Girls' Angle Bulletin

August 2010 • Volume 3 • Number 6

To Foster and Nurture Girls' Interest in Mathematics

An Interview with Lillian Pierce, Part 1
Lightning Learns Pool
Anna's Math Journal
The Adventures of Emmy Newton
Down to the Wire
Summer Fun Solutions:
Sets, Fibonacci numbers, Pool
Drawing, and more

From the Director

Girls' Angle is turning three this September. Countless people have helped Girls' Angle get to where it is today. To the members, member's parents, mentors, the Support Network, the advisors and directors, the Women In Mathematics video presenters, the Bulletin contributors, the donors, and many others, Thank You all!

Still, there is so much more to do and I hope that we improve on many fronts. One area I'd like to concentrate on improving is feedback for the Bulletin. This magazine is primarily intended for all girls. So, girls, we welcome you to tell us what kind of math you want us to address in these pages. What we will provide is high quality voices in mathematics who will address the content you want us to address so that you can rest assured that what you read in these pages is mathematically sound.

One last thing: Katy Bold has been reliably producing her column *Math In Your World* since Volume 2, Number 1. This issue, we're skipping her column in order to celebrate her marriage! Congratulations, Katy!

All my best,
Ken Fan
Founder and Director

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

Individuals

Marta Bergamaschi
Charles Burlingham Jr.
Anda Degeratu
Eleanor Duckworth
Julee Kim
Toshia McCabe
Alison Miller
Suzanne Oakley
Mary O'Keeffe
Beth O'Sullivan
Elissa Ozanne
Patsy Wang-Iverson
Anonymous

Institutions

The Mathematical Sciences Research Institute
Science House

Corporate Benefactors

Big George Ventures

Girls' Angle Bulletin
The official magazine of
Girls' Angle: A Math Club for girls
Electronic Version (ISSN 2151-5743)

girlsangle@gmail.com

This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Grace Lyo
Executive Editor: C. Kenneth Fan

On the cover: Fifteen is a Triangular Number by Ken Fan.

An Interview with Lillian Pierce, Part 1

Lillian Pierce received her doctoral degree in mathematics from Princeton University. She just completed a year at the Institute for Advanced Study in Princeton and this fall, she will be a Marie Curie Fellow at Oxford.

…what is the way to understand a problem or a proof to this depth? By getting your hands dirty: don't just watch someone else take apart that car. Do it yourself!

Ken: It's a real treat for me to interview an analytic number theorist. First of all, what is number theory? And, what is *analytic* number theory?

Lillian: At its heart, number theory is what it says it is: the study of numbers. "What is there to know about numbers that we don't already know?" people ask me. "Lots" is the answer to that. My favorite example of something we still don't know has to do with prime numbers: a prime number is a positive integer that is only divisible by itself and 1. So the first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, … We know that there are infinitely many prime numbers[1]. But let's look more closely at that list of the first few primes: two interesting things already pop out: 2 is the only even prime number. (All other even numbers are divisible by 2, so can't be prime!) And if you look closely at that list you'll see that there are several pairs of prime numbers that are precisely two apart: 3 and 5, 5 and 7, 11 and 13, 17 and 19. These are called **twin primes**. The question is: are there infinitely many pairs of twin primes? We don't know! This is one of the most famous unsolved problems in number theory.

Here are a few more number theoretic questions: Can every even number greater than 2 be written as a sum of two (odd) primes? Can you write every positive integer as a sum of four squares? How many prime numbers are there smaller than X? These questions are aesthetically attractive to mathematicians, especially because they are so easy to state. In fact, because these problems can be stated simply in terms of the natural numbers we all know and love, they have been admired and pondered over for centuries, by mathematicians all over the world. But these are also very hard problems! The first problem is still unsolved, while the last two were finally solved in 1770 and 1896, respectively. So those give you an idea of the flavor of number theoretic questions.

I work in a particular part of number theory called analytic number theory, so I try to answer number theoretic questions by analytic methods. What are those like? Analytic techniques bring in the use of functions and ideas related to the calculus that you learn in high school: integration, differentiation, infinite series. One of the most important analytic tools is Fourier analysis, codified by the French mathematician Fourier in around 1807: at its heart, Fourier analysis proposes that every "well-behaved" function can be made up out of sums of sine and cosine waves. This allows us to break a function down into its component sine and cosine waves, and this is a very powerful tool that is used throughout analytic number theory.

Ken: What got you interested in analytic number theory?

Lillian: The story of how I started working in analytic number theory stretches back a long way. When I arrived as a freshman at Princeton, I took the first class that future math majors always

[1] Watch Ellen Eischen's Women In Mathematics video for a presentation of Euclid's proof of this fact. – Editor.

took in those days: MAT 215, An Introduction to Real Analysis. This is the class where you learn how to prove, rigorously, all the theorems you enjoyed using in high school calculus (like the fundamental theorem of calculus). The professor of the class was Elias M. Stein— I didn't know this at the time, but he is one of the most famous analysts of the last 50 years, and not only is he a remarkable mathematician but also a wonderful teacher. As it happened, I loved that class, and so when Professor Stein taught a class the next year on Fourier analysis, I took it too. I loved that class even more! And so I decided to take every class that Prof. Stein taught while I was at Princeton: Complex Analysis, Real Analysis, Special Topics in Real Analysis, and so on. I enjoyed all of my math classes— topology, algebra, Galois theory, graph theory, logic— but it was really the analysis that seemed to fit best with my way of thinking. When I was doing abstract algebra (the theory of groups, rings, and fields) I felt like I was trying to stack pebbles, and since the pebbles were round and smooth, this was hard to do, and sometimes the stacks of pebbles just fell over. (In mathematical terms, when I tried to prove statements, I wasn't satisfied with how concepts fit together, and didn't feel like my ideas lined up into as tight and solid a proof as I wanted.) But when I was thinking about concepts in analysis, I pictured mechanical machines, with cogs and wheels that fit together precisely. Once I had assembled the machine (my proof), I could turn the wheels and see how everything clicked into place and whirred and spun and....produced the theorem. Perhaps part of the reason that analysis seemed so clear to me was Professor Stein's teaching— during my years at Princeton (with co-author Rami Shakarchi) he wrote a series of books for undergraduates, the Princeton Lecture Series in Analysis, which are now highly regarded textbooks. I also did two independent projects with Prof. Stein: a junior paper and a senior undergraduate thesis. And it was in my undergraduate thesis that I first encountered analytic number theory, because Professor Stein suggested I work on a topic that was new and surprising in 2001: the connection between the zeroes of the Riemann zeta function and so-called "random matrices."

I realized that doing mathematics, that intense mental exploration resulting in something with more precision and perfection than anything in "the real world," gave me a pleasure that no other academic pursuit did.

I didn't realize it at the time, but Prof. Stein had just set me on a new path toward number theory that changed my mathematical life. When I graduated from Princeton as an undergraduate in 2002, I knew that I wanted to go to graduate school and become a mathematics professor. I also thought that I would do my Ph.D. in harmonic analysis and become an analyst. However, I had the chance to study at Oxford University in England for two years before returning to the US to do a Ph.D. As it turns out, Oxford has a deep tradition in number theory, in which I had a new interest. So while I was at Oxford, I decided to study number theory, and I ended up working with a greatly respected number theorist named Roger Heath-Brown.

Ken: Can you explain one of your own contributions to the field?

Lillian: The integers have a very nice property: given an integer, there is one and only one way to factor it into a product of prime numbers. That is, each integer has a "unique prime factorization," up to the ordering in which you write the prime factors. So, for example, $24 = 2 \times 2 \times 2 \times 3$, and that is the only way 24 can be broken down into prime numbers. (We consider $2 \times 2 \times 2 \times 3$ and $2 \times 3 \times 2 \times 2$ to be "the same" since they both contain 3 factors of 2 and 1 factor of 3.) This property might seem obvious, and in a way it is, because we live in the world of integers, and this is what we are used to. But there are other mathematical "worlds" where unique factorization does not hold.

The study of abstract algebra introduces us to these other worlds. The real numbers are an example of a mathematical construct called a **field**: a set of numbers within which we can do addition, subtraction, multiplication, and division. The integers are an example of a different mathematical construct called a **ring**: a set of numbers within which we can do addition, subtraction, and multiplication, but not necessarily division. (By this I mean that if you add, or subtract, or multiply, two integers together, you get another integer. But if you divide one integer by another, for example 3/5, you might not get an integer, and so you might leave the "ring of integers.") Mathematicians have devised ways to study the properties of fields and rings in great generality. Just like the integers lie inside the real numbers, inside every field is a ring, called the "ring of integers," because they act like our integers do. Or almost— depending on the field, the ring of integers might not have unique factorization! One of the interesting questions is exactly this: what special properties must a field have for its ring of integers to have unique factorization?

This information is encoded in something called the **class number** of the field. Each field possesses a class number, which is a positive integer between 1 and infinity. The class number is like a label (called an **invariant**), just like the year of your birth is a form of label for you. If you know someone was born in 1995, you'd know how old she is now, what year she is in school, what she's probably studying in math at school, what songs she probably heard on the radio when she was 10, what world events she's seen unfold during her life, and many other things. Similarly, if you know the class number of a field, you know a lot about it. In particular, if a field has class number equal to 1, we know that its ring of integers does have unique factorization! So it is a big question in number theory to classify which fields have class number 1. We still don't know how to do this.

The problem is this: currently mathematicians understand how to label fields in terms of another invariant, called a **discriminant**. This is kind of like a height—let's think of it as the height of a person. If I tell you that someone is 5' 4" tall, can you tell me how old they are? I bet not! I'm about 5' 4" but so is my mother, and you might be too! And we're all different ages. Similarly, currently mathematicians know how to label fields by their "heights" (discriminants), but they don't know how to say much about the class number of the field, simply from knowing its height, unless they gather much more information. (So if you know that I'm 5' 4" but also know that I graduated from college in 2002 you might come much closer to translating this into my true age.) What we'd like to be able to do is translate knowing the discriminant of a field directly into understanding its class number. This is something that I worked on when I was at Oxford. (Specifically, I proved an upper bound for the "3-part of the class number of a quadratic field," in case you want to hear the fancy name for it!) This may sound like a very abstract question, but it is important enough that it's been studied for hundreds of years— Gauss[2] even worked on it, in the early 1800s. And we still only understand the tiniest tip of the iceberg.

Ken: When I first met you, you were a graduate student in mathematics at Princeton. Now, you have a doctoral degree in math. Could you please tell us something of the history of your Ph.D. thesis? How did you come up with the ideas in it?

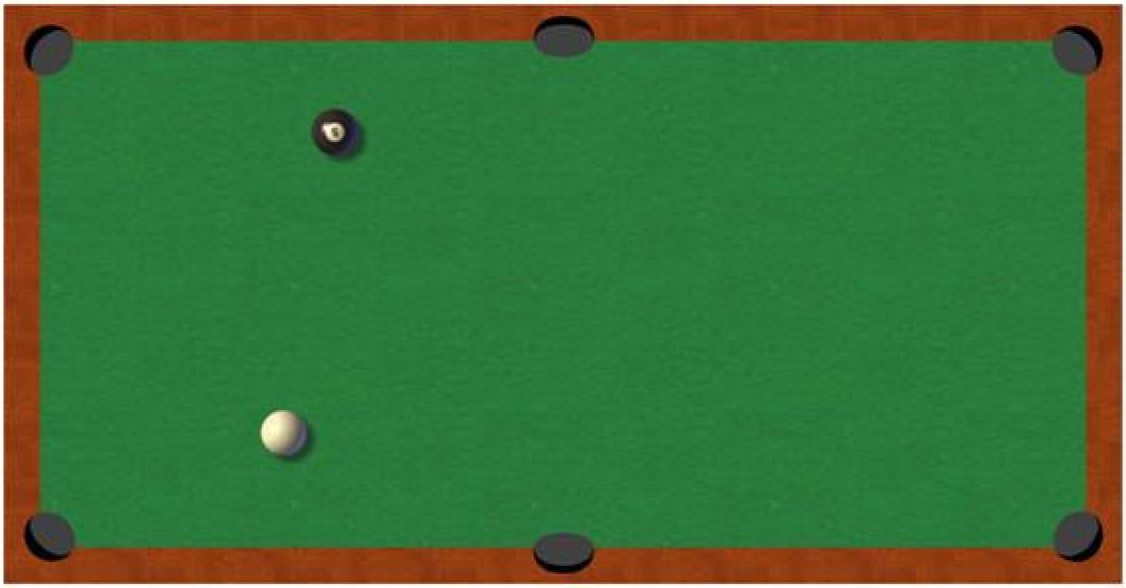
To be continued…

[2] Carl Friedrich Gauss (1777-1855) wrote one of the foundational books of number theory, the *Disquisitiones Arithmeticae*.

Lightning Learns Pool

by Lightning Factorial

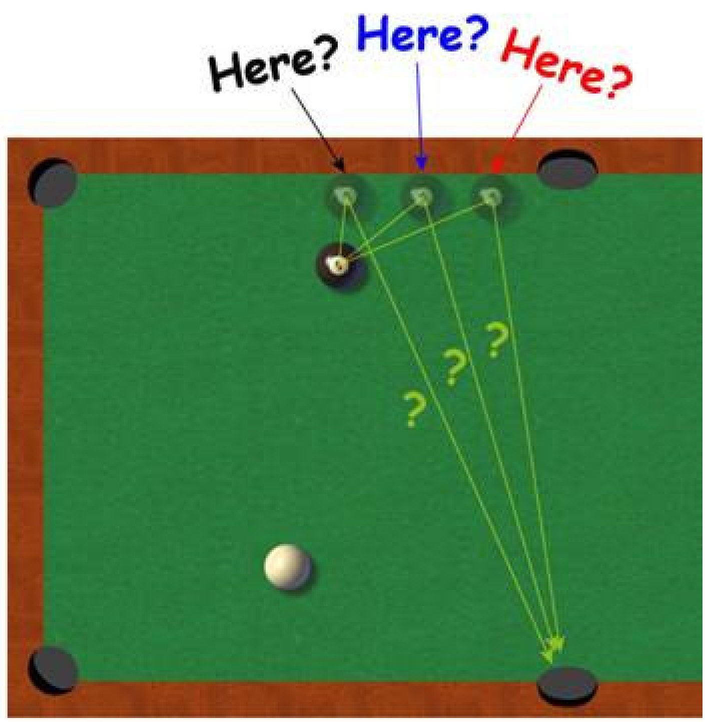
I've always wanted to learn how to play pool. The balls are nicely colored and its fun to watch them bounce around the table. So I decided to take lessons…



Things were going fine, until I encountered the situation shown at left.

I had to hit the white cue ball into the black 8 ball and try to sink the 8 ball into one of the six pockets.[1] But, it didn't look possible to sink the 8 ball in the side pocket nearest to it because the angle looked too sharp. Nor did it look possible to sink the 8 ball into the nearest corner pocket. At least, it sure didn't look very easy to do that. I turned to my coach.

"Hey, coach! What am I supposed to do? It doesn't look possible to sink the 8 ball."



"Lightning, this is a perfect set up for a bank shot! Try to sink the 8 ball into the lower side pocket by bouncing it off the top curb."

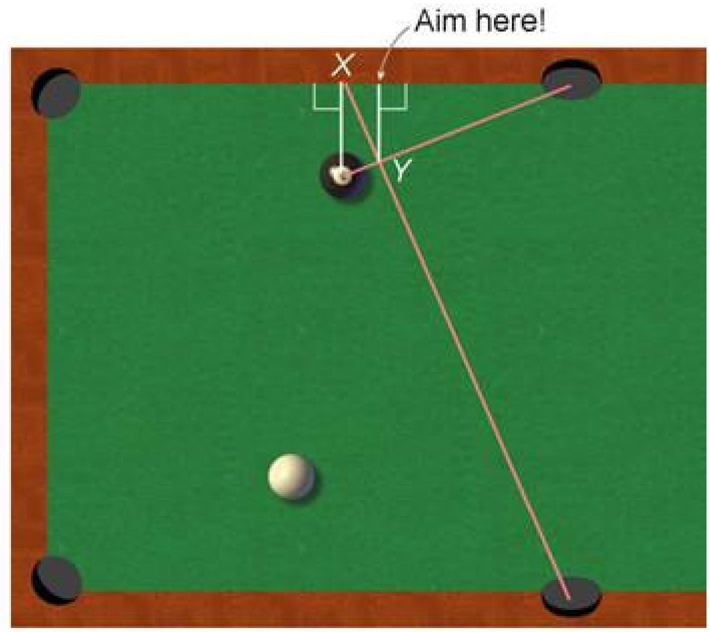
"Neat!" I thought. "But where should I try to make the 8 ball hit the top curb?" I wondered aloud.

I know from Grace's Summer Fun problems (see page 28) that the 8 ball will bounce off the curb at the same angle that it hits the curb. But how does one locate this special point that balances the incoming and outgoing angles?

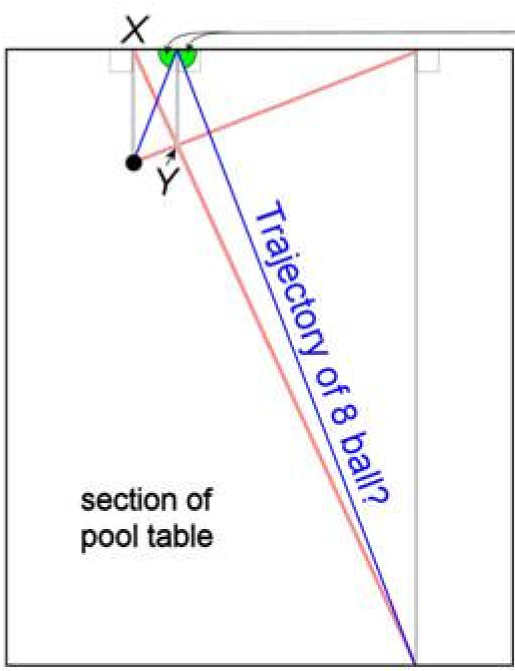
"Good question!" exclaimed my coach. "Let me show you a neat trick to figure that out.

[1] Although we don't explain how to play pool here, this situation arose during a game of "8 ball."

"Here's a simple way to get a good sense of where to aim," continued my coach. "It's not absolutely perfect, but it's a very good place to start. Imagine two lines on the pool table. One of them connects the center of the 8 ball to the center of the pocket *opposite* to the pocket you want to sink the ball in. The other connects the center of the destination pocket to the point marked X in the figure at right. The location of X is where the line from the middle of the 8 ball meets the curb at a right angle. These two lines, drawn in red in the figure, intersect at the point Y. From the intersection point Y, draw the line segment perpendicular to the curb. Aim so that the 8 ball will hit the curb exactly where this perpendicular touches the curb."

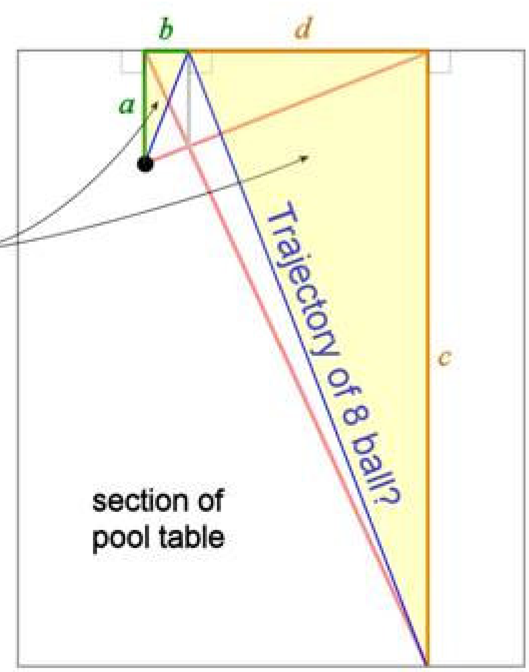


That's very interesting, I thought to myself. But how does that jive with my notion that the ball will bounce with equal incoming and outgoing angles? To find out, I drew an idealized geometric figure of the situation.



In other words, with the lines set out as directed, I have to show that these two angles are equal.

Now, those two angles are equal if and only if these two yellow right triangles are similar to each other. After all, once the measure of one acute angle of a right triangle is specified, the measures of all three angles are fixed by the fact that one of the angles is 90° and the sum of the angles of any triangle is 180°.

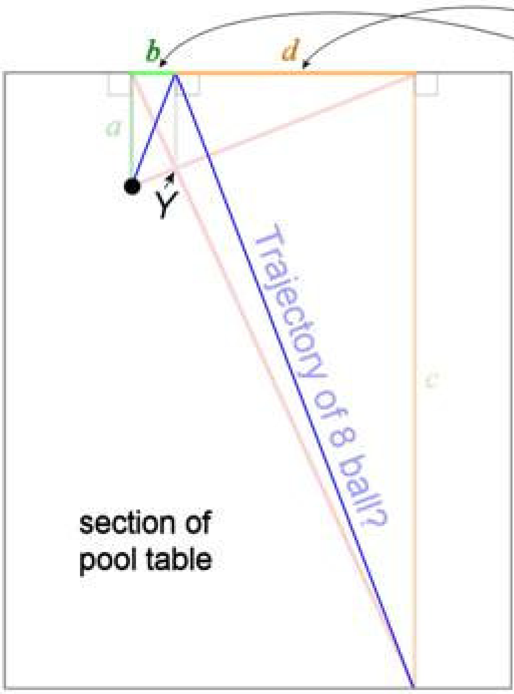


On the other hand, the two yellow right triangles are also similar to each other if and only if

$$a : b = c : d.$$

If this equation can be established, that would show similarity of the two yellow right triangles, which would in turn show that the incoming and outgoing angles in the proposed trajectory are in fact equal.

Now, watch what happens when the line segments labeled b and d are slid down the figure until they reach the level of point Y.

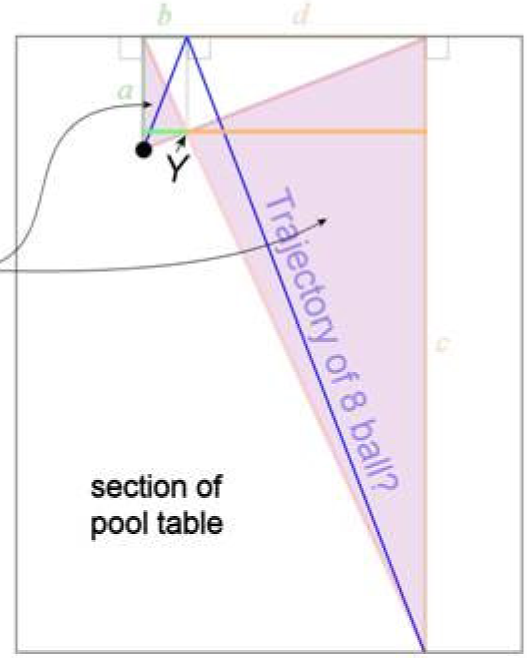


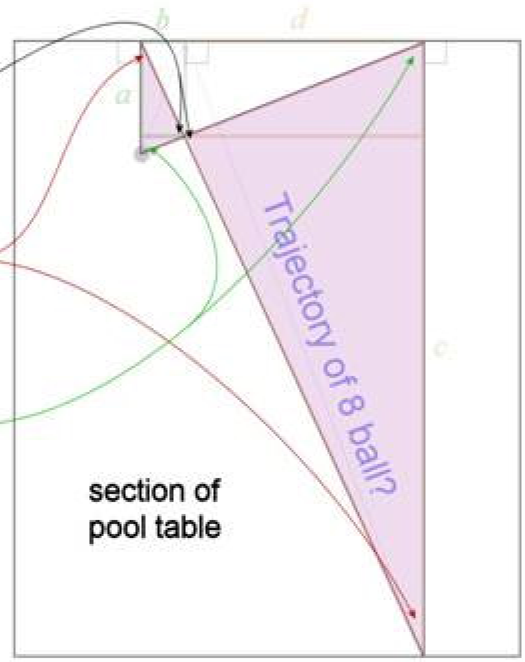
If we slide these two line segments down to be level with the point *Y*, they become altitudes of these two purple triangles.

Do the two purple triangles look similar to each other?

They are!

Do you see why?





These two angles are equal because they are vertical angles.

These two angles are equal because they can be viewed as alternate interior angles formed by a transversal crossing two parallel lines.

These two angles are equal because they also can be viewed as alternate interior angles formed by a transversal crossing two parallel lines.

Because the two purple triangles have the same angles, they are similar to each other. Therefore, because the ratios ***a*** : ***b*** and ***c*** : ***d*** are the corresponding ratios of height to base in each triangle, they are equal by similarity. In this way, we can conclude that the incoming and outgoing angles of the proposed trajectory of the 8 ball are, in fact, equal.

So my coach was right!

Err…well, my coach cautioned that this was not a perfect solution. That must have to do with the fact that pool balls are not idealized points. Also, when pool balls bounce on the curb, because of friction and spin, they don't always bounce so that their incoming and outgoing angles are equal. Still, it is a helpful guide.

I set out to play pool, but ended up doing math!

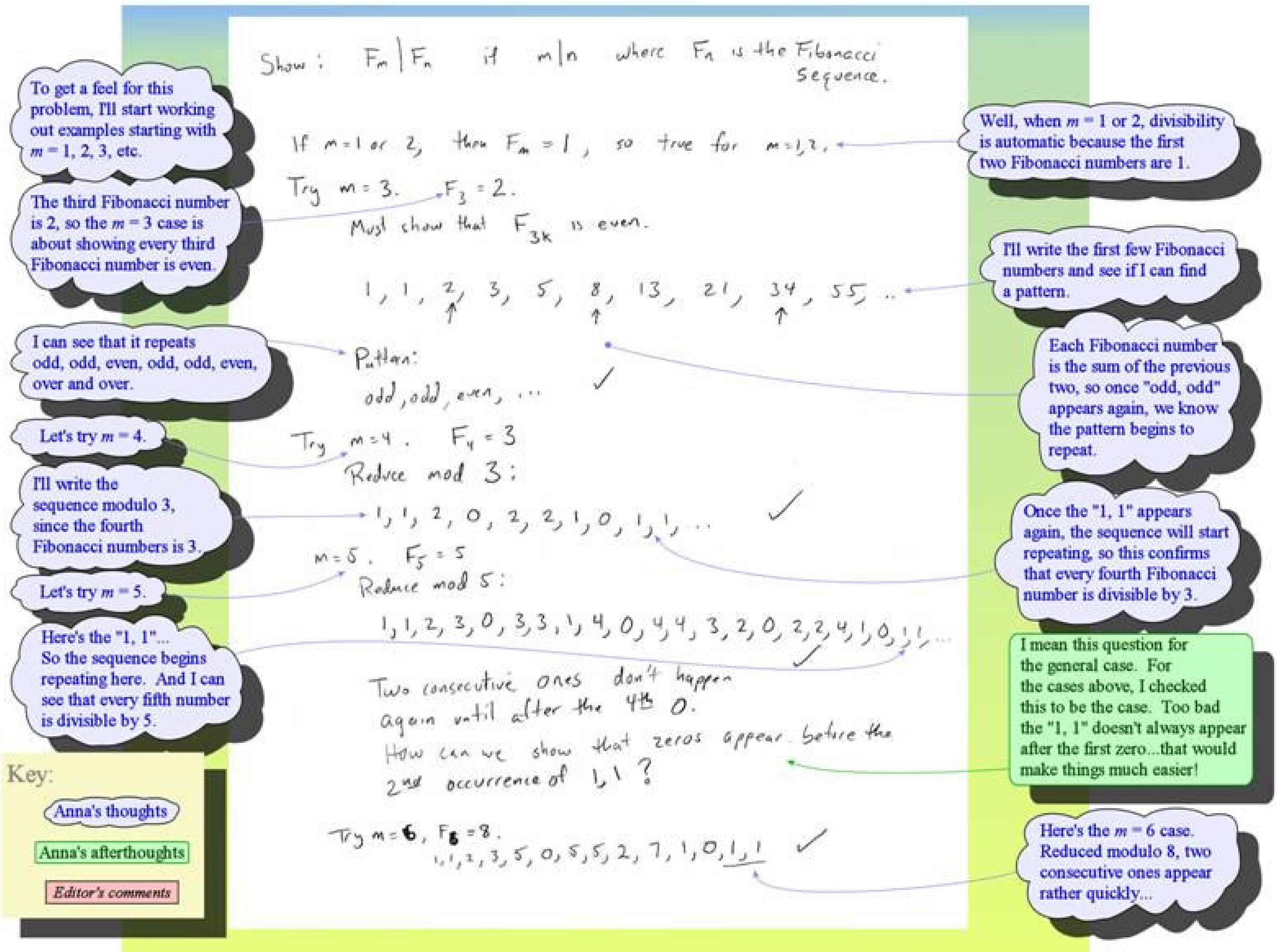
Editor: Grace Lyo

Anna's Math Journal

By Anna B.

In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles. Her process is not unlike that of even the most heralded mathematician: mathematics is a journey of discovery on which every mathematician will inevitably follow many wrong turns, believe many incorrect facts, and encounter many mysteries.

Anna tackles problem 3 from Ben Boyajian's Fibonacci number Summer Fun problem set on page 31. Anna makes ample use of modular arithmetic. For more on modular arithmetic, see *Prueba del 9* in Volume 2, Numbers 3 and 4 of this Bulletin or consult any book on number theory, such as *Invitation to Number Theory* by Oystein Ore.



I never knew this before!

(4, 6 did not appear ⇒ No Fibonacci number leaves a remainder of 4 or 6 when divided by 8.)

I actually began with the zero, then put an x. I can see why the x would repeat because the next number is the sum of the previous two, in this case $0 + x$. Then I continued the sequence a bit more, and then decided to trace backward a little bit too.

Looking over the examples I just computed, it seems that everytime there is a zero, the next two numbers are the same. That's interesting. Maybe that's a key, I should explore that further.

$$-x,\ x,\ 0,\ x,\ x,\ 2x,\ 3x,\ 5x,\ \ldots$$

Hey, wait a sec! The coefficients here appear to be the Fibonacci numbers...of course they are! The coefficients start 1, 1, and each coefficient is the sum of the previous two.

In other words...

The sequence that begins

$$x,\ x,\ \ldots$$

has nth term xF_n.

With this new knowledge, I'll write the sequence modulo F_m afresh.

$1, 1, \ldots, F_{m-2}, F_{m-1}, 0, F_{m-1}, F_{m-1}, \ldots, F_{m-1}F_{m-2}, F_{m-1}^2, 0$, etc ✓

I see! After the mth number, the first m numbers just repeat, only multiplied by F_{m-1}. Since 0 times anything is 0, there will be a 0 in the $2m$th position, and so forth. So this settles the problem!

In fact, the number just before ~~the~~ F_{km} is congruent to F_{m-1}^k modulo F_m:

I never knew this fact either!

$$F_{km-1} \equiv F_{m-1}^k \pmod{F_m}$$

Should be able to determine F_n modulo F_m for all n in terms of $F_1, \ , F_m$.

Hmmm...I should be able to express F_n (modulo F_m) in terms of just the first m Fibonacci numbers. I'll write the Fibonacci sequence modulo F_m again, only put m entries in each row only. Think...think...think...!

$0,\ 1, 1, 2, \quad , F_{m-1}, \cancel{0}$

$0, F_{m-1}, F_{m-1}, \quad , F_{m-1}^2,$

$0, F_{m-1}^2, F_{m-1}^2, . \quad , F_{m-1}^3,$

I think this should do it...

$$\boxed{\text{If } n = mk + r, \text{ then } F_n \equiv F_{m-1}^k F_r \pmod{F_m} \quad (0 \le r < m)}$$

It took a bit of thinking to come up with this... but I still don't know if there is a use for this fact. I feel as if it must hint at something deeper.

I'm not sure what to do with this fact. Actually, now that I've settled the original question, I'm wondering if the result is "if and only if"...I'll think about that instead.

Is it true that $F_m | F_n$ if <u>and only if</u> $m | n$?

Actually, what Anna is doing here is often a very good idea. If you have managed to show that some statement P implies Q, it's often a good idea to check to see if Q implies P, in other words, check if the ***converse*** *is true too.*

Assume $m > 2$ (because the case $m = 2$ is definitely not if and only if!)

Funny...maybe that formula can be used to resolve this question!

Suppose $n = mk + r$, $0 < r < m$.

We know $F_n \equiv F_{m-1}^k F_r \pmod{F_m}$

Want to show $F_{m-1}^k F_r \not\equiv 0 \pmod{F_m}$

F_m can't divide F_r because $F_r < F_m$ (since $r < m$)

?

Hm. Well. I'm stuck. I think I'll take a break and come back to this later. But at least I got the original question.

Can you help Anna show this? Send your thoughts to girlsangle@gmail.com.

AB 8.18.10

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

The Adventures of Emmy Newton

Episode 6. *The Music of the Spheres*

by Maria Monks

Last time: Emmy and Melissa were trying to make their way past the obstacles in the basement of their school to pick up a package from the Principal's office to bring to Mr. Wheel's classroom. They were greeted by Amy, who was coming the other way from the Principal's office with the package in her hand, and the three girls made their way past the last obstacle together. Meanwhile, the large pile of wooden planks in one of the rooms in the basement had caught fire, and the fire alarm was ringing. Emmy and Melissa hurriedly joined the crowd of students filing out the building...

Emmy knew they should tell a teacher that there was a real fire in the basement. She spotted Mr. Mann, the gym teacher. "Mr. Mann! Mr. Mann, there's a..."

"Keep moving, Emmy, keep moving, we have to evacuate and take roll."

"But there's a..." said Emmy and Melissa simultaneously.

"Keep moving, we have to evacuate!"

"Come on," said Emmy to Melissa, "We'll tell someone when we line up." They kept moving. The students and teachers of Euler Elementary School walked along their usual fire alarm exit route: out the front doors, across the parking lot, and onto the grass near the woods, where the teachers made sure everyone was accounted for. Ms. Spinner, the second grade math teacher, loved to count by sevens, so she always instructed the students to line up in rows of seven. The students all knew this by now, and they started lining up in rows of seven, facing the school with their backs to the trees. Emmy ended up in the back of her row, peeking around the heads to look at the building.

"Very good, very good," said Ms. Spinner, bustling up and down the rows, "Let's all get into rows of seven so we can take roll... Kyle, no rows of eight now, move to your left... ok, very good..."

The students, after lining up, started talking and whispering to each other across the rows. "Do you see smoke?" said one first-grade girl. "I think it's a real fire!" her friend squeaked.

"Ms. Spinner! Ms. Spinner, is it a real fire?"

"Now, now," said Ms. Spinner, and continued counting the students by sevens. "21, 28, 35, ..." She passed Emmy and Melissa's row, still counting, and Emmy interrupted her.

"Ms. Spinner," called Emmy, "There is a real fire, I saw it!"

"I did too!" said Melissa.

"Now now, don't go scaring the first graders."

"No, really..."

Ms. Spinner moved on and kept counting. "49, 56, 63, ..."

"How many students are absent today, Ms. Spinner?" asked Elizabeth as the teacher passed.

"Why none," said Ms. Spinner. "It's a perfect day, isn't it! So I'll just count to make sure we have the right number of students out here." She continued on. The other teachers, blissfully unaware that there was indeed a fire in the basement of the school, decided to join in and began to line themselves up in rows of seven. Emmy looked down towards the last of the rows of the students and teachers and saw that there were five left over in the last line. "Something's wrong," said Emmy.

"You mean that our school is on fire?" said Melissa. "Of course something is wrong."

“No, something is wrong with the number of people in the last line,” said Emmy. “I remember that there are 343 students and 28 teachers and staff in Euler Elementary. Ms. Spinner said that there are no students absent today, and if any teacher was absent they’d call in a substitute teacher, so the number of people here should be divisible by 7.”

“How did you check that so fast?” asked Melissa, trying to add 343 and 28 and divide the result by 7 in her head to see if there was any remainder. “Well, 343 is 7 cubed, so it’s 7 times 49... I memorized that,” Emmy added at Melissa’s quizzical look. Melissa sighed, and Emmy continued, “Although you could also figure out that 343 is divisible by 7 by noticing that if you add 7 to it, you get 350, which is easily divided by 7 - it’s 7 times 50. So if you sorted 350 people into rows of 7 and took away one row, you’d have 343 people left, and they are still sorted into rows of 7. So, 343 is also divisible by 7.”

“Ah, right,” said Melissa. “And then there are 28 teachers, who form 4 more rows of 7.” Her eyes lit up. “Ohhh... we’re really using the fact that if two numbers are divisible by 7, then their sum, or their difference, is also divisible by 7. So, since 343 and 28 are divisible by 7, the total number of people, 343 + 28, is divisible by 7!” That was a much cooler way to figure it out than doing long division.

“Exactly,” said Emmy, “And if you put Ms. Spinner in that last row, there are only six. Someone is missing...” she squinted over at the teachers.

“Where’s Mr. Wheel?” said Melissa, standing on her toes to see above the heads of the students. “He usually goes for a walk in the woods when we have a fire drill, but he always waits until roll has been called and everyone is accounted for. I didn’t see him at all this time.”

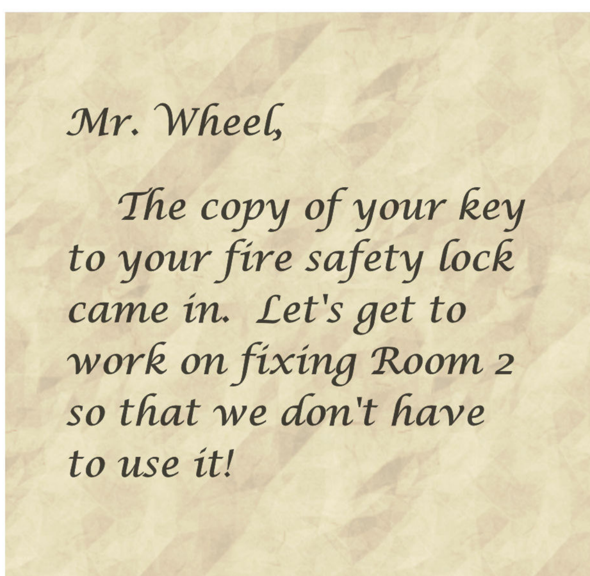
Emmy’s eyes lit up. “I think I know what’s going on,” she said. “Mr. Wheel would be exactly the kind of person to set up something like that in the basement, to challenge us, as a final exam or something. I bet he’s in there trying to put out the fire!”

Emmy started pacing back and forth in her small space between the two adjacent rows of chattering students, only able to take two small paces in either direction before turning around. “What do you think could have been in that package?” She stopped. “Amy,” she called.

Amy was two rows down from her, staring fixedly at the school. “Amy, do you still have that package?”

Amy turned, still wide-eyed, and nodded wordlessly. “Pass this to Emmy,” she whispered to Katy, who looked at the package, shrugged, and passed it along.

Emmy ripped open the package. Inside the package was a key, along with a note from the Principal. The note read,


Mr. Wheel,

The copy of your key to your fire safety lock came in. Let's get to work on fixing Room 2 so that we don't have to use it!

"The Principal!" said Emmy. "I didn't count the Principal! He's not around either. He usually is the last to exit the building in a fire drill, so the teachers won't realize he's gone either! He and Mr. Wheel must be in there trying to put out the fire, but neither of them has the key!"

"We have to get it to them!" said Melissa. "Let's go!"

"No, I'll just go. You stay here. It's too dangerous for both of us to go, and I think I know where their fire safety thing is. Plus, everyone is more likely to notice if two of us disappear, and we don't want to cause more chaos!"

"Are you kidding me? There's no way I'm not coming," said Melissa. She looked around. All the other kids were still talking to each other, not looking at Emmy or Melissa. The neat rows of seven that Ms. Spinner already counted had started to break down into amorphous groups.

"All right," said Emmy, who didn't want to waste time arguing. "Let's go through the woods." The two girls snuck back into the trails behind them and started running.

Emmy knew the trails well, as she always walked the long way out of the school at the end of the day, out the back door and through the woods to get to the far side of the parking lot where her mom picked her up. Melissa, who took the bus home, followed close behind as the trails wove through the woods, across fallen logs and over rocks, all the way to the back entrance of the school.

As they ran up to the door, it burst open before they even reached the handle. It was Mr. Wheel. He looked disheveled and agitated, and his right sleeve was singed. "Emmy, Melissa! There you two are! We've been looking all over for you. And where's Amy?"

"She's out on the grass, she's fine," panted Emmy. "We have the package from the Principal that you needed!" She held out her hand, holding the key.

"Thank goodness!" He took the key and ran back inside.

Emmy and Melissa looked at each other, still gasping for air. "Well, we finally delivered the package to Mr. Wheel!" said Emmy. Melissa laughed as she caught her breath.

Emmy peeked through the back door of the school. The hallway smelled of smoke. Mr. Wheel was running down the hall. The Principal was nowhere to be seen. Mr. Wheel suddenly stopped, and appeared to jam the key into the side of the wall. He pulled outwards and a section of the wall opened like a door. It was a big door, and after opening about 30 degrees from the wall, it got stuck. He pulled harder, but it didn't move. He tried to fit through, but the gap was too small.

"Emmy! Melissa! Come here!" Emmy and Melissa ran down the hallway. "Emmy, can you fit through the door?"

Emmy slivered sideways and just barely fit through the heavy door. She appeared to be in a closet of sorts, one that she never knew existed. It must have blended with the wall when the door was closed. She blinked and her eyes slowly adjusted to the dim light.

"Ow! Melissa, you're stepping on my foot."

"Sorry!" Melissa had come through the door as well.

"Do you see a big switch?" called Mr. Wheel.

Emmy turned around. The switch was behind her, in the OFF position. "Yeah!"

"Great. Turn it on!"

Emmy and Melissa grabbed the lever simultaneously and pulled down. They heard a loud hissing noise.

"Ok, you two go back outside with your class. Go! I'm going to go get the Principal."

Emmy and Melissa ran back outside, coughing from the smoke. They headed back along the trail and out to where they had been standing before. No one had seemed to notice them...

Except for Amy, who was still looking around wide-eyed. She turned when Emmy and Melissa popped out of the woods. "What's going on?" said Amy.

“I’m not sure, but I think everything is all right,” panted Emmy. “Mr. Wheel was in there, and we helped him pull a big switch that supposedly extinguished the fire. He went to get the Principal... there they are!” Mr. Wheel and the Principal were walking across the parking lot, looking relieved.

The principal looked at his watch and muttered something to Mr. Wheel. Mr. Wheel nodded. Emmy glanced at her own watch. There were only about 30 minutes left in the school day.

“Recess until the end of the day!” announced the Principal loudly. There was a roar of excitement, and the students ran off to the fields to the side of the parking lot, breaking out into games of soccer, basketball, and capture the flag. The teachers looked at each other in surprise, and some of them went up to the Principal to ask why he interrupted their lesson plans. Their eyes grew wide and they looked from him to the school building and back again as he began to explain.

Emmy and Melissa walked over to join Mr. Wheel looking down at his feet. “Mr. Wheel...” said Emmy, about to ask him what exactly was going on.

Mr. Wheel looked up. “Emmy! Melissa! The girls who saved Euler Elementary! Come on,” he gestured. “Let’s go for a walk on the trails, and I’ll tell you what’s going on.”

The three of them walked back into the woods and Mr. Wheel began to talk. “The building is fine,” he began. “The fire didn’t spread very far, and the lever you two pulled turned on our new backup sprinkler system in the basement in case of a fire.

“The mathematical obstacles in the basement were being set up as a final exam for the math classes,” he continued. “They could even be adjusted to different difficulties for different grades. We were almost done constructing it, but we were having some technical difficulties with Room 2 - the room with the wooden poles.”

“Yeah, they ended up crashing behind us and catching fire!” said Melissa.

“Right - we intended for some of the poles to be raised or lowered into the ground depending on which class was attempting the challenge. But the platforms that raised and lowered tended to overheat, causing a major fire hazard. So, before we even attempted to turn them on, we installed the new fire safety system, and locked it shut so that no one would accidentally flood the basement. We made a few copies of the key to open the lock, and the Principal wanted one key delivered to me. I sent you two to the office to pick it up, but I was so engrossed in teaching that I had forgotten all about it!” He shook his head and rubbed his hands in his hair. “When the bell rang for classes to change, Ms. Worthwright called my classroom to ask where you two were. It was then that I remembered, and realized what must have happened.

“The door to the basement stairway was blocked off all year as we were setting up the puzzles, but today the workers who were double-checking the fire safety system left it open. I figured that two young girls with as much curiosity as you could not possibly resist going down the extra flight of stairs!”

“We actually didn’t realize we went down too far until we were already there,” said Emmy, “We were working on your problems, and lost track of how many flights we went down!”

Mr. Wheel laughed. “I must be having a bad influence on you. Math can be addictive like that. In any case,” he went on, “I got a substitute to sit in for my class, I went down the stairs, and I then had a 50 percent chance of choosing the door that you went through - remember, the first room was on the corner, and you could go through either door if you solved the riddles. Of course, I chose the wrong door.”

“So there were even more puzzles the other way?” asked Emmy.

"Yep, and I'll save that for your actual final exam," said Mr. Wheel with a smile. "But it took me quite a while to make my way through the other obstacles, and I had no idea how far along you were. I then got myself stuck at one puzzle."

"What was the puzzle?"

"It's a secret, and don't worry, I'll fix it before finals come around." Emmy and Melissa looked at each other nervously. "But don't worry, it's not dangerous in the least. I just made it too difficult, and the puzzle is a bit different every time you enter... anyway, I was stuck on it for a while. But I made it through, and by that point I realized that you had gone the other way around the courtyard. The fire alarm then rang, and my worst fears were confirmed. I ran back to the first room and felt heat from the second room where the wood was on fire, and called your names. I didn't want to open the fireproof door for fear that the fire would spread, so I could only hope that you weren't trapped in there. As everyone evacuated, the Principal realized what must have happened, and he went down the opposite staircase to look for you."

"Oh yes, we made it out shortly after the fire alarm sounded," said Emmy, "So he must have missed us."

"Well, thank goodness you made it out... I'm quite impressed, actually," said Mr. Wheel. "That last puzzle was not an easy one. Anyway, we now had a conundrum; Amy had my key to the fire safety system, and for all we knew, she could have been trapped in a pile of blazing wood!

"The Principal kept looking for you guys down in the basement, and I decided to run out to the field to see if any of the other math teachers had their key on them. The other teachers also were given a copy of the key, see, but I didn't know where they put them. That was when I saw you two with the key!" He breathed a sigh of relief. "It's a good thing the walls that we installed between the rooms were fireproof, and the walls and ceiling of the basement are made of stone. Otherwise, we never would have had time to put out the fire before it spread and burned down the rest of the building."

Mr. Wheel sighed again, rubbed his hands in his hair, and shook his head. "What a day!"

"Yeah," said Emmy. "I have to say, though, your puzzles were a lot of fun to solve!" Melissa nodded in agreement.

"Well, I'm just happy you're safe. Safety is the one thing that is more important than math problems," said Mr. Wheel with a smile. Emmy and Melissa giggled.

They had apparently made a loop in the woods, because they were back out on the parking lot again. Emmy's mom was sitting in her car in the usual spot, waiting to pick her up. "See you tomorrow, Melissa!" she said. "Bye, Mr. Wheel!"

"Emmy!" her mom greeted her as she climbed into the car. "How was your day?"

"It was great!" said Emmy. "You'll never believe what we did in Mr. Wheel's class today!"

Things were back to normal, but Emmy sensed that there would be plenty more adventures to come.

Editor: Grace Lyo

Down to the Wire

by Julia Zimmerman

Clara scuffed her soles along the sidewalk, stepping purposefully on every crack. She knew it was bad luck to do that, but she felt too dejected to care. *As if my luck could get any worse,* she thought. To reinforce the gravity of the situation, she dragged out a resigned sigh.

Finally, her best friend took the bait: “Okay, I’ll bite - what’s the matter? It’s only the second week of school, and I really don’t think anything so terribly disastrous can have happened already.”

Clara shot Elizabeth a dark glare. Usually she enjoyed Elizabeth’s dry humor, and they made a great Laurel and Hardy, but this was a serious matter. “What date did *you* draw to present your Great Scientist project? And how far have you gotten on yours?”

Elizabeth frowned. “I drew next Wednesday. I’ve searched for my scientist, Jane Goodall, online, and I’ve gotten some books from the library. Why?”

“Because *I* drew *tomorrow,* and all I’ve done so far is the decorative border on the poster board!”

“Clara! You knew there was a chance you’d be the first one to present; besides, we got the assignment last Friday!”

“I know, I know... but *Broadway: Greatest Performances* was on, and you know how I feel about show tunes! Besides, I thought a 1 in 26 chance of going first was totally worth the risk.”

Elizabeth rolled her eyes. Leave it to Clara to think four hours of contrived fictional situations set to cheesy music was worth trading for 20% of a semester grade. Still, Clara *was* her best friend, so she said, “Well, I want to help, but I’ve got softball practice until 5:30, so how ’bout you get working - on something *besides* the decorative border - and then bike to my house around 6:00 and we’ll pull something together. Okay?”

Clara threw her arms around Elizabeth. “You’ve *saved* me!” Paraphrasing Scarlett O’Hara from *Gone with the Wind*, she added, “As God is my witness, I’ll never procrastinate again!” Elizabeth laughed and waved good-bye.

When Clara got home, she threw herself into her task with gusto (that’s how she did most things); she soon found herself in the library, surrounded by books on computers and the navy. She stayed that way, curled up with a book in her lap, until her ringtone startled her out of her concentration.

“Hello? Oh, hey, Elizabeth. What time is it? Already?? I’ll be right over - sorry I’m late. No, I’ve been reading the whole time! Yes, of course reading *relevant* books. Yeesh.” Quickly, she gathered her backpack and notebook and headed to Elizabeth’s.

Elizabeth barely had time to get the door open before Clara was off, talking a mile a minute about her scientist: “You’re going to love this, Liz, it’s so cool! She wrote the A-0 system for UNIVAC and --”

“Clara! Wait! I don’t even know who ‘she’ is yet! You’re not making any sense! Can you start over and go more slowly?”

Clara tried to slow down, but her thoughts still seemed to fly out of her mouth in disjointed phrases: “Right. Well, ‘she’ is Grace Hopper… a rear admiral in the Navy during the ’40s, ’50s, ’60s, ’70s, and ’80s. She married Vincent Hopper in 1930… in 1928 she graduated from Vassar with degrees in math and physics… then went to Yale for graduate school in math. Oh, Vincent was chair of the NYU English department. She developed the first compiler…”

Elizabeth frowned. *Compiler*, she thought, *what’s that*? But Clara was going so fast, she couldn’t find a convenient moment to interrupt her friend and ask.

Clara continued her torrent, "…invented the term 'debugging,'- that's because she traced the breakdown of a computer to a *real bug* that got caught inside the machine! So she fixed the computer by removing it…de*bug*ging…ha ha…get it?- worked on the Mark I, II, and III programming staff…the Mark I, II, and III? Oh those are computers… She even turned down a full professorship at Vassar so that she could take a Navy contract to work at Harvard's Computation Lab. Grace Hopper helped develop a new computer language called COBOL…"

"Hold on! Hold on! That's a little better," said Elizabeth. "But, what's a compiler?"

Clara sketched the function of a compiler: "Oh, a compiler? Well, to understand what a compiler is, you first have to understand that computers are basically machines that manipulate bits of information. A bit is a tiny piece of data that can hold only the single value 0 or 1. So computers come with a language, sometimes called 'machine language' that manipulates these bits, or small collections of bits, in various ways. However, humans find it very difficult to write programs in this machine language because humans tend to think in more conceptual terms and not in terms of zeros and ones. So 'higher level' languages were created to enable humans to write programs using a more conceptual approach. A compiler takes instructions written in such a high-level language and translates them into the machine language. In other words, a compiler typically plays the role of translating from a human-friendlier language to a machine-friendlier language."

"Oh, cool! This presentation is going to be great!" Elizabeth grinned at Clara, and tossed her a marker. "Let's discuss the facts we want to convey and the order we think would be clearest!"

An hour and a half later, the girls had worked out the order and content of their presentation, and they'd moved them from Clara's thoughts to the words on their poster. They wanted their classmates to know that Grace Hopper had served for many years in the Navy, advancing the emergent field of computer science by delineating concepts such as compilers and machine-independent programming languages. She had taught at Vassar and worked at the Harvard Computation Lab, and authored and co-authored many papers. Her early excellence in math and physics - she'd enrolled at seventeen in Vassar and done graduate work in mathematics at Yale - were indicative of the creativity and intelligence she would later apply to the field of computer science. As a pioneer in the new field, she faced unanticipated problems, such as the shut-down of a giant computer when a moth became trapped inside it (leading to the term "de-bugging"). The US Government was so appreciative of her service that they named the naval warship USS Hopper after her and awarded her the Defense Distinguished Service medal upon her retirement. She received many other awards, including the National Medal of Technology, and buildings have been dedicated to her or named in her honor.

Grace Hopper gives her autograph and a "nanowire" to a Navy programmer in 1985.

Naval History & Heritage Command photo NH 96931

Clara and Elizabeth decided that they would re-enact one of her most well-known methods for explaining the nanosecond. Since Hopper was at the forefront of the burgeoning computer science field, many times she faced audiences who were not familiar with how computers worked. To explain why satellite communication was not instantaneous, and to explain why computers needed to be small to be fast, she would pass out pieces of wire just under a foot in length to her audience. She referred to the wires as "nanoseconds," because their length was the maximum distance that light can travel in one nanosecond. She would then show the audience a huge coil of wire almost a thousand feet in length, which represented the maximum distance light can travel in a microsecond.

The next day, near the end of the presentation Elizabeth got up from her desk and opened the door for Clara; Clara went out into the hallway to retrieve her red radio-flyer wagon. On the wagon, there was a length of rope Clara and Elizabeth had made by tying shorter pieces together (they hadn't been able to find a thousand-foot coil of wire, though they'd tried).

"This," she said, "represents one microsecond. There are a billion nanoseconds in a second, and there are one million microseconds in a second, so there are one thousand nanoseconds in a microsecond. Look at the piece of wire in your hand. Now imagine that the wire is thinner in diameter than one of your hairs, and that it is coiled up inside a computer. The smaller the computer is, the smaller the wires connecting all the parts. And the smaller the wires, the less time it takes for the transmission of information, and the faster it will run. That's why computers need to be small to be fast. As to why satellite communication takes time, imagine all the distance between a satellite and earth. Light travels faster than anything else we know of. So even if we transmitted information using light, the nanosecond wire represents the furthest distance the information could travel in a nanosecond. See why it can't be instantaneous?"

Their teacher smiled at Clara, who felt relieved and even proud at the conclusion of her presentation. "Clara, would you please explain the relationship between distance, speed, and time, more plainly?"

Clara, who felt she'd been perfectly clear, remembered what Elizabeth had said when she'd first burst into Elizabeth's house, spewing disjointed facts about Grace Hopper. Grace Hopper had used this teaching aid to make an abstract concept that relied on her personal knowledge and experience understandable to a wide variety of people; this was Clara's chance to do the same thing.

"Well," she started hesitantly, "it's a direct consequence of the formula 'distance equals rate times time.' That's exactly the concept Grace Hopper was relying upon when she came up with her nanosecond wire. The speed of light in a vacuum is constant. That means that for light, distance traveled in a vacuum is directly proportional to time. Thus, distances can be measured in terms of the time it would take light to traverse that distance. Astronomers use light years, for instance. The nanosecond wire has length equal to the distance that light can travel in one nanosecond. There is nothing we know of that travels faster than light in a vacuum. That's what makes Grace Hopper's nanosecond wire so relevant. Her nanosecond wire represents the maximum distance a piece of information can travel in one nanosecond."

Clara's teacher nodded at her. "Well said!"

Clara had a vision of her classmates as the audience at the Oscar's, wearing designer gowns and priceless jewels. Should she give an acceptance speech? There were so many people to thank! But her classmates began to clap, so Clara decided to give only one delicate bow and a gracious smile to the class before returning to her seat. After all, being understated could be so classy! Visions of Audrey Hepburn in *Breakfast at Tiffany's* ran through her head.

On their way out of the building that afternoon, Elizabeth said, "Wow, Clara, nice job!"

"Grace Hopper inspired me to learn more about math and computer science. She was so creative and hardworking!" Clara, who had decided sometime in fifth period that maybe being understated wasn't quite her thing, added, "Just like some might say that I am, with my plays. Did I mention that I'm writing a musical about her life? *Amazing Grace*. It's going to have *lots* of cool stuff, like in this one scene, dancing Turing machines! And, of course, *pizzazz*."

As Clara spun on the sidewalk, singing the opening phrases of her musical and scattering her books all over the ground in the process, Elizabeth smiled. "Pizzazz," she said to herself, "of course."

This red curve represents the maximum distance light can travel in one nanosecond.

Summer Fun!

In the last issue, we invited members to submit solutions to a number of Summer Fun problem sets.

In this issue, solutions to many of the problems are provided. These solutions will sometimes be rather terse and, in some cases, are more of a hint than a solution. We prefer not to give completely detailed solutions before we know that most of the members have spent time thinking about the problems. The reason is that *doing* mathematics is very important if you want to learn mathematics really well. If you haven't tried to solve these problems yourself, you won't gain as much when you read these solutions.

Solutions that are especially curt will be indicated in red. Please do not get frustrated if you read a solution and have difficulty understanding it. If you run into difficulties, we are here to help! Just ask!

If you haven't thought about the problems, we urge you to do so *before* reading the solutions. Even if you cannot solve a problem, you will benefit from trying. Even more, it will become easier to read other people's solutions after you've tried to solve the problems yourself.

With mathematics, don't be passive! Get active!

Move that pencil and move your mind! Your mind may just end up somewhere no one has been before.

Also, the solutions presented are *not* definitive. Try to improve them or find different solutions.

Set Inclusions

by Julia Zimmerman

In all these problems, we will use capital letters of the English alphabet as they appear below:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

An object is symmetric about a line if half of its image can be reflected about that line to form the whole image. For example, this letter **T** is symmetric about a vertical line. An object has 180° rotational symmetry if half of its image can be rotated 180° about a point to form the whole image. For example, this letter **N** has 180° rotational symmetry about a point halfway along its diagonal.

Some Set Related Terminology

- If an element x is in a set p, then we write "$x \in p$".
- If every element in a set q is also in set p, then we say that "q is a subset of p" and write "$q \subseteq p$".
- If $q \subseteq p$ but $q \neq p$, then we say that "q is a proper subset of p" and write "$q \subset p$".

Note: Some people use the symbol "$\subset$" for subset and put a small "$\neq$" beneath the "$\subset$" symbol to stand for proper subset.

1. What fraction of the alphabet is symmetric about a line? What fraction of the alphabet has 180° rotational symmetry?

Let a be the set of every capital letter of the alphabet.
Let b be the set of capital letters which are symmetric about some line.
Let c be the set of capital letters which are symmetric about a vertical line.
Let d be the set of capital letters which are symmetric about a horizontal line.
Let e be the set of capital letters which have 180° rotational symmetry.
Let f be the set of capital letters which have 180° rotational symmetry about a point that is not part of the image of the letter.

2. Are any of these sets defined above subsets of any of the other sets? For example, is $b \subseteq c$? Is $c \subseteq b$? How do you know?

3. Are any of these sets *proper* subsets of any of the other sets?

4. Is it true that each of the sets a, b, c, d, e, and f is a subset of itself? Why or why not?

For problems 5 and 6, let p, q, and r be sets.

5. If $p \subset q$ and $q \subset r$, then is it true that $p \subset r$? Is it true that $p \subseteq r$? If $p \subseteq q$ and $q \subseteq r$, then is it true that $p \subset r$?

6. Suppose $p \subseteq r$ and $q \subseteq r$, and we know that the $x \in p$. What other sets necessarily contain x?

7. For each integer m, define S_m to be the set of multiples of m. In other words, S_m contains the numbers km for all integers k. When is $S_m \subseteq S_n$? When is $S_m \subset S_n$? When is $S_m = S_n$? What elements are in both S_m and S_n?

Summer Fun!

Solutions (Julia Zimmerman and Ken Fan)

1. There are 14 capital letters which are symmetric about a line:

A C D H I K M O T U V W X Y

If you included B and E, it may be acceptable. In the type we provided, the middle bar of the B and E are just slightly above the central position, so we didn't count them. This means that 7/13 of the alphabet (as provided) is symmetric about a line. By contrast, 7/26 of the alphabet (as provided) has 180° rotational symmetry.

2. We have $f \subseteq a, b, c, d, e$. Also, $e \subseteq a$, $d \subseteq a$, $d \subseteq b$, $c \subseteq a$, $c \subseteq b$, and $b \subseteq a$.

3. In fact, every set inclusion in the solution to problem 2 is a proper set inclusion. To see this, one checks that for each inclusion, the two sets do not contain the same elements.

4. Yes, this is true by definition.

5. The answers are yes, yes, and no, respectively. For the first question, if we have $p \subset q$ and $q \subset r$, then every element of p is in q and every element of q is in r. This means that every element of p is also in r. Because both inclusions are proper, it means that there exists an element in r which is not in q. This element cannot be in p either, because if it were, it would be in q since $p \subset q$. Therefore $p \subset r$. The second question follows because if $p \subset r$, then, by definition, we also have $p \subseteq r$. For the third question, the answer is no because it is possible that $p = q = r$, in which case we would have $p \subseteq r$ but not $p \subset r$.

6. We are given that $p \subseteq r$. By the definition of subset, if x is in p, then x is in r. However, we cannot conclude that x is in q. For example, suppose $p = \{0\}$, $q = \{1\}$, and $r = \{0, 1\}$. Then p, q, and r satisfy the conditions of the problem. If we take $x = 0$, then we see that x is in p, but x is not in q.

7. We have $S_m \subseteq S_n$ whenever m is a multiple of n. To see this, suppose $S_m \subseteq S_n$. Note that $m \in S_m$, so $m \in S_n$. This means that $m = nk$ for some integer k, i.e. m is a multiple of n. Conversely, if m is a multiple of n, then we can write $m = nk$ for some integer k. But this means that $m \in S_n$. Every element of S_m can be written as mj for some integer j, and since $mj = nkj$, we see that every element of S_m is also in S_n, which shows that $S_m \subseteq S_n$.

If $S_m \subset S_n$, from the last paragraph, we know that $m = nk$ for some integer k. For the inclusion to be proper, there must be some element nj in S_n such that nj is not in S_m, in other words, such that nj is not a multiple of m. If n is divisible by m, no such element exists, but if n is not divisible by m, then we can take $j = 1$. We conclude that $S_m \subset S_n$ if and only if m is a multiple of n but not equal to n or $-n$.

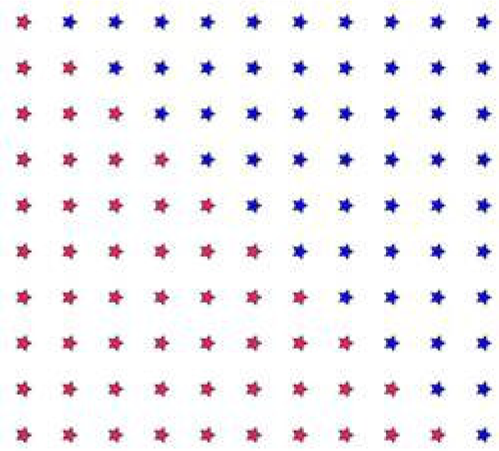
If $S_m = S_n$, then we have both $S_m \subseteq S_n$ and $S_n \subseteq S_m$. This means that n is a multiple of m and m is a multiple of n, and this can only happen if $m = n$ or $m = -n$.

An element is in both S_m and S_n if and only if it is a number which is a multiple of both m and n. In other words, the union of S_m and S_n is equal to S_a where a is the least common multiple of m and n.

Summer Fun!

Adding Up Numbers

by Helen Wong



For this problem set, you might find it helpful to watch my Women In Mathematics video on the Girls' Angle website.

1. The picture at right has red and blue stars arranged in 10 rows and 11 columns. How many red stars are there in each row? Can you see how it can be used to explain Gauss' trick to show that $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \frac{1}{2}(10)(11)$?

2. All together, the seven Harry Potter books contain a total of 4100 pages. Amy is an incredibly gifted girl (and extremely quick at reading), but it takes her a while to get hooked onto the books. The first night, she reads only the first page. The second night, she is a bit more interested and manages to read the next two pages. The third night, she starts where she left off and reads the next three pages. She continues this way, so that on the *n*th night, she reads *n* new pages. What page will she be on at the end of one week? How long will it take her to finish all seven books?

3. What is the sum of the first five *odd* numbers $1 + 3 + 5 + 7 + 9$? The first six odd numbers? The first seven odd numbers? Do you see a pattern?

4. Jo wants to collect Silly Bandz. She and her aunt have struck a deal. For the first week that Jo gets an A in class, her aunt will give her one new bracelet. If Jo keeps up her A for a second week, her aunt will give her two new bracelets at the end of the second week. If Jo keeps it up for three weeks in a row, her aunt will give her four new bracelets at the end of the third week. Because Jo's aunt worries that Jo will forget about their deal, she writes down her promise using a chart:

Week	New bracelets
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512

Week	Total bracelets
1	1
2	3
3	7
4	15
5	
6	
7	
8	
9	
10	

Help Jo and her aunt fill in the chart for the total bracelets! Is there a pattern? Can you find a formula?

Summer Fun!

Solutions (Ken Fan)

1. There is 1 red star in the first row, 2 in the second, 3 in the third, and so on. Thus, the total number of red stars in the figure is equal to 1 + 2 + 3 + … + 10. The number of blue stars in the diagram is the same as the number of red stars, and together, all the stars form a 10 by 11 rectangular arrangement. Such a rectangular arrangement has 10×11 total stars, so the number of red stars must be half of this. Thus, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \frac{1}{2}(10)(11)$.

2. At the end of n days, she will have just finished reading page number $1 + 2 + 3 + \ldots + n$. Since there are 7 days in a week, at the end of a week, she will have just finished reading page number $1 + 2 + 3 + \ldots + 7 = 0.5(7)(8) = 28$.

To figure out how long it will take her to read all seven books, we have to find the smallest integer n such that $1 + 2 + 3 + \ldots + n \geq 4100$. In other words, for what n is $\frac{n(n+1)}{2} \geq 4100$? Multiplying both sides by 2, this becomes $n(n + 1) \geq 8200$. If you know the quadratic formula, you can use that to get at the answer directly. But if you don't, you might reason that $n(n + 1)$ is roughly n^2 and the square root of 8200 is about 90. This will be too small, but it will be close to the correct answer. We compute 90(91) = 8190 and 91(92) = 8372. So the answer is that on the 91st day, she will have finished all seven Harry Potter books.

3. See **Mouse**'s solution on page 11 of volume 2 number 1 of this Bulletin.

4. Here's the chart filled in:

Week	New bracelets
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512

Week	Total bracelets
1	1
2	3
3	7
4	15
5	**31**
6	**63**
7	**127**
8	**255**
9	**511**
10	**1023**

Notice that if you add one to every entry in the "Total bracelets" column, you get the entries in the "New bracelets" column, only shifted by one row.

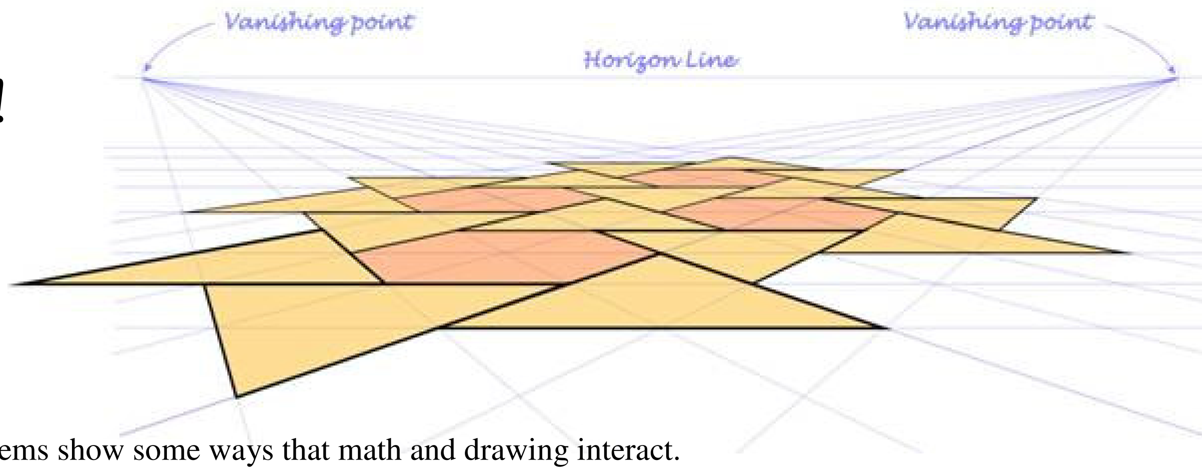
The "New bracelets" column lists the powers of 2. The entry in the nth row is 2^{n-1}. Therefore, the entry in the nth row of the "Total bracelets" column is $2^n - 1$.

To prove this rigorously, you have to show that $1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$. There are many ways to do this. One way is to interpret it as a binary addition problem. If you have a different way, feel free to share it with us!

Summer Fun!

Draw!

by Ken Fan



These problems show some ways that math and drawing interact.

1. Make a scale drawing of the floor plan of your home.

2. A circle in a perspective drawing will always look like some sort of ellipse. Here's a standard way to draw an ellipse: Take a string. Place your drawing on top of something that you can put a thumb tack into, such as a scrap of soft wood. Push two tacks through the paper into the backing. Tie the ends of the string to the tacks leaving slack. Now, take a pencil and use the tip of the pencil to hold the string taut. Draw around the tacks keeping the string taut and the pencil vertical at all times. The result is an ellipse! What kind of ellipse do you get if you tie both ends of the string to the same tack?

It's challenging to draw perfect geometric figures like circles and straight lines with a free hand. But if you have a compass for making circles and a straightedge to make straight lines, many geometric figures can be drawn with great accuracy. The next three problems all utilize a compass and straightedge.

3. Think of a way to draw a perfect equilateral triangle with the aide of a compass and straightedge.

4. Now try drawing a perfect square.

5. How about a perfect regular pentagon? Can you draw a perfect regular pentagon using a compass and straightedge? Here's a handy fact: $\cos 72° = \dfrac{\sqrt{5}-1}{4}$.

6. In Vermeer's *The Art of Painting* (see page 31 of the last issue of this *Bulletin*), there is a perspective drawing of a checkered floor. Make a perspective drawing of a tiled floor of your own design. If you're not sure which one to try, I'll suggest starting with a brick pattern or a tiling with equilateral triangles. As an example, at the top of this page is a perspective drawing of a tiling that consists of regular hexagons and equilateral triangles. Auxiliary lines used to construct the drawing have been left in place.

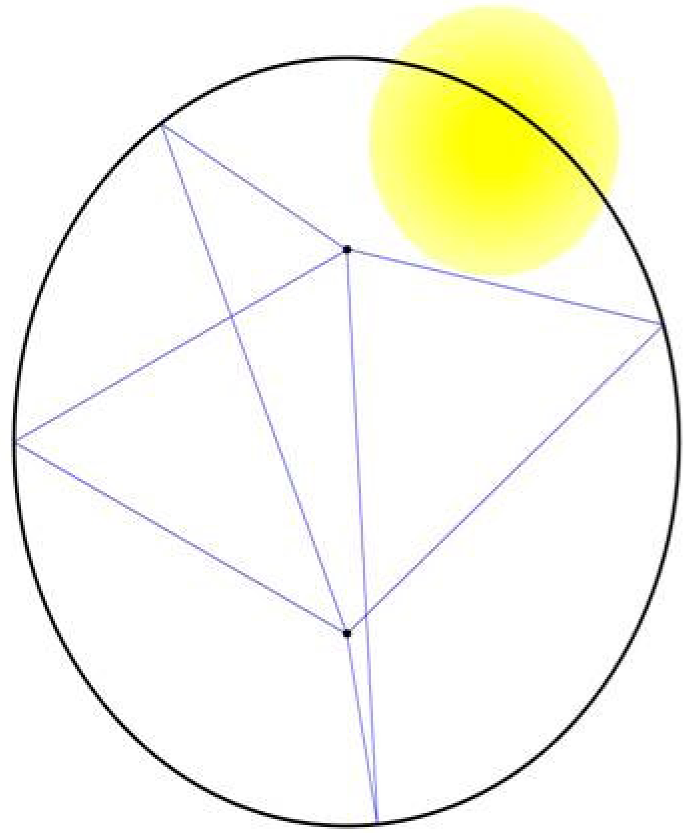
Send your drawings to girlsangle@gmail.com!

Summer Fun!

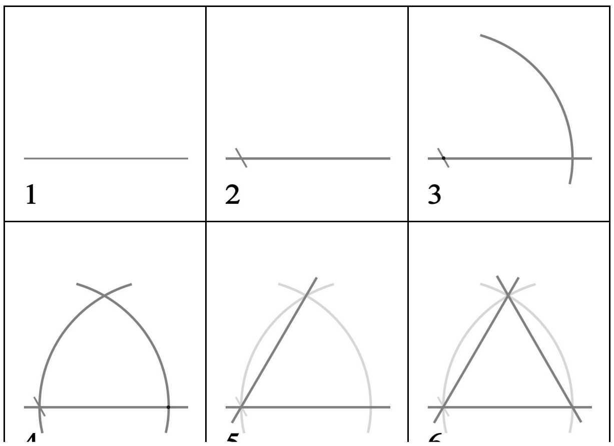
Solutions (Ken Fan)

Answers could vary for all of these problems.

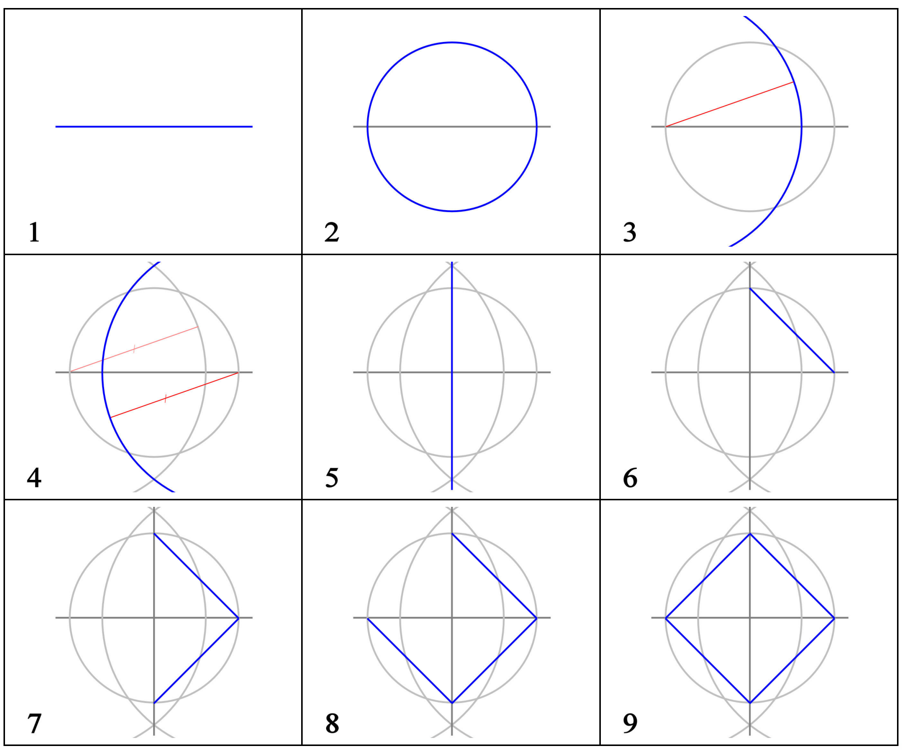
2. An example is shown at right. Experiment with different separations between the tacks and different string lengths.



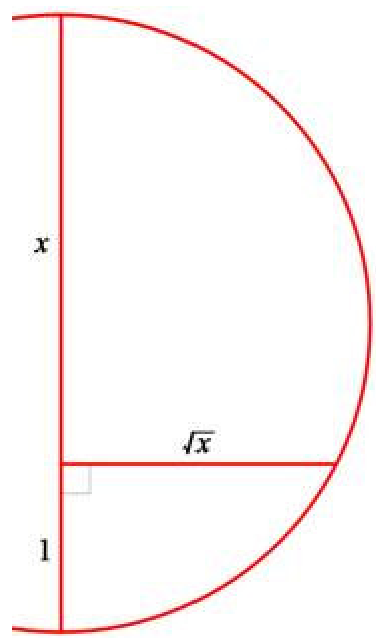
3.



4.



5. Here's a small hint:



6. The answer is about 6 feet. It's not really possible to be much more accurate without having a better reproduction or access to the original painting.

Summer Fun!

The Factorial

by Lightning Factorial

This Summer Fun problem set explores properties of the factorial.

1. Complete the following table.

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24						

2. Show that the number of trailing zeros in $n!$ is given by

$$\sum_{k=1}^{\infty} \left\lfloor \frac{n}{5^k} \right\rfloor = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{5^2} \right\rfloor + \left\lfloor \frac{n}{5^3} \right\rfloor + \ldots$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x. This formula was used in last issue's *Anna's Math Journal*.

3. What is the coefficient of $abcd$ in $(a + b + c + d)^4$?

4. Ten books all by different authors are found shelved in no particular order. What is the probability that they are ordered alphabetically by the authors' last names?

Wilson's theorem states that if p is a prime number, then $(p - 1)! + 1$ is divisible by p. The next few problems outline a way to prove this. Fix a prime number p.

5. Check that Wilson's theorem is true for $p = 2$ by direct computation.

6. Assume from now on that $p > 2$ and let m be a positive integer less than p. Consider the first $p - 1$ multiples of m: $m, 2m, 3m, \ldots, (p - 1)m$. Show that no two of these multiples of m share the same remainder when divided by p. In other words, let km and lm be two of these multiples with $1 \le l < k < p$. Show that p does not divide the difference $km - lm$.

7. Use 6 to conclude that there is exactly one positive integer $k < p$ which makes the product km leave a remainder of 1 when divided by p.

8. Show that there are exactly 2 positive integers x less than p that make $x^2 - 1$ divisible by p. (Recall that $p > 2$.) What are they?

9. Use 7 and 8 to show that $(p - 1)!$ can be written as 1 times $p - 1$ times a product of $\frac{p-3}{2}$ numbers that each leave a remainder of 1 when divided by p. From this, show that $(p - 1)! + 1$ is divisible by p.

Summer Fun!

Solutions (Lightning Factorial)

1.

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	**120**	**720**	**5040**	**40320**	**362880**	**3628800**

2. Here's a sketch. The number of trailing zeros in a number is equal to the highest power of 10 that divides that number. Suppose 2^x is the highest power of 2 that divides the number and 5^y is the highest power of 5 that divides the number. Then the highest power of 10 that will divide the number is the minimum of x and y. So the problem is equivalent to figuring out the highest powers of 2 and 5 that divide $n!$. If 2^x is the highest power of 2 that divides $n!$ and 5^y is the highest power of 5 that divides $n!$, one can show that $y < x$, so the number of trailing zeros in $n!$ will be equal to y. The formula provided is a formula for this number y.

3. The coefficient of $abcd$ in $(a + b + c + d)^4$ is 4!.

4. There is only 1 way these books can be ordered alphabetically by author's last name. However, there are 10! ways to order them. With no reason to assume that any way of ordering these books is any more likely than any other way, the probability is $\frac{1}{10!}$.

5. When $p = 2$, the expression $(p - 1)! + 1$ evaluates to $(2 - 1)! + 1 = 2$, and 2 is divisible by 2.

6. Suppose km and lm leave the same remainder when divided by p, where, without loss of generality, we assume $1 \le l < k \le p - 1$. It follows that p must divide $km - lm = (k - l)m$. However, both $k - l$ and m are less than p so p, being prime, cannot divide either and so it cannot divide their product. This contradiction shows that the remainders of $m, 2m, 3m, \ldots, (p - 1)m$ upon division by p are all distinct.

7. Problem 6 tells us that the $p - 1$ numbers $m, 2m, 3m, \ldots, (p - 1)m$ each leave a different remainder when divided by p. Because p cannot divide any of these numbers, none of these remainders can be zero. But there are only $p - 1$ nonzero remainders. Therefore, each of the possible remainders is achieve exactly once. Thus, there is exactly one positive integer $k < p$ for which the product km leave a remainder of 1 when divided by p.

8. Suppose p divides $x^2 - 1 = (x + 1)(x - 1)$. Because $p > 2$, we must have that p either divides $x + 1$ or p divides $x - 1$. If p divides $x + 1$, then $x = p - 1$. If p divides $x - 1$, then $x = 1$. (Recall that we are restricting x to positive integers less than p.)

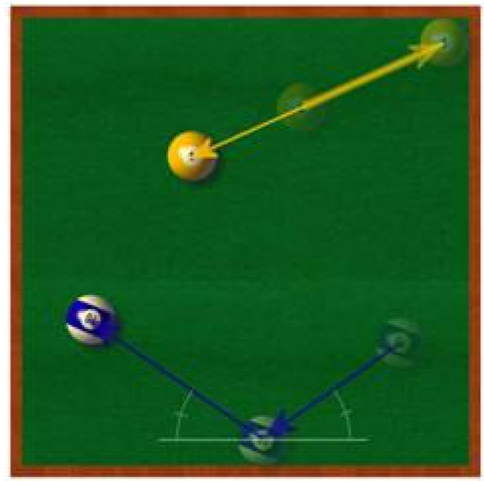
9. Problems 7 and 8 tell us that each number from 2 to $p - 2$ (inclusive) can be paired with a unique different number also between 2 and $p - 2$ (inclusive), such that their product leaves a remainder of 1 when divided by p. Note that if x is paired with y under this scheme, then y will be paired with x because $xy = yx$. This means that the product of the numbers from 2 to $p - 2$ (inclusive) will leave a remainder of 1 when divided by p. Therefore $(p - 1)!$ will leave a remainder of $p - 1$ when divided by p, and so it follows that $(p - 1)! + 1$ is divisible by p.

Summer Fun!

Pool

by Grace Lyo

Casi is spending the summer at her parents' cabin in the woods. There aren't very many kids around, nor is there a computer with an internet connection. There is, however, a square pool table, some pool balls, and a cue stick.



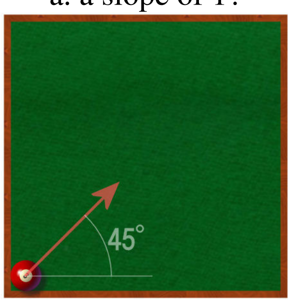
What's pool? In case you've never played pool before, a pool ball is a hard ball about the size of a peach. The ball is placed on the pool table, and a cue stick, which is a long, straight stick, is used to hit the ball in a precise direction. This is called "shooting" the ball. The ball then rolls on the table until it reaches the edge. The edge of the table is upraised (the upraised part is called a "curb"), so that when the ball runs into it, it bounces.

Casi's puzzle. By experimenting, Casi notices that whenever the ball hits the curb, it bounces as in the picture at right. If the ball bounces off a side curb, the two indicated angles are equal. If the ball bounces from the corner, it comes back along the same line that it went in. She also notices that if she hits the ball hard, it bounces around the table and sometimes comes back to where it started.

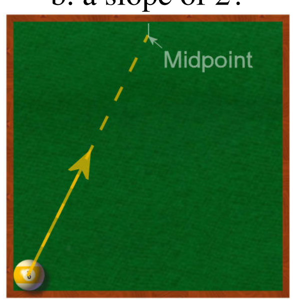
She decides to systematically shoot the ball from the bottom left corner of the table and count how many bounces are required before the ball returns to the starting place (if it ever does). She hopes that if she keeps track of the slope/angle at which she shoots the ball and the corresponding number of bounces, she can find a pattern. Let's figure out what she discovers.

1. How many times will the ball bounce before returning to the start position if she shoots with…

a. a slope of 1?



b. a slope of 2?



c. a slope of 3?

d. a slope of n, where n is an arbitrary positive integer?

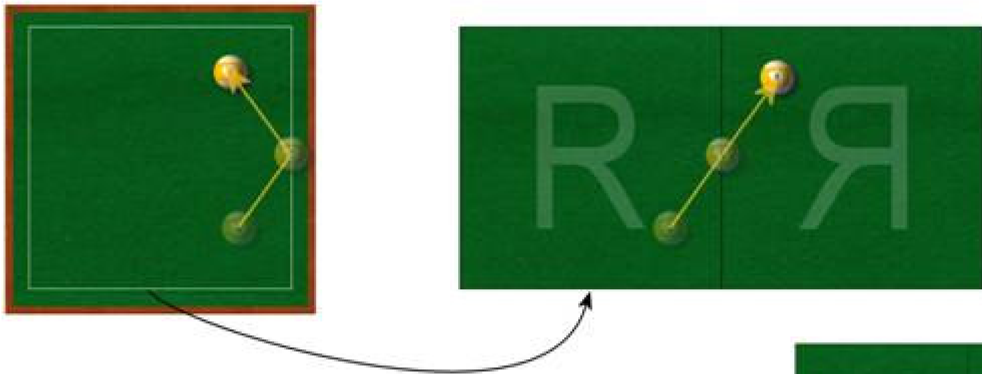
2. Do the same thing as in question 1, but with slopes 1/2, 1/3, ..., $1/n$.

3. Now she tries positive rational numbers (numbers n/m, where n and m are positive integers).

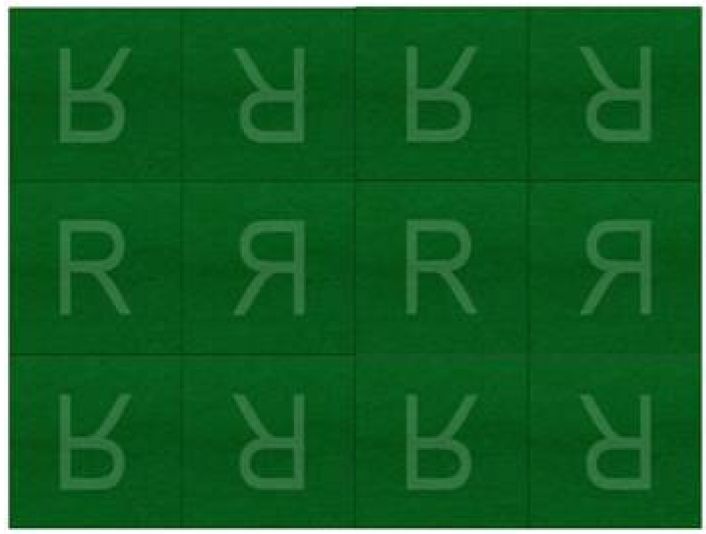
4. What if she shoots the ball at a 60° angle? If you're having trouble, you can make an educated guess by doing your computations with rational numbers that are better and better approximations for $\sqrt{3}$. I.e., first use the approximation 2 for $\sqrt{3}$, then try 1.7, then 1.73, etc.

Summer Fun!

Through the looking glass. (Don't read this before answering questions 1-4!!) Now Casi gets an interesting idea. She decides to draw a picture of the pool table and the ball rather than shooting with the actual pool table. The interesting part is that she will not only draw a picture of the pool table, but also a picture of its mirror image immediately adjacent to it, as in the picture at left.



When she draws a picture of the ball's trajectory, she can draw a straight line instead of a bent one. It's as if the ball passes through a mirror into "mirror image land" rather than bouncing. She can, in fact, fill an entire plane with pictures of the table and its mirror image.

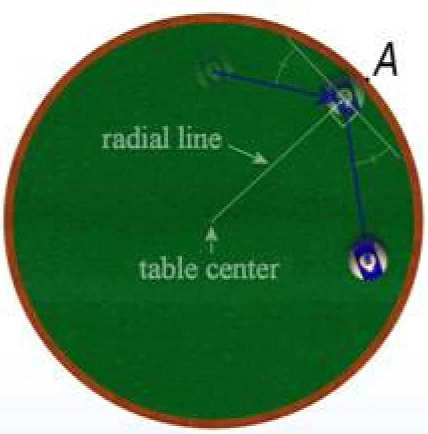


5. On the drawing of mirror images, draw the path of the ball that is shot with a slope of 1.

6. On the drawing of mirror images, draw the path of the ball that is shot with a slope of 2.

7. Revisit questions 1-4 using this trick.

Use your imagination. Now Casi begins to think about ways to change the problem a little bit. (Before reading on, spend a little time thinking about ways to change the problem yourself). For instance, she could try moving the starting point of the ball. She could also try changing the shape of the table. She decides to figure out what happens when the table is a circle.



How the ball bounces. When the ball hits the curb, say at point *A*, it acts just as it would act if the rounded curb were replaced by its tangent at that point. If you don't know what a tangent is, look at the diagram at left. It depicts what happens when the ball hits the curved curb.

Exercises. Casi decides to place her ball at the very bottom point of the circle.

8. Suppose she shoots the ball at a 45° angle to the tangent. How many times will the ball bounce before returning to the starting position? Try some other angles of your choice.

9. For what angles will the ball eventually return to where it started?

Summer Fun!

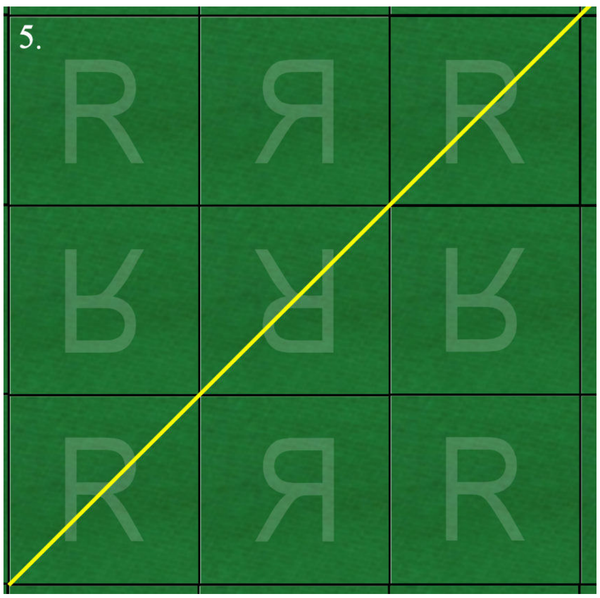
Solutions (Ken Fan)

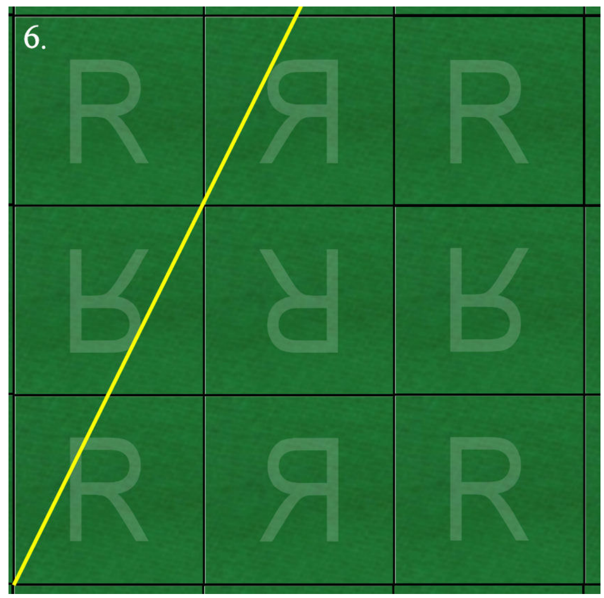
1. a. 1 bounce. b. 3 bounces. c. 5 bounces. d. $2n - 1$ bounces.

2. When the slope is $1/n$, the ball will bounce $2n - 1$ times before returning.

3. Let's assume, without loss of generality, that n/m is written in lowest terms. In other words, n and m have no common factors other than 1. Then, the ball will bounce $2m + 2n - 3$ times before returning.

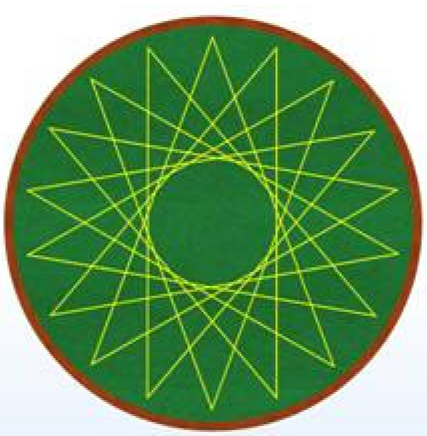
4. The ball will never return!





7. Although the answers will, of course, be the same, in the plane of mirror images, a bounce happens whenever the path crosses over a grid line. Does this view help you calculate the answers to the first four problems?

8. At a 45° angle, the ball will bounce three times before returning. It's path will form a square.



9. Angles that are a rational number of degrees will result in the ball returning. At right the path of a ball launched at a 70° angle (from the horizontal) is shown.

There is quite an extensive mathematical literature on billiard ball problems. For instance, see *Mathematical Time Exposures* by I. J. Schoenberg.

Summer Fun!

Fibonacci Numbers

by Ben Boyajian

The Fibonacci sequence is defined recursively by $F_1 = 1$, $F_2 = 1$, and, for $n \geq 3$, $F_n = F_{n-1} + F_{n-2}$.

1. Complete the following table of Fibonacci numbers:

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
1	1	2	3	5	8						

2. Which of these Fibonacci numbers are even? Which are multiples of 3? Which are multiples of 5? Can you predict in general which Fibonacci numbers will be multiples of 2, 3, or 5? Can you prove your assertions?

3. Prove that if n is a multiple of m, then F_n is a multiple of F_m.

4. Now consider non-Fibonacci numbers. For example, which Fibonacci numbers are multiples of 4? How about multiples of 7? (Hint: $F_6 = 8$ is the first Fibonacci number that is a multiple of 4, and $F_8 = 21$ is the first Fibonacci number that is a multiple of 7).

5. Can you predict in general which Fibonacci numbers are multiples of k, where k is an arbitrary positive integer? Can you prove your assertions?

6. Complete the table of greatest common factors of F_n and F_m at right. What patterns do you notice? Can you prove your assertions?

Table of Greatest Common Factors of F_n and F_m

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
F_1	1	1	1	1	1	1	1	1	1	1	1	1
F_2	1	1	1	1	1	1	1	1	1	1	1	1
F_3	1	1	2	1	1	2	1					
F_4	1											
F_5	1											
F_6	1											
F_7	1											
F_8	1											
F_9	1											
F_{10}	1											
F_{11}	1											
F_{12}	1											

8. Define a sequence by $L_n = F_{2n}/F_n$. Find the first several values of L_n. What do you notice about this sequence? Can you prove your assertions?

9. Consider sums of Fibonacci numbers. Let $S_n = F_1 + \ldots + F_n$. Compute the first few values of S_n. Can you find an explicit formula for S_n? (Hint: Recall the explicit formula for the Fibonacci sequence. See volume 3, number 1 of this *Bulletin*.)

10. Let O_n be the sum of the first n odd Fibonacci numbers and let E_n be the sum of the first n even Fibonacci numbers. Compute the first few values of O_n and E_n. Can you find explicit formulas for O_n and E_n?

Summer Fun!

(Selected) Solutions (Ken Fan)

These problems are quite challenging. If you have trouble proving these, don't be discouraged. Because of space limitations, the hints we give often point to very terse ways to prove things. Terse proofs often fail to shed light on why things are true, so it's always a good idea to try to devise your own proofs. Even if you don't succeed, you'll gain more insight.

2. From the table, F_3, F_6, F_9, and F_{12} are even, F_4, F_8, and F_{12} are multiples of 3, and F_5 and F_{10} are multiples of 5. Here, we will use **modular arithmetic** to show that F_k is a multiple of 3 if and only if k is a multiple of 4. If we write down the Fibonacci sequence modulo 3, we find the sequence: 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, … Because each Fibonacci number depends only on the previous two Fibonacci numbers, the reoccurrence of 1, 1 in this sequence means that the sequence begins repeating itself. So we can see that, modulo 3, the Fibonacci sequence repeats 1, 1, 2, 0, 2, 2, 1, 0 over and over, and from this we can see that only every 4th Fibonacci number is a multiple of 3. For multiples of 2 and 5, see *Anna's Math Journal* on page 9.

3. See *Anna's Math Journal* on page 9.

4. Combine the given hint with problem 3.

6. One might notice that the greatest common factor of F_n and F_m is equal to F_k, where k is the greatest common factor of n and m. Hint: It is rare that there be only one way to prove something and this case is no exception. One common way of proving this fact is to utilize the identity: $F_{m+n} = F_{m+1}F_n + F_mF_{n-1}$, which can be proven by induction. Another way to prove this fact is to use the fact that Anna discovered in *Anna's Math Journal* on page 9. There, she found that if $n = mk + r$, where $0 \le r < m$, then $F_n \equiv (F_{m-1})^k F_r \pmod{F_m}$, where we take $F_0 = 0$. In addition to this fact, note that consecutive Fibonacci numbers are relatively prime. (To see this, observe that if d divides both F_{a+1} and F_a, then d must also divide $F_{a+1} - F_a = F_{a-1}$. But then d also divides F_{a-2}, and so it must divide F_{a-3}, and so on, until we conclude that d must divide F_1. But $F_1 = 1$, so $d = 1$.) Anna's observation says that $F_n = F_{mk+r} = cF_m + (F_{m-1})^k F_r$, for some constant c. From this, we see that if p divides F_n and F_m, then it must divide $(F_{m-1})^k F_r$, and using the fact that F_{m-1} and F_m are relatively prime, this means p must divide F_r. Conversely, if p divides F_m and F_r, then it must divide F_n. Thus $(F_n, F_m) = (F_m, F_r)$. Continue the argument by noting a parallel with the Euclidean algorithm (see *The Euclidean Algorithm* in Volume 2, Number 6 of this Bulletin).

8. The first few values of L_n are 1, 3, 4, 7, 11, 18, … The sequence L_n also satisfies the Fibonacci recurrence relation $L_{n+1} = L_n + L_{n-1}$. Here's a hint for how to prove this: Use the explicit formula for the Fibonacci numbers and the fact that $x^2 - y^2 = (x + y)(x - y)$.

9. There are many ways to simplify this sum. One way is to express it in terms of two geometric series using the explicit formula for the Fibonacci numbers.

10. Again, there are many ways to simplify these sums. One way is to express the sum E_n in terms of two geometric series. If you know S_n and E_n, then you can determine O_n because $S_{2n} = O_n + E_n$.

Summer Fun!

Calendar

Session 7: (all dates in 2010)

September	9	Start of the seventh session!
	16	
	23	
	30	Tanya Khovanova
October	7	
	14	
	21	
	28	
November	4	
	11	Veteran's Day – No meet
	18	Anita Suhanin and Noam Weinstein
	25	Thanksgiving - No meet
December	2	
	9	

Author Index to Volume 3

Anna B.	3.11, 4.12, 5.11, 6.09
Anna Boatwright	1.10, 2.11
Katy Bold	1.17, 2.16, 3.17, 4.20, 5.19
Ben Boyajian	5.29, 6.31
Doris Dobi	1.15
Kara Donohue	5.07
Ellen Eischen	4.03
Lightning Factorial	4.07, 5.26, 6.06, 6.26
Ken Fan	1.12, 2.13, 2.25, 3.08, 3.13, 3.19, 4.09, 4.22, 5.10, 5.25, 5.32, 6.24
Noah Fechtor-Pradines	4.14
Allison Henrich	1.03
Cathy Kessel	2.06, 3.03
Grace Lyo	5.27, 6.28
Maria Monks	1.20, 2.18, 3.22, 4.23, 5.13, 6.11
Ina Petkova	1.06
Lillian Pierce	6.03
Bjorn Poonen	3.14, 4.17, 5.15
Amy Tai	2.03
Helen Wong	5.03, 5.24, 6.22
Julia Zimmerman	5.23, 6.16, 6.20, 6.21
Fern	1.26
Ilana	3.29
Rowena	4.29

Key: n.pp = number n, page pp

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) comes with membership. We are working hard to lower the cost of the Bulletin. Until we do, however, nonmembers can receive the printed version by becoming a Bulletin Sponsor. Please contact us if interested.

The Bulletin targets girls roughly the age of current members. Each issue is likely to contain some material that feels very challenging or difficult to understand. If you are a member or Bulletin Sponsor and have any questions about the material, feel free to ask us about it!

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

How do I join? Membership is granted per session and includes access to the club and extends the member's subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle president? Ken Fan is the president and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls’ Angle: A Math Club for Girls

Membership Application

Applicant’s Name: (last) ______________________________ (first) ______________________________

Applying For (please circle): Membership Bulletin Sponsorship

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: ________________ Email: ______________________

For **membership**, please fill out the information in this box. **Bulletin Sponsors** may skip this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you’d like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter’s name in any way. Do we have permission to use your daughter’s image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls’ Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls’ Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a one session membership
  - ☐ Bulletin Sponsorship: $60-99 Subscribing. $100-999 Bronze. $1000-2999 Silver. $3K+ Gold.*
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls’ Angle**. Mail to: Girls’ Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

*Silver and Gold sponsors may specify a recipient for their sponsored copies of the Bulletin.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

Girls' Angle

A Math Club for Girls

Advertising Section

The print version of the Girls' Angle Bulletin is advertisement free.

The appearance of advertising in the electronic version of the Girls' Angle Bulletin is neither a guarantee nor an endorsement by Girls' Angle, Inc. of the product, service, or company or the claims made for the product in such advertising. The fact that an advertisement for a product, service, or company has appeared in the electronic version of the Girls' Angle Bulletin shall not be referred to in collateral advertising.

As a matter of policy, Girls' Angle, Inc. will sell advertising space in the electronic version of the Girls' Angle Bulletin when the inclusion of advertising does not interfere with the mission or objectives of Girls' Angle, Inc.

Advertising in the electronic version of the Girls' Angle Bulletin only occurs in a specially designated advertising section and may not occur alongside other content. Girls' Angle, Inc., in its sole discretion, retains the right to decline any submitted advertisement or to discontinue posting of any advertisement previously accepted. By submitting ads for consideration, all advertisers agree to these principles, as amended from time to time.