Girls' Angle Bulletin

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To Foster and Nurture Girls' Interest in Mathematics

From the Director

In its first three years, over forty people have contributed to the Bulletin, ranging from our members to professional mathematicians. Eleven different women in mathematics have been interviewed. Today, we have readers all over the world.

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In this issue, I'm excited to welcome back former Girls' Angle mentor Cammie Smith Barnes with her new column *Errorbusters!* Cammie has a doctoral degree in mathematics from Harvard. Today she is faculty at Sweet Briar College.

Mahalo to Renee Kee Maeda, Linda Molyneux, and Andrew Fulop for rushing to help me secure images of Haleakalā!

Finally, don't miss the new ambigram by Scott Kim on the bottom of page 20. His masterful designs have inspired ambigrams by our members and affiliates. I hope you enjoy them!

All my best,
Ken Fan
Founder and Director

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Grace Lyo
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Haleakal*ā. Photo by Linda Molyneux. Can Haleakalā be seen from the shores of Oahu?

An Interview with Lillian Pierce, Part 2

This is the concluding half of our interview with Oxford University Marie Curie Fellow Lillian Pierce.

Ken: When I first met you, you were a graduate student in mathematics at Princeton. Now, you have a doctoral degree in math. Could you please tell us something of the history of your Ph.D. thesis? How did you come up with the ideas in it?

My goal as a person is to keep the same pleasure in observing and learning about the world that I felt as a child.

Lillian: After I finished a masters in number theory at Oxford, I returned to Princeton to do my Ph.D., but at this point I was in a quandary. I knew that my way of thinking about mathematics was fundamentally best suited to analytic methods, and so I wanted to learn more about harmonic analysis, of which Fourier analysis is a part. But currently the field of analysis is mainly focused on problems related to partial differential equations, and I realized that I wasn't as interested in PDEs [Partial Differential Equations] as I was in number theoretic questions. How could I combine my two interests? Fortunately, Professor Stein pointed me towards a new area of research that he had begun pursuing only a few years earlier: discrete analogues in harmonic analysis. The first place to start was to read what other people had done in the field, so as to understand the important questions and the existing techniques. There aren't many papers in this field yet, compared to more well established fields, so it was easy to tell what I needed to read and understand, compared to a field where there might be hundreds of relevant papers. But it was also challenging because the history of the field was relatively shallow.

Where do new ideas come from? I suppose this varies from person to person. I find that it is really important for me to understand existing ideas that have worked on related problems. When I say "understand," I mean understand to the depth of being able to take the proof apart and rebuild it with my eyes closed and both hands tied behind my back. I think that in math we have to have extraordinarily high standards for understanding things. After all, you wouldn't set out to build a new, sleeker, faster car without taking a look inside an existing car…and possibly disassembling and reassembling that old car, and other cars you find inspiring, many times!

And what is the way to understand a problem or a proof to this depth? By getting your hands dirty: don't just watch someone else take apart that car. Do it yourself! Instead of just reading a paper or a proof, I try to recreate every single step, with pen and paper. By the time I've understood a proof to the extent that I can assemble it in my head and run through the concepts without needing to refer to the original proof, I then consider that method to be one of my tools. (You have probably started accumulating a toolbox of mathematical tools also—everything you learn deeply can go into that toolbox.) Often, new ideas are simply extensions or compositions of existing ideas. In fact, putting together two old ideas in a new way can be a new idea, and sometimes even a striking innovation.

There is an alchemy in arriving at a new idea that is hard to describe— I think that key components of this process are (a) understanding the problem you want to solve (b) understanding some ideas people have used in similar situations (c) actively struggling with the problem in order to isolate the hardest components (d) trying all your existing tools and seeing exactly how they succeed or fail (e) then stir and stir all these ideas in your head in a peaceful environment, and if you're lucky, an inspiration will pop up. And finally, don't give up. As Samuel Beckett once wrote: "Try again. Fail again. Fail better."

Ken: Now, you're working at the Institute for Advanced Study. What's your daily life like at the IAS?

Lillian: I am officially finished with being a student, 23 years after entering kindergarten! That sounds like a long time, but those years flew by. In fact, I loved being "in school"— in the sense of learning, sharing, teaching— so much that I have decided to spend the rest of my life in the academic environment. In reality, even though I'm no longer technically a student, my life this year is much like my life last year: I spend my days trying to understand math other people have done, and trying to explore my own personal mental landscape of mathematics and find new facts about math that have never been illuminated before. So part of a typical day is spent reading papers other mathematicians have published (reading means writing, also, because I try to work through the ideas of the paper myself). And part of the day is spent trying to apply these ideas, or ideas they have inspired, to problems I'm interested in. Much of my days are spent alone, in quiet, thinking about problems I am working on, but this quiet and peaceful time is seasoned by seminars and meetings with my mentors and collaborators. In order to understand difficult ideas better, I get advice from mentors and ask questions of my peers, so I am still learning all the time. One of the best ways to keep up with current math research is to go to seminars, so in the afternoon I often attend a seminar at which a mathematician describes research he or she is doing. (Mathematicians fly all over the country giving seminars like this— it's a lot of fun to go visit another university and talk to all the mathematicians about what you are doing, and what they are doing.) At the IAS there are no students, but later on I'll be working at a university with students, and then my days will also be livened by teaching lectures, which I greatly enjoy!

The more people who learn some math, the better!

Ken: Do you remember when you decided to become a mathematician? What attracted you to the subject?

Lillian: My attraction to mathematics is so deep that it's hard to explain it. One of my earliest memories is of watching my mother balance her checkbook. As I remember it, she was propping the checkbook up against the steering wheel of the car as she wrote and she showed me what she was writing to keep me amused; maybe we were waiting for my older brother and sister to get out of school. My mother, an exacting teacher of penmanship, has unusually beautiful handwriting and I recall the variegations of the blue-grey ink of her fountain pen as she wrote out numbers on this particular occasion. I also remember having the startling realization as she wrote the numbers that *the numbers were communicating to each other*! Putting numbers in proximity to each other *meant* something, and the way they interacted dictated which numbers you should write down next… This was absolutely stunning to me. Who knows what simple arithmetic my mother was doing, but to me it was as beautiful as a chord progression in one of my favorite Bach concerti. We think this might have been when I was about 4 years old; we're not sure.

In kindergarten I spent some time at a Montessori school, and I remember being given little pamphlets of math problems. I loved these! I have no idea what was in them, but I do remember that I loved filling out the books so much that I filled out all the books the teachers had, and then there wasn't much more for me to do…and I dropped out of kindergarten. After that, for much of the rest of the time before college, my mother educated me at home. I remember having the thought when I was about 8 that I wanted to become something in math, not a math teacher…but…I searched for a fancier word…a math *professor*! I had never met a professor at that point in my life and I don't know where I got this idea. Over the next 10 years I had a lot of other ideas about what I wanted to do (…civil rights law, neurosurgery…) but when I showed up at Princeton for college, I decided to be a math major, because I enjoyed math so much, but also to complete the premedical courses and go to medical school afterward. Ultimately, even though I loved the premedical courses (Molecular Biology! Chemistry! Physics!) I realized that doing mathematics, that intense mental exploration resulting in something with more precision and perfection than anything in "the real world," gave me a pleasure that no other academic pursuit did.

Ken: I understand you recently became a mother. Congratulations! Are you planning to play an active role in the math education of your child?

Lillian: Of course! For babies and children, simply interacting with the world is a tremendously important and pleasurable learning experience. My goal as a person is to keep the same pleasure in observing and learning about the world that I felt as a child. It is great to be reminded of the curiosity, ambition, confidence, and determination that a baby shows in mastering the skills of navigating her world. I am continually impressed by the way my daughter, now one, is observing the world, and the way she consistently seeks out the most challenging thing she can do, and practices that skill, with gradually increasing success, until she masters it. ("Now I'm going to stand up! Plop. Ok, now I'll try again. Plop. Ok, this time I'm sure I can do it! Plop. Let's try that again…") I hope that I can help her keep this kind of confidence and determination in learning mental skills as well as physical ones.

Instead of just reading a paper or a proof, I try to recreate every single step, with pen and paper.

After all, solving a hard math problem is similar to learning how to stand up. In particular, it is crucial to have the confidence to try out a new idea, watch it fail ("Plop!"), without (too much) disappointment or embarrassment, and then try out something else. So while I don't think I'll educate my child exclusively at home the way I was educated, I'll definitely try to craft a family environment in which learning, exploring, failing, trying again in good humor, is as natural as breathing. And since I love thinking about how things work (physical as well as mathematical), I'm sure I won't be able to stop myself from puzzling aloud with Pip about the inner workings of all sorts of objects around us…

Ken: There aren't very many women in mathematics today. Do you think there is gender bias against women in the field of mathematics?

Lillian: This is a tough question and brings up a lot of subtle issues. First of all, I think there are more women in mathematics than we realize. I recently read that 48% of all college math majors in the US are women! It makes me so happy to think of all these people majoring in math! All these women will surely apply their mathematical knowledge throughout the rest of their lives, whether or not they pursue a job that looks mathematical in the strictest sense. The more people who learn some math, the better!

But it is true that I know many more male mathematicians than female ones, if I consider all the mathematicians I know of all ages. If you ask someone "Why are there so few female mathematicians?" the most common answers you get will probably sound something like either "because girls aren't as good at math" or "because there's discrimination against women in math and science." Let's not waste time on the first answer, which I think is absolutely false. (If you want some data, you can check out the book "The Mathematics of Sex: How Biology and Society Conspire to Limit Talented Women and Girls." But it's probably a better use of your time simply to go back to reading your favorite math book!) As for the second answer people commonly give, I think that there is some truth to it. I think one of the most subtle, and hence most insidious aspects of discrimination affecting girls who love math is stereotyping. Technically speaking, a stereotype is a widely held belief that gives a standardized and oversimplified concept about members of a certain group. That's a fancy definition, but it's easier to spot a stereotype in practice, such as "boys are good at running" and "girls are good at cooking." The concept "girls aren't as good at math" is also a stereotype.

Now, I said stereotypes are insidious, and here's why: by definition, they are widely held beliefs, and so we grow up surrounded by stereotypes, and if something is very familiar, it is sometimes hard to realize that we need to re-evaluate it for truth. So instead of trying to evaluate the common stereotypes I mentioned above, which might be hard for us to evaluate truly objectively, let's make up some new ones. "People with green eyes read very fast." "People with long thumbs are good at singing." "People with curly hair prefer tuna salad to peanut butter and jelly." Wait a second, you are saying—none of these make any sense! And you're right. It doesn't necessarily make sense to assume that a biological trait, such as eye color, finger length, or hair curliness —which a person has no control over—will determine their preferences and skills.

But making stereotypes seems to be common throughout human cultures. Why do humans make stereotypes about groups if the stereotypes are so often inaccurate (and ultimately harmful)? This is an important question in the field of sociology, and I can't give a full answer, because that's not my field of expertise. But here's how I think about it: at the root of stereotyping lies the basic skill of pattern finding. Pattern finding is a great skill to have as a mathematician, but it is also crucial to the way all kinds of animals learn. If you were living in a cave many thousands of years ago, it would have been important to be able to recognize edible from inedible berries, plants, and fungi. If you spotted a saber-toothed tiger, it would have been a good idea to apply the stereotype "saber-toothed tigers like to eat tasty morsels like me" and run in the other direction. Now maybe it wouldn't have been fair to that particular saber-toothed tiger, who might not have been hungry that day, but it was a reasonable pattern for you to apply to a carnivorous hunter: that stereotype linked genuinely related traits. (And it would have been important for the survival of the saber-toothed tiger to spot you as fitting the pattern of tasty morsels.)

But the problem with many of the stereotypes that we apply to each other today is that they don't actually link relevant traits, on average. Having XX chromosomes doesn't cause a greater talent for kneading bread—just like having curly hair doesn't necessarily make someone reach for the tuna salad. So it would be a good idea if we could just stop applying senseless stereotypes to each other. Here's where another aspect of human culture comes in, though: we tend to learn our culture from the people around us, who learned the culture from the people around them…it's very hard to eradicate existing stereotypes. It can also be hard to go against them: if my parents had always made me PB&J sandwiches because I had straight hair, and made my sister tuna sandwiches because she had

When I say "understand," I mean understand to the depth of being able to take the proof apart and rebuild it with my eyes closed and both hands tied behind my back.

curly hair, and I "knew" that all people with straight hair loved PB&J, and everyone looked at me as if there was something wrong with me when I asked for tuna, I'd probably end up sticking with PB&J.

Of course, that example sounds very silly. But let's change the words around: if my parents had always given me dolls, but gave my brother a chemistry set, and I knew that all girls loved dolls, and everyone made fun of me when I tried to play with the chemistry set, and told my brother he must not mean it when he asked for a doll, then I'd probably stick with the dolls and he'd probably stick with the chemistry set. Luckily, my parents didn't give me only dolls—nor my brothers only chemistry sets. We all played with dolls, and we all climbed trees, and we all learned to bake bread, and we all played with engineering kits and microscopes. (And I'm not sure that any of us really liked tuna.) Even more important, when I was growing up, I never thought it was strange that I was good at math, just like my older brother, and I was never embarrassed to say that I loved math!

Now that we know how inaccurate and harmful stereotypes can be, what can we do? First of all, we can try not to apply them to others. Second of all, we can try to notice when they are applied to us. One of the best ways to "disarm" a stereotype is to remember actively that it is inaccurate or false. This simple technique has been studied by social psychologists, and it is actually quite effective at canceling out the negative message of the stereotype! (If you're interested in reading about some experiments, look up research on "stereotype threat.") If you're interested in learning more about identifying stereotypes about gender that affect both boys and girls negatively, I like the book called "Beyond Guns and Dolls: 101 Ways to Help Children Avoid Gender Bias."

There is an alchemy in arriving at a new idea that is hard to describe— I think that key components of this process are (a) understanding the problem you want to solve (b) understanding some ideas people have used in similar situations (c) actively struggling with the problem in order to isolate the hardest components (d) trying all your existing tools and seeing exactly how they succeed or fail (e) then stir and stir all these ideas in your head in a peaceful environment, and if you're lucky, an inspiration will pop up. And finally, don't give up.

Ken: Do you have any advice on learning mathematics for our club members?

Lillian: Enjoy math! Especially, enjoy challenges! Math is just like any other skill: practice counts. Also, have courage and confidence in your abilities, and don't shy away from failing to solve something immediately: try out your ideas, make mistakes, learn from your mistakes. The *struggle* itself is one of the most important parts of your practice. Try again! Fail again! Fail better!

Ken: Thank you so much for this interview!

Haleakalā

by Ken Fan
edited by Jennifer Silva

When you stand at the beach, the horizon is about three miles distant. Boats that sail away will seem to disappear below the horizon. If a ship sailed on for 10 miles, how tall would its masts have to be for it to remain visible to the people left behind? What if it sailed on for 100 miles? Then how high would its masts have to be to remain visible?

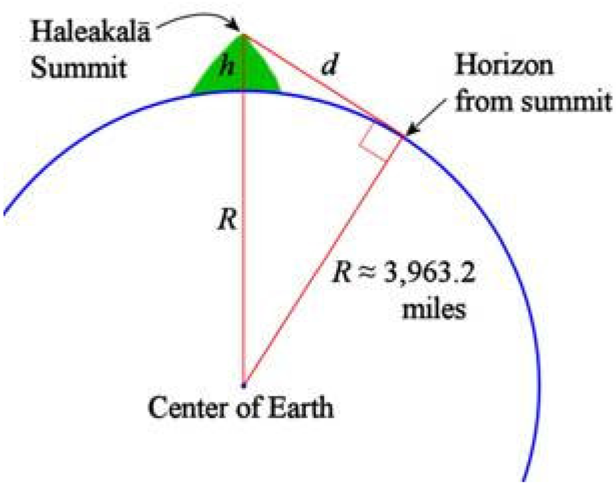
Answers to such questions can be used to settle a small debate among residents of Hawaii, America's 50th state. To understand this debate, we have to know a little bit about Hawaii's geography. Hawaii is an archipelago in the Pacific Ocean. Most islanders inhabit one of the seven youngest islands, with the majority residing on Oahu. The islands are famous for volcanoes, among other things. In fact, three quarters of Maui, the second largest of the islands, is part of one volcano named *Haleakalā*, or, Home of the Sun.

Photo courtesy of NASA/GSFC/JPl, MISR Team

Even though Haleakalā is more than a hundred miles from Oahu, some say that on a very clear day, you can stand on Oahu's shores and see the summit of Haleakalā faintly looming over the horizon. Many stare, but see only blue sky and proclaim that there is nothing else to see. But others insist that the volcano is there, though very faint because of all of the atmosphere in between. Can these people really see Haleakalā or are they just a bunch of liars? Think of Maui as a really big boat and Haleakalā as a ten thousand foot high mast. Is it possible to see Haleakalā from more than a hundred miles (or half a million feet!) away?

Mathematics to the Rescue

First, observe that if you can see the summit of Haleakalā from the shore, then someone on Haleakalā should be able to see you, albeit with the aide of a telescope. So we can answer the question of whether Haleakalā is visible from the shores of Oahu by determining how far away the horizon is to someone standing on the summit of Haleakalā.



Let's draw a diagram. Drawing to scale would be difficult because the Earth is so enormous compared to even something as big as Haleakalā, so we will not worry about making a scale drawing. The line of sight from the summit to the horizon just touches the Earth: it is **tangent** to the Earth at the horizon. This means that the radius from the center of the Earth to the horizon makes a right angle with the line of sight[1]. In other words, the red figure in the diagram at left is a right triangle. One leg is the radius of the Earth. The other is the distance from the

[1] We're making the simplifying assumption that the Earth is a perfect sphere. In reality it isn't, but it's quite close.

summit to the horizon. The hypotenuse has length equal to the radius of the Earth plus the altitude of the summit. The relationship between these three lengths is perfectly expressed by the Pythagorean theorem: $(R + h)^2 = R^2 + d^2$. We solve for d and find that

$$d = \sqrt{h^2 + 2Rh}\,.$$

All we have to do to settle the debate is substitute actual values for the radius of the Earth and the altitude of Haleakalā's summit and compute d. When you do this calculation, you have to take care that you use the same units for all the distances. I will compute everything in units of miles.

The radius of the Earth is approximately 3,963.2 miles. The height of Haleakalā's summit above sea level, according to the Haleakalā National Park service, is 10,023 feet, which is about 1.9 miles. Using our formula for the distance to the horizon (and a calculator), I find that d is approximately equal to 122.7 miles. So … the people who claim to be able to see Haleakalā from the shores of Oahu are right! On a very clear day, you really should be able to see it looming over the horizon, flanked by the tips of Molokai and Lanai.

A special message to readers in Hawaii:

If one day you're at the beach on Oahu and happen to see Haleakalā in the distance, please snap a photo and send it to us!

Mahalo!

Let's take a closer look at our formula for the distance to the horizon: $d = \sqrt{h^2 + 2Rh}$. Whenever you derive a formula that represents something physical, it's a good idea to see how it agrees with your intuition and to do some quick checks to make sure that the formula is reasonable. For example, because this formula represents an actual distance, the units should work out properly. Do they? Inside the square root, the units of h^2 and $2Rh$ are both $(\text{length})^2$, so when you take the square root, you do get units of length.

Photo courtesy of Andrew Fulop

What happens when $h = 0$? This represents having your eyes right at sea level, such as when you're swimming. What should d be if the formula is correct? Does it give the value you expect? How about when h gets bigger and bigger? What do you expect for the value of d as h tends to infinity? Does the formula behave as you expect?

Can you invert the relationship between d and h? For example, how high must your eyes be if you want to see a buoy that is floating 10 miles away? How high must you be if you would like to be able to see Hawaii and Maine without having to do anything but turn your neck? Would this be possible from the International Space Station?

Often, it isn't necessary to know something with a high degree of accuracy. When precision isn't important, it is often possible to develop a simplified version of the formula that is not exactly correct, but still of practical use. When h is small compared to R, the value of h^2 is small compared to $2Rh$. This means that for heights that are, say, one hundredth the radius of Earth (which is still higher than airplanes fly), $d \approx \sqrt{2Rh}$. Using this formula, we can deduce that if your eyes are h inches above sea level, the horizon will be about $\frac{7}{20}\sqrt{h}$ miles away.

Finally, notice that if you don't know the radius of the Earth, but you do know how far the horizon is when viewed from a certain height, then you could use the formula (assuming the Earth is a sphere) to compute the radius. Have you ever measured something so big? Can you devise an experiment to accomplish this? If you do, let us know what value you get!

Brownie Points

by Tigran Ishkhanov and Grace Lyo

Let's play a game! It's called "brownie points."

Delicious, delicious brownie

The goal of the game is to get enough brownie points to win a scrumptious brownie. This brownie has delicious chocolate chunks in it. It's delicately crisp on the outside, and moist and chewy on the inside. Mmmmmmm.

How much is this wonderful brownie worth? Well, it's really good, so it's going to cost you 100 points. Here's how you collect points.

Rolling a die

I have a pair of standard, 6 sided dice. You're going to roll them, and I'm going to give you the sum of the two numbers in brownie points: if you roll a 2 and a 6, you get 8 brownie points.

Now here's the catch: I'm obviously not going to give you these brownie points for *free*! In order to play this game, you have to pay me. My dice game will cost you 6 brownie points. This means that if you decide to play and you roll a pair of 1s, then you now have -4 brownie points (boo hoo). Should you play?

(Think about this problem a bit before reading further.)

It's often a good idea to start by making the problem a bit easier. We can start by getting rid of one of the dice. In this case, there are only 6 numbers that we can roll instead of 11. We can simplify further by replacing the die with a fair coin, where we think of tails as being worth one point and heads as 2.

How many brownie points should it cost to play the game with the coin? Well, if it only costs 1 brownie point, then you're definitely always going to choose to play the game; sometimes you'll get an extra point and sometimes you'll just break even. On the other hand, if it costs 2 brownie points, then you're definitely never going to choose to play the game! So the price should be somewhere between 1 and 2 brownie points. But what exactly should it be?

To understand this, we need to think carefully about what the question is really asking. The question is really asking how, in general, to decide whether to play a game like this given a particular price. In other words, what is a strategy that will win you brownie points *in the long run*? Implicit here is that we have to think about what happens if you play the game multiple times. Think about the case where we play the game 10 times before reading on.

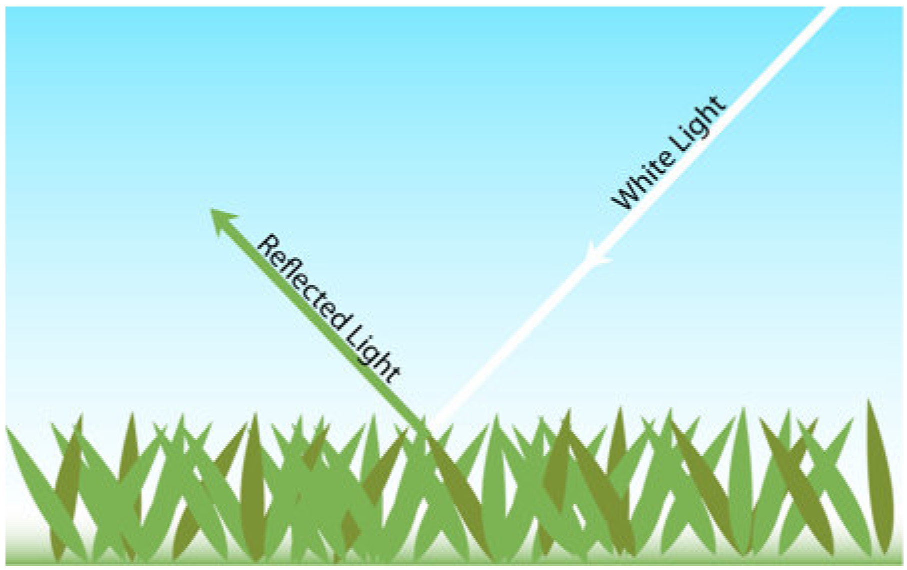
Continued on page 21.

An Introduction to ***Additive Color***

By Toshia McCabe

Every day, televisions and computers provide a myriad of images that capture our attention and shape our view of the world. But have you ever wondered how they create such beautiful and realistic color images?

To answer this question, lets step back and think about how color is created and perceived. Everything we see is light that is either reflected or generated by a source. For example, you can see your desk, your books, and the blackboard in school because your eye detects the light that is reflected from them. Your eyes can also see self-luminous objects such as traffic signals, candle flames, and fireworks because they emit light.



The sun is the most powerful light source in our solar system. Believe it or not, a ray of sunlight contains all the colors that we can possibly see! When sunlight strikes a surface, such as grass, some of the sun's colors are absorbed by the grass and the rest is reflected. This reflected light is what we see as green.

To look at it another way, you can think of colored light as sunlight (sometimes called **white light**) minus some of its original components. Our eyes have sophisticated sensors called **cones** that are sensitive to certain colors of light. We typically[2] have three types of cones in our eyes: one that detects red, one that detects green, and one that detects blue. How is it, then, that we can see orange, yellow, maroon, and gold? The answer is that our cones respond in *varying degrees* to different colors. For example, if you look at a purple iris, both the blue cones and red cones will send strong signals to your brain. Since purple irises are a little more blue than red, your blue cones will probably respond more than the red cones. If we look at something less colorful like a sidewalk, all of our cones will be active to a slightly different degree. As a matter-of-fact, all three types of cones are active pretty much all the time (except when it's really dark)! It would be very rare for any one type of cone to respond in isolation.

Let's return to our original question of how it is that TVs and computers create realistic color images. Since our eyes contain red, green, and blue sensors that detect all colors, do you think it's possible to recreate many of the colors we see in the world by mixing red, green, and blue light? It turns out that you can mix different colored lights together to create new ones. Furthermore, if you choose red, green, and blue as your primary colors, you can create a wide variety of distinct colors.

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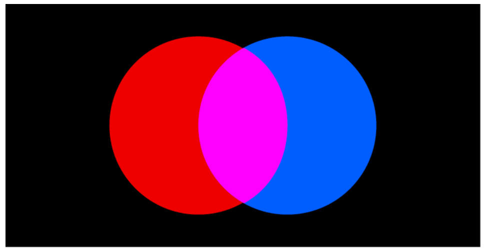
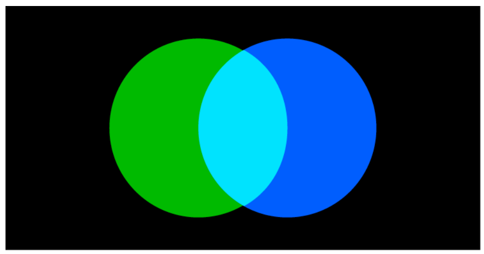
[2] Some people are color-blind and have fewer cone types. Also, there may be some people who have a fourth cone type. Dr. G. Jordan and Dr. J. Mollon of the University of Cambridge found evidence that some women have the ability to detect color differences that are consistent with having a fourth cone cell type. See "A study of women heterozygous for colour deficiencies" in Vision Research, 1993, July, volume 33(11), pp. 1495-1508.

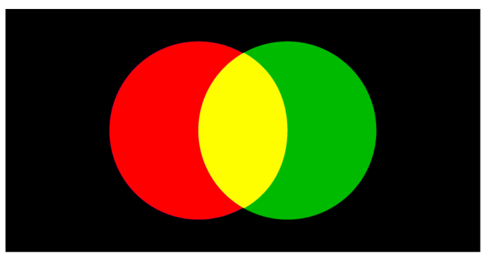
Here's how it works. Imagine that you have red, green, and blue spotlights that have dimmer switches to allow for a range of intensities between off (0%) and full on (100%). If we overlap the red and blue lights (both at 100%) a nice shade of magenta will appear in the overlap region.



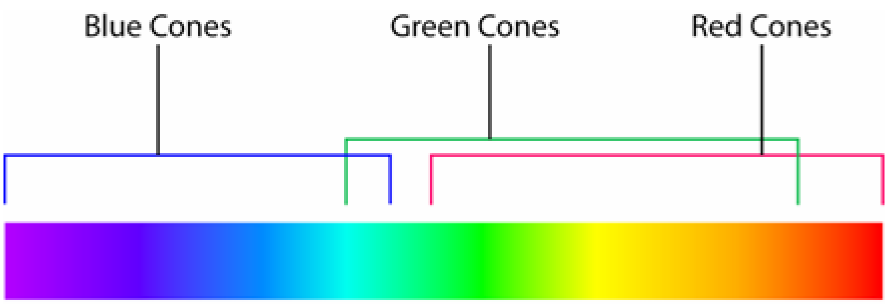
Similarly, if we overlap green and blue lights (again at 100%), we will see a beautiful aqua color.



And if we were to overlap the red and green lights, we would see yellow!



Huh? How did *that* happen? It turns out that when you mix red light and green light, both the red and green cones respond to give you the sensation of yellow. To think about it another way, look at the color spectrum below. It represents the range of colors that humans can see. Notice that yellow falls between red and green. The brackets above the color spectrum show the range of colors detected by each type of cone. There's a huge overlap between the red and green cones, and yellow is right in the middle of it!



This brings up another question. Did your art teacher ever tell you that yellow is a primary color? Furthermore, isn't green considered to be a secondary color (the mixture of blue and yellow)? Your art teacher is actually correct. Yellow is considered a primary color (and green a secondary color) when you create new colors by mixing paint. Remember, objects and painted surfaces *absorb* light. This means that any color made by mixing two or more paints will be darker than any of the colors you started with. If you mix red and green paint together, guess what the new color will look like? Brown! However if you overlap colored *lights*, the new color will always be brighter than any of the starting colors. This means that the paint primaries are different to make up for the fact that as you mix different paints together, the resulting color will get duller and duller.

I've shown you a few examples of additive color mixing, but there are many, many more colors you can make with different combinations of red, green, and blue intensities. What do you think happens when all three lights are set to 100%?

TVs and computer displays rely on additive color mixing to create dazzling and realistic pictures. They also rely on the fact that our eyes tend to blur things that are really tiny and close together. In this case, the red, green, and blue lights are not placed *on top* of each other (as in our examples above). Instead, tiny red, green, and blue rectangles are placed *beside* each other. The rectangles are so small that they appear to merge into tiny little points when we look at them from normal viewing distances. Here's a really close-up shot of a computer screen displaying a red letter "R", green letter "G", and blue letter "B" against a gray background.

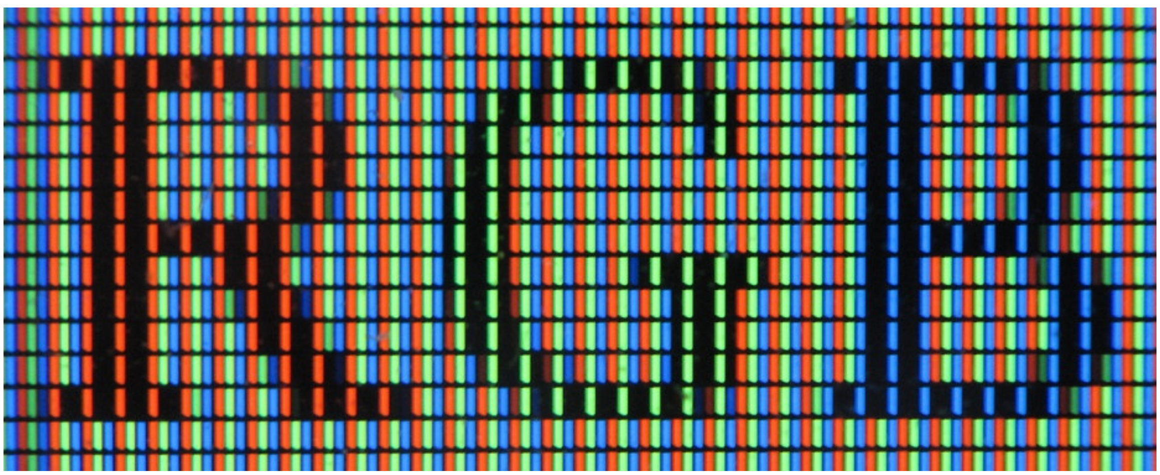


Photo by Luís Flávio Loureiro dos Santos/Courtesy of Wikipedia

If you look at this picture and squint your eyes, the effect will become even more evident. Pretty amazing isn't it?

Digital displays can't actually produce an infinite range of red, green, and blue intensities ranging from 0% to 100%. Most displays receive image information that is delivered in 8 bit chunks of computer memory. To understand this, you can think of an 8 bit chunk (called a byte) as an 8-digit binary number. What's the largest number we can represent in a byte? That number would contain a 1 in all 8 positions of the byte:

$$11111111_2 = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 255_{10}$$

The total number of values we can represent in a byte (ranging from 00000000 to 11111111) is $2^8 = 256$.

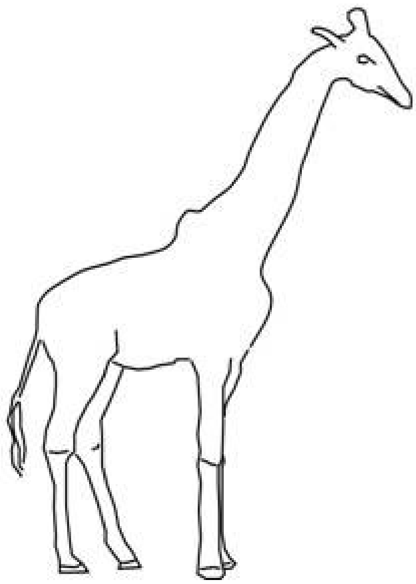
What does this all mean in terms of red, green, and blue points of light on a display? It means that each colored rectangle can be set to any of 256 distinct intensities. This gives us a color palette that has $256 \times 256 \times 256 = 16{,}777{,}216$ colors!

Color mixing can be a lot of fun! If you're interested in experimenting with additive color mixing, you can do so with any graphics or photo editing program.

Similarity, Part I: Recognize the Giraffe

by Ken Fan
edited by Jennifer Silva

This dialogue is from a recent Girls' Angle meet between a mentor and a group of members. The mentor held up the drawing below and asked, "What is this?"



Members: It's a giraffe!

Mentor: How do you know it's a giraffe?

Members: Because it has a long neck.

Mentor: Really? Let's see just how long this neck is.

The mentor took a ruler and gave another to one of the members.

Mentor: Here's a ruler. Could you please measure the length of your neck while I measure the length of this neck [pointing at the drawing]?

The member measured the length of her neck.

Member: My neck is a little more than 2 inches long.

Mentor: And I get about an inch for this neck…it's not even half the length of your neck! How can you say this neck is *long*?

Another member: Because…it *is* long…compared to its legs!

That's the key! The neck is long *compared to the legs*.

In other words, we recognize the drawing as a drawing of a giraffe not because of the absolute length of the neck, but the relative length of the neck compared with its body. The length of the giraffe's neck in the drawing is nowhere near as long as the length of the neck of a real giraffe. However, the *ratio* of the length of the neck to the length, say, of a leg in the drawing *is the same* as what you would find if you computed the analogous ratio for a real giraffe.

When the ratios of corresponding lengths in two figures are always equal, the two figures are said to be **similar** to each other. Our ability to recognize the drawing as a giraffe helps explain why the notion of similarity is so basic. In fact, our brains are designed to very quickly recognize when two figures are similar to each other. Sometimes, our brain will even equate two similar objects, as when we point to a photo and say, "That's me!" In truth, that three inch little image of you *isn't* really you!

If you want to make a recognizable sculpture of a friend, the key is to make sure that all ratios of corresponding lengths are equal. In other words, the portrait must be geometrically similar to the real person. If you have a nagging feeling that something is off, it means that the ratios of some key lengths are incorrect. Perhaps it's the width of the mouth compared to the width of an eye.

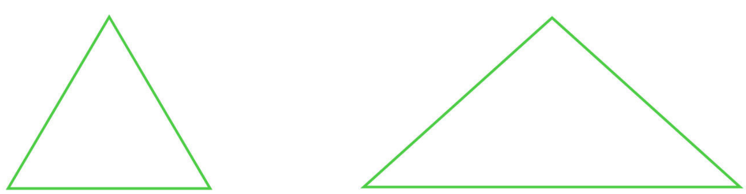
You can probably sense that these three drawings are different, but can you figure out exactly how?

Mathematicians have developed the habit of examining new concepts in their simplest settings. That's because experience has shown that understanding of complex situations can often be achieved by analyzing the simplest cases. Let's try to find the most basic interesting figures to use to begin our study of similarity; portraiture is way too complex. People's faces involve so many subtleties: look in the mirror and watch how much your expression changes with the slightest widening of eyes and parting of lips!

Could the simplest figure with which to begin our study of similarity be a single point? All points are identical in size and shape, but similarity doesn't become interesting until you have shapes of different sizes. Consequently, points are not interesting with respect to similarity.

What about two points? Two points define a unique line segment, though if we look at two line segments, we see that we can always view one as a portrait of the other (remember: mathematical line segments have no thickness). Another way of thinking about this is that you can make two line segments appear identical by positioning them at appropriate distances from your eyes. For instance, if one segment is twice as long as the other, then holding the longer one twice as far away from your eyes as the shorter one will make them appear identical. Thus, all line segments are similar to each other. So we need still more complexity to begin studying similarity.

How about three points? Three points typically define a triangle. Are there two triangles that are not similar to each other? Sure! Now we're getting somewhere:



Two non-similar triangles.

In fact, given enough time and space, one could draw infinitely many triangles no two of which would be similar to each other.

Thus, we'll begin our systematic study of similarity by looking at triangles. In particular, let's ask ourselves the following question: What methods can we use to determine whether or not two triangles are similar to each other? Think about that and tune in next time for an answer!

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

In this installment, Anna continues her investigation of divisibility properties of Fibonacci numbers.

F_n = Fibonacci Sequence, $F_1 = F_2 = 1$.

> If $n = mk + r$, then $F_n \equiv F_{m-1}^k F_r \pmod{F_m}$
> $(0 \le r < m)$

This is the result I got last time. I want to think about it more. I feel there is something more here.

I was thinking about how I proved this, and it dawned on me that you can't have two consecutive zeros when you reduce the Fibonacci sequence modulo F_m.

Note: $F_{m-1} \not\equiv 0 \pmod{F_m}$.

If it were, then because $F_{n+1} = F_n + F_{n-1}$, once you have two consecutive numbers both divisible by F_m, all Fibonacci numbers would be too. But $F_1 = 1$.

Hey, actually you can't have consecutive zeros modulo any integer > 1, for the same basic reason!

Actually, F_{m-1} and F_m must be relatively prime!

If $d \mid F_{m-1}$ and $d \mid F_m$, then $d \mid F_{m-2} \Rightarrow d \mid F_{m-3}$, etc.

eventually $d \mid F_1$ but $F_1 = 1$. $\Rightarrow d = 1$.

That means that consecutive Fibonacci numbers are relatively prime. Neat.

> $(F_{m-1}, F_m) = 1$ for all $m > 1$

I wonder what the fact that consecutive Fibonacci Numbers being relatively prime says about my earlier observation...

$$F_n \equiv F_{m-1}^k F_r \pmod{F_m}$$

$$F_n - F_{m-1}^k F_r = c F_m, \quad c \text{ some integer.}$$

If $d \mid F_n$ and $d \mid F_m$, then $d \mid F_{m-1}^k F_r$.

Since $(F_{m-1}, F_m) = 1$, if $d \mid F_{m-1}^k F_r$, then $d \mid F_r$.

Summary: If $n = mk + r$ with $0 \le r < m$, then $d \mid F_n$ and $d \mid F_m \Rightarrow d \mid F_r$.

Key:
- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

Pick n, m with $n > m$.

Can write $n = mk + r$ with $0 \le r < m$.

Then if $d \mid F_n$ and $d \mid F_m$, $d \mid F_r$.

Euclidean algorithm.

> This all reminds me of the Euclidean algorithm. Let me review that in my mind here.

$$n = mk + r_1 \qquad 0 \le r_1 < m$$

$$m = r_1 k' + r_2 \qquad 0 \le r_2 < r_1$$

$$r_1 = r_2 k'' + r_3 \qquad 0 \le r_3 < r_2$$

$$\vdots$$

until $r_x = r_{x+1} k_{x+1}$.

r_{x+1} = gcd of n and m

> In the Euclidean algorithm, you divide one number into another and get a remainder. Then you divide the remainder into the divisor, and keep going. Eventually, you arrive at the greatest common factor of the original two numbers.

If $d \mid F_n$ and $d \mid F_m$, then $d \mid F_{r_1}$.

then $d \mid F_{r_2}$, $d \mid F_{r_3}$, …, $d \mid F_{r_{x+1}}$

> So if I repeatedly apply my observation from the last page, I can conclude this.

So **If $d \mid F_n$ and $d \mid F_m$ then $d \mid F_{(m,n)}$.**

Since (F_n, F_m) divides both F_n and F_m, we must have $(F_n, F_m) \mid F_{(m,n)}$.

> Actually, since that works for any divisor *d*, it has to work for the greatest common divisor too!

Proposition: $(F_n, F_m) \mid F_{(n,m)}$.

> This is cool!

$d \mid F_m$, $d \mid F_n \Rightarrow d \mid F_r$

Converse? $d \mid F_r \Rightarrow d \mid F_m$ and $d \mid F_n$?

> I wonder if the converse of the divisor fact is true.

Example: $8 = 3(2) + 2$

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

$F_8 = 13$, $F_3 = 2$ $F_2 = 1$

> This example isn't a good one because $F_2 = 1$, which divides everything!

Ex: $11 = 4(2) + 3$

$F_{11} = 89$ $F_4 = 3$ $F_3 = 2$

$2 \nmid F_4$ Converse not true.

> The converse isn't true. Actually, it seems like wishful thinking to think the converse could be true! There are too many ways to get a fixed remainder.

AB 10.18.10

Key:

- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

Ambigrams are presentations of words or phrases that include some kind of symmetry. We present a number of ambigrams by Girls' Angle members, mentors, and affiliates. On the last page, we conclude with three ambigrams by ambigram master Scott Kim. You can find more of his ambigrams on his website or in his book *Inversions*.

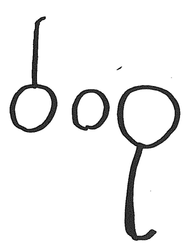
Nilla

Molly

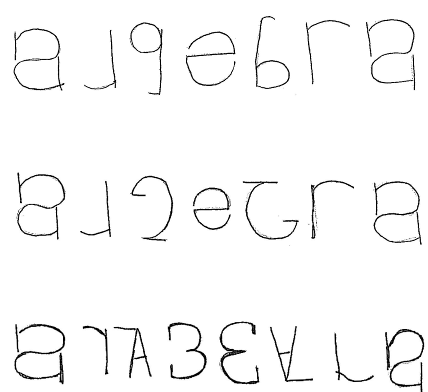
Nina

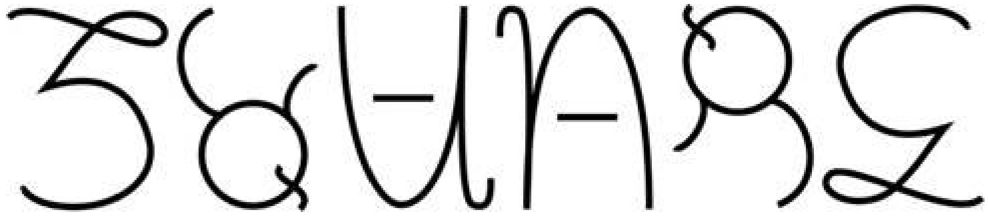
Pixie

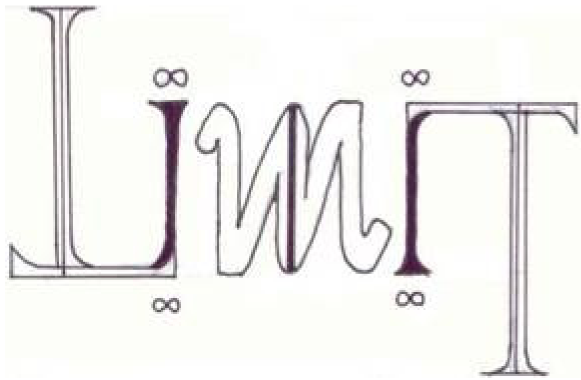
Bico

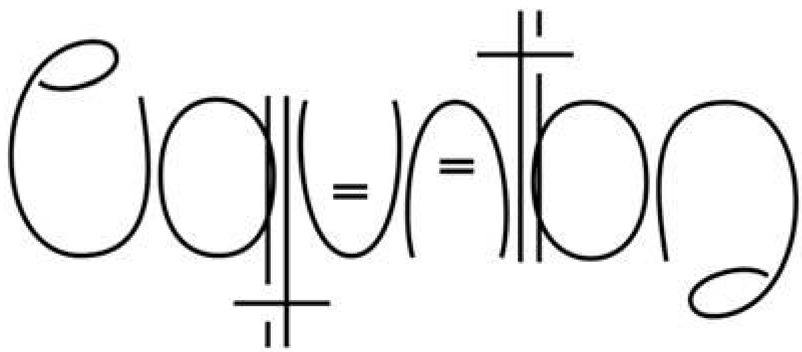


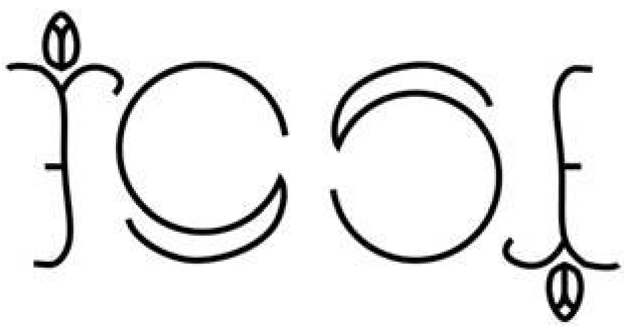
Atata

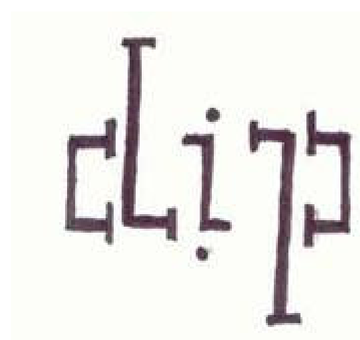












“Square”, “Rose” – Toshia McCabe, “Later” – Liz Simon,
“Equation” – Ken Fan, “Limit”, “Clip”, “Calculus” – Keren Gu

- Scott Kim

- Scott Kim

- Scott Kim

Brownie Points

Continued from page 10.

10 pennies

I have 10 pennies in my pocket, each of which is equally likely to land on heads as on tails in a coin toss. I'm going to shake them around in my hand and dump them on the table. Then we're going to count the number of heads and the number of tails. For every head, you get 2 brownie points; for every tail, you get 1.

We anticipate seeing around 5 heads and 5 tails. Sometimes there'll be 4 heads and 6 tails, or 7 heads and 3 tails. We'll even see all 10 coins come up tails every so often. But if we shake up the pennies and dump them on the table enough times, keeping track of the total number of heads we've seen and the total number of tails we've seen, then the ratio of heads to tails will get closer and closer to 1.

Now consider the case where there are exactly 5 heads and 5 tails. If each head gives you 2 brownie points and each tail gives you 1, you get 15 brownie points total. So if I want to break even, I should charge 15 brownie points for 10 coin tosses. That averages out to 1.5 points per coin toss.

We've just computed something called the **expected value**. When I flip a single fair coin with tails = 1 and heads = 2, the expected value is 1.5. The terminology here is a little bit misleading, because "expected value" makes it sound like you would "expect" to get 1.5 brownie points after a single coin toss. In reality, you can be completely sure that after a single coin toss you will *never* get 1.5 brownie points! The real reason that the value is described as "expected" is that if you played the game enough times, you would expect to get an average of 1.5 brownie points per toss.

Now that we've looked carefully at an easier situation, let's go back to the game with a single 6-sided die. Is it easier to think about now?

(Think about this for a while before moving on.)

We're going to use the same trick that worked so well with the coin toss game. Let's imagine that we have 60 dice (60 instead of 10 because it's divisible by 6). When we roll all 60 of them, we anticipate seeing roughly 10 occurrences of each number, 1 through 6. If each number does come up 10 times, then we would get 10 + 20 + 30 + 40 + 50 + 60 = 210 brownie points in total. This means I should charge 210 brownie points to roll 60 times, which is an average of 3.5 brownie points each time. If I charge 3 brownie points to play, you definitely want to take me up on that offer!

Now can you answer the original question? Send your answers to girlsangle@gmail.com!

Related questions

1. What if one of the dice is biased in favor of 6? What if you are as likely to roll a 6 as you are of not rolling a 6, but the numbers 1, 2, 3, 4, and 5 are equally likely as each other?

2. Suppose you have 2 dice. You roll them, and if the numbers match, then you get 10 times the product of the two numbers in brownie points; if the numbers don't match, you get nothing. (If you roll 2 sixes you get 360 points!)

3. What if you take the sum of 3 dice instead of 2? How about 4 dice?

4. (Warning: This problem is quite difficult.) Suppose you have a single die. You roll it and can decide whether to take that number of brownie points or to roll again. If you choose to roll again, you have to take the number of brownie points that you roll the 2nd time, even if it's smaller than what you rolled the first time. How much should this game cost to play, and what should your strategy be?

5. Try to come up with an example of your own.

Real-World Games

There are actually adults who play games like this for a living. (They get paid to play these games!) When you pay 3.5 brownie points for the opportunity to roll the dice, you're paying for the right to earn more brownie points in the future provided that something that you can't completely predict (but think you have a hunch about) goes well. A "right" that you can buy in this way is called an "option."

Here's an example of how a particular type of option — a "stock" option — works. There are lots of companies in the world. These companies are all worth a certain amount of money, and that amount fluctuates up and down all the time. Anybody, including you or me, can buy a very, very small piece of a company called a "share." If I buy a share of Company X today, it might cost me, say, $306. Then, if I hold onto the share and sell it a month later, the price might have changed to $325. I will have made a profit of $19. The problem is, I can also lose money. A lot of money. If a month from now a share of Company X is only worth $100, I would be out $206.

If I don't like the possibility of losing large sums of money, I can buy an option instead. An option is the right to buy shares for a specific price at some specific point in the future. For instance if I want to buy the right to buy a share of Company X for $300 one month from now, it would cost me some relatively small amount like $12. If the price of Company X goes up to $325, then I can exercise my right to buy my share for $300 and then immediately sell it for $325, giving me a net profit of $13. If on the other hand, the price of Company X goes down to $100, then I obviously don't want to buy a share of Company X at $300, because I can get it for so much cheaper. The right that I spent $12 to have becomes worthless, but at least I haven't lost $206! Options give you a way of limiting your losses in advance.

There's one important way that buying an option is different from buying the right to roll the dice in our game. In our game, we know exactly what the parameters are. We know exactly how likely we are to roll any of the 6 numbers. In the real world, things aren't that certain. Nobody can actually predict whether an option will be worth something someday.

Where's My Brownie?!

Instead of working with dollars, you and I are working with brownie points. You can buy a right (an option) to roll the dice (to buy a share) in hopes of getting more brownie points (in hopes of getting more dollars).

If after reading this article, you're up for the challenge of winning your brownie, you'll have to come to the Girls' Angle meet on November 18 to get it. It will be soooooo delicious!

Errorbusters!

by Cammie Smith Barnes

As a mathematics professor at a small liberal arts college, I teach math to students at a wide variety of levels, in courses such as trigonometry and precalculus, multivariable calculus, introduction to mathematical proofs, group theory, and beyond. Certain common mistakes and misconceptions appear in their work again and again, sometimes even creeping into the homework and exam solutions of my upper-level students! In each issue of this column, I will choose one of the more prevalent errors and discuss how you can remember to avoid it.

Let us start by considering one very popular mistake. It's one of my all-time favorites—or should I say, least-favorites?— and I'm always on the lookout for it. I call it "**dropping the minus sign**".

Suppose that we want to subtract one quantity from another, but that the quantity we're subtracting has more than one term. For instance, consider the subtraction problem

$$8 - (2 + 3)$$

which could represent, say, that a neighbor, Alicia, bought eight apples at the farmers' market and then gave some out to her friends: two apples to Betty, and three apples to Carolyn. The subtraction problem tells us how many apples Alicia still has:

$$8 - (2 + 3) = 8 - 5 = 3.$$

Recall that the parentheses around the sum 2 + 3 tell us that, strictly speaking, by the order of operations (parentheses, powers, multiplication and division, addition and subtraction), we must evaluate this sum **before** we perform the subtraction.

Yet, suppose that we would like to simplify the expression by removing the parentheses, but without actually evaluating the sum. We can do this by distributing the minus sign to every term that was inside the parentheses, like so:

$$8 - (2 + 3) = 8 - 2 - 3.$$

We do this because we need to subtract off both the 2 and the 3; it wouldn't be correct just to subtract one or the other, since Alicia gave away first two apples, then three apples.

Seems straightforward, right? Well, far too often, I see the equivalent of the following **mistake**. Can you spot the error?

$$8 - (2 + 3) = 8 - 2 + 3 = 6 + 3 = 9.$$

Surely the evaluation **cannot be correct**, as otherwise Alicia would have somehow gained an apple, instead of giving away five! Reading the string of equations left to right, the second + should be a –.

"But what is the point?" you

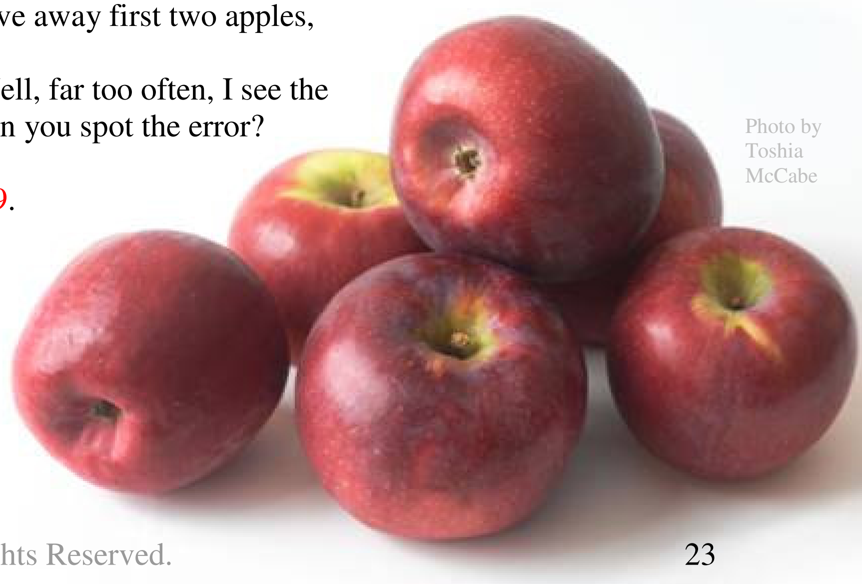


Photo by Toshia McCabe

might ask. "Why not just simply avoid the error by always doing the operation or operations **inside** the parentheses first?" If the expression that you wish to simplify involves something more complicated than just numbers, then you may not have a choice! For example, suppose that we want to simplify the following expression:

$$(5x + 4) – (2x + 7).$$

Due to the variable x, which occurs in both pairs of parentheses, we cannot simplify the sums inside the parentheses. Instead, we must simplify the entire expression by removing the parentheses and distributing whatever is necessary to the terms that were inside them.

Because there is a minus sign in front of the rightmost pair of parentheses, we must distribute this minus sign to both the $2x$ and the 7. Meanwhile, there is nothing in front of the leftmost pair of parentheses, and removing them doesn't change the order of operations that we must perform, so we can go ahead and omit these parentheses without consequence. So

$$(5x + 4) – (2x + 7) = 5x + 4 – 2x – 7.$$

Combining like terms (that is, putting all the terms with an x together and all the terms with just numbers together), we get

$$5x + 4 – 2x – 7 = 5x – 2x + 4 – 7 = 3x – 3.$$

Hence, $(5x + 4) – (2x + 7) = 3x – 3$.

The common **error** that occurs all too often is the following:

$$(5x + 4) – (2x + 7) = 5x + 4 – 2x + 7 = 3x + 11.$$

Now do you see why we must be vigilant and never drop a needed minus sign? There is a big difference between $3x – 3$ (correct) and $3x + 11$ (incorrect).

Let's try an even trickier example. Suppose that we want to solve the following equation for x:

$$2(x + 2) – (3x – 4) = 5(x – 3) – 1.$$

This was an actual question on one of my exams!

The first step in solving such an equation is to do what we need to do to remove the pairs of parentheses and simplify both sides. We need to be careful to distribute the minus sign outside the second pair of parentheses to both the $3x$ and the -4:

$$2x + 2(2) – 3x – (-4) = 5x – 5(3) – 1.$$

Recall that -(-4) = +4, so by simplifying each side further, we get

$$2x + 4 – 3x + 4 = 5x – 15 – 1.$$

Combining like terms, we obtain

$$-x + 8 = 5x – 16.$$

Adding x to both sides, we now have

$$-x + 8 + x = 5x - 16 + x,$$

or, in other words,

$$8 = 6x - 16.$$

Since we need to get the variable x on one side by itself, we must add 16 to both sides, to get

$$8 + 16 = 6x - 16 + 16,$$

which gives

$$24 = 6x.$$

Finally, we divide both sides by 6 to obtain

$$\frac{24}{6} = \frac{6x}{6}$$

or rather, the solution $4 = x$.

In attacking this question, however, several students fell into the following **trap**. Starting with the same equation,

$$2(x + 2) - (3x - 4) = 5(x - 3) - 1,$$

these students fell for the temptation to drop the parentheses without properly distributing the minus sign, giving them the **incorrect** new equation:

$$2x + 4 - 3x - 4 = 5x - 15 - 1.$$

Further solving this new equation yields $x = \frac{8}{3}$, not at all the solution that I had in mind!

So the moral of the story is to be extra careful when removing parentheses, and keep an eye out for any tricky minus signs! Here are a few more examples for you to try, with both a right and a wrong solution. Circle the correct solution and cross out the incorrect one! (Answers are provided on page 30.)

1. Simplify $(4 + 8) - (7 + 2)$. Choose the correct solution:
   A. $(4 + 8) - (7 + 2) = 4 + 8 - 7 + 2 = 12 - 7 + 2 = 5 + 2 = 7$
   B. $(4 + 8) - (7 + 2) = 4 + 8 - 7 - 2 = 12 - 7 - 2 = 5 - 2 = 3$
2. Simplify $(3x - 5) - (4x - 15)$. Choose the correct solution:
   C. $(3x - 5) - (4x - 15) = 3x - 5 - 4x + 15 = -x + 10$
   D. $(3x - 5) - (4x - 15) = 3x - 5 - 4x - 15 = -x - 20$
3. Simplify $(6x - 4 + 1) - (2x + 8 - x)$. Choose the correct solution:
   E. $(6x - 4 + 1) - (2x + 8 - x) = 6x - 4 + 1 - 2x - 8 + x = 5x - 11$
   F. $(6x - 4 + 1) - (2x + 8 - x) = 6x - 4 + 1 - 2x - 8 - x = 3x - 11$

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 7 – Meet 1 – September 9, 2010

Mentors: Jennifer Balakrishnan, Ryan Heffrin, Lauren McGough, Jennifer Melot, Liz Simon, Rediet Tesfaye, Bianca Viray, Fan Wei, Drew Wolpert

Our icebreaker activity for the first meet of the seventh session was about sorting. Members sorted themselves with respect to data such as alphabetically by last name or increasing order of age. In this way, they also had to get to know one another.

Here are three problems from this meet:

1. Write the numbers from 1 to 10 in a row. You are allowed to change these numbers by adding one to one of the numbers and subtracting one from another. You may do this as many times as you wish. The goal is to produce 10 fives in a row. Can this be done? If so, how? If not, why not?

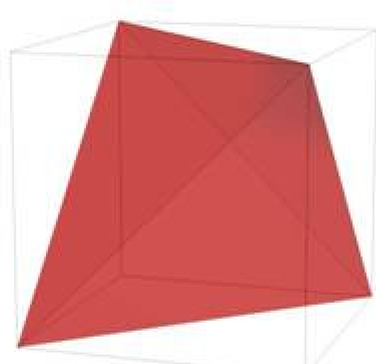
2. From a picture of a tennis match taking place at the US Open tennis tournament, deduce the distance between the baseline of the tennis court and the back wall, given that the tennis court is a regulation tennis court.

3. You have 10 balls. Each ball is one of two possible weights. Using a balance scale at most 3 times, can you determine whether or not all the balls are the same weight?

Session 7 – Meet 2 – September 16, 2010

Mentors: Jennifer Balakrishnan, Keren Gu, Ryan Heffrin, Jennifer Melot, Ellen Rice, Liz Simon, Carolyn Stein, Rediet Tesfaye, Bianca Viray

A number of girls worked on billiard ball problems found in Grace Lyo's Summer Fun problem set in Volume 3, Number 6 of this Bulletin. These problems are about the properties of paths that billiard balls follow when they bounce around on a square or circular billiard table. Some girls began thinking about billiard ball paths in a cube.



Consider a cube. Let A be a vertex. Let B, C, and D be the three vertices that one can travel to from vertex A by traveling exactly two edge lengths along the edges (without doubling back). The four vertices A, B, C, and D form the vertices of a regular tetrahedron. Can you show that a billiard ball that starts off moving within the boundary of this regular tetrahedron remains forever confined to it? Here, assume that the billiard ball has no size.

Session 7 – Meet 3 – September 23, 2010

Mentors: Jennifer Balakrishnan, Keren Gu, Jennifer Melot,
Liz Simon, Rediet Tesfaye, Bianca Viray

A couple members thought about parity and addition. Integers come in two flavors, even and odd. The two members came up with this definition of even: An integer is even if and only if it can be split into two equal (integer) parts. They then showed that the sum of two even integers is, again, even. I'd like to share their proof here rewritten using standard notation from algebra.

Let A and B be two even numbers.

Because A is even, we can write $A = N + N$, where N is an integer, by definition. Similarly, we can write $B = M + M$, where M is an integer.

We can then compute

$$A + B = (N + N) + (M + M) = N + N + M + M = N + M + N + M = (N + M) + (N + M).$$

The last expression shows that $A + B$ can be split into two equal integer parts, each of size $N + M$. Therefore, $A + B$ is also even.

Essentially the same proof can be used to show that if A and B are both divisible by d, then so is their sum.

Session 7 – Meet 4 – September 30, 2010

Mentors: Jennifer Balakrishnan, Ryan Heffrin, Jennifer Melot,
Liz Simon, Rediet Tesfaye, Bianca Viray

Special Visitor: Tanya Khovanova, Research Affiliate, MIT

Tanya gave a presentation about invariants. After discussing casting out nines, she introduced a "magic apple tree" which, curiously, only produced oranges and bananas. This exotic tree behaved as follows:

> If you plucked 2 bananas, an orange would grow back.
> If you plucked 2 oranges, an orange would grow back.
> If you plucked a banana and an orange, then a banana would grow back.

She then posed this question: If there are 100 oranges and 100 bananas to start with, and then after several people pluck fruit the tree ends up with 1 fruit, what kind of fruit is it?

After the girls solved this, she then had them play a game which begins with the numbers 1 through 2005 written on the board. You are allowed to replace any two numbers with the absolute value of their difference. The question is, can you fill the board with all zeroes?

But the larger question is, what does this board game have to do with the magic apple tree?

Session 7 – Meet 5 – October 7, 2010

Mentors: Keren Gu, Samantha Hagerman, Ryan Heffrin, Jennifer Melot,
Liz Simon, Carolyn Stein, Inna Zakharevich

For part of the meet, some girls played a round of Math Tag. In Math Tag, each player picks a unique integer between -40 and 40. Then, 6 twenty-sided dice are rolled. Players have one minute to come up with an expression that involves all 6 numbers that are rolled and evaluates to one of the numbers picked by another girl. If an expression evaluates to a girl's number, that girl has been "tagged" and is eliminated from the game. Play continues until only one girl is left. If you try this game, whatever you do, don't tag yourself!

Modular arithmetic has been an ongoing thread at Girls' Angle this session. Fix a modulus n and an integer x. How does the sequence x, $2x$, $3x$, $4x$, … behave modulo n? How does the sequence x, x^2, x^3, x^4, … behave module n? These two questions are fundamental number theory questions and worth examining in great detail. (They are also worth thinking about without reducing modulo n.)

If you don't know what a modulus is and want to know, ask us about it at the club!

Quick Primer on Modular Arithmetic

When we work "modulo n", what we mean is that we are considering two integers a and b to be the same if and only if n divides $a - b$. This is sometimes written

$$a = b \pmod{n}.$$

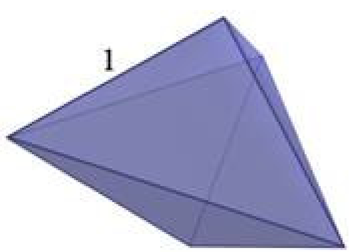
Can you show that:

1. We have $a = a \pmod{n}$.
2. If $a = b \pmod{n}$, then $b = a \pmod{n}$.
3. If $a = b \pmod{n}$ and $b = c \pmod{n}$, then $a = c \pmod{n}$.

Also, can you show that if $a = b \pmod{n}$ and $c = d \pmod{n}$, then both $a + c = b + d \pmod{n}$ and $ac = bd \pmod{n}$.

Session 7 – Meet 6 – October 14, 2010

Mentors: Keren Gu, Samantha Hagerman, Ariana Mann, Jennifer Melot,
Liz Simon, Rediet Tesfaye, Bianca Viray, Inna Zakharevich



Some girls constructed a polyhedron with 6 faces all congruent to an isosceles right triangle using techniques of modular origami. If the length of the leg of one of these right triangles is one unit long, what is the volume of the resulting solid?

What symmetries does this object have? What are the various distances that exist between pairs of its vertices? How does this figure relate to the image of the tetrahedron inside the cube from Meet 2?

The figure above has six congruent triangular faces. Let's take a step back and consider solids which have four congruent triangular faces. What shapes are possible? For what triangles is it possible to make a four-sided polyhedron all of whose faces are congruent to the given triangle? Can you express the volume of the resulting tetrahedron in terms of the side lengths of the triangle?

Session 7 – Meet 7 – October 21, 2010

Mentors: Jennifer Balakrishnan, Christine Breiner, Keren Gu, Samantha Hagerman, Ryan Heffrin, Jennifer Melot, Liz Simon

Special Visitor: Jane Kostick, Woodworker

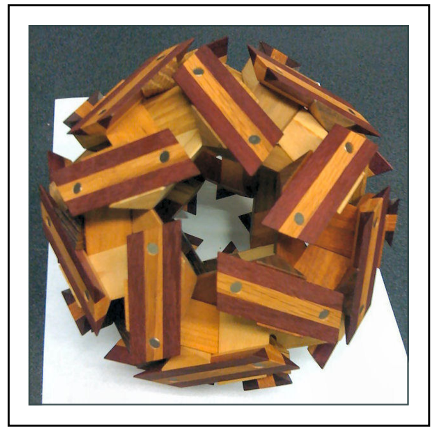
A little bit of calculus has been appearing at the club this session. In meet 7, one calculus topic that came up was to prove the product rule for differentiation:

$$(fg)' = f'g + fg',$$

where f and g are differentiable real valued functions defined on the set of real numbers.

Here's a follow-up question: What is the nth derivative of fg?

We also did a mini-presentation on similarity. See page 14.



Jane brought in a bunch of geometrically inspired puzzles, mostly made out of wood. Using the parts from three "6-axis" puzzles, one girl made the object shown at right.

Session 7 – Meet 8 – October 28, 2010

Mentors: Samantha Hagerman, Ryan Heffrin, Ariana Mann, Jennifer Melot, Emily Peters, Liz Simon, Rediet Tesfaye, Fan Wei, Siyao Xu

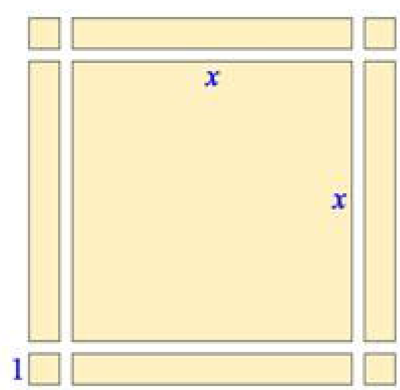
One member began thinking about hypercubes. If we let $F_{k,n}$ denote the number of k-dimensional faces on an n-dimensional hypercube, she was able to show that

$$F_{k,n+1} = F_{k-1,n} + 2F_{k,n},$$

when $k \geq 0$ and $n \geq 0$. For example, a square has 4 vertices and 4 edges, so $F_{0,2} = 4$ and $F_{1,2} = 4$. Using the recurrence relation above, we can compute that $F_{1,3} = F_{0,2} + 2F_{1,2} = 4 + 2(4) = 12$, and, indeed, a cube has 12 edges.

Here's another way to see this. Examine the figure at right. How does this figure relate to the algebraic expression

$$(x + 2)^2?$$



If you understand that, then ask, how do the numbers $F_{k,n}$ relate to the coefficients of $(x + 2)^n$?

If you enjoy thinking about higher dimensional objects, here's another problem for you: What are higher dimensional analogues of the triangle and tetrahedron?

Calendar

Session 7: (all dates in 2010)

September	9	Start of the seventh session!
	16	
	23	
	30	Tanya Khovanova, Research Affiliate, MIT
October	7	
	14	
	21	Jane Kostick, woodworker
	28	
November	4	Valerie Gordeski, Raytheon
	11	Veteran's Day – No meet
	18	Anita Suhanin and Noam Weinstein
	25	Thanksgiving - No meet
December	2	Lenore Cowen, Tufts University
	9	

Session 8: (all dates in 2011)

January	27	Start of sixth session!
February	3	
	10	
	17	
	24	No meet
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Girls' Angle thanks Microsoft and MITX for naming Girls' Angle as a beneficiary of the 2010 Women's Leadership Forum which was held at the Microsoft New England Research and Development Center in Cambridge, Massachusetts.

Here are answers to the *Errorbusters!* problems on page 25.

1. B
2. C
3. E

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) comes with membership. We are working hard to lower the cost of the Bulletin. Until we do, however, nonmembers can receive the printed version by becoming a Bulletin Sponsor. Please contact us if interested.

The Bulletin targets girls roughly the age of current members. Each issue is likely to contain some material that feels very challenging or difficult to understand. If you are a member or Bulletin Sponsor and have any questions about the material, feel free to ask us about it!

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

How do I join? Membership is granted per session and includes access to the club and extends the member's subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For (please circle): Membership Bulletin Sponsorship

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: ________________ Email: ______________________

For **membership**, please fill out the information in this box. **Bulletin Sponsors** may skip this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a one session membership
  - ☐ Bulletin Sponsorship: $60-99 Subscribing. $100-999 Bronze. $1000-2999 Silver. $3K+ Gold.*
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

*Silver and Gold sponsors may specify a recipient for their sponsored copies of the Bulletin.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

Girls' Angle

A Math Club for Girls