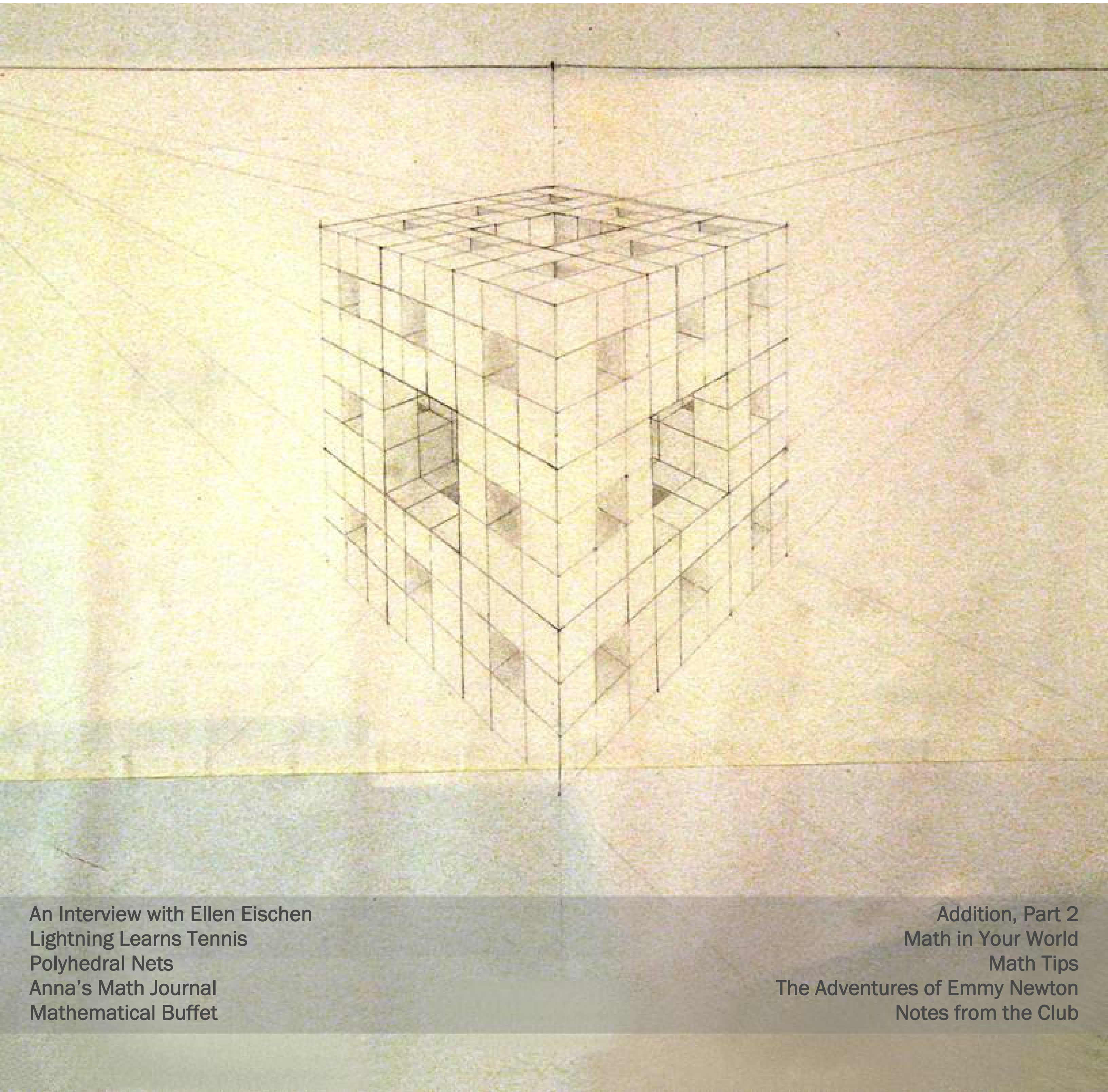
Girls' Angle Bulletin

April 2010 • Volume 3 • Number 4

To Foster and Nurture Girls' Interest in Mathematics



From the Director

A number of parents have asked if Girls' Angle will continue during the summer. We do hope that some day, Girls' Angle will be open all year round.

But, in the meantime, remember that members can communicate with us throughout the summer. Feel free to write to us about anything related to math! You can even ask us for math problems. We'll be very happy to give you some! In fact, our June issue will have a new batch of Summer Fun problem sets, and, as with any math problem, members are encouraged to submit their solutions. Members should feel free to contact us about mathematics at any time, not just during the summer.

The sixth session of Girls' Angle is drawing to a close. It was quite a special session. We had more members than ever before, and each meet was filled with more math than ever before. We also had more mentors per meet than ever before, so we were able to average a little less than two students per mentor. Let's carry this momentum through the summer and into session seven!

All my best,
Ken Fan
Founder and Director

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Girls' Angle Bulletin
The official magazine of
Girls' Angle: A Math Club for girls
Electronic Version (ISSN 2151-5743)

girlsangle@gmail.com

This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Level 2 Menger Sponge* by **Rowena**. Place your eye 5 ⅓" over the center point on the horizon line for a perfect perspective view!

An Interview with Ellen Eischen

Ellen Eischen is Boas assistant professor of mathematics at Northwestern University. She presented a proof of the infinitude of prime numbers in the second Girls' Angle Women In Mathematics video which can be viewed on our website.

Ken: Hi Ellen, thank you for making our second WIM video and for agreeing to do this interview.

Ellen: I think that interviewing mathematicians is a great idea. I look forward to reading the interviews of other mathematicians.

Ken: You seem to love prime numbers! Was thinking about prime numbers what got you into mathematics?

Ellen: Yes. In fifth grade, we learned about prime numbers in math class. I had a hard time with the definition at first, because I thought that there should be a pattern. In fifth grade, I was so stressed about not finding the simple pattern I assumed existed that I actually cried the night before the test on prime and composite numbers! I was also confused about unique factorization, which seemed surprising to me. I kept thinking about properties of numbers in general for years after that. To go to sleep at night, I would try to answer my own questions about numbers, which felt disjoint from the math classes at school. Sometimes, though, I would stay up late with a flashlight under my bedspread, trying to prove things about numbers.

Ken: In the WIM video, you showed a page from the notebook you kept in high school [see page 6]...Do you still keep notebooks like that today?

Ellen: Yes, I do have a math journal, but it is completely different from the notebook I kept in high school.

In high school, I wanted to hide my mathematical interests from my peers, because I wanted to fit in. So my journal was a place I wrote about things (like math!) I would not dare discuss with anyone else.

Ask questions. Be organized. Write down as precisely as possible what you are trying to figure out. Get lots of sleep.

By the end of my sophomore year at Princeton, I had stopped writing about math in my journal. By that point, I was spending lots of time on math in my classes. Also, as a math major, I obviously was not trying to hide the fact that I liked doing math. Furthermore, I found I had to think about something other than math to go to sleep, since I spent lots of time on it everyday at that point.

Just before I began to write my dissertation, someone suggested I keep a math notebook to record any progress I made each day. Now, I type my mathematical thoughts and progress in a journal on my computer. I have literally hundreds of pages of typed journals on my computer. My journals now are highly organized. Since I type my journal, it's easy for me to search for something six months later to recall how I proved something. In addition to entries recording my daily progress, I have a section containing all the problems I think might be interesting to work on in the future and ideas I have about them. When I have an idea for a problem, I add it to that section. My journals also include notes from meetings with my advisor, the questions I had about those meetings, the questions I planned to ask him, and his answers to those questions. I have a section containing particularly helpful advice I have received. I also have lots of sections with data that would probably be boring to an outsider but is useful to me.

I generally do not share my journals with people, though.

Ken: Do you find mathematics easy?

Ellen: I found mathematics easy through my first year of calculus. (However, the mathematics I did in my journal was hard, in the sense that I would think about problems for weeks or longer without necessarily arriving at an answer.)

Don't be afraid of what people think of you, and don't compare yourself to others too much.

Since then, though, I have found it challenging. In college, it took me a while to realize that one needs to work hard to do well in mathematics and that the fact I found it hard did not mean I had suddenly become bad at mathematics.

Ken: Do you have any advice for how best to learn mathematics?

Ellen: I think I will be figuring this out for the rest of my life. However, here are some things I find useful: Ask questions. Be organized. Write down as precisely as possible what you are trying to figure out. Get lots of sleep. Sleep is important for doing mathematics.

Ken: Can you explain to us what your first published theorem is about?

Ellen: My first published theorem that I proved independently was in the field of graph theory. I did this the summer after my junior year of college, while attending a research program for undergraduates. The problem I studied has applications to airline scheduling.

Basically, my theorem classified certain graphs[1] (certain "tripartite graphs", for those who know some graph theory) that can be pulled apart into two smaller graphs that look the same and contain a path of a specified length.

Ken: When you get stuck on a problem, what kinds of things do you do to try to get unstuck?

Ellen: I write down the problem as precisely as I can, and I write down precisely what I *do* understand. I then try to refine the question to make it even more specific.

If I'm still stuck, I might talk to someone about it. Also, I might exercise or think about something else for a while in case my brain just needs a break.

Ken: Have you ever made mistakes in math?

[1] The "graphs" that Ellen refers to here are networks of nodes with connecting edges.

Ellen: Yes, but none of them are particularly interesting stories.

Ken: What is one of the most memorable experiences you have had in mathematics?

Ellen: I had never thought about this before. Probably, though, I would pick meetings with John Conway[2] during my freshman spring at Princeton. He spent several hours meeting with me, teaching me about Bernoulli numbers and discussing some questions I had written in my journal in high school but had never shared with anyone. It was exciting because it was the first time I had ever seriously discussed number theory with someone.

Ken: Do you have any hobbies aside from math?

Ellen: Yes! I enjoy running, swimming, hiking, being outside, singing, cooking, and trying new foods. I also really enjoy meeting new people!

Ken: Do you have any advice for the girls that come to Girls' Angle?

Ellen: Be patient with the mathematics. It's hardest when you are stressed about it. Don't be afraid of what people think of you, and don't compare yourself to others too much. Also, ask lots of questions! Keep in mind that if someone knows more about a mathematical topic than you do, it just means they have taken a course or read a book that you haven't. Knowing about a particular topic does not make them "better at math" than you.

Ken: Are there any things that have surprised you or been different from what you expected life to be like as a mathematician?

Ellen: I am surprised by how cool people seem to think it is when they hear I am a mathematician. I would have expected the opposite response. Recently, the guy next to me on the plane asked what I did. When he found out I do mathematics, he exclaimed, "Wow, awesome!"

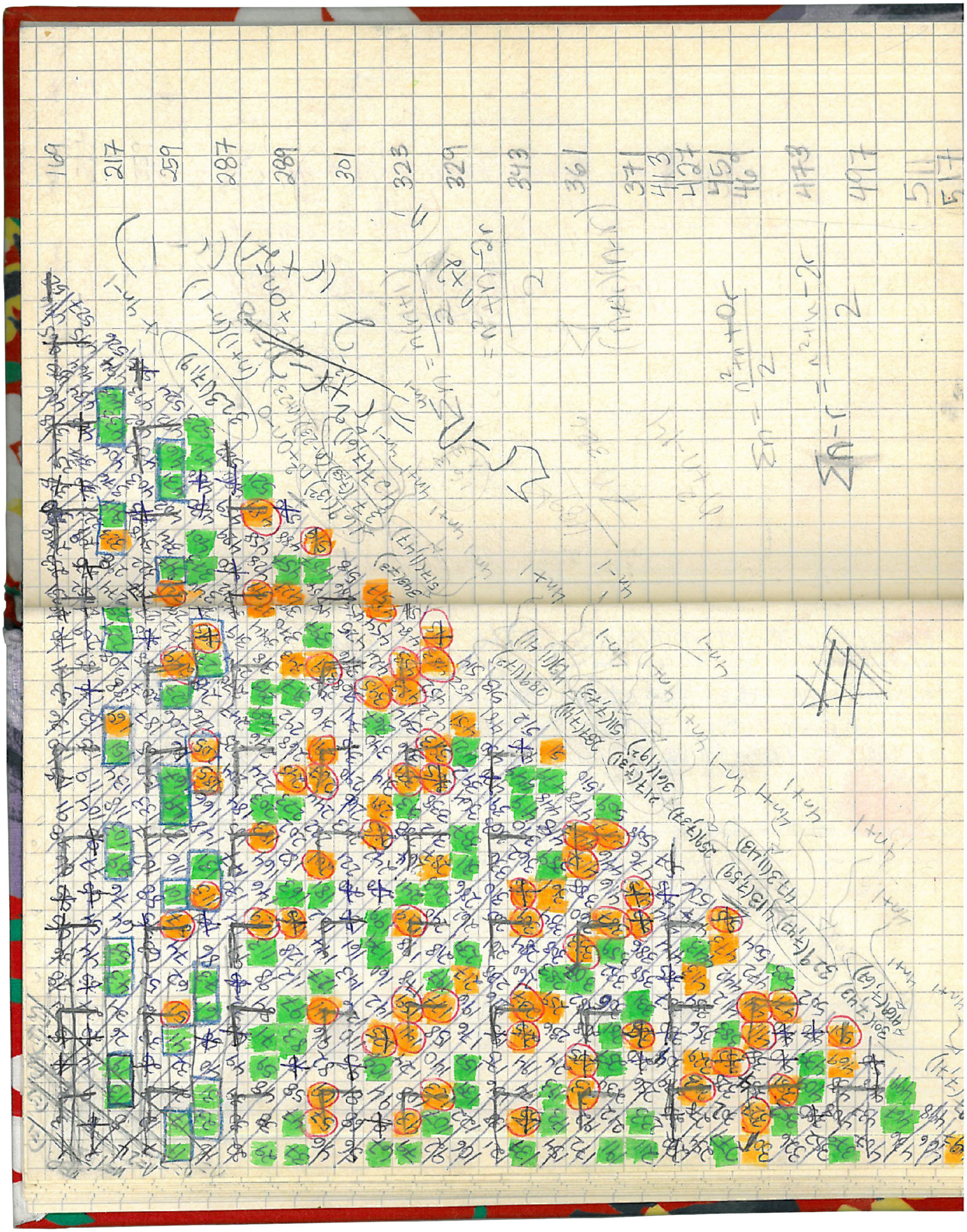
Another surprising thing is how much mathematicians travel. This year, mathematics has taken me to Copenhagen, Boston, Maryland, Hawaii, and Ontario. Early in 2010, I will go to Vancouver, British Columbia and probably Hawaii.

Ken: That's a lot of distant places! If you ever find yourself in Boston again, please consider visiting Girls' Angle!

In Ellen's Women In Mathematics video, she showed pages from one of the notebooks she kept when she was still in school. With her permission, we have reproduce them on the next page.

How are the positive integers arranged on the page? What do the green highlighted numbers represent? Are there any rows that have no green highlight in them? What do you think the orange highlighted numbers are? Send your thoughts to girlsangle@gmail.com.

[2] John Conway is a professor of mathematics at Princeton University.



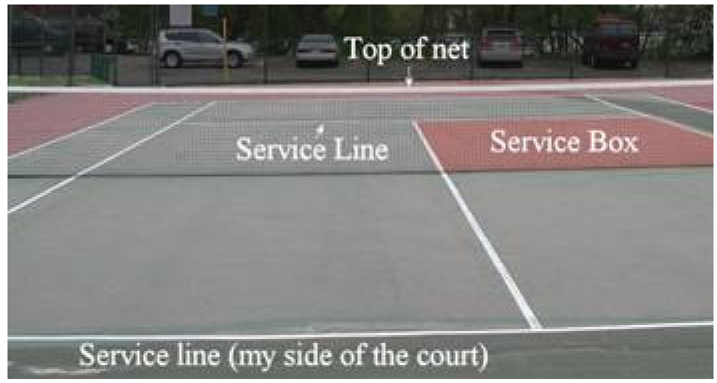
Two pages from one of the personal math notebooks of Ellen Eischen.

Lightning Learns Tennis

by Lightning Factorial

I've always wanted to learn how to play tennis. What an amazing game!

So I signed up for tennis lessons. Things were going well, until it came to the serve. What a difficult motion! What's more, from where I was supposed to stand, I couldn't even see the target, a rectangular region on the court known as the **service box**.



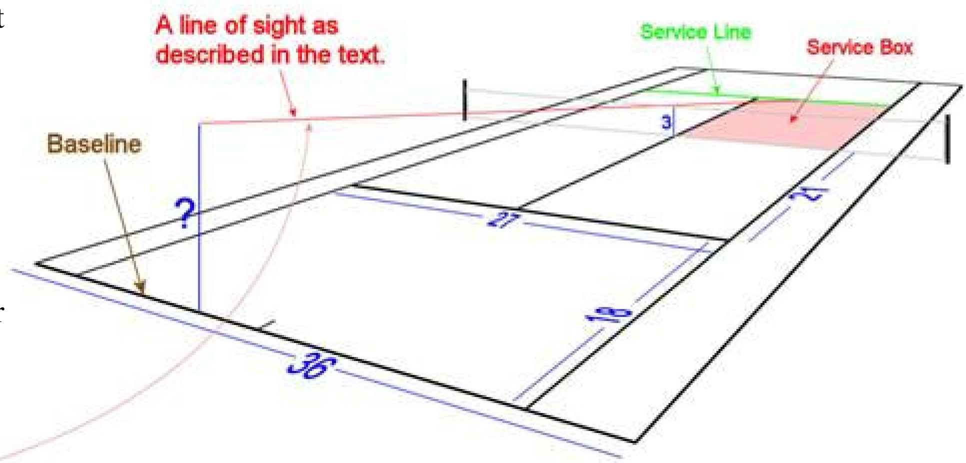
View from the baseline.

"It's behind the net," advised my coach. I looked lower, and, sure enough, there through the mesh of the net was the service box.

"Impossible!" I exclaimed. "How am I supposed to hit a target that's completely blocked by a net?" But suddenly, I was struck with an idea. From my eye level, the service box is submerged below the net, but during the serve, the ball is tossed up. Perhaps from the vantage of the tossed ball, one can see at least part of the service box above the net. Maybe, *that's* why people toss the ball and hit it as high up as they can when they serve. I decided to investigate.

First, I looked up the official measurements of a tennis court. See the diagram below.

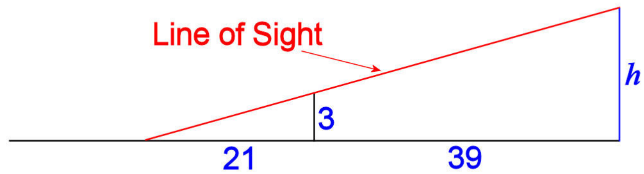
What I wanted to know is just how high you have to be before you can see the service line above the net from the baseline. It turns out that the net isn't a uniform height. It's higher at the posts than it is at the center. To make things easier for myself, I'll aim over the center of the net, where the height is 3 feet.



If I imagine a line segment that goes from a point on the service line through the top of the middle of the net all the way to a point above the baseline, how high would this endpoint hover over the baseline? This line segment represents a line of sight. If the ball is higher than the endpoint over the baseline, the ball would be able to "see" into the service box.

Each point on the service line produces a different line of sight. So, which point on the service line should I use? One common feature of all these line segments is that they all belong to the same plane, namely, the plane that contains the service line and passes through the top of the middle of the net. In other words, it's the plane of the triangle whose base is the service line and whose apex is the top of the middle of the net. Because this plane is not tilted to one or the other side of the tennis court, it doesn't matter which line segment I use to compute the height I'm interested in. All choices will produce the same answer. So I'll take the point at the so-called "T" of the service line, which is the midpoint of the service line. For brevity, I'll refer to this point as the "T" point.

Here's a side view of the situation.



Drawing is not to scale.

I want to compute the height labeled h in the diagram. Do you see how to do that?

One way is to compute the slope of the line of sight in two different ways. Recall that the slope of a line is given by the ratio of the "rise" over the "run" between a pair of points on the line. I'll compute the slope using two different pairs of points. One pair will be the "T" point and the endpoint of the line of sight above the baseline. The other pair will be the "T" point and the top of the middle of the net. Setting these slope computations equal, we get the equation:

$$\frac{h}{39+21} = \frac{3}{21}$$

Solve this and you'll find that h is $7\frac{4}{7}$ feet, which is a little less than 7 feet 7 inches. No wonder the service line looks submerged below the top of the net! Not even 6' 3" tennis pro Akgul Amanmuradova would be able to see the service line above the net from the baseline.

If you think of another way to find h, please tell us about it at girlsangle@gmail.com!

Armed with this knowledge, I measured my arm and racquet and found that I get about 2.5 feet of extra height if I reach up high with my racquet. I'm taller than 5 feet 1 inch, so I'm in luck! I can play tennis!

Wait a minute. I've seen children play tennis who are less than 5 feet tall.

"Hey, coach, I just don't get it. Even at my height, the ball can only barely see the service box during my serve…and people who are less than 5 feet…how do they serve? Do they jump?" I asked incredulously.

"Lightning? Haven't you ever heard of gravity?..."

"Err…uh…I was analyzing the game of tennis if it were to be played on the International Space Station."

"…and there's spin too!"

"Outdoor tennis on the International Space Station, to be precise..."

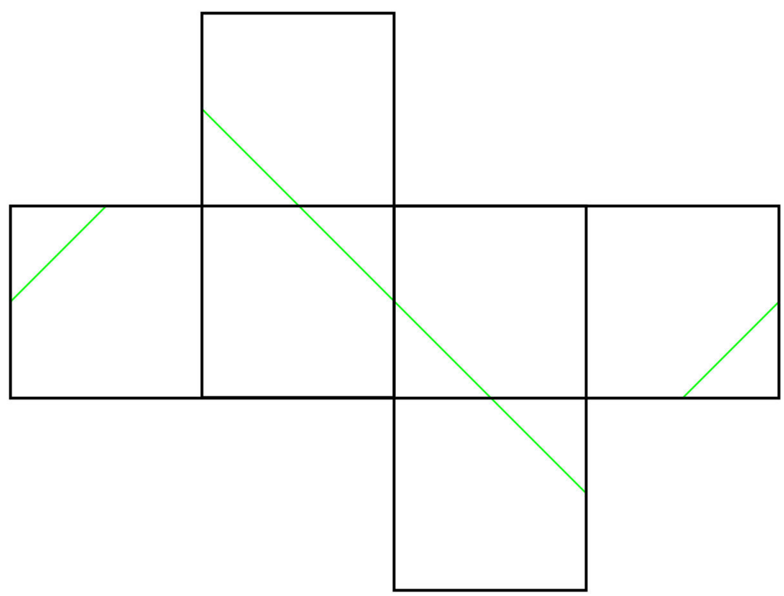
I set out to play tennis, but ended up doing math!

Polyhedral Nets

A polyhedral net can be used to build and think about polyhedra using a piece of paper.

Take a polyhedron and cut along some of its edges. Keep cutting until you can fold the faces flat. If you can do this in such a way that the result is one connected piece with no overlapping faces, then you have produced a **net** of that polyhedron.

If you're having trouble seeing the three-dimensional images induced by the stereo pairs in the Mathematical Buffet on page 14, you can use these nets to make paper models of the Platonic solids. To start, here is a net for a cube. Can you predict what will happen to the green lines before you fold this net up into a cube?



Here are some questions:

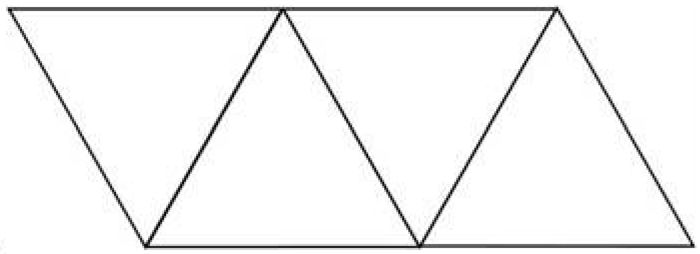
1. It's not always possible to "unfold" a polyhedron so that the result has no overlapping faces. Can you think of an example?

2. A shape is called **convex** if the line segment connecting any two points inside the shape is also contained inside the shape. For example, a triangle is convex, but a chevron is not. It has been conjectured that the surface of every convex 3D polyhedron has a net, but it has never been proven. Can you prove it?

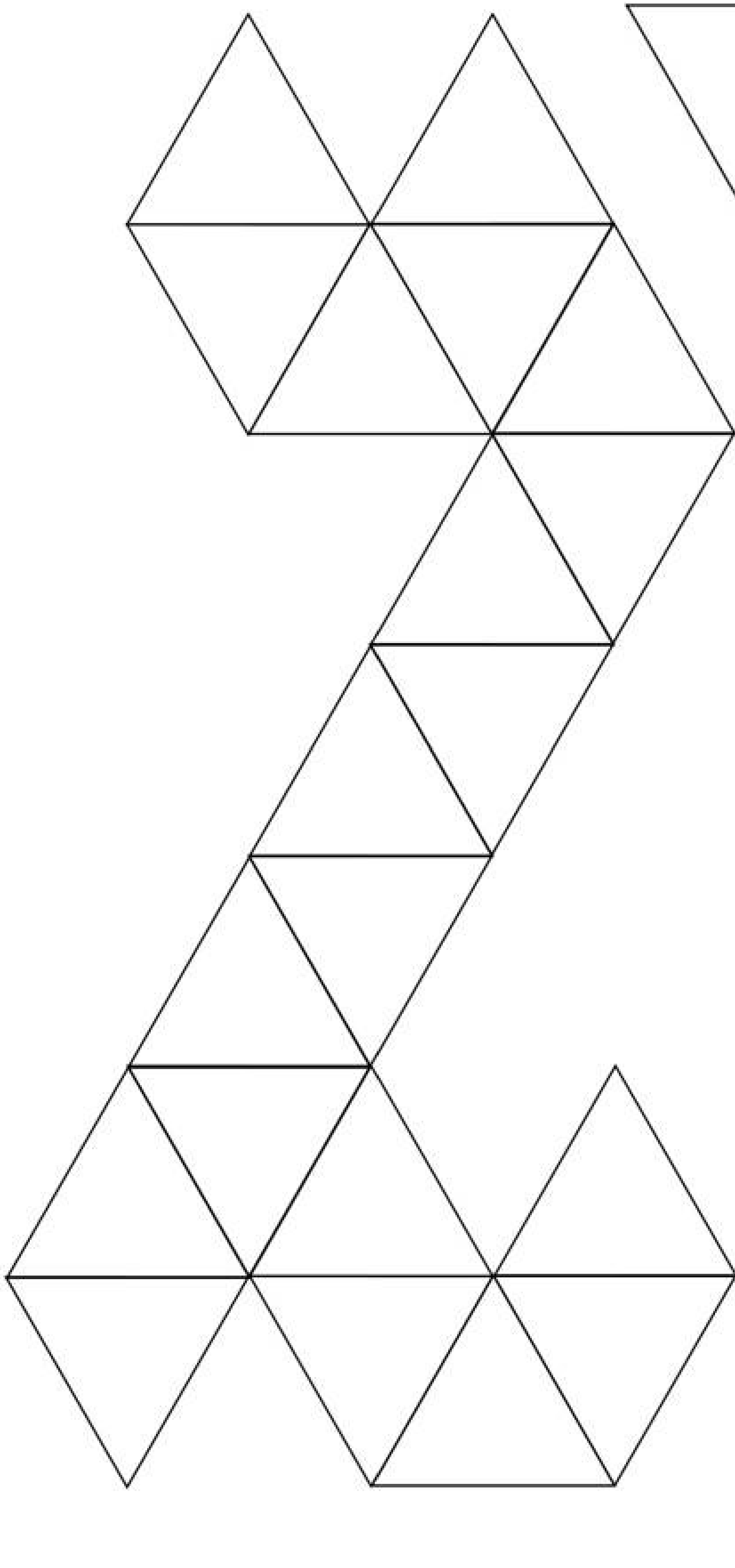
3. How many different nets are there that represent a cube? How about an octahedron?

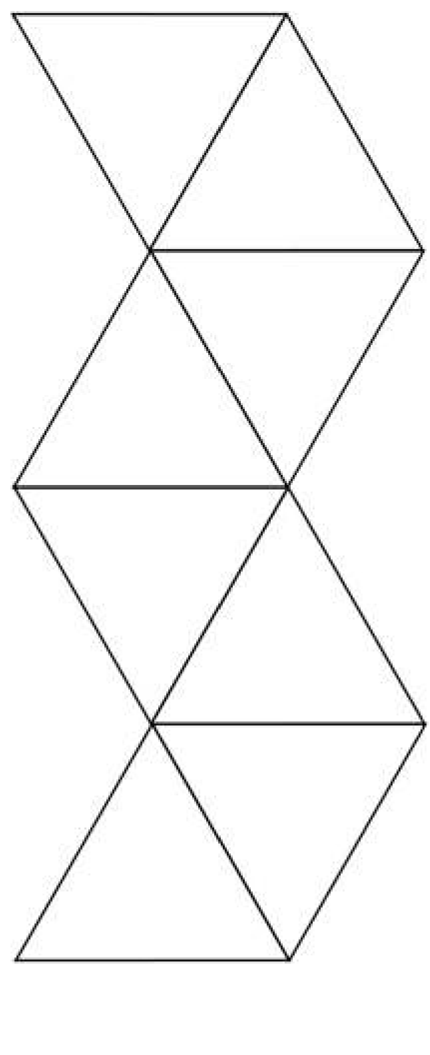
4. If you travel along the surface of a dodecahedron (see the net on page 11), what is the minimum distance between two diametrically opposed vertices?

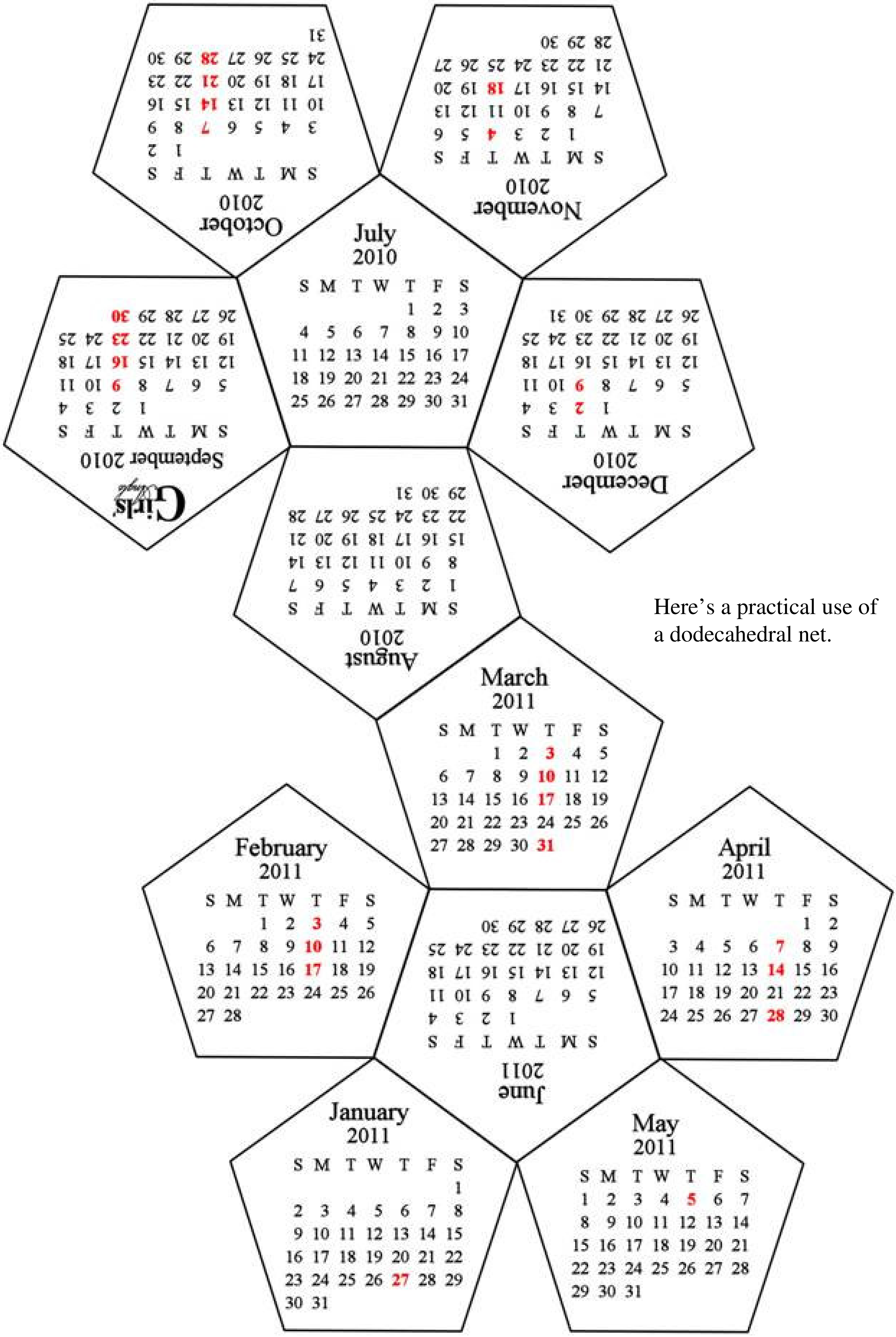
5. Does every net correspond to exactly one polyhedron? Is it possible to fold up two copies of a net and produce two different polyhedra?



More Nets







Here's a practical use of a dodecahedral net.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

In this installment, Anna tackles **Billy-bob-joe-bob-jim**'s problem from the last issue. She asked for the number of paths from the upper left to the lower right corner of her special street map when you are not allowed to cross the same street more than once.

First, I'll copy the map and try to just list all paths directly. "S" is the start and "E" is the end.

Hmm. It's pretty confusing. I need to think of a strategy so I can systematically find all the paths.

I'll try to always turn right if I can, and if I can't, I'll go straight, otherwise I'll take a left.

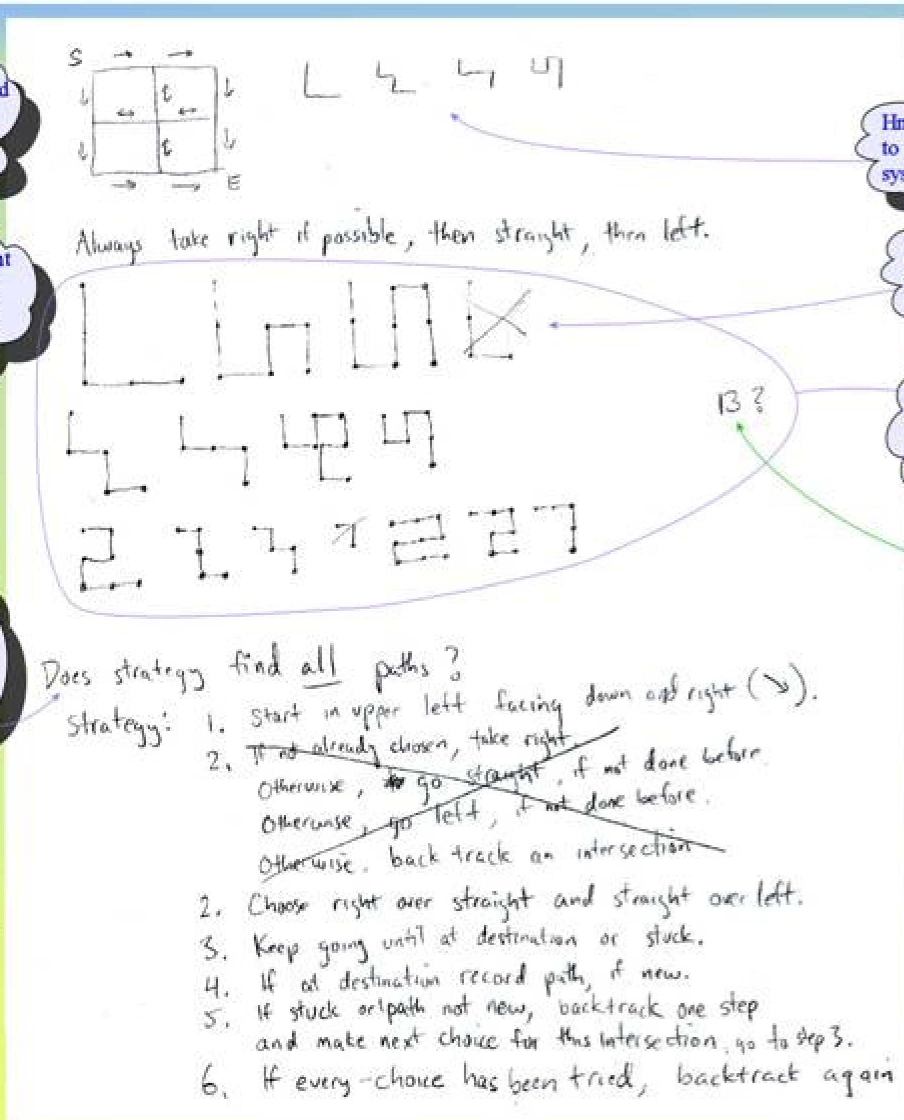
I'm crossing this out because I see there are no more paths that start off this way.

I found 13 paths, but I'm not confident in this answer. I think I should try to make the strategy more precise.

I later found this answer to be incorrect. I'm glad I didn't just go with the first answer I got! In retrospect, had I thought about the symmetry of the problem, I could have seen that the answer had to be even and that would have informed me right away that this answer was wrong. Symmetry can be so useful!

It'll be really important to know if the strategy finds every path. I better write this question down so I don't forget about it later!

Hmm. I can already tell this algorithm has problems. I'll think about it and try to fix it up.



Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

I've thought about it and am ready to rewrite the algorithm.

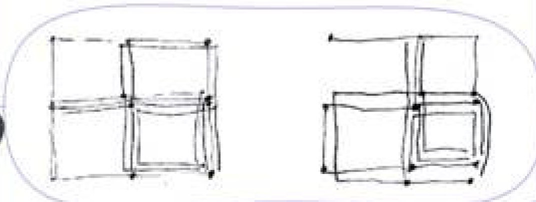
New algorithm:

1. Start at upper left facing ↘.
2. Choose a direction that obeys street directions, has not been tried before, and choose right over straight (if possible) and straight over left (if possible)
3. If no choice is possible skip to step 5.
4. Walk down choice to next intersection. If not at final destination go to step 2.
5. If at final destination, record path.
6. Backtrack one street and go to step 2.

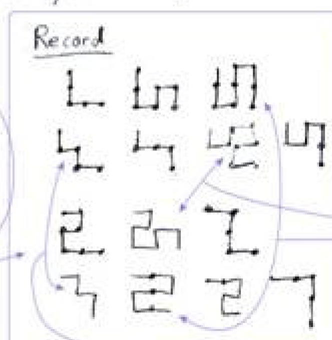
I need to come back later and think about whether this algorithm finds all the paths. It could be useful to have such an algorithm if I consider more complex maps someday. Maybe I'll even come up with a crisper algorithm too.

I forgot to say in step 6 that if you can't backtrack anymore, then the algorithm terminates.

I'll try to carefully carry out this algorithm. Here I'll put scratch work.



And here, I'll record the paths that I find.



Looking at these paths, I can see a symmetry that results from the map itself being symmetric. I'll pursue this thought in a moment. I don't know why this didn't occur to me before!

14 total?

This time, I found 14 paths! Is this correct, or did I make a mistake? And...can I be sure that this algorithm finds all of the paths?

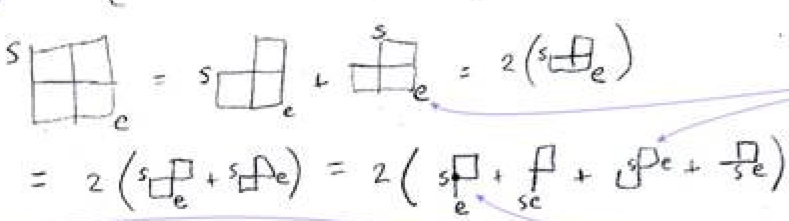
Does algorithm find all paths?

I think it does ... probably can prove by induction on the number of streets.

I think the algorithm finds all paths and that this could be proven by induction on the number of streets. I'll worry about this later, but I'll make note of it here so I don't forget.

Symmetry { paths that start ↓ } ⟷ { Paths that start → }

Here's the symmetry I noticed after finishing the algorithm. It makes me realize I can count paths according to how they begin or end.



These maps are submaps of the given map. Each map diagram here stands for the number of paths from "S" to "E", subject to the restrictions in the problem.

~~= 2(3 + 1 + 2 + 2) = 16 ?~~

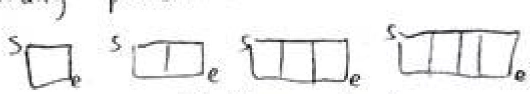
Great. Now I got 16... yet another answer! Let me double check this work here.

Oh, I see the mistake. This submap only has 2 paths, not 3, because one of the ones I counted violates the direction of certain one way streets.

= 2(2 + 1 + 2 + 2) = 14 ✓

So I get 14 again. I'm pretty confident 14 is the answer.

How many paths in:



etc ?

all 2-way streets

I wonder if there's more interesting math here. Maybe this family of maps would be interesting to study.

AB 4.17.10

Key:

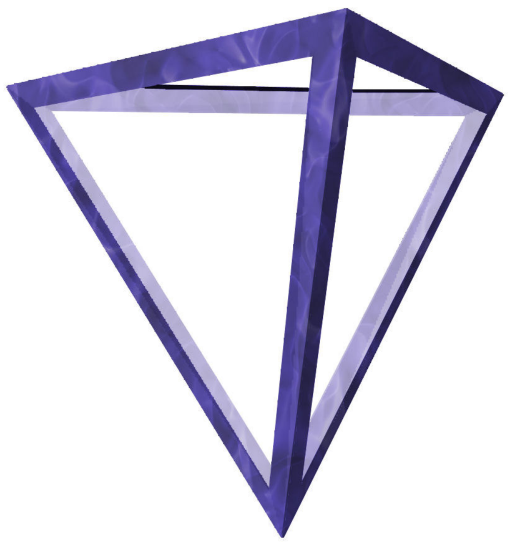
Anna's thoughts

Anna's afterthoughts

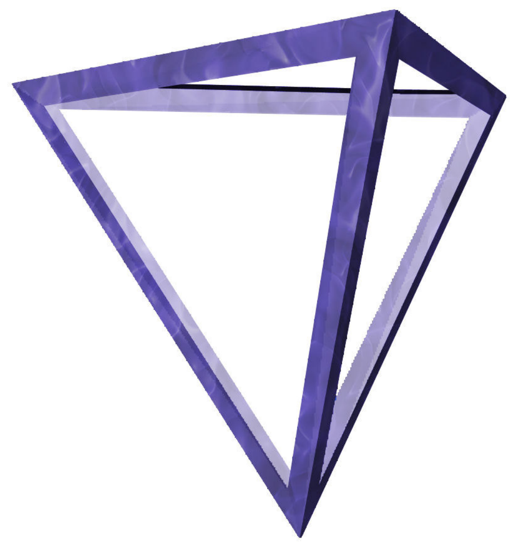
Editor's comments

Mathematical Buffet

Stereo Pairs of the Platonic Solids designed by Noah Fechtor-Pradines



Tetrahedron



Tetrahedron

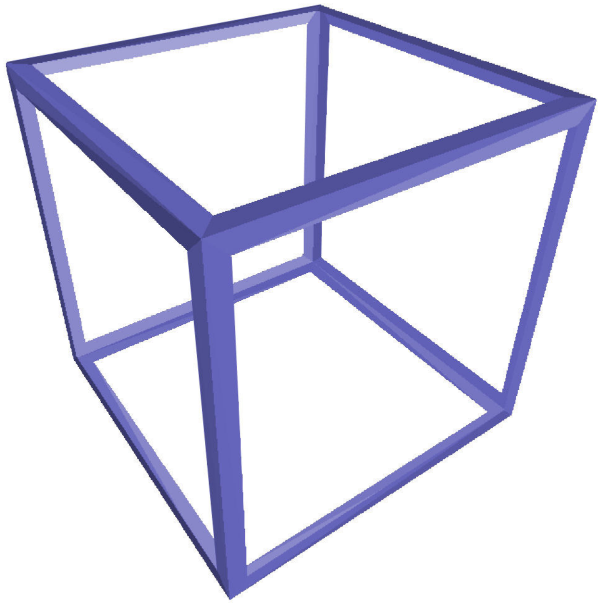
Our two eyes don’t see the same thing. Because they’re not in the same place, they each see the world from a slightly different vantage point.

In perspective drawing, everything is drawn for a viewer with one eye. So when we look at a perspective drawing, we have to choose which eye to view it with.

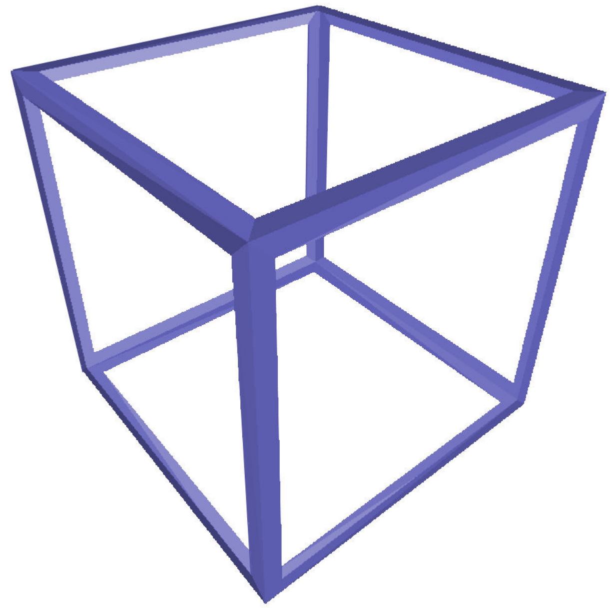
In a stereo pair, two images, one for each eye, are placed side by side. The left image shows what the right eye would see if there were a true 3D object in front of you and the right image shows the same object as it would be seen by the left eye. If you can cross your eyes so that your right eye receives the left image while your left eye receives the right, then you’ll be rewarded with the illusion of seeing an actual 3D object!

On this and the following two pages, Noah presents stereo pairs of the Platonic solids that he created for this *Bulletin*. To view them, hold the page at a comfortable viewing distance and cross your eyes so that your left eye is looking at the right image and your right eye is looking at the left image. If you’re having trouble, it sometimes helps to hold a pencil in front of the image and focus on that, and then gradually remove the pencil out of your field of view. If you’ve never seen stereo pairs before, don’t worry if it seems to take a while to get the hang of it.

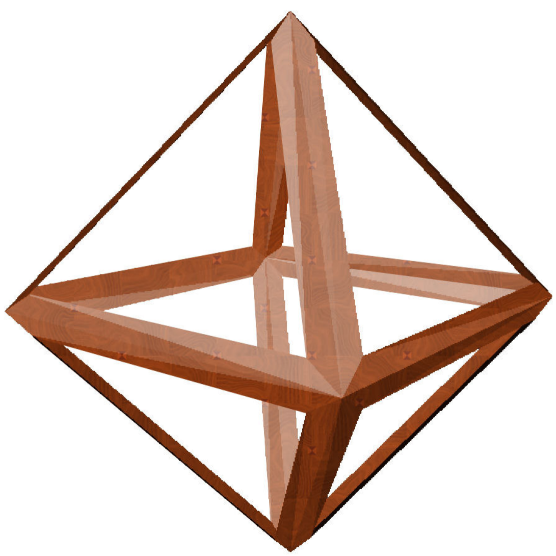
- Editor



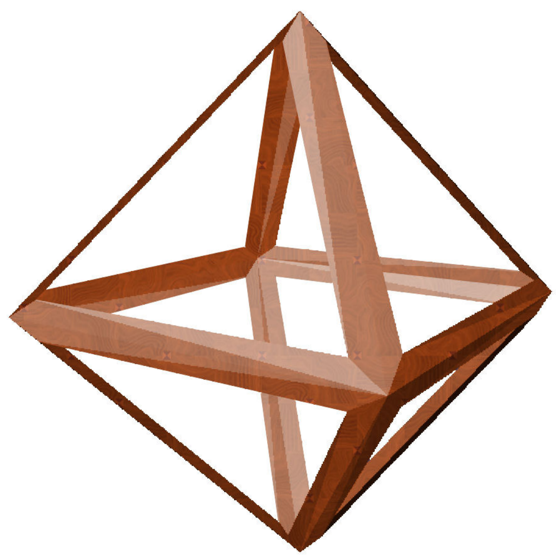
Cube



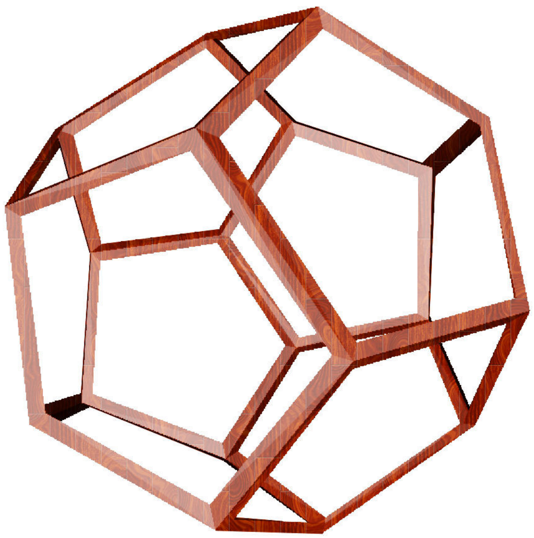
Cube



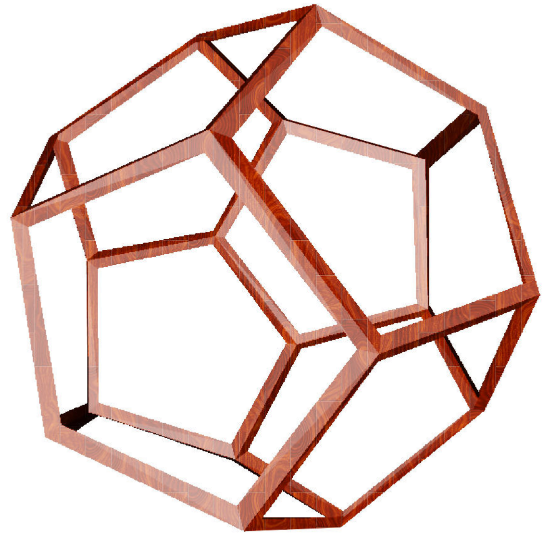
Octahedron



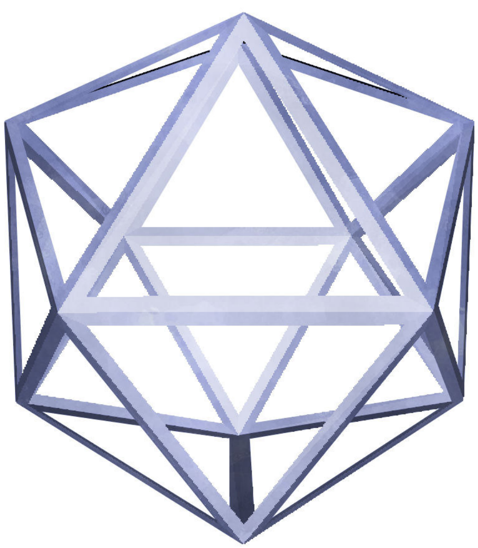
Octahedron



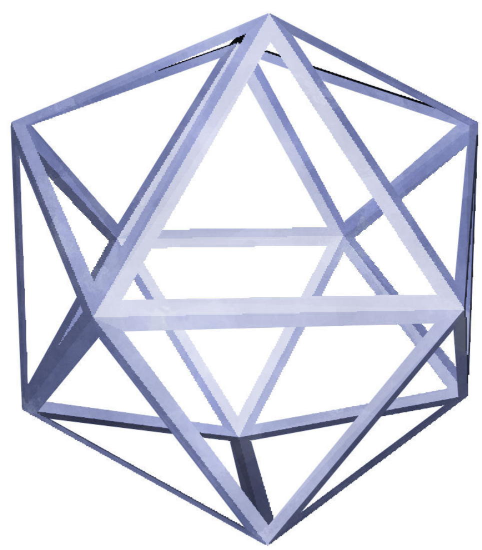
Dodecahedron



Dodecahedron



Icosahedron



Icosahedron

Addition
Part 2

by Bjorn Poonen


Photo by Toshia McCabe

Last time, you told me:

> For any numbers m and n, the expression $m + n$ means the number of elements of $A \cup B$, where A is any m-element set and B is any n-element set such that A and B are disjoint.

Yes, and you can use this to add sizes of infinite sets too.

What does that mean, the size of an infinite set?

You need to introduce new "numbers", called **cardinal numbers**, for measuring the sizes of infinite sets. For example, the size of the set

$$\{0, 1, 2, 3, 4, \ldots\}$$

of nonnegative integers is called $\aleph_0$ ("aleph-zero"). Any other set that can be matched up with this set also has size $\aleph_0$.

Wait. The set

$$\{0, 2, 4, 6, 8, \ldots\}$$

can be matched up with your set $\{0, 1, 2, 3, 4, \ldots\}$ like this:

$$\begin{array}{ccc} 0 & \leftrightarrow & 0 \\ 2 & \leftrightarrow & 1 \\ 4 & \leftrightarrow & 2 \\ 6 & \leftrightarrow & 3 \\ 8 & \leftrightarrow & 4 \\ & \vdots & \end{array}$$

So what you just said would mean that my set $\{0, 2, 4, 6, 8, \ldots\}$ also has size $\aleph_0$.

Yes, that's right.

But that's ridiculous! Your set has a lot more elements than mine! They can't both have size $\aleph_0$!

Maybe *you* think it's ridiculous, but that's just because you're assuming that infinite sets behave like finite sets do. It's true that if S is a subset of a *finite* set T and $S \neq T$, then the size of S is strictly less than the size of T. But this doesn't hold for infinite sets. Your example proves this!

Anyway, what I was saying is that the same definition of addition works for adding cardinal numbers. For example, you can calculate $\aleph_0 + 3$ this way.

Is 3 a cardinal number?

Yes, any number that is a possibility for the size of a set is a cardinal number. So 0, 1, 2, 3, and so on, are all cardinal numbers, as are $\aleph_0$ and some even bigger ones.

OK, let me try to compute $\aleph_0 + 3$. Wait; my calculator doesn't have a button for $\aleph_0$!

Calculators aren't good for everything. Even if it did have a button for $\aleph_0$, you'd probably want to think about the answer it gave you, to check that it made sense. For example, if the calculator told you that $\aleph_0 + 3$ was 5, would you believe it?

No. . . I think the answer should be a lot bigger than that. If the calculator told me 5, I would probably think that I mistyped $\aleph_0 + 3$ as $2 + 3$. OK, forget the calculator. I'll use your definition. I need to pick a set of size $\aleph_0$ and a set of size 3. So let $A = \{0, 1, 2, 3, 4, \ldots\}$; this has size $\aleph_0$. And let $B = \{1, 2, 3\}$; this has size 3.

Cat: MEOW!

Oh yeah, I forgot to make them disjoint. So let me keep the set $A = \{0, 1, 2, 3, 4, \ldots\}$, but choose $B = \{\text{frog, cow, FluffyFur}\}$ instead.

It's not the 3-element set that *I* would have chosen, but it does work.

I think FluffyFur likes it too. Anyway, then $\aleph_0 + 3$ is the size of the set

$$A \cup B = \{0, 1, 2, 3, \ldots, \text{frog, cow, FluffyFur}\}.$$

Wait; am I allowed to rearrange the elements?

Sure. When describing a set, the ordering of the elements doesn't matter. All that matters is which elements are in the set, and which are not.

OK. Then

$$A \cup B = \{\text{frog, cow, FluffyFur}, 0, 1, 2, 3, \ldots\}$$

and this can be matched up with $\{0, 1, 2, 3, 4, \ldots\}$ like this:

$$\begin{array}{rcl} \text{frog} & \leftrightarrow & 0 \\ \text{cow} & \leftrightarrow & 1 \\ \text{FluffyFur} & \leftrightarrow & 2 \\ 0 & \leftrightarrow & 3 \\ 1 & \leftrightarrow & 4 \\ 2 & \leftrightarrow & 5 \\ & \vdots & \end{array}$$

So $A \cup B$ has size $\aleph_0$. This means that $\aleph_0 + 3 = \aleph_0$.

Right!

But then if I cancel $\aleph_0$ from both sides, I get 3 = 0!

It turns out that subtraction involving infinite cardinal numbers is not defined, so you are not allowed to cancel both sides the way you would if they were finite numbers. So there is no contradiction here.

It still doesn't look right to me. If I start with $\aleph_0$ and add 3, I should end up with something bigger than $\aleph_0$.

That would be so if you had a finite number instead of $\aleph_0$. But this is just another situation where infinite cardinal numbers behave differently from finite ones.

If infinite cardinal numbers behave differently from finite numbers, how do I know that your definition of addition is correct for them?

It's not really a matter of being "correct" or not. There is no pre-existing notion of addition for infinite cardinal numbers to check it against. So you have to use other criteria to decide whether a definition is good or not. What makes my definition good is that

(1) it is simple,

(2) it is the same definition that works for finite numbers, and

(3) it satisfies most laws of addition that hold for finite numbers. For instance, with my definition, the commutative law $m + n = n + m$ holds whether or not m and n are finite.

You are free to make up a different definition if you want, but it would probably have more problems than mine. So I'm sticking with my definition.

Next time: FluffyFur and friends discover that there is a problem with the definition when it comes to adding negative numbers and other kinds of numbers.

Editor: Grace Lyo

More on $\aleph_0$

In this dialogue, Bjorn showed that the set of nonnegative integers has the same cardinality as the set of nonnegative even integers.

Can you show that the set of integers also has the same cardinality as the set of nonnegative integers?

Can you show that the set of rational numbers has the same cardinality as the set of integers? Recall that a rational number is any number that can be expressed as the ratio of two integers.

Can you show that if S and T have the same cardinality and T and U have the same cardinality, then S and U have the same cardinality?

Send your answers to girlsangle@gmail.com!

Likely or Not?

By Katy Bold

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Yahtzee is a dice game for two or more players that has some similarities to poker. Players roll dice and earn points based on the outcomes of the roles. Points can be scored for various combinations of numbers on the dice, including three of a kind, four of a kind, or a full house. The highest point value roll is for a Yahtzee, when all five dice show the same number. If you play a game of 13 rounds with a friend, is it surprising that one of you gets a Yahtzee?

For each player's turn, she gets three rolls before scoring the dice. Because analyzing this question for the full game of Yahtzee is rather involved, we will answer a similar question for a simplified game of Yahtzee.

If each player gets one roll per turn, what is the probability of seeing a Yahtzee (five of a kind)?

When all the outcomes are equally likely, the general formula for the probability of an event is:

$$\frac{\text{Number of ways the event can occur}}{\text{Total number of outcomes}}$$

Let's start by determining the probability of rolling a Yahtzee on a single turn. There are six different faces on a die (labeled 1, 2, 3, 4, 5, and 6). A roll of a Yahtzee is 5 of a kind, so there are 6 possible Yahtzees: all ones; all twos; all threes; all fours; all fives; or all sixes. So, the number of ways to roll a Yahtzee is 6.

It takes a little more work to count the total number all of the possible rolls. Let's start by imagining that we have only one die. It has six possible values, each equally likely.

Now consider that we have two dice. If the first die is a 1, there are six possible values for the second die: 1, 2, 3, 4, 5, or 6. If the first die is a 2, there are again six possible values for the second dice. The pattern continues, so for each of the 6 possible values of the first die, there are 6 possible values for the second die:

Die 1	**Die 2**
1	1, 2, 3, 4, 5, 6
2	1, 2, 3, 4, 5, 6
3	1, 2, 3, 4, 5, 6
4	1, 2, 3, 4, 5, 6
5	1, 2, 3, 4, 5, 6
6	1, 2, 3, 4, 5, 6

In total, there are $6 \times 6 = 36$ possible rolls with two dice.

Let’s start listing out some of the possible rolls for 3 dice:

Die 1	Die 2	Die 3
1	1	1, 2, 3, 4, 5, 6
1	2	1, 2, 3, 4, 5, 6
1	3	1, 2, 3, 4, 5, 6
1	4	1, 2, 3, 4, 5, 6
1	5	1, 2, 3, 4, 5, 6
1	6	1, 2, 3, 4, 5, 6

If the first die is 1, the second die can take the values 1 to 6, and for each of those pairs, there are six values for the third die. In total, there are 36 rolls with the first die taking the value 1. There are six different possibilities for the first die, and for each of those six values, there are 36 possible combinations of dice 2 and 3. In total, there are $6 \times 36 = 216$ possible rolls of three dice.

What is the pattern for the number of possible rolls, depending on the number of dice?

Number of dice	Number of possible rolls
1	$6 = 6^1 = 6$
2	$6 \times 6 = 6^2 = 36$
3	$6 \times 6 \times 6 = 6^3 = 216$
4	?
5	?

Going back to the probability of rolling a Yahtzee on a single roll, we compute:

$$\frac{\text{Number of ways the event can occur}}{\text{Total number of outcomes}} = \frac{6}{6^5} = \frac{1}{6^4} = \frac{1}{1296}$$

So, the probability of rolling a Yahtzee on a single roll is one out of 1,296.

Now let’s return to our original question. In the simplified version, one game consists of 26 rolls. Let $Y = 6$ be the number of ways a Yahtzee can occur on a single roll and let $T = 6^5$ be the number of outcomes possible on a single roll of 5 dice. We just computed Y/T. With 26 rolls, there are T^{26} possible outcomes. How many of these involve a Yahtzee? The number that involve a Yahtzee is equal to the total number of outcomes minus the number of outcomes that *do not* involve a Yahtzee. On a single roll, there are $T - Y$ outcomes that are not Yahtzee. In 26 rolls, there are $(T - Y)^{26}$ outcomes that do not involve a Yahtzee. So the final answer is

$$\frac{T^{26} - (T-Y)^{26}}{T^{26}} = 1 - (1 - \frac{Y}{T})^{26} = 1 - (\frac{1295}{1296})^{26} \approx 0.01987 .$$

Think about it:

- Which is a more likely roll– four of a kind, or a Yahtzee (which is five of a kind)? Why?
- How many ways can you roll a four of a kind?
- If your friend rolls a Yahtzee, are you more or less likely to roll a Yahtzee on your next turn, or is there no difference?
- How likely is it that for an entire game, you get a Yahtzee on the first roll of every turn?

Math Tips

by Ken Fan

Here's an algebra problem that was recently given in a school: Solve $\frac{3x}{2} - \frac{x}{8} = 11$ for x.

And here's one student's solution:

$$\begin{aligned} \frac{3x}{2} - \frac{x}{8} &= 11 \\ \frac{24x}{16} - \frac{2x}{16} &= 11 \\ \frac{24x - 2x}{16} &= 11 \\ 24x - 2x &= 186 \\ 22x &= 186 \\ x &= 8 \end{aligned}$$

Even though this student arrived at the correct answer, there are mistakes. Can you find them?

Perhaps you noticed that the student combined fractions by using the common denominator 16 when, in fact, the student could have used 8 (which is the least common multiple of the two denominators). That's true, but it isn't wrong to use 16.

Did you notice that the student multiplied 11 by 16 incorrectly? The product of 11 and 16 is 176, not 186. But in the last step, the student also divided 186 by 22 incorrectly, and, amazingly, ended up with the correct answer!

Though multiplication is one of the earliest operations we learn and one of the operations that we get a lot of practice performing, the act of multiplying two numbers is error prone. Even professional mathematicians carelessly multiply incorrectly on a regular basis. In fact, this is true of numerical computation in general, not just multiplication.

So, to reduce errors, try to compute less!

In the above problem, for example, was it really necessary to compute the product of 11 and 16? Let's take up the solution from the 3rd equation, but this time refrain from calculating the product:

$$\begin{aligned} \frac{24x - 2x}{16} &= 11 \\ 24x - 2x &= 16 \cdot 11 \\ 22x &= 16 \cdot 11 \\ x &= 8 \end{aligned}$$

To get to the last step, notice that 22 is 2 times 11, so when you divide the right hand side by 22, the elevens cancel and all you have to do is divide 16 by 2. Not only was it unnecessary to compute the product, by refraining, we were spared from an error-prone division task later!

So here's the tip: Before you begin a laborious, error-prone computation, think about whether you really need to do it! You might save yourself a lot of time and errors.

The Adventures of Emmy Newton

Episode 4. *The Answer in the Song*

by Maria Monks

Emmy and Melissa are making their way through the basement of their school in order to get to the Principal's office. In the last episode, they came across a small door near the ceiling of one of the rooms, constructed a ladder-like ramp up to the door, and Melissa climbed up and opened the door...

"Check it out!" said Melissa, poking her head through the little door.

Emmy looked up. "What's on the other side?" But Melissa didn't hear her. She climbed into the room and out of sight.

"Melissa?" No response. Emmy shook her head, climbed up the wobbly ramp and peered beyond the door. It was very dark, but there appeared to be a spiral sliding board leading down into the next room. She couldn't see what was at the bottom of it, or anything else in the room, for that matter. "Melissa, are you down there?" said Emmy.

"Yeah, I'm here," she called, "Just slide down - ow!" Melissa's voice came from below. "I just bumped into something. What the heck is on these walls?"

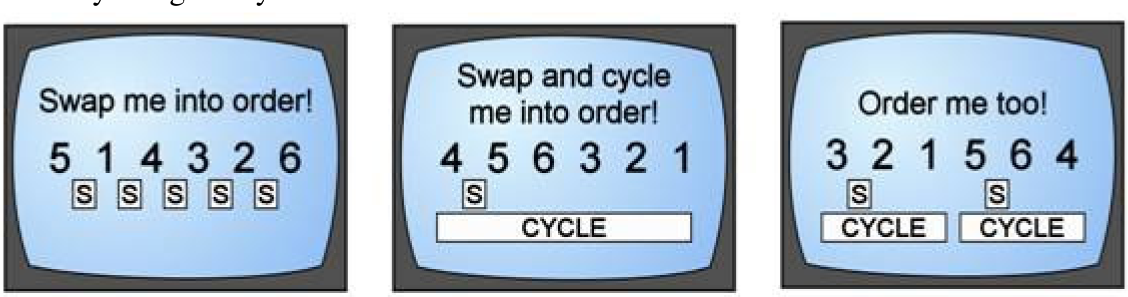
Emmy slid down the sliding board and landed on the cold basement floor. She coughed; it was rather musty. It was also completely dark except for the dim light that shone through the little door above them.

Emmy could hear Melissa's cautious footsteps from a few feet away. "Bump into anything like a light switch yet?" Emmy asked. Her eyes were starting to adjust to the lighting, and she noticed a thin chain or rope of some sort hanging from the ceiling, illuminated by the light that shone through the little door on the wall. Emmy got up, walked over and pulled down on the chain. "This should do something."

Sure enough, the room began to light up- rows of little lights turned on one by one in rapid succession, along the ceiling and down the corners of the walls, three large monitors on the back walls turned on, and strange music filled the room. It was a quiet cacophony of sounds, as if someone had turned on several radios at once and tuned them all to different channels.

Melissa looked around, wide-eyed, at the glowing lights, at the black monitors, at the speakers on the back wall that she had bumped into, and at the plastic spiral sliding board they had just slid down. She then saw the brown wooden door next to the monitors. It was padlocked, just like the doors in the first room. She tried the handle. Naturally, it was locked.

Emmy, however, was not paying any attention to the majority of her surroundings. She was instantly intrigued by the numbers and instructions on the three monitors on the wall:



Emmy peered closer. "It looks like a touchscreen." She pressed the leftmost button labeled "S" on the first monitor. The 5 and the 1 above it switched places and glowed brighter than the other numbers on the screen. The sequence now read:

1 **5** 4 3 2 6

"Aha!" said Emmy. "These S buttons - S for 'swap', I suppose - switch the position of two adjacent elements in the sequence."

"Oh, it's going to be easy so sort them into order then," said Melissa. "We already got the 1 in place now. Let's move the 2 to the left!"

Melissa reached over and pressed the fourth S button, then the third, then the second, forming the sequences:

1 5 4 **2** **3** 6

1 5 **2** **4** 3 6

1 **2** **5** 4 3 6

on the screen.

"Nice!" said Emmy. "Let's do the same thing for the 3." She pushed the fourth and third buttons:

1 2 5 **3** **4** 6

1 2 **3** **5** 4 6

Now only the 4 and the 5 needed to be swapped. "You do the honors," said Emmy, and Melissa grinned. She loved the feeling of satisfaction she got upon putting last piece of a puzzle like this into place. She pressed the button between the 4 and the 5, and the monitor blinked three times. The text changed to "You win!" and the background glowed bright orange.

"Listen," said Emmy, "I think I'm starting to be able to hear words in the music," said Emmy.

Melissa listened closely. The music did seem to be a bit more coherent, like one voice was singing something sweet and pure but the jumble of noise was still drowning it out. "I bet we just sorted one of the tracks of this song into order," said Melissa. "Let's do it for the others."

She walked over to the second monitor. "I wonder what this button does?" said Melissa.

She pressed the button labeled CYCLE. The sequence instantly changed to

1 4 5 6 3 2

Melissa Blinked. "What the heck did it do?"

"It cycled them," said Emmy. "That is, it moved each number one place to the right, but wrapped around at the end. See, if we press it again, the 2 will wrap around and move to the first position, and everything else will move to the right one spot." She pushed it again:

2 1 4 5 6 3

"Oh!" said Melissa. "But can you really sort the sequence using only this one sort button and one cycle button?"

"Yes," said Emmy, pushing the CYCLE button again so that the sequence now read

3 2 1 4 5 6

Emmy paused and tilted her head at the screen. "I wonder if it's possible to rearrange a sequence any way you like using just this cycle and swapping in the first two positions?"

"I… think so," said Melissa slowly. She stared at the screen and nodded. "I think you can make any swap you like."

Emmy looked at her friend. It was rare that Melissa had a mathematical insight that Emmy didn't see first, but when she did, it was often quite ingenious. "How?" asked Emmy.

"Well," said Melissa, "Say we want to swap two numbers that are next to each other, like on the first screen. Like, here we want to move the 1 to the left just like last time, and so we want to swap the positions of the 2 and the 1."

"Right, but the S button would swap the 3 and the 2, not the 2 and the 1."

"That's ok," continued Melissa. "we can first hit the CYCLE button until the 2 and the 1 are lined up with the S…" she pressed the CYCLE button 5 times, "hit the S…" she pressed the S button, "and now cycle back to where we started." She pressed CYCLE one more time:

3 1 2 4 5 6

"That's awesome!" said Emmy. "You're right. So now we can just press S," she pressed the S button, "and the 1 is now in place." The screen read:

1 3 2 4 5 6

"Yeah, so now we just have to switch the 3 and the 2!" said Melissa. She pushed CYCLE five times, then tapped the S button once. "You do the honors," she said to Emmy with a triumphant grin.

"My pleasure," said Emmy. She pushed the CYCLE button and the monitor blinked three times. The girls could now almost discern the words of the song filling the room, but it was still drowned out by a cacophony of disorder above it.

Emmy walked over to the last monitor. "All right, we got the hang of this now. I'll take the right half and you take the left."

Melissa looked, gave Emmy a nod, and the two girls pushed the buttons on the third screen until the numbers were in order and the monitor blinked three times. Suddenly, all the tracks of the music were lined up and three voices were singing in harmony.

"Cool. Now what?" said Melissa. "Shh!" said Emmy. "Let's listen to what they're saying."

The two girls listened closely to the lyrics of the song that filled the room:

The travelers have come today!
They fixed us, so we're here to say:
To go continue on their way,
They must first crack the code!

The door would simply like to show
A list of six, all in a row
All different, and yet cannot show
When in the third screen's mode.

What's more, a single button press
In Screen 1's mode, no more, no less,
Would sort the sequence, I confess,
Within this little ode!

"What's an ode?" said Melissa, looking around the room with a puzzled gaze.

Emmy laughed. "It just means a little lyrical song. The song was talking about itself."

The music began to repeat and Emmy began to think. "OK, so the code is going to be some ordering of the numbers 1 through 6." She began to pace around the room. "It cannot show when in the third screen's mode… on the third screen, you can only mix up the first three and the last three numbers, so 1, 2, and 3 must be on the left half and 4, 5, and 7 are on the right."

Melissa thought about it and nodded. "And it said you can sort the sequence with a single button press in the first screen, so it's just a single switch away from being in order. Hey!" she realized. "If we start with the numbers in order and we switch 4 and 3, then it also can't be on the third screen, so I think that's it!"

"Yes!" agreed Emmy. "So the sequence is 1, 2, 4, 3, 5, 6." She walked to the door and punched in the numbers. The lock clicked.

To their surprise, the door swung open before they even got to turn the handle. "Emmy, Melissa!" said the figure in the doorway. "There you two are!"

TO BE CONTINUED…

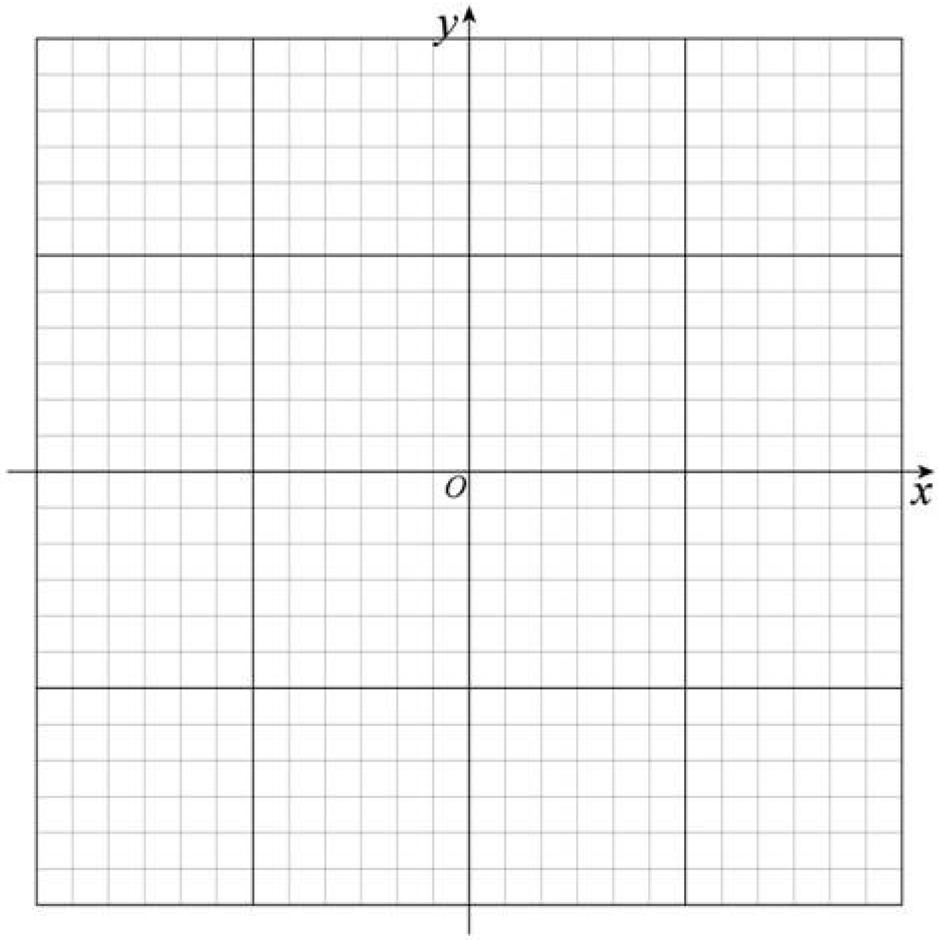
Editor: Grace Lyo

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 6 – Meet 5 – March 4, 2010

Mentors: Lauren Cipicchio,
Grace Lyo,
Ariana Mann,
Jennifer Melot,
Maria Monks,
Charmaine Sia



While some girls continued work making perspective drawings, other girls constructed a drawing defined by equations. Perhaps you'd like to try too?

For each formula below, find all pairs (x, y) where x and y are real numbers that make the formula evaluate to zero. Plot these points on the graph at right. You should be able to identify the resulting drawing.

If you make up your own drawing in this manner, send us your formulas!

1. $(x - 3)^2 + (y - 3)^2 - 1$

2. $x^2 + 6x + y^2 - 6y + 17$

3. $(x + 2y - 16)^2 + |2y - 12| + |2y - 14| - 2$

4. $| x + 4 | + | x + 2 | + y^2 - xy + \dfrac{x^2}{4} - 16y + 8x + 62$

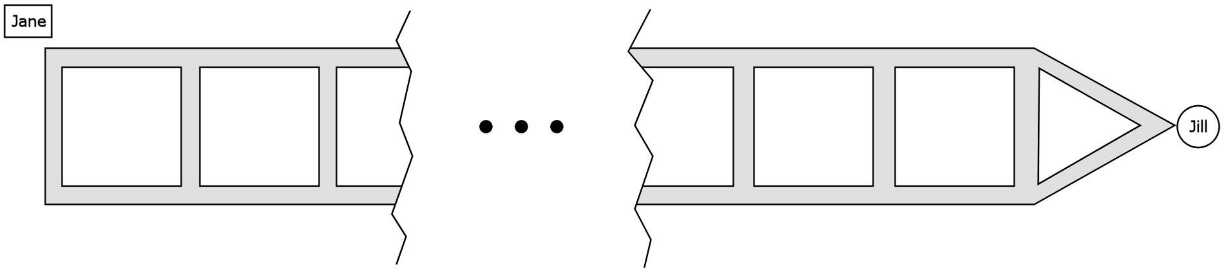
5. $|x^2 + y^2 - 25| + |y| + |y + 5| - 5$

Session 6 – Meet 6 – March 11, 2010

Mentors: Aliaa Barakat, Lauren Cipicchio, Rachel Fraunhoffer,
Lauren McGough, Charmaine Sia, Liz Simon, Julia Yu

Often one math question will inspire another. **Billy-bob-joe-bob-jim**'s problem (see the previous issue of this *Bulletin*) inspired the following path counting problem:

Jane wants to walk to Jill's house.



Jill's house is at the tip of a triangular block as shown in the map. Before the triangular block, there is a row of square blocks. There are n vertical streets in total. All of the streets are two-way streets. Jane will walk in such a way that she never traverses a street more than once.

Let the distance between any two adjacent intersections be 1 unit.

If $n = 1$, how many paths can Jane follow that are of length 1? 2? 3? etc.
If $n = 2$, how many paths can Jane follow that are of length 1? 2? 3? etc.
If $n = 3$, how many paths can Jane follow that are of length 1? 2? 3? etc.

What is the *general* answer?

Session 6 – Meet 7 – March 18, 2010

Mentors: Rediet Abebe, Lauren Cipicchio, Ariana Mann, Charmaine Sia, Liz Simon,

One of this meet's demonstrations was to show how to cut a bagel in half. Of course you can just cut it in half the standard way, but we cut the bagel into two halves that ended up being linked together like two links in a chain. Can you figure out how?

Session 6 – Meet 8 – April 1, 2010

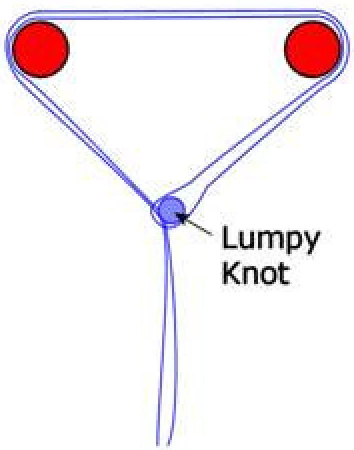
Mentors: Rediet Abebe, Lauren Cipicchio, Lauren McGough, Jennifer Melot,
Maria Monks, Charmaine Sia, Carolyn Stein, Julia Yu

Special Visitor: Cynthia Rudin, MIT Sloan School

Cynthia talked about her work creating a ranking system to rate the most fire-prone manholes in Manhattan. The problem is that every now and then, a manhole catches on fire, or worse, explodes. She created a program that took in all the past incident data and ranks the manholes in terms of safety. Using these rankings, the city of New York could gain a better sense of where to send maintenance crews. The reason for using a computer program instead of just doing it by hand? There are over 50,000 manholes on Manhattan! As new incidents are reported, the program adjusts its formula...that is, it learns from new information.

A number of girls thought about intersections of geometric objects. For instance, what can you get by intersecting a plane with a cube? If the plane passes through the cube parallel to a face, the intersection will be a square. Can you see how to get an equilateral triangle? What is the maximum number of sides of an intersection of a plane and a cube? For a hint, see page 9.

Two of our youngest members, 8 year olds **Bob** and **Dill Pickle**, tried to figure out how to leash a dog to two poles in such a way that removal of either pole sets the dog free. After some experimentation, the two came up with a surprising and very interesting quasi-solution. They put a lumpy knot in the leash that made the leash consist of two loops. The longer loop was attached to the dog. They pulled the free loop taught and wrapped it around the two poles hooking the end back over the lumpy knot (see the figure at left). If held taught, the lumpy knot served as a catch for the loop preventing the dog from running away. When either pole is removed, the resulting slack would cause the loop to loosen and the lumpy knot would fall out setting the dog free! Clever!



But why do I call this a “quasi-solution” and not a solution? The reason is that if the dog does not keep the leash taut, the resulting slack would have the same effect as removal of a pole. If removal of a pole is intended to release the dog, then the dog should be able to get free by letting the tension out of the leash.

Still, the idea is clever and worth exploring further. Can the same idea be extended to work in a similar way with 3 or more poles? What is the minimum length of string needed?

<u>Session 6 – Meet 9 – April 8, 2010</u>

Mentors: Rediet Abebe, Lauren Cipicchio, Grace Lyo, Jennifer Melot,
Maria Monks, Charmaine Sia, Carolyn Stein, Drew Wolpert, Julia Yu

Some girls began studying nets of polyhedra. See page 9.

<u>Session 6 – Meet 10 – April 15, 2010</u>

Mentors: Rediet Abebe, Lauren Cipicchio, Rachel Fraunhoffer,
Maria Monks, Charmaine Sia, Carolyn Stein, Fan Wei, Julia Yu

Special Visitor: Elissa Ozanne, MGH and Harvard Medical School

Elissa talked about using math to analyze risk factors in breast cancer. She showed the girls a web-based applet where a woman can enter information about herself and have her risk of getting breast cancer assessed. She discussed some of the mathematical concepts used to create the applet.

Making a Perspective Drawing of a Level 2 Menger Sponge

by **Rowena**

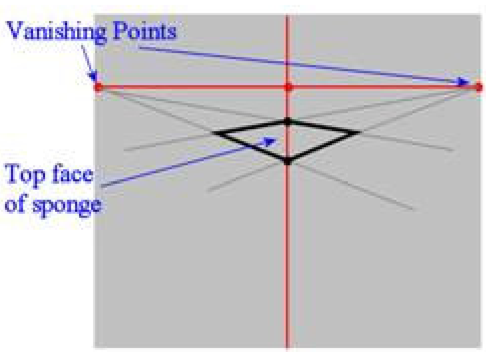
Drawing a perfect perspective drawing of a cube requires several special vanishing points, mathematically found so that when we look at the drawing it looks like all the sides are the same, even though in the drawing they aren't (it looks kind of warped if you don't look at it right).

A Menger sponge is actually based on a cube, not a sponge. It's called a sponge because it has lots of holes, like a real one. A Menger sponge can be explained as follows: Start with a regular 3 by 3 by 3 cube. A level 0 Menger sponge is a solid cube. Now remove the center blocks of each face and the cube at the very center. That's a level 1 Menger sponge. A level 2 Menger sponge is like a level 1, except that each remaining block is replaced with a scaled down level 1 Menger sponge. You can keep doing this process of replacing cubes with scaled down level 1 Menger sponges ad infinitum, and if you do, you get a Menger sponge.

A perspective drawing is a drawing that looks like a 3 dimensional object when looked at from a certain vantage point. Ken said, when I first started with the project, that the hardest part about drawing this would be making sure that the drawing represents a cube. He was definitely right.

A perspective drawing of a cube generally needs 3 vanishing points, one for each set of parallel edges. A vanishing point is a guiding spot for making your drawing. It is where lines that represent parallel lines in space converge in the drawing. For example, the point directly in front of the vantage point is where if a person walked straight away along a perpendicular to the drawing plane, the person would vanish. We ended up needing one more vanishing point, but I'll tell about that later.

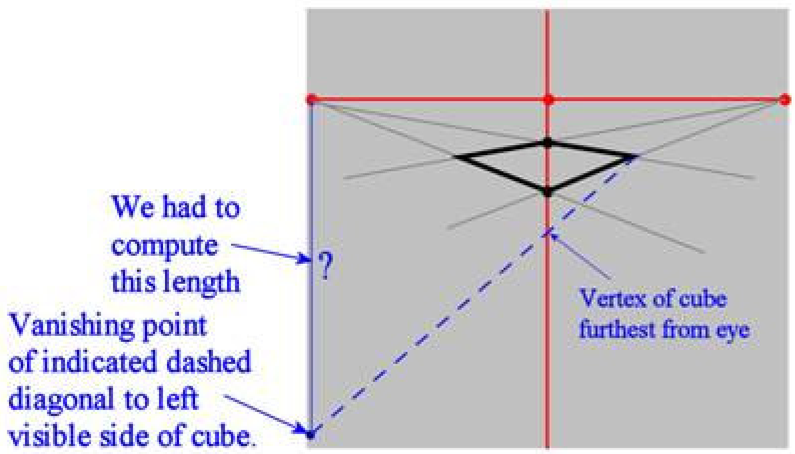
We wanted to draw the sponge situated so that it was sitting on a flat surface with one of the diagonals of the top face parallel to the plane of the drawing. The four vertical edges are parallel to the drawing plane, so they do not converge to a vanishing point in the drawing. In the drawing, those remain vertical and parallel to each other. The 8 horizontal edges of the cube form two sets of parallel edges which each require a vanishing point. We put these two vanishing points at the very edges of the paper. We divided the distance between the two points in half and marked the center point at the middle. Before we drew the center dot, we drew a line between the two side dots to create the horizon line.



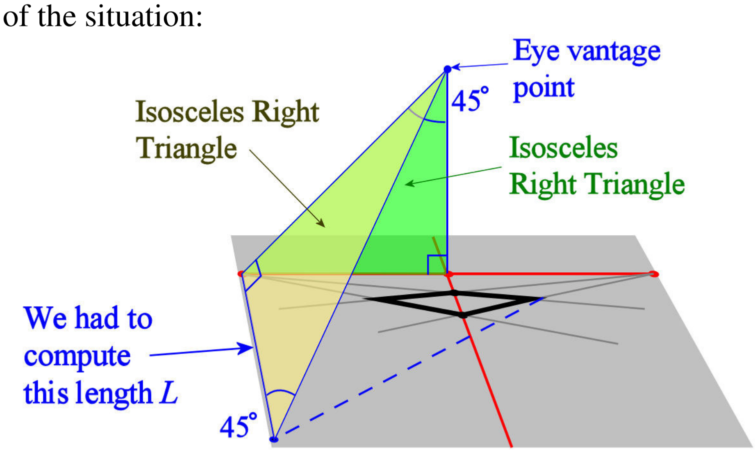
We drew a vertical line through the center point through the entire piece of paper. On this centerline we chose two dots reasonably close together and close to the top. These would be the closest and farthest corners of the top face of the sponge. We used a ruler to draw line segments on the lines connecting the vanishing points on the horizon with these two dots. All of these lines formed a quadrilateral. This quadrilateral was the top face of our sponge.

Now we had to figure out how long to draw the vertical edges of the sponge. There are three faces that we cannot see. The diagonal of the right, hidden side face that passes through the rightmost vertex of the top face would pass through the bottom of the furthest (hidden) vertical edge. So, if we can draw this diagonal, we can figure out the length of that vertical edge. Once one vertical is drawn, the others can all be drawn too using the vanishing points on the horizon line. To draw this diagonal, we sought its vanishing point.

Now, to find the vanishing point of the relevant diagonal of the right, hidden side of the cube, position your eye at the vantage point (above the middle of the horizon line) turn your head 45º to the side (so that you are looking along a line parallel to the top edge of the right, hidden side, then 45º down (so that you are looking along a line parallel to the relevant diagonal). The 45º angle is determined by the geometry of a square. It is the angle between a side of a square and a diagonal. We built a 3D protractor out of folded paper, rulers, and tape to help us with this, but it was too clumsy to use. After fumbling around trying to get it to work, we decided to use math instead.



Here's another view of the situation:



Now, this means that the desired length is a leg of the yellow isosceles right triangle. The other leg of the yellow triangle is the hypotenuse of the green isosceles right triangle. One leg of the green triangle is the distance between the left vanishing point on the horizon line and the center, which, in our case was 12 inches. Using the Pythagorean theorem, the desired length, labeled L in the figure, satisfies $L^2 = 12^2 + 12^2$, so $L = 12\sqrt{2}$ inches. This is very close to 17 inches. Knowing L, we can mark the vanishing point of the relevant diagonal. We can then draw in the dashed line in the figure above to find the vertex of the cube furthest from the eye, which lets us complete the rest of our cube. [For the final drawing, see the cover! –Editor]

Calendar

Session 6: (all dates in 2010)

January	28	Start of sixth session!
February	4	
	11	
	18	No meet
	25	Jericho Bicknell, Waltham Fields Community Farm
March	4	
	11	
	18	
	25	No meet
April	1	Cynthia Rudin, MIT Sloan School
	8	
	15	Elissa Ozanne, Harvard Medical School and MGH
	22	No meet
	29	
May	6	

Session 7: (all dates in 2010)

September	9	Start of the seventh session!
	16	
	23	
	30	
October	7	
	14	
	21	
	28	
November	4	
	11	Veteran's Day – No meet
	18	
	25	Thanksgiving - No meet
December	2	
	9	

Girls’ Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls’ Angle? Girls’ Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls’ interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls’ Angle mentors, the Girls’ Angle Support Network, the Girls’ Angle Bulletin and Community Outreach.

Who are the Girls’ Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls’ Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls’ Angle Bulletin? The Girls’ Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) is free for members and can be purchased by others. Please contact us if you’d like to purchase printed issues.

What is Community Outreach? Girls’ Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members’ efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 2 ways: **membership** and **active subscription** to the Girls’ Angle Bulletin. **Membership** is granted per session and includes access to the club and extends the member’s subscription to the Girls’ Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. **Active subscriptions** to the Girls’ Angle Bulletin allow the subscriber to ask and receive answers to math questions through email. Please note that we will not answer email questions if we think that we are doing the asker’s homework! We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of “catching up with the group” doesn’t apply. Note that you can receive the Girls’ Angle Bulletin free of charge. Just send us email with your request.

Where is Girls’ Angle located? Girls’ Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For: ☐ Membership
☐ Active Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________
(Parent/Guardian Signature)

Membership-Applicant Signature: ____

☐ Enclosed is a check for (indicate one) (prorate as necessary)
- ☐ $216 for a one session membership
- ☐ $50 for a one year active subscription
- ☐ I am making a tax free charitable donation.

☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

Girls' Angle

A Math Club for Girls