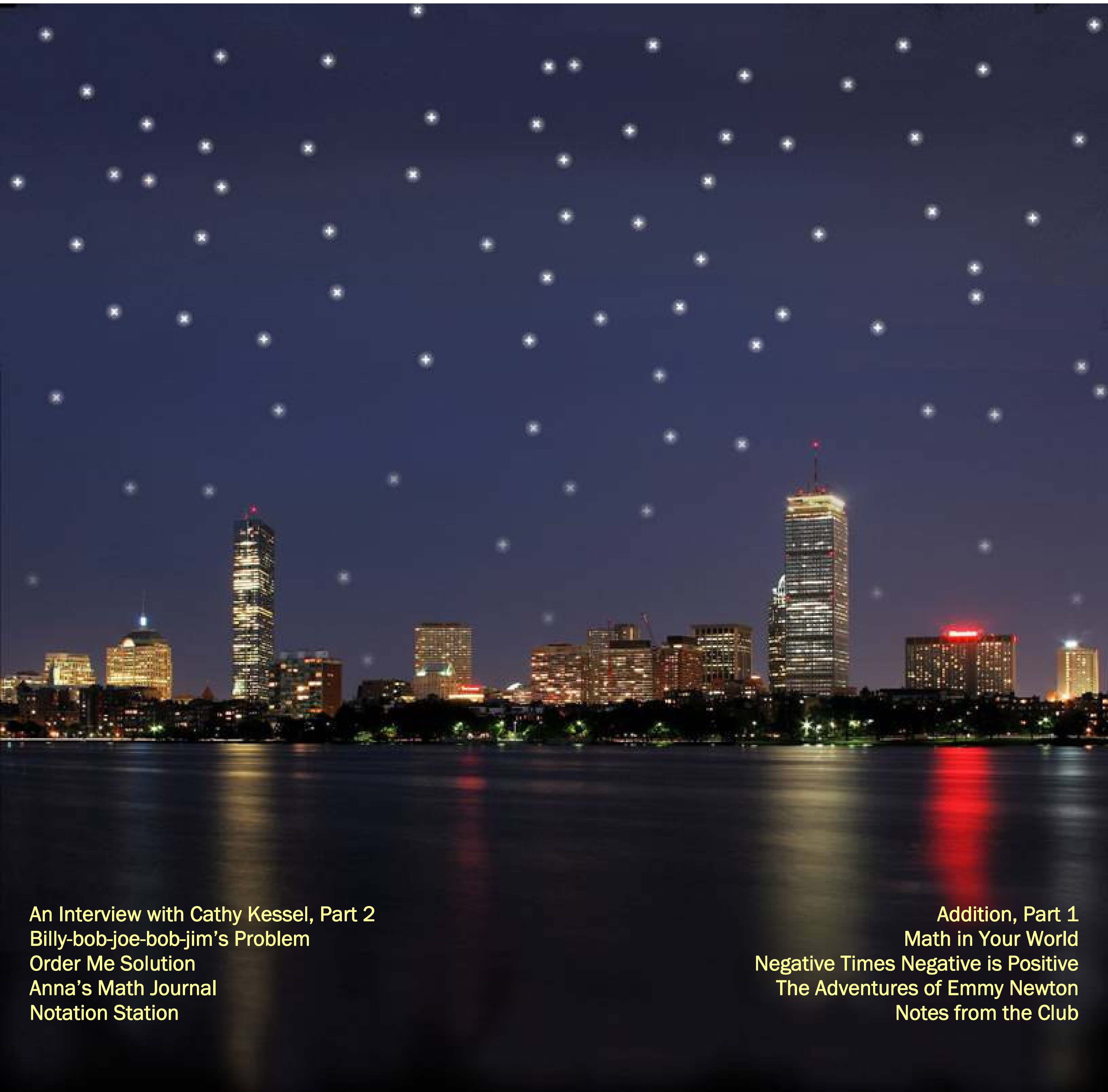
To Foster and Nurture Girls' Interest in Mathematics



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From the Director

Today, the Bulletin is the product of many contributors. In addition to our regular columns, problems, and club notes, this issue highlights the work of two members. One poses a problem and the other presents a solution.

We open with the conclusion of Cathy Kessel's two-part interview. Thank you, Cathy, for your thoughtful and detailed responses!

Last issue, we announced that Claude Shannon Professor of Mathematics at MIT, Bjorn Poonen, joined the Girls' Angle Advisory Board. Dr. Poonen is a number theorist. He has published scores of papers on a wide variety of sophisticated mathematical topics. So it is quite an extraordinary treat that he was willing to write an article about addition for the Girls' Angle Bulletin! I hope you enjoy these words from a great modern mathematician about perhaps the most fundamental and basic of mathematical operations.

And to our members: remember, this is your magazine: *You* guide its contents!

All my best,
Ken Fan
Founder and Director

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Stellar Operations*. Design by Toshia McCabe and Ken Fan. Boston skyline courtesy of wikimedia.org.

An Interview with Cathy Kessel, Part 2

Cathy Kessel recently served as president of the Association for Women in Mathematics, a nonprofit organization created to encourage women and girls to study and to have active careers in the mathematical sciences, and promote equal opportunity and the equal treatment of women and girls in the mathematical sciences. This is the second half of her interview with Girls' Angle.

Ken: In 2004, the AMS conducted a survey and found that at ten of the top math departments in the US, less than 6% of the tenured faculty were women. Do you think there is an explanation for this statistic? If so, what is your explanation?

Cathy: I think it's not a simple story.

First of all, I'll give some reasons why statistics about tenured professors are so often mentioned in discussions of women and mathematics. Many mathematicians have academic positions—although there have been some notable exceptions. In academe, tenure is, essentially, a guarantee of job security that comes with recognition that one's academic work is deemed worthwhile. It's not always true that the best mathematicians are at the top departments nor that every faculty member of a top department is wonderful. However, the top departments do try to hire the best mathematicians.

We need to encourage everyone to do mathematics! Everyone should be able to deal with the mathematics that occurs in their lives—and to be able to enjoy mathematics.

In the United States (and many other countries), proportions of women in many top positions are very small. In government, we're still waiting to have a woman as president. Nancy Pelosi became the first female speaker of the house in 2007. But, the American Mathematical Society, which is the major society for research mathematicians in the United States, had its first female president (Julia Robinson) in 1983 and another (Cathleen Morawetz) in 1995.

In business, very few women are heads of large corporations. According to the March 2009 issue of *Fortune Magazine*, only 2.4% of Fortune 500 CEOs are women. Compared with that, it seems encouraging that women are 5% of the tenured faculty in the "top ten" mathematics departments. In 1991, women were only 4 out of 303 tenured faculty members in those departments. In 2004, they are 16 out of about 300. Going from 1% to 5% in thirteen years might seem very slow, but other surveys suggest that change in mathematics and the sciences might be speeding up in the 21st century.

In 2002 and 2007, the chemist Donna Nelson did two surveys of the "top 50" departments in various scientific fields, including mathematics. (I'm putting "top 50" in quotation marks because these classifications are always debatable.) She and her survey team asked the chairs of these departments for the numbers of tenured professors and of assistant professors. In 2002, Nelson found that women were 19% of assistant professors in those mathematics departments. Five years later, in 2007, that statistic was 28%. I find that change quite impressive. In the U.S., women earn about 30% of the PhDs in mathematics and they are now getting entry-level positions in top 50 departments at about the same rate. Nelson's statistics show similar trends for fields such as physics, chemistry, and engineering—the percentage of female assistant professors in the top 50 has increased and is approaching (sometimes exceeding) the rate at which they earn PhDs.

Surveys of mathematics departments in general, not just the top 10 or top 50, show a similar trend. Every five years, the Conference Board of the Mathematical Sciences surveys a sample of mathematics departments. This survey asks department chairs how many tenured and tenure-track[1] professors are in their departments. The most recent of these was in 2005. This table [see the table below] gives the numbers of women in tenured and tenure-track faculty positions in mathematics departments, as classified by highest degree granted. (Calculations of percentages are mine and given in parentheses.)

	Fall 1995	Fall 2000	Fall 2005
Tenured women (percentage of tenured faculty)			
PhD-granting departments	317 (7%)	346 (7%)	427 (9%)
MA-granting departments	501 (15%)	608 (19%)	532 (21%)
BA-granting departments	994 (20%)	972 (20%)	1373 (24%)
Tenure-track women (percentage of tenure-track faculty)			
PhD-granting departments	158 (20%)	177 (22%)	220 (24%)
MA-granting departments	235 (29%)	276 (32%)	337 (33%)
BA-granting departments	748 (43%)	517 (32%)	693 (28%)

The numbers of women in tenured and tenure-track faculty positions in mathematics departments, as classified by highest degree granted.

The percentages of women in each of these categories are increasing—except for tenure-track positions at BA-granting mathematics departments. But, notice the increase in the number of tenured women at these departments between 2000 and 2005.

That's just a little bit of context and history. As I said, I think it is not a simple story. There are all sorts of factors. Some affect women only, but others affect everybody, but seem to have a heavier impact on women.

One factor that may affect women more is the two-body problem. If a mathematician is part of a couple, there's a good chance that the other member of the couple is a mathematician or scientist. In one study, 41% of married male mathematicians were married to a scientist, mathematician, or engineer—which increased to 78% for married female mathematicians. I think that everyone suspects that many mathematicians, particularly female mathematicians, are part of a two-mathematician couple, but statistics about this are hard to find. One suggestive piece of evidence comes from a survey of physicists which found that 43% of married female physicists were married to other physicists and over 68% of married female physicists were married to scientists.

[1] Tenure-track professors are in positions that can lead to tenure.

Finding jobs for both members of such a couple in the same geographical location has been known for a long time among mathematicians as the two-body problem, after the physics problem of the same name. In the past, these couples sometimes lived in different cities so both could have jobs, or lived in the same city while one (often a wife) had part-time jobs or no job at all. However, universities are starting to become more aware that many mathematicians and scientists are part of a two-body situation—and are becoming more creative about finding jobs for both bodies.

Couples have also been very creative in searching for jobs. Ellen Spertus, a computer scientist, has an interesting web site called "How to Do a Massive Two-Body Job Search" which is about her job search in 1997. She ended up with a position at Mills College, which is in the San Francisco Bay Area, and her partner (who later became her husband) got a job in Mountain View—also in the Bay Area. I see from her web page, that in addition to being an associate professor at Mills, she's now a part-time software engineer at Google, which is in Mountain View.

Another factor that may affect women more than men are issues that concern work and family, often called "work–family balance." Tradition and studies suggest that family issues—taking care of children, doing housework, and so on—tend to be the responsibility of women more than men. However, that situation is changing. For example, men with children now spend more time on childcare than in the past.

Some authors note that talking about "work–family balance" suggests that work and family occupy separate spheres and are in conflict—time spent on work detracts from time with family and vice versa. They point out that some interactions between family and work are beneficial. Individual creativity or institutional change can transform "work–family conflict" into a strength rather than a weakness. One example is given in an AWM *Newsletter* article entitled "Do Dust Bunnies Multiply According to the Fibonacci Sequence?" (The answer is yes at the author's house because she's figured out a way to make the first two members of the sequence be zero.)

In thinking about biology and genetic inheritance, it may be useful to ask if indeed there are differences, are they differences that make a difference?

A third factor is what I'll call "environment" which concerns mentoring, transparency of expectations, climate, and so on. Sometimes conditions are such that women don't always have access to information as readily as men do. Some recent publications discuss the "mysterious career paths" and "labyrinths" that women need to traverse without a map and with little guidance in order to get to top positions in corporations. It may not always be possible (or necessary) to put such guidance in writing, however, doing so may make a difference. A study of science departments found that departments which had written guidelines for graduate students on matters such as course of study, exams, and other expectations tended to have a larger percentage of women who earned Ph.D.s.

The two-body problem, work–family interaction, and environment are factors that may affect both men and women, but, on average, seem to affect women more. It's important to put "on average." Some men may be affected by one or all of them and some women may not be affected by any. And, the changing statistics for the percentage of female professors of mathematics suggest that these factors aren't quite the obstacles they used to be.

On the other hand, habits and ways of thinking often change very slowly—and those are associated with the fourth factor, gender schemas. This is the term described by the psychologist Virginia Valian as "a set of implicit, or nonconscious hypotheses about sex differences." These affect our expectations about women and men—and girls and boys—with respect to many things, in particular, with respect to mathematics. These may cause differences in how women and men are judged, whether they are considered for particular honors, invited to give talks, or simply whether they are even noticed or remembered. I heard from a graduate student once that she asked one of her professors for a letter of recommendation when she was applying to graduate school. She had taken a course from him the semester before and earned an A. It was a small undergraduate class—but he didn't remember that she took the course. Busy professors may have a sneaking sympathy for him, because for them the events of the previous semester can seem like the distant past, but gender schemas may played a role in this professor's lack of recall.

Being systematic in various situations can help to reduce the effect of these unconscious expectations. For example, if the professor had gone through the list of the students in the course and made a few notes about the ones who got As for future reference (just in case he needed to write letters of recommendation or nominate students for awards), then he might have been more likely to remember her.

Ken: Do you think, as some prominent psychologists have suggested, that there are innate biological differences that could explain why girls tend to lose interest in math at a faster rate than boys?

Cathy: *Do* girls lose interest faster than boys?

Conjectures about biological differences affecting women's intellectual abilities have been made for centuries, sometimes by very eminent people. The forms of the conjectures have changed over the centuries and, so far, they seem to have had a limited shelf life. That's not a reason to dismiss the current ones, but it does induce skepticism.

A study of science departments found that departments which had written guidelines for graduate students on matters such as course of study, exams, and other expectations tended to have a larger percentage of women who earned Ph.D.s.

In thinking about biology and genetic inheritance, it may be useful to ask if indeed there are differences, are they differences that make a difference? It used to be that just being female was enough to get a person excluded from university study. That was certainly a biological difference that made it much harder to maintain an interest in mathematics, let alone learn it—but perhaps not the kind of difference that we think about today.

Currently, neuroimaging studies of brain activity are one source of speculations. I'm definitely no expert in this field and I don't know if any studies have found robust gender differences in various kinds of brain activity. But, if they exist, how do we interpret them? It’s not always obvious to scientists—although it may seem straightforward to popular writers.

Ken: What do you think are good ways to encourage more girls to do mathematics?

Cathy: We need to encourage everyone to do mathematics! Everyone should be able to deal with the mathematics that occurs in their lives—and to be able to enjoy mathematics. It’s very sad to see how fearful people can be about this subject and that so often it’s perceived as so limited and boring.

Part of this is due to our curriculum and the way teachers do—or don’t—learn mathematics. There is a lot of concern about both among mathematicians—and a lot of energy devoted to trying to make them better. Improving school mathematics may be particularly important for girls for two reasons. First, because they may (in general) have fewer opportunities outside of school for learning about mathematics. Second, many elementary teachers—about 90% of whom are female—often have not received adequate mathematical educations and sometimes fear mathematics. That’s not a happy situation for these teachers and their students. Moreover, such teachers are not good role models for girls and may help to reinforce stereotypes about women’s lack of capacity for mathematics.

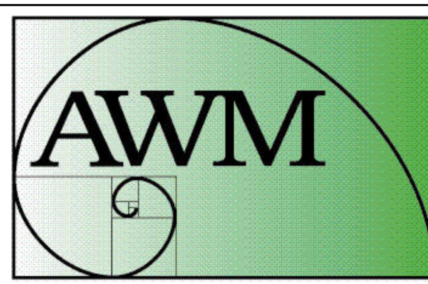
Psychological research suggests that beliefs about the nature of talent and ability are important too. I don’t claim that such beliefs are stated explicitly, in fact, I think they’re often extremely murky and incoherent. However, many people behave in ways that are consistent with such beliefs. It took me years to find out about the notion of a “math mind.” That’s (more or less) the idea that some people have “math minds,” some don’t, and if you don’t, then you will have trouble with middle and high school mathematics. According to this view, any struggle to understand a mathematical idea indicates that you don’t have a “math mind.”

…many elementary teachers—about 90% of whom are female—often have not received adequate mathematical educations and sometimes fear mathematics.

In contrast, I think that mathematical ability is developed and that understanding isn’t necessarily immediate. We aren’t born knowing much mathematics (to put it mildly)—or how to read, or how to write. Learning often takes some effort. Given the sorry state of mathematics education in the United States, the effort required to learn school mathematics can be immense. So, I’m not surprised that many people seem to believe only the very exceptional can learn school mathematics, let alone more advanced mathematics.

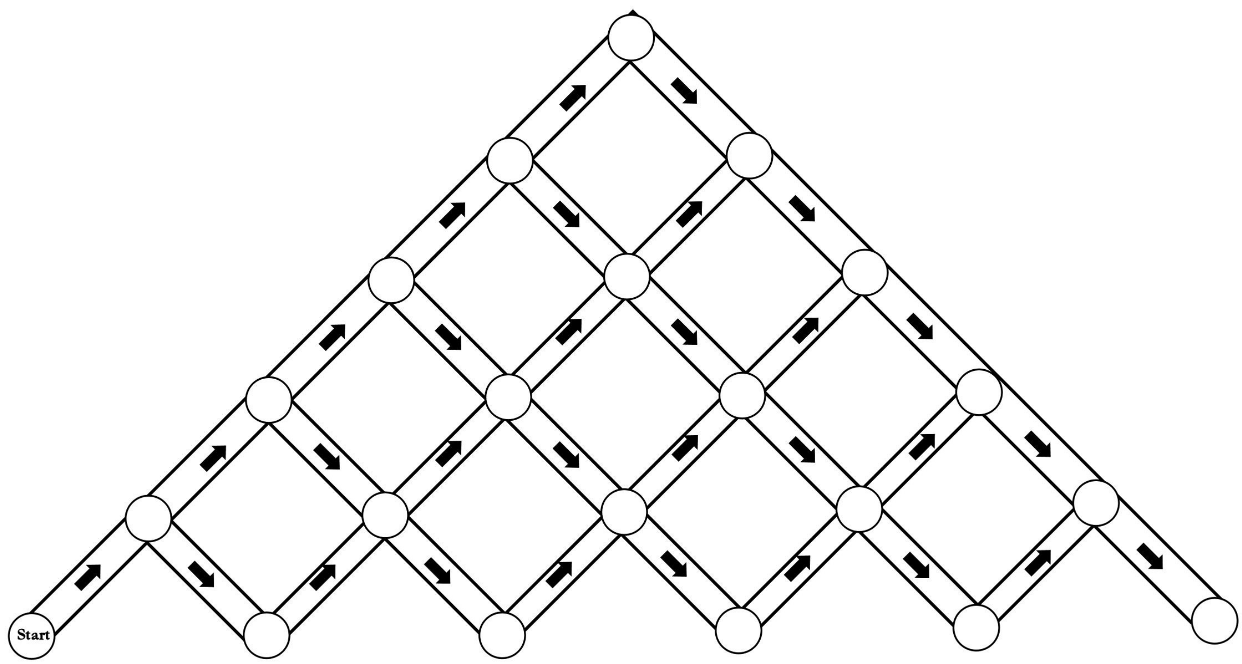
Ken: Thank you for this wonderful interview!

The logo for the Association for Women in Mathematics is shown at right. Being the logo of a mathematical association, it has mathematical significance. What mathematics do you see in the logo?

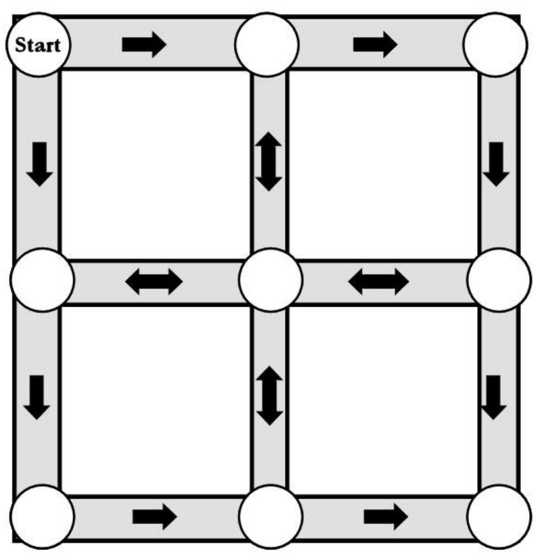


Billy-bob-joe-bob-jim's Problem

At the club, some girls worked on counting the number of different ways to get from one intersection to another in a network of one-way streets. Try this with the following street map:



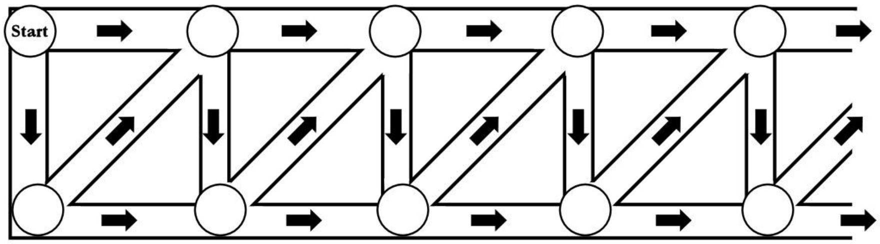
Club member **billy-bob-joe-bob-jim** finished a number of such problems. Maria Monks, the mentor who was working with **billy-bob-joe-bob-jim**, suggested that they each dream up their own street maps and exchange them as a challenge for each other. Here is the street map that **billy-bob-joe-bob-jim** left for Maria to solve:



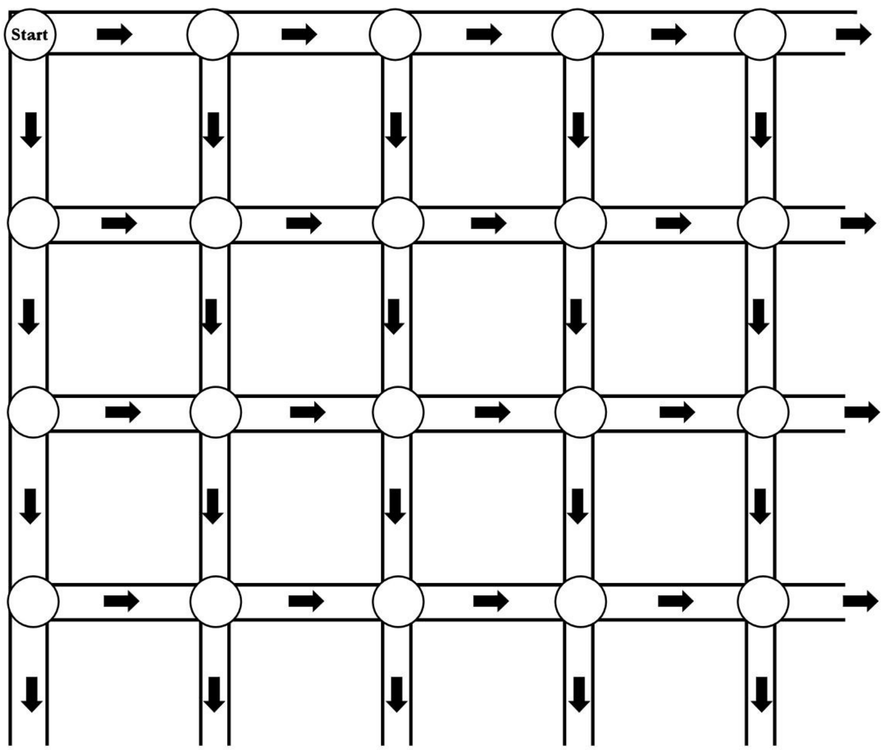
How many ways are there to get from the intersection labeled "Start" to each of the other intersections? For this problem, you may use each street at most one time. (Here, a "street" refers to a segment of road between two adjacent intersections.)

If you enjoy counting paths in networks of streets, try these:

1

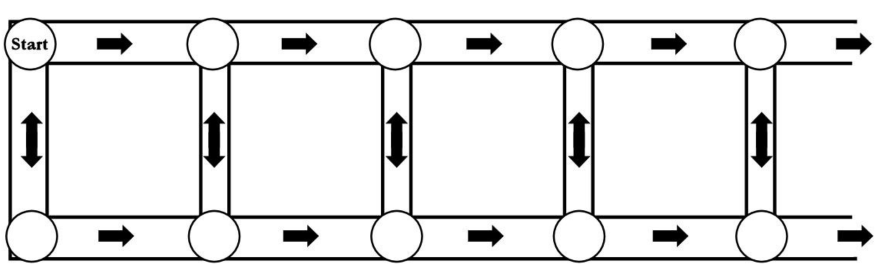


2



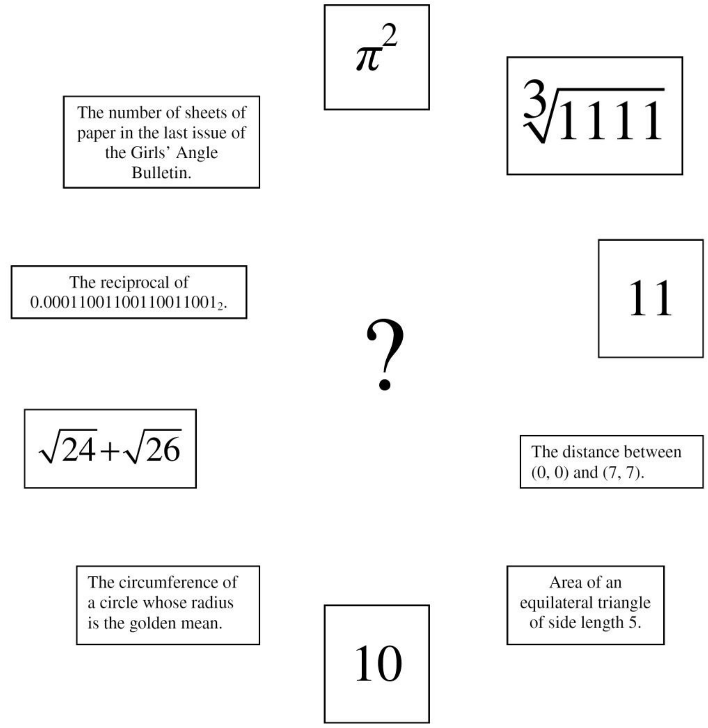
3

In this one, there are two-way streets, but you may cross a two-way street at most once.



Send in your answers to girlsangle@gmail.com!

Order Me Solution



Here are the answers to the ordering problem from the last issue. The problem was to place the 10 numbers shown at right in order from least to greatest *without the aide of a calculator*.

The answer is:

π^2 < the distance between (0, 0) and (7, 7) < $\sqrt{24}+\sqrt{26}$ < 10 < the reciprocal of 0.00011001100110011001_2 < the circumference of a circle whose radius is the golden mean < $\sqrt[3]{1111}$ < area of an equilateral triangle of side length 5 < 11 < the number of sheets of paper in the last issue of the Girls' Angle Bulletin.

The number which caused the most difficulty was the reciprocal of 0.00011001100110011001_2. Notice, however, that this number is close to, but less than $0.0\overline{0011}_2$. Let $x = 0.0\overline{0011}_2$. We can convert this number to a fraction in the following way:

If you're having trouble following this argument, you may need to review the binary number system.

$$\begin{aligned}
100000_2x &= 11.\overline{0011}_2 \\
10_2x &= 0.\overline{0011}_2 \\
100000_2x - 10_2x &= 11_2 && \text{(subtract the second line from the first)} \\
11110_2x &= 11_2 \\
x &= \frac{11_2}{11110_2} \\
x &= \frac{1_2}{1010_2} && \text{(reduce the fraction to lowest terms)}
\end{aligned}$$

Since the decimal number 10 is equal to 1010_2, we find that $x = 0.1$ in decimal, so $1/x = 10$. If we invert a (positive) number that is less than one tenth, the result will be greater than 10.

To complete this particular problem, we have to compare this number to the others. There are many ways to do this, and you should try to find one for yourself. You'll probably come up with a different method because people have different preferences when it comes to computation. In any case, it is important to keep in mind that what follows is ad hoc. Because

$$0.1 - 1/2^{20} < 0.00011001100110011001_2 < 0.1,$$

we know that

$$10 < \text{the reciprocal of } 0.00011001100110011001_2 < 1/(0.1 - 1/2^{20}).$$

Notice that $1/(0.1 - 1/2^{20}) = 10(1/(1 - 10/2^{20})) < 10(1/(1 - 1/101)) = 10(101/100) = 10.1$, which is less than the next smallest number in the list. (The number 1/101 was chosen exactly because it is greater than $10/2^{20}$ and, when $10/2^{20}$ is replaced by 1/101, the resulting expression is easy to compute by hand.)

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Recently, a friend said to me, "Twice a prime can be a perfect nth power only when the prime is equal to 2. But what about one plus twice a prime? When can that be a perfect nth power?" I had no idea, so I decided to investigate.

$2p+1 = a^n \quad , \; n>1 \; , \; p$ prime

Actually, thinking about just adding one to a prime seems interesting. I'll think about that first.

$p+1 = a^n \quad , \; n>1 \; , \; p$ prime

Primes are defined in terms of number of factors. Numbers with exactly two factors are prime. That gives me an idea. I'll try to isolate the prime and study factors.

$p = a^n - 1$

I'm not sure what to do about the variable exponent. I'll simplify by looking at the case $n = 2$.

$n=2: \quad p = a^2 - 1$

I know how to factor this. It's a special case of the identity $a^2 - b^2 = (a+b)(a-b)$.

$= (a+1)(a-1) \quad \Rightarrow a = 2$

$\Rightarrow p = 3$ is prime

If both the factors $a+1$ and $a-1$ are greater than 1, this will be composite, not prime. So a has to be 2.

$3 + 1 = 2^2$

Hey, a solution! Let's try $n = 3$!

$n=3: \quad p = a^3 - 1$

If $a = 1$, then $a^3 - 1 = 0$. That means that $a - 1$ divides evenly into $a^3 - 1$.

$= (a-1)(a^2+a+1)$

$$
\begin{array}{r}
a^2+a+1 \\
a-1\,\overline{)\,a^3 \qquad -1} \\
\underline{a^3-a^2} \\
a^2+0 \\
\underline{a^2-a} \\
a-1 \\
\underline{a-1} \\
0
\end{array}
$$

This is my work for computing $a^3 - 1$ divided by $a - 1$.

$\Rightarrow a = 2$

$p = 4+2+1 = 7$

$7 + 1 = 2^3$

Again, a must be 2! And, it turns out that p is 7 which is prime. So that's another solution!

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

I think I'm ready to tackle the general case.

General Case: $p = a^n - 1$

$$= (a-1)(a^{n-1} + a^{n-2} + \cdots + a^2 + a + 1)$$

From the cases $n = 2$ and 3 I guessed that $a^n - 1$ factors like this. You can check that this is correct. Multiply it all out carefully and you'll find that many terms cancel.

Just as before, this factorization shows that p can be prime only if a is equal to 2.

$\Rightarrow a = 2$

$p = 1 + 2 + 2^2 + \cdots + 2^{n-1}$

Solution: If $p = 1 + 2 + 2^2 + \cdots + 2^{n-1}$ is prime, then $p+1$ is 2^n and these are the only solutions.

It came down to figuring out for which n, this expression is a prime number.

I had no idea, so I made a table to see if I could see any patterns.

When is $1 + 2 + 2^2 + \cdots + 2^{n-1}$ prime?

n	0	1	2	3	4	5	6	...
2^n	1	2	4	8	16	32	64	...
$1+2+2^2+\cdots+2^n$	1	3	7	15	31	63	127	...

I've underlined the prime numbers, but I don't see a pattern. Hmmm. Well, I'll think about this later. Now, I'll get back to my friend's original question.

$2p + 1 = a^n$ ($n > 1$, p prime)

$2p = a^n - 1$

$$= (a-1)(a^{n-1} + a^{n-2} + \cdots + a^2 + a + 1)$$

I can proceed pretty much the same as before...I can reuse the factorization of $a^n - 1$. The only difference is that the left hand side is twice a prime instead of only a prime.

$a = 2$? not possible (even $\neq$ odd)

$a = 3$? $2p = 2(3^{n-1} + 3^{n-2} + \cdots + 3^2 + 3 + 1)$

$p = 3^{n-1} + 3^{n-2} + \cdots + 3^2 + 3 + 1$

$a > 3$: RHS composite.

I should have said that the right hand side is composite *even after it is divided by 2*, which it is!

Fancy that...before, it was a sum of consecutive powers of 2. Now, it's a sum of consecutive powers of 3!

When is $1 + 3 + 3^2 + \cdots + 3^{n-1}$ prime?

n	0	1	2	3	4	5	6	...
3^n	1	3	9	27	81	243	729	...
$1+3+3^2+\cdots+3^n$	1	4	13	40	121	364	1093 ?	...

Afterward, I checked that 1093 is, in fact, a prime number! So, that's another solution:

$2(1093) + 1 = 3^7$

When is $kp + 1$ a perfect n^{th} power? p prime, $k \geq 1$.

AB 2.17.10

Anna managed to reduce the problem to determining which sums of consecutive powers of 3 are prime, but couldn't figure out which ones are. Can you shed any light on this matter? And what about the even more general question stated here? Send your solutions to girlsangle@gmail.com.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Notation

by Ken Fan

Through time, some ways to notate mathematical ideas have become standard. Learning the notational conventions is important in order to be able to communicate. The best way to learn new notation is the same way we learned the alphabet: repetition. Practice makes perfect!

Multiplication

$$8 \times 5$$

The "cross" notation introduced by William Oughtred in 1631. Because of the potential confusion with the letter "x", the cross is rarely used in expressions with variables.

$$8 \cdot 5$$

The dot and the cross are equivalent, although if there are variables, the dot is preferred because it won't be confused for a variable.

$$8(5) \qquad (8)(5)$$

Parentheses can also be used to show multiplication. Though rare, writing "(8)5" is also possible.

$$8(-5)$$

When multiplying negative numbers, one must use notation to make clear that a product is desired. Without parentheses, that is, if written "8 -5", it would mean "8 minus 5".

$$8x \qquad ab$$

Juxtaposition. When variables are present, dots and parentheses are often omitted, unless the parentheses are required to force a specific order of evaluating the operations, as in "$6(a + b)$".

$$812$$

Juxtaposition cannot be used when you multiply two numbers because you can't tell what numbers are being multiplied. In the above, we see the number 812. It cannot mean "8 times 12" or "81 times 2".

$$8 \cdot x \qquad (a)(b)$$

If you want to include a dot or parentheses, you can.

Addition
Part 1

by Bjorn Poonen

Photo by Toshia McCabe

What is the meaning of 5 + 3?

It's 8! Even my cat knows that.

You're not really answering my question. I'm asking not about the value of 5 + 3, but about what it means to add 5 and 3. How about this:

> The expression 5 + 3 means the total number of grapes that you have if you have 5 grapes in your left hand and 3 grapes in your right hand.

What if I also have one grape in each ear?

Well, then the total number of grapes you would have would be 10... but that's not what I had in mind. Let me change my answer:

> The expression 5 + 3 means the total number of grapes that you have if you have 5 grapes in your left hand and 3 grapes in your right hand and no grapes anywhere else.

The same approach can be used to explain what 4 + 5 means, or what 7 + 2 means, or what 0 + 8 means, or what 12 + 3 means, or what…

Are you going to go on like this forever?

I don't have to. Fortunately, there is a way to say all these infinitely many statements at once.

How?

By using **variables** that can stand in for any numbers:

> For any numbers m and n, the expression $m + n$ means the total number of grapes that you have if you have m grapes in your left hand and n grapes in your right hand and no grapes anywhere else.

This says that no matter what two numbers you choose ("For any numbers m and n"), the rest of the sentence will be true when you replace m and n by those two numbers. So this one sentence contains all the special cases that I was listing before, and all the cases I didn't get to.

Your definition doesn't work for 2873 + 148.

Why not?

Because there is no way that you could fit 2873 grapes in your left hand!

OK, well then I can use the same definition with grains of sand instead of grapes.

What about 92096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274 + 194912983367336244065664308?

What about it?

You won't be able to use your definition no matter what kind of objects you use, because my first number is more than the number of atoms in the universe! Ha ha ha!

Who said I had to use *physical* objects?

Well, you said you were going to have them in your hands.

OK, let me give you a better version of the definition. If A and B are two sets, then the **union** $A \cup B$ is defined as the set of elements that belong to either A or B or both.[1] In other words, $A \cup B$ is obtained by taking all the elements of A together with all the elements of B. For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c\}$, then $A \cup B = \{1, 2, 3, 4, 5, a, b, c\}$. Now what I want to say is that 5 + 3 means the number of elements of $A \cup B$, where A is any 5-element set and B is any 3-element set such that…

That's not right! If A is the 5-element set $\{1, 2, 3, 4, 5\}$ and B is the 3-element set $\{4, 5, 6\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6\}$, and this has only 6 elements. So you are saying that $5 + 3 = 6$!

You interrupted me before I finished my sentence. I was about to say that A and B should be chosen to be disjoint.

What does "disjoint" mean?

It means that there is no element that belongs to both A and B. For example, $\{1, 2, 3, 4, 5\}$ and $\{a, b, c\}$ are disjoint. Your sets $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6\}$ were not disjoint because, for instance, 4 belongs to both A and B.

OK, here is my sentence (*don't interrupt this time!*):

The expression 5 + 3 means the number of elements of $A \cup B$ where A is any 5-element set and B is any 3-element set such that A and B are disjoint.

How do you know that the number of elements of $A \cup B$ doesn't depend on *which* 5-element set A and *which* 3-element set B you use?

[1] The expression $A \cup B$ is pronounced "A union B".

That's a good question, because if a different pair of sets could give a different value for the number of elements of $A \cup B$, then 5 + 3 would not have a well-defined value, at least not by my definition. Fortunately, this doesn't happen; I can prove to you that it is well-defined:

> Suppose that A' is another 5-element set and B' is another 3-element set such that A' and B' are disjoint. Because A and A' both have 5 elements, it is possible to choose a matching between the elements of A and the elements of A'. Similarly, it is possible to choose a matching between the elements of B and the elements of B'. Putting these matchings together gives a matching between $A \cup B$ and $A' \cup B'$, so $A \cup B$ must have the same number of elements as $A' \cup B'$.

OK, I'm ready to give a general definition, in which 5 and 3 are replaced by arbitrary numbers m and n:

> For any numbers m and n, the expression $m + n$ means the number of elements of $A \cup B$, where A is any m-element set and B is any n-element set such that A and B are disjoint.

This definition gives the right answer even if you start putting food in your ears.

Can you use this to attach a meaning to 0 + 0?

Sure. Choose A to be the empty set $\varnothing$. (In fact, that is the only possible choice of a set with 0 elements.) Choose B to be the empty set $\varnothing$ too.

Wait; are those disjoint?

Yes, because there is no element that belongs to both of them. You can't even find an element that belongs to *one* of them, because the empty set has no elements!

So let me continue. The union $A \cup B$ also is the empty set, because there are no elements belonging to either A or B. So, according to the definition, 0 + 0 means the number of elements of $A \cup B = \varnothing$; that number is 0. Thus $0 + 0 = 0$.

Cat: Purrrr…

OK, if FluffyFur is satisfied, I'm satisfied.

> Next time: FluffyFur and friends try adding sizes of infinite sets. Later they realize that there are some kinds of numbers for which the definition doesn't work.

Editor: Grace Lyo

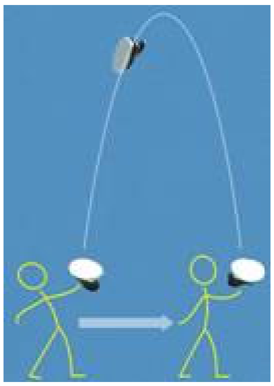
MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Sighted: A Parabola in the South Pacific

By Katy Bold

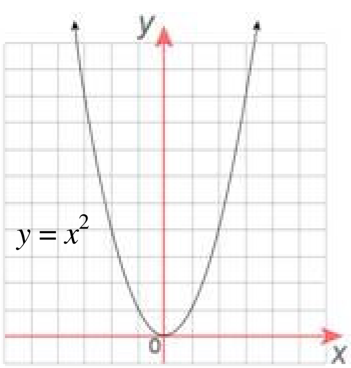
At a performance of the musical, *South Pacific*, I saw a parabola! South Pacific is a love story on an island in the South Pacific during World War II– not the typical setting for a parabola. For most of the play, the U.S. sailors on the island are not involved in the war, and they have time to lounge around and cause trouble.



One sailor strolls across the stage and nonchalantly tosses his hat in the air. He keeps walking and catches the hat a few steps later. The hat lands perfectly in his hand, and he seems to make no special effort to catch it. Was this sailor skilled at hat tossing? Did he practice for hours– getting his pace just right so that he can catch the hat at just the right moment?

No! This sailor had no special skills– he simply relied on Newton's First Law of Motion:

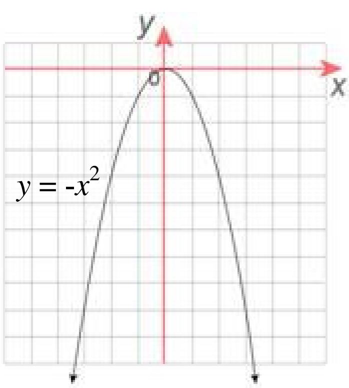
> An object in motion will stay in motion unless acted on by another force.



Before he tosses the hat, both the sailor and the hat move with the same speed in the horizontal direction. When he tosses the hat straight up, he changes the vertical speed of the hat but not the horizontal speed. The hat continues moving forward, and its horizontal speed is the same speed as the moment it left the sailor's hand.

While the sailor looked pretty cool with his hat toss, I was more impressed to see the shape of the curve traced out by the hat– a **parabola**.

> **Think about it**
>
> Newton's First Law of Motion doesn't seem true in some cases. For example, if you roll a heavy ball on carpet, it eventually stops. Why does Newton's Law *appear* to fail here?
>
> If the sailor started running just *after* releasing the hat into the air, would he still easily catch it?



You may have seen parabolas when studying math before. The most common parabola in math class might be $y = x^2$. The hat parabola is essentially just flipped over: $y = -x^2$. See the graphs at left. But, there are infinitely many possible parabolas that we can draw– they can be narrower or wider, and they can be shifted up/down or right/left. The general equation for a parabola, or quadratic, is

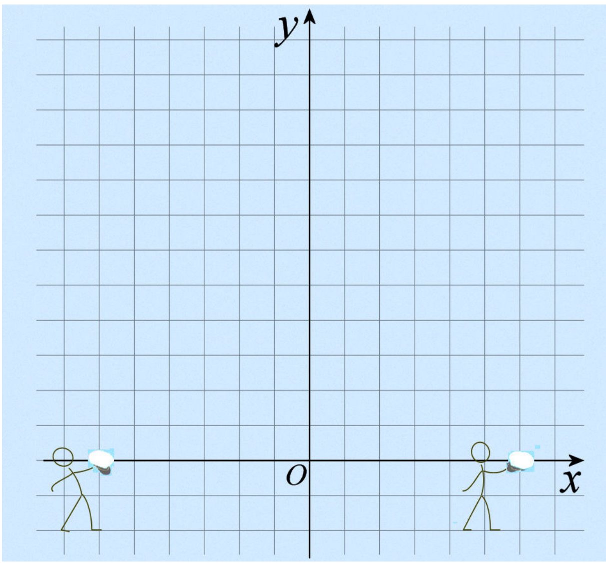
$$y = ax^2 + bx + c, \qquad (1)$$

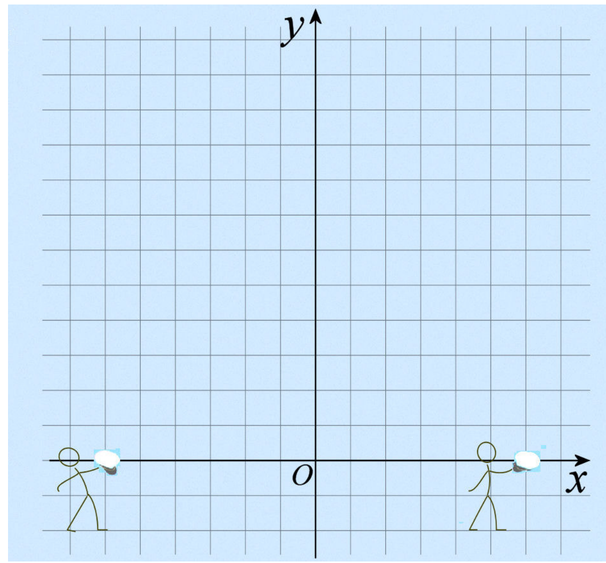
where a, b, and c are constants.

You could also write the equation like this:

$$y = d(x - r_1)(x - r_2),$$

where d, r_1, and r_2 are constants. The two values r_1 and r_2 are called the **roots** of the equation– x-values that make $y = 0$. We could consider the two points where the sailor tossed and caught the hat as roots of the parabola. If I told you the locations of these two points, could you find the exact parabola traced out by the hat? Or are there several possible parabolas? Draw what the parabola might look like. If you think there's more than one possibility, use the second graph.





These two roots determine some features of the parabola, but without knowing how high the hat went, we do not know the exact shape of the parabola. This is an example of why *three* conditions are needed to determine a unique parabola. Notice in the general equation for a parabola (1) that there are *three* constants: a, b, and c.

For straight lines, only 2 points are needed to uniquely identify the line. For the parabola, we just saw that we need 3 points. How many points do you think are needed to uniquely determine a cubic equation, which is an equation of the form $y = ax^3 + bx^2 + cx + d$, where a, b, c, and d are constants? What pattern do you see about the general form of the polynomial equations and the number of conditions needed to uniquely identify the curve? In the table below, a, b, c, and d are constants.

Type of Equation	General Form	Number of Conditions	Example Set of Conditions
Linear	$y = ax + b$	2	slope + y-intercept
Quadratic	$y = ax^2 + bx + c$	3	2 roots + height
Cubic	$y = ax^3 + bx^2 + cx + d$	?	

Take It To Your World

- Try tossing a ball straight up into the air while you walk. If you can juggle, try walking while you juggle. Be sure to try this in an area with lots of open space.
- Are you able to easily catch the balls? Why?

Negative Times Negative Is Positive

by Ken Fan

How are negative numbers handled when multiplied?

Let's settle this question once and for all!

If x is	positive	**and y is**	positive	**then xy is**	positive.
	positive		negative		negative.
	negative		positive		negative.
	negative		negative		positive.

But, *why* is a negative times a negative positive?

In a way, this is a peculiar question because it "forgets" that *we* are the creators of mathematics!

However, before getting too philosophical, let's first be practical. Multiplication is important because it is used often. That means that it's a good idea to know the table above. You can quickly memorize it by repeating it to yourself often. It may help to remember that the product of two numbers of the *same sign* is positive, whereas the product of two numbers of *opposite signs* is negative:

Same Sign:	**Positive**
Opposite Signs:	**Negative**

Now let's get back to the "why".

To answer this, we have to go to a very specific place, but not a place that represents an actual physical location or even a specific point in history. We have to go to a specific place in the logical development of mathematics. Specifically, we have to go to a point located *after* negative numbers have been defined, but *before* it was decided how to multiply with them.

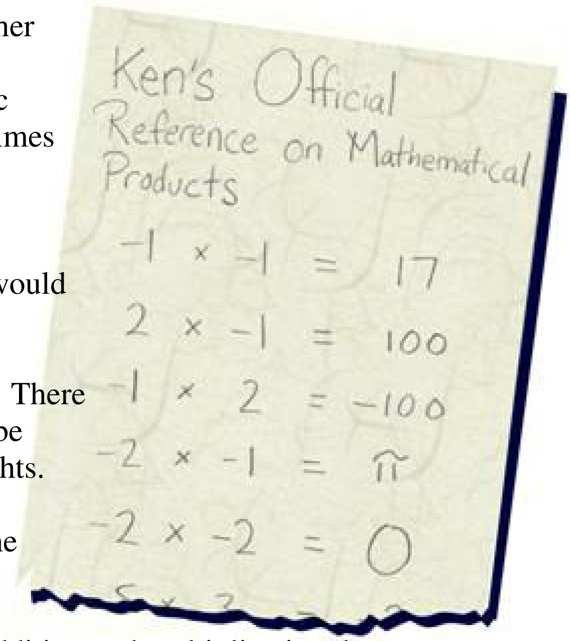
The story of how negative numbers came to be is fascinating, but not the subject of this article. Let's take it for granted that negative numbers have been introduced as the **additive inverses** of the positive numbers. That is, to each positive number, there exists a unique negative number, which, when added to it, produces zero. Let's also assume that multiplication has been defined for nonnegative numbers.

At this special place in the logical development of mathematics, our task is to figure out how to multiply with negative numbers. *That* is what this article is about! We know how to multiply *non*negative numbers, but we don't know how to multiply *negative* numbers. At this stage, because multiplication involving negative numbers *has not even been defined*, there is no way to even determine the product of two negative numbers let alone explain why the result is positive. In order to answer, "what is a negative times a negative?" we first have to *extend* the definition of multiplication so that it includes negative numbers.

So, now we ask, "How might we go about extending multiplication to the negative numbers?"

Because we are the creators of mathematics, we are free to extend multiplication in whatever way we wish!

However, some ways are more useful or more meaningful than other ways. You *could* just assign values to all the products involving negative numbers in an arbitrary way. You could create a gigantic volume of products. Every time someone wanted to compute -3 times -8, they would have to pull down *Ken's Official Reference on Mathematical Products* and look up the answer! Why, if these products became widely adopted, a whole industry of "product experts" who earn their keep by memorizing chunks of the book would be created!



But of course, no one would care about Ken's *Official Reference*! There would be no "product experts". After all, the products wouldn't be anything more than a snapshot of Ken's random, whimsical thoughts.

In order to extend multiplication in a useful or meaningful way, the extension must reflect some kind of organizing principle.

Briefly, the laws of arithmetic are the commutative and associative laws of addition and multiplication, the distributive law, and the existence of the additive and multiplicative identities 0 and 1.

When restricted to nonnegative numbers, addition and multiplication do not produce random numbers. Instead, addition and multiplication exhibit many patterns. These patterns reflect an underlying structure, and this structure is summed up in the **laws of arithmetic**. If you're interested in the laws of arithmetic, please look them up or ask about them at the club! I won't discuss the commutative, associative, and distributive laws of arithmetic here. Instead, I'll finally get to the point of this article:

A meaningful way to extend multiplication to negative numbers is to try to do so in such a way that the laws of arithmetic remain valid.

Let's see what happens if we try!

We know that 1 + (-1) = 0, by definition of negative one; this is what it means for negative one to be the additive inverse of one.

Let a be a nonnegative number. If we want the distributive law to hold, we must have $a(1 + (-1)) = a(1) + a(-1)$. Substituting 0 for 1 + (-1), we get $a(0) = a(1) + a(-1)$. We know how to multiply the parts involving nonnegative numbers. Simplifying those parts yields $0 = a + a(-1)$. So, if we want the distributive law to hold, this last equation *forces* us to declare that $a(-1)$ be the additive inverse of a. If a is positive, then the additive inverse of a is negative, and this illustrates how we are forced to have, at least in this case, a positive times a negative come out negative. Also notice that we must have (0)(-1) = 0.

Let's see if demanding that the laws of arithmetic hold forces us to pick a specific value for the product (-1)(-1). The distributive law would say that (-1)((-1) + 1) = (-1)(-1) + (-1)(1). Again, (-1) + 1 = 0, so this becomes (-1)(0) = (-1)(-1) + (-1)(1). Insisting that the commutative law of multiplication hold, we would require that (-1)(0) = (0)(-1), and we just saw that we are forced to declare (0)(-1) = 0. For the same reason, we are forced to have (-1)(1) = -1. So, we further simplify (-1)(0) = (-1)(-1) + (-1)(1) to 0 = (-1)(-1) + (-1). Adding 1 to both sides, we arrive at:

$$1 = (-1)(-1).$$

So, as soon as we say, "let's try to extend multiplication to the negative numbers in such a way that the laws of arithmetic remain valid", the value of the product -1 times -1 is destined to be 1. That is, we can choose how we wish to extend multiplication to the negative numbers, but once we choose to extend in such a way that the laws of arithmetic remain valid, the value of (-1)(-1) becomes fixed.

So, the answer to the question, "why is a negative times a negative positive?" is this: It is so if we wish that the laws of arithmetic (which were deduced in the realm of nonnegative numbers) remain true. Because the structure codified by the laws of arithmetic make addition and multiplication so useful and interesting, people were and are quite satisfied to extend multiplication in this way.

If, in the future, someone devises an alternative way to extend multiplication to the negative numbers in a way that is useful but does not obey the laws of arithmetic, that would be really interesting! But, to avoid confusion, such an extension would likely be given a new name and the word "multiplication" would be preserved as the name of that extension that does obey the laws of arithmetic.

Let's summarize the contents of this article. I want to be explicit about some of the things that I have done and some of the things that I have not done. What I have done is to indicate how, if we choose to extend multiplication to negative numbers in such a way that the laws of arithmetic hold, then we are forced to make a multiplication where a negative times a negative is positive. However, one thing I have not done is to carry this process to completion. I only showed that -1 times -1 would have to be +1. Also, I haven't shown that it is actually possible to extend multiplication to the negative numbers so that the laws of arithmetic hold. To do that, you have to carefully define an extension of multiplication to the negative numbers and then establish that when so defined, all the laws of arithmetic are valid. This task is not so daunting as it might seem if you've never done something like that before. The reason is that the laws of arithmetic don't leave you with any choice about how to define the products! Try it and see!

By the way, people have been adding and multiplying with both positive and negative numbers for a very, very long time. Everyone alive was born into a world where such sums and products were being computed every day, all over the planet. As a result, there are people who have forgotten that negative numbers are a human creation. To these people, what I said earlier about being free to extend multiplication in any way we please may seem to be just plain incorrect. But, you really can! The challenge is to do so in a way that people find useful. It should be no surprise that people use a multiplication where a negative times a negative is positive. After all, people use useful things!

Can you make up your own special operation on numbers and establish its properties?

Feel free to send any thoughts to girlsangle@gmail.com.

Editor: Grace Lyo

The Adventures of Emmy Newton

Episode 3. *The Ramp of Pythagoras*

by Maria Monks

Last Time: Emmy and Melissa found themselves trapped in the southeast corner of the basement of their school as they were trying to find their way to the Principal's office. They figured out the passcode to unlock a door that led north, and walked cautiously into the next room...

Emmy blinked. This room was much more brightly lit.

Her eyes gradually adjusted to the light, and slowly the room came into focus. It was more like a long hallway than a room, in fact. This made sense, Emmy thought, since they should be directly underneath the east hallway that runs north-south on the first floor of the square school building. The floor had the same brown-and-white checkered tiling pattern as the rest of the school, the same familiar fluorescent lights hanging from the ceiling, but the walls were bare unpainted grey stone.

The hallway was anything but empty. Three parallel rows of tall rectangular wooden beams standing vertically in rubber stands lined the entire hallway, like a forest of branchless trees. The beams were about five inches wide and five inches deep and of various heights, some only a few feet high and others nearly reaching the ceiling. On one side of each beam, there were also horizontal grooves cut into the beam at evenly spaced heights.

Emmy began to walk slowly down one of the aisles formed by the rows of beams, looking around at them. Each one had a number stamped on to it in black ink, and all the beams seemed to be different sizes. In fact, the larger numbers seemed to correspond with larger beams. A tall beam that nearly reached the ceiling had the number 14 written on it, and another beam numbered 2 barely came up to her waist.

"Wow..." she said, and saw a fog rising from her breath. In her awe she hadn't even realized that it was getting very cold in the basement. She looked back at Melissa, who was still standing by the doorway, shivering and staring fixedly down the hallway. "Are you ok?"

Melissa pointed. "You didn't see that yet?"

Emmy looked. There was another wall blocking the opposite end of the hallway, and in the distance Emmy could make out words written on the wall in bright violet ink. "You have made it this far. Continue on!" with an arrow pointing to the left, to what appeared to be a door on the wall.

The door, however, didn't seem to be touching the ground.

"Let's go take a closer look," said Emmy. The girls walked, wide-eyed, to the end of the hallway. Their footsteps rang out in the empty silence. Emmy gripped her notebook more tightly under her arm.

They reached the opposite wall, and found that there was indeed a door– a small, square door that looked just barely big enough for an average-sized person to crawl through... up near the ceiling. There was no staircase or ladder leading up to the door, and the girls certainly couldn't reach it from the ground, even if they jumped. A black number 12 was written on the wall underneath the door.

"What's this?" said Melissa. There was some sort of bracket on the ground in front of the door, several feet away from the wall, that appeared to be made of the same rough material as the stands that were holding the wooden beams. There was also a number 5 written on the ground between the contraption and the wall.

“It’s bolted to the ground,” noted Emmy, “And up at the bottom of the door is something similar, bolted to the wall.” There was also a bracket of some sort underneath the door.

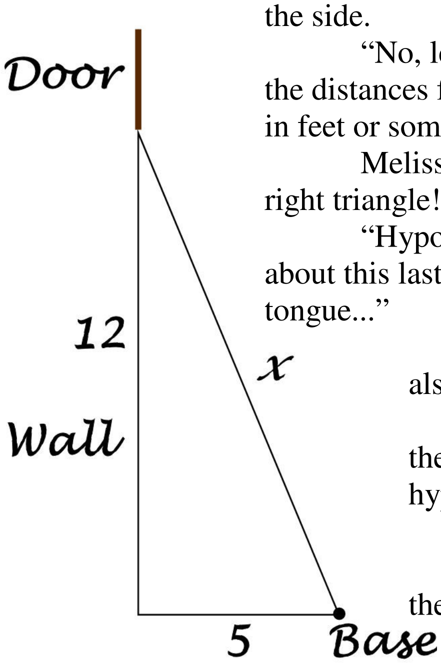
Emmy looked at the entire setup, from the inaccessible door to the bracket on the ground to the sea of vertical wooden posts filling the hallway, and realized that there might be a way up to the door after all. “If we could find a wooden beam that is just the right length, from this base on the ground,” she pointed to the bracket bolted to the ground, “diagonally up to the other bracket just below that door, we could probably climb up it safely, without it sliding out from underneath us! They even have those grooves that we can use as handholds.”

Melissa looked at the beams warily. “Are you sure one of those beams would be strong enough to hold us? And what’s to prevent it from slipping sideways? These bracket things don’t look like they’ll hold one of the beams all that tightly in place.”

Emmy looked closer at the brackets, thinking. She walked over to the nearest beam, measured its width against her hand, and used the width to measure the bracket on the ground. “It looks like this base is wide enough to hold exactly three beams. If we put all three in place, they probably wouldn’t slip from side to side. So now we just need to find three beams of the right length.”

Melissa looked around. “Do you see a tape measure or a yardstick anywhere? These beams are all different sizes. How are we going to figure out which ones are the right length? I guess we could start trying them...” She started examining the posts, trying to estimate which one would be a good fit to make a ramp up to the door.

Emmy thought there was more to it than that. These numbers must mean something… She flipped to a new page in her notebook and sketched the wall, the floor, and the ramp from the side.



Emmy’s sketch.

“No, let’s think about this,” said Emmy. “I’ll bet these numbers 5 and 12 are the distances from the contraption to the wall, and from the floor to the door, maybe in feet or something.”

Melissa looked at her diagram. “Hmm, with the ramp in place, it forms a right triangle! Isn’t there a way to figure out the hypothesis from the legs?”

“Hypotenuse,” corrected Emmy, “And yes, I think Miss Wright taught us about this last year. It’s called... ah, I can’t remember the name, it’s on the tip of my tongue...”

“The Pythagorean theorem?” said Melissa. “I think Mr. Wheel also reviewed it at the beginning of the year.”

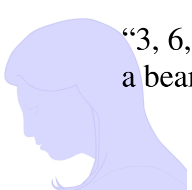
“That’s it!” said Emmy, “I remember it now. In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse. So in our case, if the ramp has length x, then...”

She wrote on her paper: $x^2 = 5^2 + 12^2$.

“Ah,” said Melissa, “5 squared is 25, and 12 squared is 144, so they add to 169.”

“Which is the square of 13!” Emmy finished. “So x is 13. We need three beams of length 13. Let’s find them!”

She headed down the aisle, reading the numbers off the first row. “3, 6, 9, 12, hmm, it looks like there’s one missing here.” Emmy looked down on the spot where a beam should have been. On the ground was written:



0

15 is too big!

"Strange," muttered Emmy. She shook her head and resumed walking. "3, 6, 9, 12, 0 again, with the same message: '15 is too big'. It looks like the heights are counting by 3, but then they stop at 12 and repeat."

"Let me see if I can see a pattern in the second row." Melissa started to walk down the next aisle. "1, 6, 11, 1, 6, 11 again... they're counting by 5's, but again they're not going too far before they repeat. Hey!" she exclaimed suddenly. "On the top of the beams of height 1, there is that same message– '15 is too big'!"

Emmy looked at the beam of height 1 Melissa was pointing to, and looked up at the longer poles in the room. She remembered the pole of height 14 that almost touched the ceiling. "I bet this ceiling's 15 feet high, and that's why 15 is too big. So you couldn't continue to count by threes in the pattern 3, 6, 9, 12, 15, because a pole of height 15 wouldn't fit in this room... wait a minute!" Something about the sequences of heights was nagging at her. She began to pace back and forth. "What's 11 + 5?"

"16," said Melissa, looking at her friend in anticipation. When Emmy started asking easy questions like that, she knew it meant that her friend just had a great idea.

"Of course, which is 1 mod 15! These are arithmetic sequences taken mod 15!"

"Huh?" said Melissa. "What's mod?"

"It's a different way of doing arithmetic," said Emmy, "when you're only dealing with numbers less than a certain special number called the modulus, in this case 15."

"What do you mean, a different way of doing arithmetic?" asked Melissa. "I never heard of this."

"I read about it in a book once. Adding and multiplying mod 15 is similar to adding and multiplying the usual way, except when you're done, you have to take the remainder when you divide it by 15 to see what's left over. This way, your answer always comes out to be less than 15."

Melissa shook her head. "You lost me."

Emmy took out her notebook. "Here, let's look at your sequence: 1, 6, 11, 1, 6, 11, and so on. The sequence is formed by adding 5 to the previous number each time, but taken mod 15. So 1 + 5 is 6, 6 plus 5 is 11, and 11 plus 5 is 16, but 16 is too big. You can't fit a pole of height 16 in this room, so instead, we take the remainder when we divide it by 15, which is..."

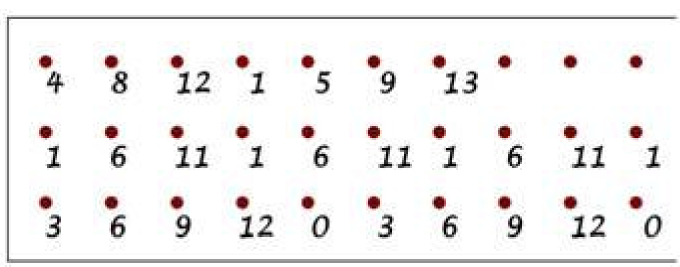
"Oh, it's 1," said Melissa. "So you're saying we basically just wrap around when we get to 15. We count like this: 11, 12, 13, 14, 0!" She giggled and continued, "1, 2, 3, 4..."

"Exactly!" said Emmy. "And it works for the first sequence too. We count by 3's starting from 3, and get 3, 6, 9, 12, 0, 3, 6..."

"Ok, ok, I get it," said Melissa. "So like, 14 + 3 is 17, but if we're doing this new arithmetic mod 15 it would be 2, because 17 is two too high."

"Right," replied Emmy. "More importantly, it means that neither of these rows is going to have any 13's in them." She started sketching the layout of the poles on her notebook. "Can you tell me what's in the third row?"

Melissa started reading the numbers. "4, 8, 12. This one's counting by 4's! 1, 5, 9, 13– here's a 13!" Emmy stopped filling in the numbers on her sketch and ran over to see. "Then if we walk 15 more posts down in this row, we should find another 13."



"How do you know that?" asked Melissa.

"Well, if we add 4's to 13 fifteen times, that's the same as adding 4×15 to 13. But remember, we wrap around every 15."

"Ah, right, because adding 4×15 is the same as if we added 15 four times, and each time we just get back to 13."

"That's another way of saying it," said Emmy. They looked at each other, high-fived for figuring out the pattern, and headed down the aisle, counting out 15 posts. Sure enough, they found another beam of height 13. They figured that, 15 more posts down, they would find the third beam that they needed.

"Ok, let's take this one to the door and see if it fits," said Melissa. The girls worked together to lift the heavy beam.

"Don't let it hit the others!" said Emmy. "We don't want to play dominoes with these things."

"Ow, Emmy, you're stepping on my foot."

"Sorry!"

They managed to lift the post and turn it sideways, and each carried one end of it as they made their way carefully to the elevated doorway. "Make sure the grooves are facing up so that we can climb it," said Emmy, and they inserted one end of the post into the left end of the base. The top of the beam fit perfectly against the left side of the bracket underneath the door.

"Success!" exclaimed Emmy. "Let's go get the next one."

They carefully picked up and turned another beam numbered 13, carried it to the door, and put it into place. The grooves in the two beams lined up perfectly. "Two down!" said Melissa, rubbing her sore fingers and arms. "On to the third."

They walked a long way down the corridor to the third beam numbered 13. "Ready?" said Emmy.

Pop.

Emmy turned to Melissa. "What did you say?"

"I didn't say anything," said Melissa.

"Then what was that sound?"

Pop! Th-thud.

They looked towards the sound, which came from the south end of the hallway. One of the smaller wooden beams appeared to have popped out of its stand and landed on the floor.

Pop!

Another came out, on the same end of the hallway. *Pop! Pop! Crash!* One by one, the posts were ejected from their bases, starting from the far end and getting closer...

"Run!" yelled Emmy, and they sprinted towards the wall with the tiny door. The crashes became louder behind them. They hit the wall, safely away from the falling forest of beams. They looked back to see the rest of the beams falling on top of one another in a heaping mess of tangled wood.

A few moments later and the hallway was silent again. Her heart racing, Emmy approached the mass of wood and shook her head. "There's no way we're digging out that number 13, let alone carrying it out of this mess."

"Let's just try to climb up to the door with two of the beams in place," said Melissa, her voice shaking. She took a deep breath and walked over to their two-thirds-complete ramp. "We're small and light– it should hold us."

She pushed the second beam to the right end of the bracket, and placed her fingers in the first set of grooves on each beam. She climbed up cautiously. The beams wobbled a bit, but stayed in place.

"Be careful," said Emmy. She walked to the base of their makeshift ladder, ready to climb up behind her friend. Melissa turned the knob and opened the door. "Emmy, you have to see this!"

TO BE CONTINUED...

Editor: Grace Lyo

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

<u>Session 6 – Meet 1 – January 28, 2010</u>

Mentors: Lauren Cipicchio, Ariana Mann, Jennifer Melot, Lauren McGough
Maria Monks, Charmaine Sia, Liz Simon, Amy Tai, Rediet Tesfaye

An underlying theme of session six is to be complete and thorough. So, our ice-breaker activity asked members to determine the complete contents of well-defined sets. Sometimes, these sets were subsets of the members themselves. In this way, members had to learn more about each other to complete the task.

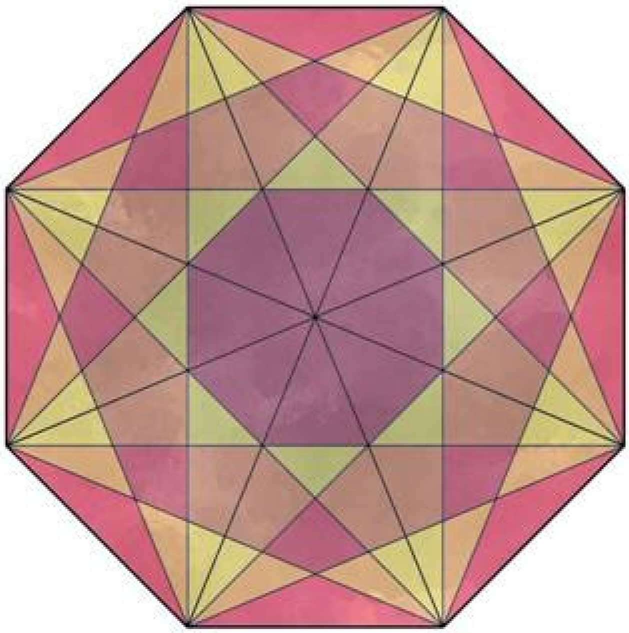
Caitlin and **billy-bob-joe-bob-jim** determined the set of prime numbers between 100 and 200 by using and ancient method known as the sieve of Eratosthenes.

It can become very challenging to identify every element of a set and there are many well-defined sets in mathematics whose contents are not yet understood. In fact, if you could determine the exact contents of every well-defined set, you would be able to solve every problem in mathematics!

Because sets occur in all branches of mathematics, the meets so far this session have been characterized by a great diversity of mathematics, including number theory, combinatorics, algebra, logic, arithmetic, and geometry.

Session 6 – Meet 2 – February 4, 2010

Mentors: Lauren Cipicchio, Rachel Fraunhoffer, Grace Lyo, Ariana Mann, Lauren McGough, Charmaine Sia, Liz Simon, Rediet Tesfaye, Drew Wolpert, Julia Yu



Some girls determined the number of intersections formed by all of the diagonals of a regular octagon. How many are there? How many regions are there? How many different kinds of regions are there? How many regions are triangles? How many are quadrilaterals? What is the answer to the analogous problem for other regular polygons?

The diagram at right shows all the diagonals of a regular octagon. Most interior intersections result from the intersection of two lines, but there appear to be some points where three diagonals are concurrent and one place where all four diagonals intersect. Are those places *really* places where more than two diagonals intersect? If you make a very, very accurate drawing, would you discover, for example, that the places where it seems that three diagonals intersect are in fact places where you will find tiny triangles?

What angle measures occur in this picture? What are the lengths between various intersection points?

Send in your analysis of other regular polygons to girlsangle@gmail.com!

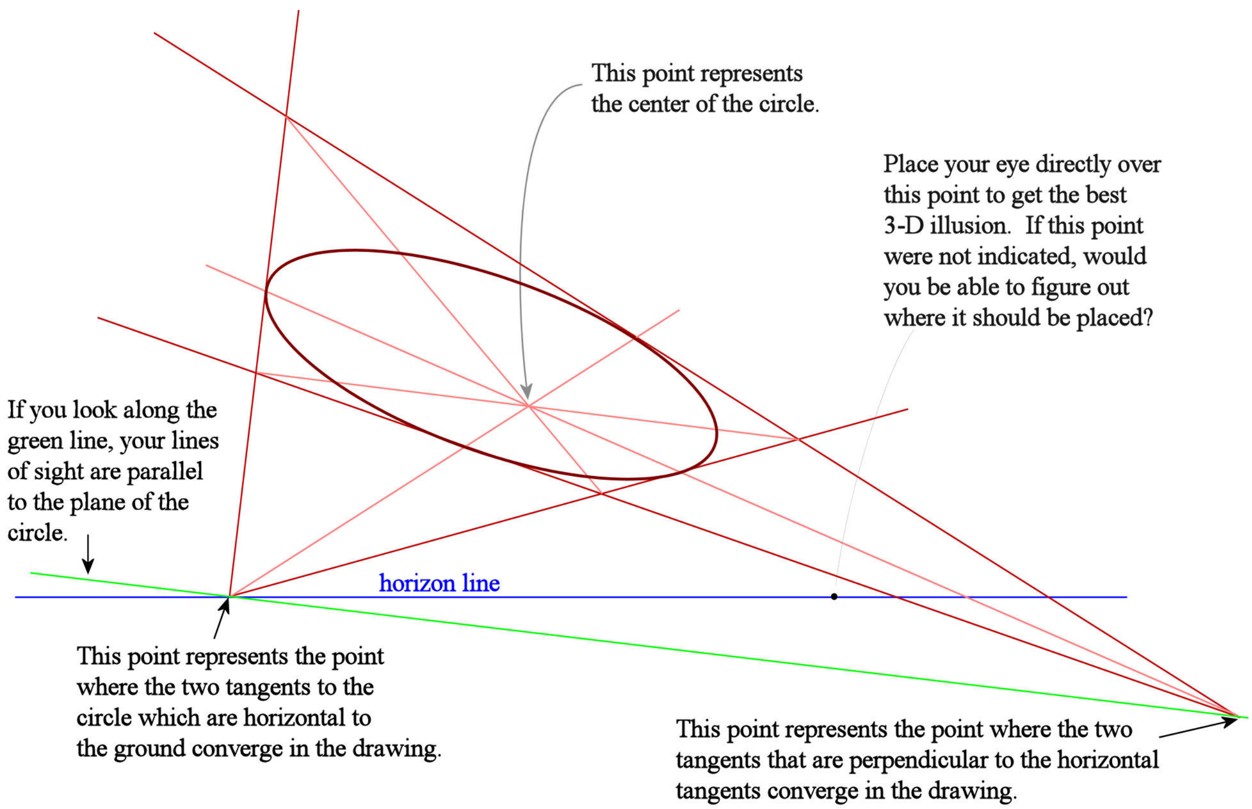
Session 6 – Meet 3 – February 11, 2010

Mentors: Lauren Cipicchio, Rachel Fraunhoffer, Grace Lyo, Ariana Mann, Jennifer Melot, Maria Monks, Charmaine Sia, Liz Simon, Rediet Tesfaye, Fan Wei, Julia Yu

Billy-bob-joe-bob-jim showed a knack for counting paths in networks of one-way streets. At the suggestion of Maria, she created her own path counting problem. See page 8.

There has been an ongoing perspective drawing station at Girls' Angle. Understanding the math behind perspective can help one create more realistic drawings. Observe just how accurately Dutch painter Jan Vermeer (1632-1675) constructs his drawing in the painting *The Music Lesson* (at right). Five lines that represent parallels in the real world all converge at a vanishing point.

modified from en.wikipedia.org/wiki/File:Jan_Vermeer_van_Delft_014.jpg



Above is a drawing that depicts a circle tilted and then rotated about a vertical axis with respect to the viewer. To maximize the illusion, you have to place your eye very close to the page. The viewing distance is critical. Change this distance and all aspects of the drawing must change with it. Do you see why?

The reason why the eye must be very close to the page is because I wanted to depict the vanishing point of tangents that are horizontal to the ground as well as that of the tangents perpendicular to the horizontal tangents. Those two vanishing points correspond to directions that are at right angles to each other. This means that your eye must be located at a point where it rotates through 90 degrees when it swivels from one of these vanishing points to the other.

To make realistic drawings of three-dimensional scenes, you must adhere to the principles of perspective geometry.

<u>Session 6 – Meet 4 – February 25, 2009</u>

Mentors: Lauren Cipicchio, Grace Lyo, Lauren McGough, Jennifer Melot, Maria Monks, Charmaine Sia, Amy Tai, Rediet Tesfaye, Julia Yu

Special Visitor: Jericho Bicknell, Waltham Fields Community Farm

Jericho introduced the club to some of the economics involved in running a farm. She focused on carrots. She talked about the importance of keeping track of worker hours, seed costs, crop output, weeds, and so forth. Each item translates into a gain or loss. She led the girls through an involved computation that showed whether the farm made money or lost money with their carrots. In this situation, math is essentially being used to help with counting something difficult to keep track of.

Solving a Quartic Equation

by **Ilana**

At the third meet of session six, some of the girls worked on finding the zeros of functions. One of the functions was the following quartic in two variables:

$$f(x, y) = x^4 + 2x^2y^2 + y^4 + 32xy + 64.$$

Can you solve this problem? On this page we present **Ilana**'s solution in her own words.

We start with the equation $x^4 + 2x^2y^2 + y^4 + 32xy + 64 = 0$.

We know that $(x^2 - y^2)^2 = x^4 - 2x^2y^2 + y^4$ because in the expression $(x^2 - y^2)(x^2 - y^2)$, you multiply each term in the first set of parentheses by each term in the second set of parentheses:

$$\begin{aligned}(x^2 - y^2)(x^2 - y^2) &= x^2 \cdot x^2 - x^2 \cdot y^2 - x^2 \cdot y^2 + y^2 \cdot y^2 \\ &= x^4 - 2x^2y^2 + y^4\end{aligned}$$

So, the original equation can be written

$$(x^2 - y^2)^2 + 4x^2y^2 + 32xy + 64 = 0.$$

All of the terms after the first term are multiples of 4, so this equation can be written

$$(x^2 - y^2)^2 + 4(x^2y^2 + 8xy + 16) = 0,$$

or, since $(xy + 4)^2 = (xy)^2 + 4xy + 16$,

$$(x^2 - y^2)^2 + 4(xy + 4)^2 = 0.$$

Now, we know a square number can never be negative. So both terms (on the left side of this last equation) must be zero.

For the first term, $(x^2 - y^2)^2 = 0$, so $x^2 - y^2 = 0$ and so $x^2 = y^2$. From this last equation, we see that

$$|x| = |y|.$$

From the second term, $4(xy + 4)^2 = 0$, we know that $(xy + 4)^2 = 0$, so $xy + 4 = 0$. Thus,

$$xy = -4.$$

The only two ordered pairs that satisfy both $xy = -4$ and $|x| = |y|$ are (-2, 2) and (2, -2).

Calendar

Session 5: (all dates in 2009)

September	10	Start of fifth session!
	17	Tanya Khovanova, mathematician
	24	
October	1	
	8	Katherine Paur, Kiva Systems
	15	
	22	Jane Kostick, wood worker
	29	No meet
November	5	
	12	JJ Gonson, Cuisine En Locale
	19	Allie Anderson, MIT Aeronautics and Astronautics
	26	Thanksgiving - No meet
December	3	Meg Aycinena Lippow, MIT CSAIL
	10	

Session 6: (all dates in 2010)

January	28	Start of sixth session!
February	4	
	11	
	18	No meet
	25	Jericho Bicknell, Waltham Fields Community Farm
March	4	
	11	
	18	Elissa Ozanne, Harvard Medical School and MGH
	25	No meet
April	1	
	8	
	15	
	22	No meet
	29	
May	6	

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) is free for members and can be purchased by others. Please contact us if you'd like to purchase printed issues.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 2 ways: **membership** and **active subscription** to the Girls' Angle Bulletin. **Membership** is granted per session and includes access to the club and extends the member's subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. **Active subscriptions** to the Girls' Angle Bulletin allow the subscriber to ask and receive answers to math questions through email. Please note that we will not answer email questions if we think that we are doing the asker's homework! We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply. Note that you can receive the Girls' Angle Bulletin free of charge. Just send us email with your request.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For: ☐ Membership
☐ Active Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

☐ Enclosed is a check for (indicate one) (prorate as necessary)
  ☐ $216 for a one session membership ☐ $50 for a one year active subscription
  ☐ I am making a tax free charitable donation.

☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

Girls' Angle

A Math Club for Girls