Girls' Angle Bulletin

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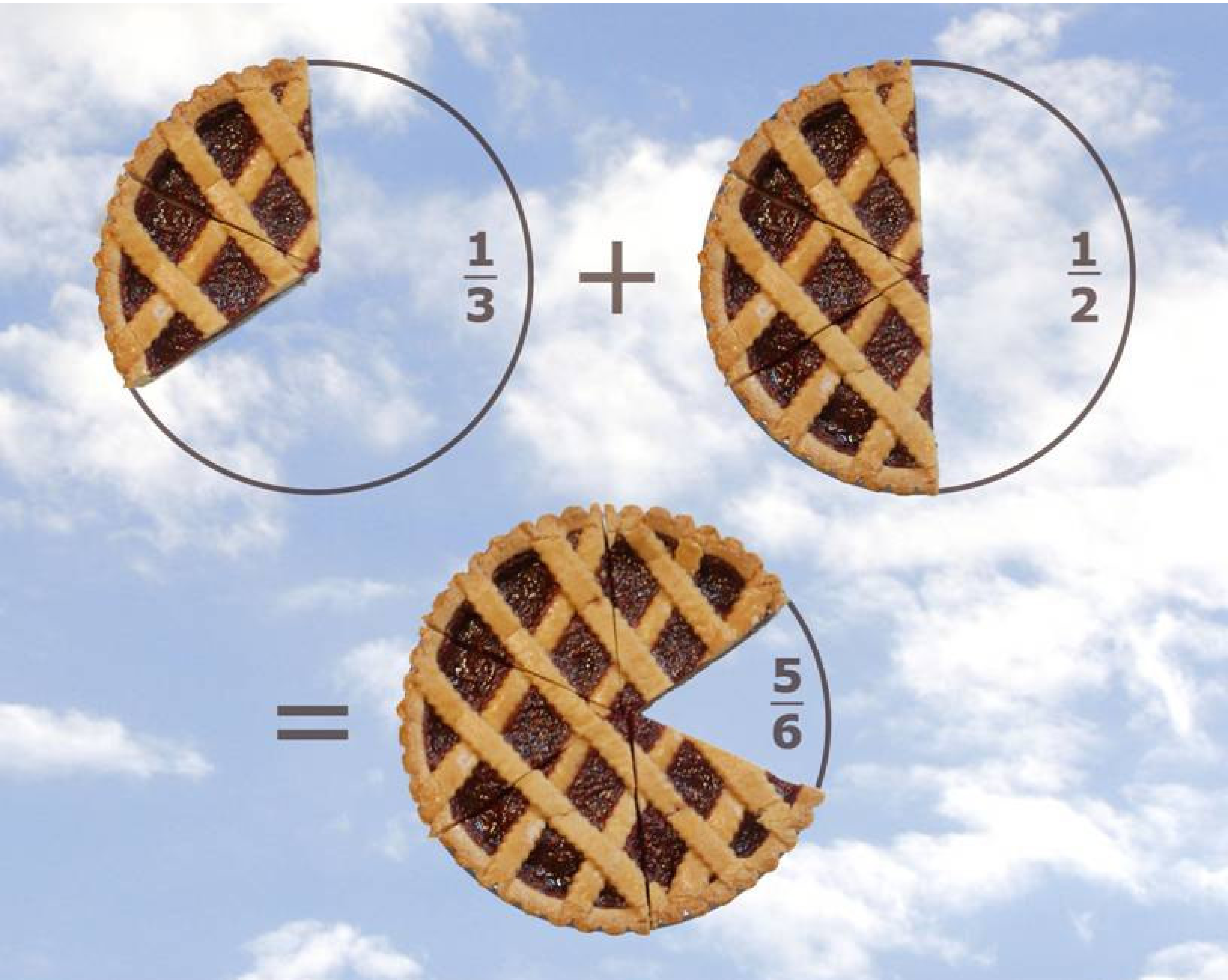
To Foster and Nurture Girls' Interest in Mathematics



From the Director

A lesser known program at Girls' Angle is Community Outreach, a service in which members of the community are invited to commission Girls' Angle's members to solve their math problems. We hope to find opportunities for our members to tackle problems whose solution will actually be used. In exchange, we ask that the girls of Girls' Angle be given credit for their work. For a recent example, please turn to page 10.

Also in this issue, we welcome new columnist Barbara Remmers. Barbara leads the Athens Area Girls Math Team in Georgia. Her members are roughly in grades 2-5 and *Coach Barb's Corner* targets that audience.

Finally, a special congratulations to Advisory Board member Elissa Ozanne who will be moving out west to take up a faculty position at the UCSF Medical Center. Elissa has been a frequent visitor to the club where she showed us how she uses math to help women survive breast cancer. We'll miss her!

- Ken Fan, Founder and Director

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Girls' Angle Bulletin

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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Grace Lyo
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Pies in the Sky by Toshia McCabe and Ken Fan.

An Interview with Erin Mullen

Dr. Erin Terwilleger Mullen is Assistant Professor of Mathematics at the University of Connecticut in Storrs. Dr. Mullen's doctoral work and early research was in the field of Harmonic Analysis. She is actively involved in math education as well and is interested in increasing the content knowledge of K-12 teachers.

Ken: Hi Dr. Mullen, thank you for agreeing to do this interview with Girls' Angle! I understand that you've recently become more involved with math education. I think that's terrific because I think that math education could use more people with backgrounds in research mathematics. I'll ask you about that later, but first, I'd like to know what got you interested in math?

Erin: I actually started out college as a chemistry major. My dad is an engineer and my mother is a neuroscientist, so I had strong role models in science, and I had plans to become a cosmetics chemist. I had always done well in math and enjoyed it, so when a math professor suggested I double major in math and chemistry, I decided to take on that challenge. As it turned out I enjoyed mathematics much more than working in a lab, so I dropped the chemistry. What I love about mathematics is that one can sit down and use her brain to come up with solutions to really cool problems.

Ken: You've done research in harmonic analysis...what is harmonic analysis? How do you think about it?

Erin: A sound wave has a particular frequency which sounds a certain way to our ear.[1] If you are listening to the radio for example, the sounds you hear are a signal composed of many sound waves with different frequencies that vary over time. Harmonic analysis studies things called functions, which are like a signal, by representing them in terms of all the different waves, or frequencies, that compose the function.

Above all, if you are not sure on how to proceed, just try something! I find that students are often afraid to try something because they think they should just know what to do. But that is often not the case. Sometimes you have to try an approach that fails to gain more insight.

Ken: How did you become interested in harmonic analysis?

Erin: My senior year of college I took an Analysis course, which provides the rigorous foundation of calculus, and an Applied Analysis course, which uses analysis to study problems in partial differential equations. An example is you take a string secured at both ends and pluck it, just like on a guitar. After the initial pluck, you want to describe how the string will vibrate. We can use harmonic analysis to describe the solution to this problem. I found this fascinating and decided to pursue harmonic analysis in graduate school.

Ken: Can you explain to us one of your own theorems?

[1] Frequency corresponds to pitch.

Erin: They are pretty theoretical and not easily stated. However, I can say that a couple of my results are related to a classical result in Harmonic Analysis about something called a Fourier series. A Fourier series is an infinite sum of cosine and sine terms each with a different frequency. For some functions, one can obtain a Fourier series by using the function to determine the coefficients of each term. In the 60s Lennart Carleson proved that the Fourier series of a function with a certain property converge back to the function, which turned out to be a very epic proof. In the 90s, Michael Lacey and Christoph Thiele found a new shorter, though not simple, proof, and some of my work is concerned with higher dimensional versions of the techniques they used called Time-Frequency Analysis.

Ken: I've never heard a proof described as "epic" before! What do you mean that?

Erin: Carleson's proof was very difficult to read and understand. Another famous mathematician, Charles Fefferman, gave another proof almost a decade later which inspired the simplest proof given by Lacey and Thiele.

Ken: When you get stuck on a problem, what kinds of things do you do to try to get unstuck?

Erin: One thing I find invaluable is to talk with a colleague about the problem. Other people can give you a different perspective on the problem which allows you to think about it in a new way. Sometimes, I find I just have to put the problem away for a bit and think about other things. I think this allows my mind to process everything and move on. Then when I return to the problem, I might see something I did not see before.

Ken: What is one of the most memorable experiences you have had in mathematics?

Erin: I would say giving my first mathematics talk. It was my final year in graduate school, and I presented my research in a departmental seminar. Although I was comfortable teaching undergraduates, I was very nervous about giving my first research talk to my peers. Would they like the research I was doing? Would I say something stupid? But overall it went very well, my colleagues were impressed, and I felt a lot of pride in passing this milestone.

Find what you love to do, whether it is mathematics or something else, and have the confidence in your abilities to do it. Setbacks or disappointments are inevitable so remember to pick yourself up and forge ahead.

Ken: I saw that you wrote a paper about something called "active learning". What do you mean by "active learning?"

Erin: Active learning places more of the responsibility of learning onto the students. Traditional lectures are inactive for the student because the teacher is giving her the information, and the student just sits there passively and does not necessarily have to do anything with the information. In active learning, teachers use different methods of instruction that force the students to think about and do something with the material they are trying to learn. An example I have done is to split the students into small groups and have them discuss homework problems they have prepared ahead of time.

Ken: Do you have advice for our members about how to best go about studying mathematics?

Erin: You probably hear this a lot, but the best thing is to just do mathematics! If you are taking a math class, work on all the homework problems. In doing this you may find you need to reread your class notes and textbook or get help from a friend or teacher which will help you to understand the problem better. Above all, if you are not sure on how to proceed, just try something! I find that students are often afraid to try something because they think they should just know what to do. But that is often not the case. Sometimes you have to try an approach that fails to gain more insight.

Ken: I always ask this question: do you think there is gender bias in mathematics today? If you think there is gender bias, do you have any advice for how to minimize the effect of it?

Erin: I think the situation has improved greatly. There are many wonderful programs, organizations, and people who encourage girls and women in mathematics. Many college mathematics departments, mine included, are understanding and accommodating on issues unique to women like maternity leave. On the other hand, you will still find individuals who do not respect a woman's mathematical abilities as much as a man's, and I do not think they always realize they are doing that. There are subtle ways in which a woman is treated differently than a man. Many young girls still grow up in a culture where they are valued for their looks and their maternal attributes and boys are valued for their intellect. That is why a program like Girls' Angle is crucial for changing these stereotypes at a young age before they become ingrained in young minds.

There is definitely still a gender gap in many science fields and mathematics. At UConn, I am the Director of the Women in Math, Science, and Engineering (WiMSE) Learning Community and one of the issues we talk about is being a woman in fields which are predominantly male. For example, in academia in these fields, women tend to have more service obligations like committee work than men. Significantly, the gap increases as you move up in seniority in the workplace or academia. For example, there are roughly the same number of males and females obtaining bachelor degrees in math. However, there are significantly less women getting Ph.D.s in math. I believe that encouraging women to pursue graduate degrees in math is an important goal.

Ken: Do you have any hobbies aside from math?

Erin: I enjoy jogging, camping, shopping, home decoration, and playing with my three young daughters. They are ages 6, 4, and 1 year old, so they keep me busy!

Ken: Do you have any advice for the girls that come to Girls' Angle?

> What I love about mathematics is that one can sit down and use her brain to come up with solutions to really cool problems.

Erin: It is a bit cliché, but if you work hard and believe in yourself, you can accomplish a lot. Find what you love to do, whether it is mathematics or something else, and have the confidence in your abilities to do it. Setbacks or disappointments are inevitable so remember to pick yourself up and forge ahead. Another piece of advice I have is to seek out mentors, male or female, who can guide you and give you advice. This person could be one of your teachers or come from a mentoring program, which you can find at many colleges. I had a wonderful mentor in my graduate advisor Loukas Grafakos. He encouraged me, believed in my mathematical abilities, and gave me advice that has served me well.

Ken: Thank you so much for this interview!

Cake Mix

by Addie Summer
edited by Jennifer Silva

"Chocolate!" I yelled back to my mom as I left the house for the bakery. I knew chocolate was my dad's favorite cake, and it was mine too, so it would be perfect for his birthday. And I needed a nice big cake. We were expecting over 30 guests.

I knew exactly where to go to get one, too: Cake Country. According to my chemistry teacher, she and her brother were both studying to be chemists, but all he cared about was whether a chemical was edible and how good it tasted if it was. So I guess it was no surprise that he became the owner of the best cake shop in town and she became my teacher.

"Don't take too long — Dad's back at 8:00," I heard my mom call out.

Cake Country's a bit of a hike to get to, but it's worth it. I hopped in my mom's car and headed for the highway.

Great, I thought to myself, this traffic stinks! I peered to the side trying to see where the bumper-to-bumper traffic ended. I could walk faster. I looked at my watch: 6:44 pm. Fortunately, Cake Country closes at 9:00 pm, I reasoned, but if I don't get there by 7:15 I'll have to figure out how to get the cake in the house without Dad finding out.

"Addie!"

"Hey, Mr. ChemCake!" That's what my chem teacher called him. "I need a big chocolate cake for my dad's birthday!"

"Oh dear … you came kinda late. There aren't any more whole cakes."

My heart sank at the news. "You're kidding! This is Cake Country! How can there be no cake?"

"I didn't say there was no cake, just no *whole* cakes. I've been serving up slices."

I approached the display case. The shelves carried partial cakes and crumbs.

"How about this rum cake? It's only missing one slice."

Rum cake? Blech! "I was hoping for chocolate. You know I love your chocolate triumph." Three concentric rings of cake made from different chocolates with chocolate syrup drizzled on top. I have no idea how he makes it!

"Hmmm. Well, how about this — why don't you take the rest of these three chocolate triumph cakes? That's all there is left. Looks to be about a whole cake, and you can have it for the price of one."

I studied the cakes. There was a quarter of one cake left, two-thirds of another, and two slices left of a third. I pointed at the two slices. "How big are those slices?"

"That one … the cake was cut into 10 slices."

Okay, I thought to myself, those two slices are two-tenths of a cake, which is the same as one-fifth of a cake. Do these three leftovers add up to a whole cake? I better add them up and find out.

I found a scrap of paper in my pocket and scribbled:

$$\frac{1}{4} + \frac{2}{3} + \frac{1}{5} =$$

I never thought I'd be doing math in a cake shop. I tried to remember how to add fractions, but I just couldn't! What are you supposed to do? Add up the numbers on top and divide by the sum of the numbers on the bottom? That would give 4 over 12. Four out of 12? That's equal to one-third … that can't be right because there's already a piece that's two-thirds right there.

Too bad the numbers in the bottom weren't all fours. If it were one-fourth plus two-fourths plus one-fourth, it would have added up to a total of four-fourths, which would be one whole cake. The problem was that the numbers in the bottom of the fractions were different.

"Mr. ChemCake? Do you remember how to add fractions?"

"Fractions? Are they edible?"

"Very funny. Seriously, do you remember?"

"Sorry, but these days I just deal in three things: dollars, cent, and slices."

I looked back at my scrap paper. I thought, Well, it's got to be less than three since that's what you'd get if there were three whole cakes. That's not much help.

I looked at the delicious cake remnants. I looked at the third cake with its two slices, each one a tenth of the original cake. Funny that I wrote 1 over 5 instead of 2 over 10, I thought, although I guess it doesn't matter really — they represent the same number.

Hmmm, 1 over 5 equals 2 over 10. And both are equal to 3 over 15 and 4 over 20. Seems that I can represent one-fifth as a fraction with any multiple of five in the bottom. So I have some freedom to alter the bottom numbers.

I can represent one-fourth using a fraction where the number in the bottom is a multiple of four. And two-thirds can be represented by a fraction with a multiple of three in the bottom.

The problem I'm having is that the numbers in the bottom are all different. If they were the same, then I could just count like I did when I pretended all the numbers in the bottom were four.

Wouldn't it be nice if there were a common multiple of all three denominators? I wondered. Oh wait! There is! I can just take their product — 4 times 3 times 5 — that's equal to 60. I should be able to represent each fraction here as a fraction with a 60 in the bottom.

Let's see. One-fourth is the same as fifteen-sixtieths. Two-thirds is the same as forty-sixtieths. And one-fifth is the same as twelve-sixtieths. So that means I have 15 plus 40 plus 12, or 67-sixtieths in total. That is 7-sixtieths more than 60-sixtieths, which means there's actually 7-sixtieths of a cake more than one whole chocolate triumph here! I completed my equation:

$$\frac{1}{4} + \frac{2}{3} + \frac{1}{5} = \frac{67}{60}.$$

"Mr. ChemCake," I announced, "if you gave me these three cakes, I'd be getting seven-sixtieths more than a whole cake, so it wouldn't be fair if you only charged me for one cake."

"No kidding?" Mr. ChemCake responded. "You've got eagle eyes — looks pretty much like one cake to me."

"I didn't eyeball it. I added the fractions. See?" I showed him my scratchwork.

"Tell you what," he said. "Go ahead and take the extra cake compliments of the house … and wish your dad a Happy Birthday!"

I guess sometimes you *can* have your cake and eat it too!

Define This Game

by Ken Fan
edited by Jennifer Silva

Define This Game is one of the more popular games that we play at Girls' Angle. All you need to play are four or more players and a good host.

How To Play

Split the players into two teams, A and B, of roughly equal size. Order the members within each team. The players should be seated in a split auditorium format with the two teams on opposite sides of the aisle and both teams facing forward.

Play proceeds in rounds in which each round Team A has a turn, then Team B has a turn to complete the round. Please keep in mind that if your team is not up, you still have to pay very close attention; you'll soon find out why!

When a team is up, one member of that team must go to the front; she will be the team's "Definer" for the round. Members of a team rotate the Definer role in the order that was determined when the teams were set up.

The Host provides only the Definer with a mathematical term (it doesn't have to be mathematical, of course, but we *are* a math club for girls!). The Definer has one minute to define the given term for her teammates. When defining the term, she must not make any gestures nor utter the term or any of its synonyms.

While the Definer is defining the term, her teammates must try to figure out what the term is. At any point, her teammates may interrupt her and state the term they believe she is defining. However, her teammates are not allowed to communicate with her in any other way. They may communicate quietly amongst themselves, but they must not influence their Definer by expressing confusion about certain points, requesting more information, or asking a question in such a way that the Definer is aware of it.

> Tip
>
> To avoid a penalty and to avoid tipping off the other team as to your thoughts, don't speak to each other - write instead!

Any violation of these rules results in a penalty of one point to the Definer's team, and its turn ends.

When the Definer's minute is up, her teammates have an additional minute to discuss amongst themselves (without the aid of the Definer) what they think the Definer was trying to define. Before the end of this additional minute, the Definer's teammates must produce a term; if they fail to do so, their turn ends and they forfeit a chance to win a point.

> Here are some good terms to use:
>
> Addition
> Angle
> Area
> Binary representation
> Binomial
> Circle
> Cone
> Congruent
> Coordinate
> Cylinder
> Cube
> Diameter
> Equation
> Even number
> Exterior angle
> Five
> Function
> Greater than
> Hexagon
> Inequality
> Infinity
> Intersect
> Isosceles triangle
> Less than
> Line
> Möbius strip
> Multiplication
> Negative number
> Number
> Odd number
> Parabola
> Parallel
> Perpendicular
> Plane
> Polynomial
> Pyramid
> Quadrilateral
> Rational number
> Real number
> Remainder
> Repeating Decimal
> Rhombus
> Sphere
> Square root
> Subset
> Tetrahedron
> Torus
> Trapezoid
> Variable
> Volume
> Zero
>
> …and so on!

Point Stealing

> Tip: Because of the possibility of a steal, the Definer's teammates must be very careful about blurting out answers prematurely!

Even if the Definer's teammates come up with the correct term or a synonym, they don't immediately win a point. The other team has a chance to steal!

There are two ways to steal the point. One is to show that the term the Definer's team came up with does not satisfy the definition that the Definer provided. The other is to produce a term with a different meaning, yet that satisfies all of the properties that the Definer provided in her definition. The other team has 30 seconds to steal the point. If the steal is successful, the other team wins the point and the turn ends.

If the other team fails to steal the point and the Definer's team has correctly determined the term (or a synonym of it), then the Definer's team earns a point. Otherwise, no point is awarded. In either case the turn ends.

When a team's turn ends, the other team gets to go. By playing in rounds, it is assured that each team has the same number of opportunities.

The Host plays a very important and quite challenging role as referee as subtle details of definitions demand careful attention! Also care must be taken in choosing a good term for each Definer. In the event that the Definer is not familiar with the term, it may be necessary to pull her aside to explain it.

Example 1	The Term: "Parallel" The Definer's words: "This is when two lines never intersect… in a plane" Her teammates: "Parallel" The other team: "Skew lines don't intersect… she didn't say 'in a plane' until later." The Host: "The intent of saying 'in a plane' was meant to rule out the skew lines case…point to the Definer's team!"
Example 2	The Term: "Less than" The Definer's words: "Opposite of more than… not equals…" Her teammates: "Less than" The other team: "The definition isn't clear- the 'opposite' of 'more than' could be taken to mean 'the complement of more than,' which would be 'less than *or equal to*.' And 'less than or equal to' is also 'not equals.'" The Host: "The other team is correct. There is some ambiguity in the term 'opposite' which could be taken as synonymous with the logical 'not.' And 'not more than' is equivalent to 'less than or equal to.' Point stolen!"

Depending on the experience level of the players, you can vary the amount of time allotted to each component of the game and change the level of difficulty of the terms that must be defined. With the time settings indicated, it takes about an hour for 12 players to play with each person getting a chance to be Definer. Please keep in mind that the score is kept just to add a little spice. I've found that it is often unnecessary to keep track of the score. The discussions that the game provokes about the definitions and terms are what's really important.

Playing this game is a fun way to explore the nature of definitions and gain an understanding of what it means for something to be well-defined. It also provides an opportunity to examine many mathematical objects and concepts in greater depth. If you try this game, please let us know about any fun moments!

Community Outreach: Tournament Design

by Ken Fan
edited by Jennifer Silva

Photo courtesy of Ellie Hawthorne

The girls' tournament seating chart in use.

Ellie Hawthorne organizes weekly gatherings of enthusiastic euchre players, many of which are senior citizens. She was searching for a player rotation that would make the tournaments as equitable as possible.

Euchre is a card game played between two teams of two. In a tournament, it would be nice to arrange things so that each player would get to partner with each of the other players as well as play against each of the other players. Preferably, the tournament would be organized so that one didn't partner with another person excessively or play against certain people disproportionately more than others.

Ellie's group has 16 players. How can the tournament seating schedule be arranged to be efficient and equitable? Ellie took advantage of Girls' Angle's Community Outreach program to find a solution, and the girls of Girls' Angle delivered!

In fact, our members found a most efficient solution. The tournament was organized into 15 rounds. In each round, four games would be played simultaneously at four different tables. Seated at each table would be four players: two teams of two. The girls arranged things so that each player would partner with each of the 15 other players exactly once and play against each player exactly twice by the end of the tournament. You can't get a more efficient and equitable scheme![1]

The players are numbered 1 through 16. Each row represents one round of the tournament, and each column represents one of the four tables. Within each row, the players at a particular table are listed as they would sit clockwise around the table. Since partners sit opposite each other when they play, the first and third players listed in each row for any given table are partners, as are the second and fourth.

Round	Table 1	Table 2	Table 3	Table 4
1	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16
2	2, 1, 3, 4	6, 5, 7, 8	10, 9, 11, 12	14, 13, 15, 16
3	2, 3, 1, 4	6, 7, 5, 8	10, 11, 9, 12	14, 15, 13, 16
4	1, 5, 9, 13	4, 8, 12, 16	3, 7, 11, 15	2, 6, 10, 14
5	5, 1, 9, 13	8, 4, 12, 16	7, 3, 11, 15	6, 2, 10, 14
6	5, 9, 1, 13	8, 12, 4, 16	7, 11, 3, 15	6, 10, 2, 14
7	1, 6, 11, 16	5, 12, 15, 2	9, 8, 3, 14	13, 4, 7, 10
8	6, 1, 11, 16	12, 5, 15, 2	8, 9, 3, 14	4, 13, 7, 10
9	6, 11, 1, 16	12, 15, 5, 2	8, 3, 9, 14	4, 7, 13, 10
10	1, 7, 12, 14	5, 3, 10, 16	9, 15, 4, 6	2, 8, 11, 13
11	7, 1, 12, 14	3, 5, 10, 16	15, 9, 4, 6	8, 2, 11, 13
12	7, 12, 1, 14	3, 10, 5, 16	15, 4, 9, 6	8, 11, 2, 13
13	1, 8, 10, 15	5, 4, 11, 14	9, 2, 7, 16	3, 6, 12, 13
14	8, 1, 10, 15	4, 5, 11, 14	2, 9, 7, 16	6, 3, 12, 13
15	8, 10, 1, 15	4, 11, 5, 14	2, 7, 9, 16	6, 12, 3, 13

The 16-player tournament seating chart devised by Girls' Angle members.

You can check that every single one of the possible partner pairings among 16 players appears exactly once in this seating arrangement. There are many patterns in this chart. How many can you find? It turns out that the girls inadvertently discovered something amazing … do you see what it is? If not, read on.

[1] Such a tournament is known as a 16 player Whist tournament in combinatorial design theory.

Euchre Tournament Seating Chart for 16 Players

Round	Table 1				Table 2				Table 3				Table 4			
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	1	3	4	6	5	7	8	10	9	11	12	14	13	15	16
3	2	3	1	4	6	7	5	8	10	11	9	12	14	15	13	16
4	1	5	9	13	4	8	12	16	3	7	11	15	2	6	10	14
5	5	1	9	13	8	4	12	16	7	3	11	15	6	2	10	14
6	5	9	1	13	8	12	4	16	7	11	3	15	6	10	2	14
7	1	6	11	16	5	12	15	2	9	8	3	14	13	4	7	10
8	6	1	11	16	12	5	15	2	8	9	3	14	4	13	7	10
9	6	11	1	16	12	15	5	2	8	3	9	14	4	7	13	10
10	1	7	12	14	5	3	10	16	9	15	4	6	2	8	11	13
11	7	1	12	14	3	5	10	16	15	9	4	6	8	2	11	13
12	7	12	1	14	3	10	5	16	15	4	9	6	8	11	2	13
13	1	8	10	15	5	4	11	14	9	2	7	16	3	6	12	13
14	8	1	10	15	4	5	11	14	2	9	7	16	6	3	12	13
15	8	10	1	15	4	11	5	14	2	7	9	16	6	12	3	13

The table above replicates the table the girls produced with some added color-coding to make it easier to see who is partnering with whom. Notice that the rounds are grouped into five sets of three rounds each where, within each set, players remain at the same table. The table below consolidates the information above by listing only the 20 distinct sets of four players that sit at a table during the tournament.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	5	9	13	4	8	12	16	3	7	11	15	2	6	10	14
1	6	11	16	5	12	15	2	9	8	3	14	13	4	7	10
1	7	12	14	5	3	10	16	9	15	4	6	2	8	11	13
1	8	10	15	5	4	11	14	9	2	7	16	3	6	12	13

Let's call each of these 20 sets of four players a "table." Notice that if you pick any two players, there is exactly one table that includes both of them. For example, the only table with both players 3 and 11 is the one comprised of players 3, 7, 11, and 15. Furthermore, if you pick two different tables, they intersect in at most one player. In fact, each row in the chart consists of four tables no pair of which intersects. Do these properties ring a bell?

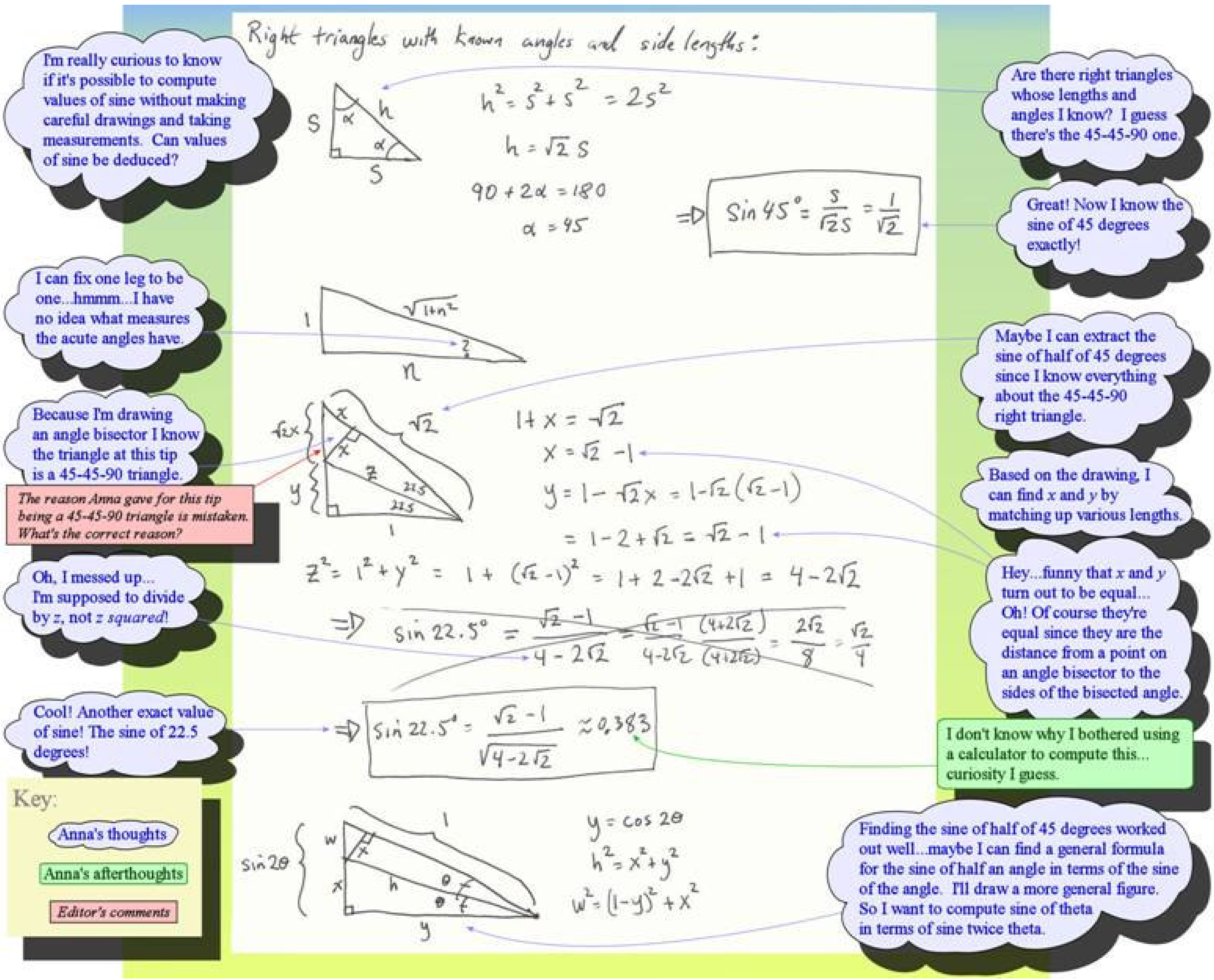
If they remind you of the properties of points and lines in a plane, it's no fluke: the chart above describes an example of a **finite geometry**. The players represent points and the "tables" are the lines. All 16 points (players) constitute a plane and the 20 lines behave just as geometric intuition would dictate for the behavior of lines in a plane: two points define a unique line and two lines either intersect or are parallel. Each row in the table partitions the plane into four parallel lines. Notice that every point is contained in exactly five lines and every line consists of four points. You can even verify Euclid's famous fifth postulate, which states that through a point outside a line, there is a unique parallel line through that point. Mathematicians will recognize this geometry as the "affine plane over the finite field with four elements."

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Intrigued by trigonometry, Anna tries to compute exact values of the sine function.



The sine of theta is *x* over *h*.

$$\sin\theta = \frac{x}{h}$$

$$x + w = \sin 2\theta$$

$$w = (\sin 2\theta) - x$$

$$((\sin 2\theta) - x)^2 = (1-y)^2 + x^2$$

Oh dear, these computations are getting tedious!

$$\sin^2 2\theta - 2(\sin 2\theta)x = (1-y)^2$$

$$x = \frac{\sin^2 2\theta - (1-y)^2}{2\sin 2\theta}$$

Oh wait! *x* over *w* is just equal to cos twice theta which is also equal to *y*!

$$\frac{x}{w} = \cos 2\theta = y$$

$$x = yw \Rightarrow w = \frac{x}{y} = \tan\theta$$

This expression for *h* seems simpler...I'll use this instead.

$$h^2 = y^2w^2 + y^2 = y^2(w^2+1)$$

I can get *h* by using the Pythagorean theorem.

$$h^2 = x^2 + y^2 = \left(\frac{\sin^2 2\theta - (1-y)^2}{2\sin 2\theta}\right)^2 + y^2$$

$$\sin\theta = \frac{x}{h} = \frac{yw}{\sqrt{y^2(w^2+1)}} = \frac{w}{\sqrt{w^2+1}} = \frac{\frac{\sin\theta}{\cos\theta}}{\sqrt{\frac{\sin^2\theta}{\cos^2\theta}+1}} = \sin\theta$$

great...I went around in a circle! This doesn't help!

OK. I'll start again. I'll try to make sure I'm making progress and not going backward!

$$\frac{1-y}{w} = \sin 2\theta = x + w$$

$$1 - y = xw + w^2$$

$$w^2 = (1-y)^2 + x^2 = w^2(x+w)^2 + x^2$$

$$w^2 - x^2 = (1-y)^2$$

$$(w+x)(w-x) = (1-y)^2$$

I can subtract this equation from this one to solve for *x*.

$$w - x = \frac{(1-\cos 2\theta)^2}{\sin 2\theta}$$

$$x + w = \sin 2\theta$$

$$2x = \sin 2\theta - \frac{(1-\cos 2\theta)^2}{\sin 2\theta} = \frac{\sin^2 2\theta - (1-\cos 2\theta)^2}{\sin 2\theta}$$

$$= \frac{1-\cos^2 2\theta - (1-\cos 2\theta)^2}{\sin 2\theta} = \frac{2\cos 2\theta - 2\cos^2 2\theta}{\sin 2\theta}$$

$$x = \frac{\cos 2\theta - \cos^2 2\theta}{\sin 2\theta}$$

These are my computations. They're a bit of a mess, but in the end, things simplified nicely. Don't use this as a model for your own work. I was just trying whatever I could think of to find *x* and *h*. I also tried to simplify expressions.

$$h^2 = x^2 + y^2 = \left(\frac{\cos 2\theta - \cos^2 2\theta}{\sin 2\theta}\right)^2 + \cos^2 2\theta = \cos^2 2\theta\left(\left(\frac{1-\cos 2\theta}{\sin 2\theta}\right)^2 + 1\right)$$

This simplifies to one, by the Pythagorean theorem.

$$= \cos^2 2\theta\left(\frac{1 - 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}{\sin^2 2\theta}\right)$$

$$= \frac{\cos^2 2\theta}{\sin^2 2\theta}(2 - 2\cos 2\theta)$$

Neat...sine theta boils down to this relatively simple expression involving cosine twice theta. Since cosine and sine are related by the Pythagorean theorem, this effectively gets what I wanted!

$$\sin\theta = \frac{x}{h} = \frac{\cos 2\theta(1-\cos 2\theta)}{\sin 2\theta} \cdot \sqrt{\frac{\sin^2 2\theta}{\cos^2 2\theta}\,\frac{1}{2(1-\cos 2\theta)}}$$

$$= \frac{\sin 2\theta}{2\cos 2\theta} = \sqrt{\frac{1-\cos 2\theta}{2}}$$

This means that I can get the exact value of the sine of any angle that measures 45 divided by a power of 2 degrees!

ABB 4.23.11

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

French Toast, Auctions, and Game Theory

By Katherine Sanden

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

How much would you pay for a slice of French toast with one bite taken out of it? $0? $1? $10? What if that bite was taken by Justin Timberlake?

In March 2000, the popular boy-band *NSYNC did a breakfast interview for Z-100, a New York-based radio station. Apparently Justin Timberlake, one of *NSYNC's lead singers, wasn't very hungry. Supposedly, when the radio station's DJ noticed that Timberlake only took one bite out of his French toast, he decided to auction it off on eBay. It sold for over $1000.

Wow. How did this happen? Do you think Z-100 could have simply listed the French toast as a fixed-price item in their online store? "FOR SALE: Justin Timberlake's leftover French toast. Price: $1000. Click here to purchase." How would they choose the price tag of $1000? No one had ever sold Justin Timberlake's leftovers before. They had nothing to compare it to. And if more than one person wanted to purchase it, there were no extra copies to sell. So, they auctioned it and let people compete to buy it.

By posting the item on eBay, Z-100 chose to sell their item using what is known as an **English auction**, or an "open ascending price auction." We call it "open" because each participant can see what the other participants have bid, and "ascending price" because each new bid has to be higher than all previous bids. Whoever places the highest bid, wins the item.

The English auction is arguably the most common type of auction today. It can be done as a silent auction, where bidders submit their bids in writing (as in a typical eBay auction), or a live auction, where an auctioneer calls out ascending prices, until all bidders except for one drop out. (Search on YouTube for "live auction" to get an idea of what this looks like.)

Besides the English auction, there are three other primary types of auctions:

- **Dutch auction:** Also known as an "open descending price auction," it is similar to an English auction except that the auctioneer begins with a high price and lowers the price bit by bit until someone is willing to pay.

- **Sealed first-price auction:** In this type, participants submit sealed bids all at once, so that no one can see anyone else's bid. The highest bidder wins the item, and pays the price she bid.

- **Vickrey auction:** Named after economist William Vickrey, this is similar to a sealed first-price auction. The highest bidder still wins the item, but instead of paying the price she bid, she pays the price the *second* highest bidder bid.

Each of these types seems quite different at first glance. But Vickrey, who used mathematics to build the foundation of modern auction theory, proved that in a world where all bidders are rational and uninfluenced by the value judgments of fellow bidders, these four types are actually *equivalent*. What does it mean for different auction types to be equivalent? It means that the seller can expect to get the same price no matter which auction method is used. Let's illustrate this with a simplified example.

Say you have a rare vintage toy that you'd like to sell. And say there are four people, P, Q, R, and S, interested in buying it. Person P values it at $50, meaning she is willing to pay $50 for the toy. Person Q values it at $40, R values it at $30, and S values it at $20. First let's observe that the English auction and the Vickrey auction will produce the same result:

- *Vickrey auction*: P, Q, R, and S each submit a sealed bid of $50, $40, $30, and $20 respectively. P wins and pays $40. Notice that in a Vickrey auction, it is in each bidder's interest to be honest. P knows that if she wins, she will only pay what the second highest bidder submitted, whether she writes $50 or $45 or $1000. So she has no motivation to place a bid lower than her actual $50. In fact, if she *does* place a bid lower than her $50, say $45, then she is taking a risk. There could be another bidder out there who values the toy at $47, and submits $47 as a bid. If this happens, the other bidder gets the toy for $45, and P would have been happy to get the toy for $47, so P misses out.

- *English auction*: P, Q, R, and S place bids until the price rises above $20, and S drops out. The bidding continues. When the price reaches $30, R drops out. Now P and Q are bidding back and forth. Q bids $40. P is willing to pay more. P can bid just over $40 – say $40.01 – and win the item now, since Q is not willing to pay more than $40. Although $40.01 is not quite equal to $40, the extra penny is a marginal cost and depends on the minimum unit of currency available.

Let's also observe that the Dutch auction and the sealed first-price auction produce the same sale price:

- *Dutch auction*: The auctioneer starts at a tremendously high price that is sure to be above the maximum value anyone would place on the toy. For example, the auctioneer could start at a price equal to the sum of all wealth in the world plus one. The idea here is that we can assume the auctioneer starts at a price that is above what *anyone* is even able to pay. Then the auctioneer recites lower prices, until the price "$50!" comes up, P bids, and wins the toy.

- *Sealed first-price auction*: P, Q, R, and S each submit a sealed bid of $50, $40, $30, and $20, respectively. P wins and pays $50.

These examples illustrate the general fact that Dutch auctions are equivalent to sealed first-price auctions and English auctions are equivalent to Vickrey auctions. But how could a Dutch auction be equivalent to an English auction or a Vickrey auction? Didn't the item sell for $50 in the Dutch auction, but only $40.01 in the English auction?

Here's the catch: Vickrey saw that in English auctions and Vickrey auctions, it pays to be honest. But in Dutch auctions and sealed first-price auctions, it does *not* pay to be honest. P knows that if she bids $45, and the next bid is only $44 dollars, she pays only $45. If she bids her honest $50, she still wins, but has to pay more. The best strategy is for each bidder to "shade" her bid; in other words, place a bid lower than what she is actually willing to pay.

How much should each bidder shade? This depends on the particular circumstances of the auction, such as the number of other bidders. Vickrey found a way to predict the amount of shading that would occur in any given auction, and proved that, due to this shading, the average revenue from a Dutch or sealed first-price auction would in fact be equal to the revenue from an English or Vickrey auction.

For further reading about this equivalence (and more), check out Vijay Krishna's book, *Auction Theory*.

Back to Justin Timberlake's French Toast

According to Vickrey's theory, we would expect the highest bidder in an eBay auction to pay an amount less than or equal to the amount at which she values the item. Does that mean the winning bidder on Justin Timberlake's French toast thought at the beginning, "Ah, I value this French toast at $1000 and I'm happy to pay that price…"? Probably not.

There are certain factors in real-world English auctions that Vickrey didn't take into account, and that often cause the highest bidder to pay too much – in other words, to pay more than the item is actually worth or to gain less than she had anticipated because the actual value of the item is less than she had thought.

For instance, Vickrey assumed that the bidders assessment of an item's value was unaffected by what other bidders thought. But in real life, a bidder might not be sure how much she's willing to pay and when she sees other people bidding higher, her perception of the item's value might change. The highest bidder on the French toast might have seen the bids go up to $400, $500, … and thought, "Wow, this item must have a very high value, because other bidders think it is worth at least $500."

Additionally, the excitement and adrenaline of an English auction – especially a live one – may cause a bidder to act without thinking. The auctioneer talks very quickly and other bidders are raising their hands – it is common for a bidder to act on an impulse, rather than a rational assessment of the item's value.

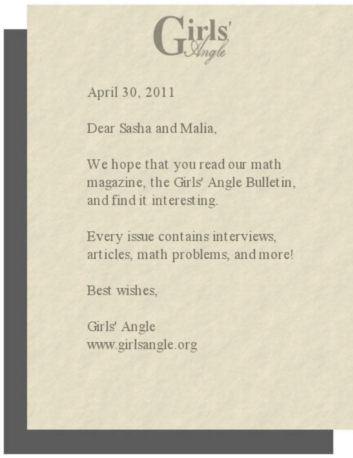
Paying too much as the winning bidder is such a common phenomenon, it has a name: the "winner's curse." If you entered an auction, what strategy would you devise to avoid falling victim to the winner's curse?

Mathematical Buffet

Rectangles by Ken Fan

The "golden" rectangle.

Rectangles! The bricks of Flatland…perhaps few care more about their rectangles than makers of stationery and painters. All rectangles shown here are drawn to scale.



Girls' Angle

April 30, 2011

Dear Sasha and Malia,

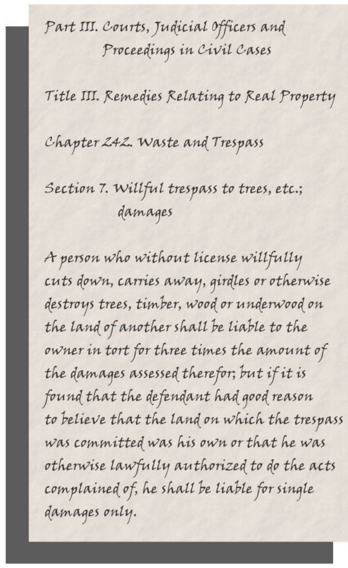
We hope that you read our math magazine, the Girls' Angle Bulletin, and find it interesting.

Every issue contains interviews, articles, math problems, and more!

Best wishes,

Girls' Angle
www.girlsangle.org

The 8.5" by 11" standard letter size. You probably have hundreds of such sheets in your home right now!



Part III. Courts, Judicial Officers and Proceedings in Civil Cases

Title III. Remedies Relating to Real Property

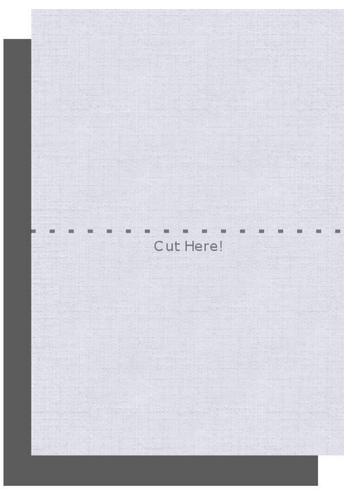
Chapter 242. Waste and Trespass

Section 7. Willful trespass to trees, etc.; damages

A person who without license willfully cuts down, carries away, girdles or otherwise destroys trees, timber, wood or underwood on the land of another shall be liable to the owner in tort for three times the amount of the damages assessed therefor; but if it is found that the defendant had good reason to believe that the land on which the trespass was committed was his own or that he was otherwise lawfully authorized to do the acts complained of, he shall be liable for single damages only.

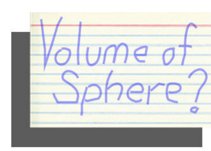
Who knows if the 8.5" by 14" legal size was born to meet lawyer's propensity for lengthy, rambling, legalese? (Don't make paper from your neighbor's trees!)

Tall rectangles are good for full length portraiture. John Singer Sargent painted this on a 43.5" by 89" canvas.

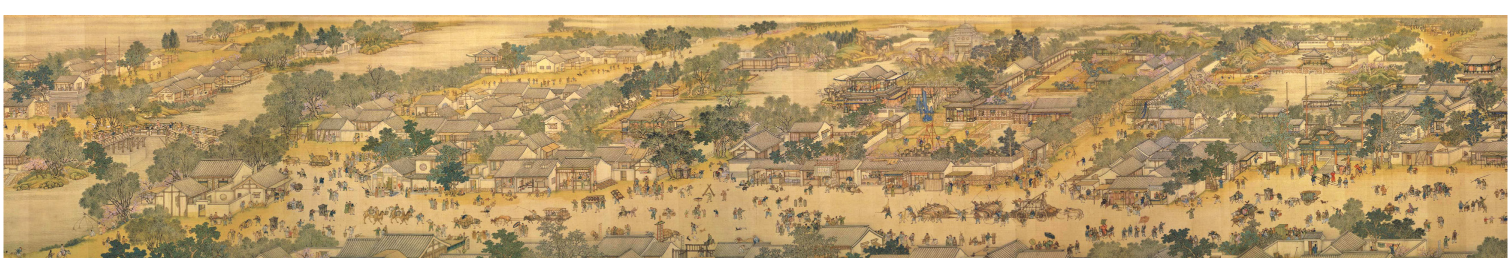


The $\sqrt{2}$:1 aspect ratio is the most common used for letters worldwide. Cut in half and get two sheets with the same proportions as the original.

Origami favors the 1:1 aspect ratio, also known as the square.



Then there's the handy 3" by 5" index card. Great for studying!



Landscape paintings often call for wide rectangles. This is but a tiny section from a copy of a 12th century painting by Zhang Zeduan. The original painting, *Along the River During the Qingming Festival*, is 17' 4" long and 9.75" high!

Errorbusters!

by Cammie Smith Barnes / edited by Jennifer Silva

Simplifying expressions is a fundamental skill that must be mastered prior to the study of calculus. Starting with a complicated expression, the goal is to reduce it to its most basic form by finding terms or factors that can be canceled. Frequently, however, students are tempted to cancel more aggressively than is acceptable. I call this error “mismatched cancellation” because the terms or factors canceled are not matched properly in the roles they play. To be canceled from a single expression, two factors or terms must play the exact opposite roles of each other.

Here is an example of this type of mistake:

$$\frac{x+\cancel{3}}{\cancel{3}x}=\frac{x}{x}$$

is incorrect. Likewise,

$$\frac{\cancel{x}+3}{3\cancel{x}}=\frac{3}{3}$$

is wrong. In the numerator of the expression on the left 3 is *added* to x, whereas in the denominator 3 is *multiplied* by x; this means that we are *dividing* the entire numerator of the expression by 3 (or, more accurately, by $3x$). Addition and division are **not** opposites of each other. Rather, addition is the opposite of subtraction, while multiplication is the opposite of division. So the two 3s are playing roles that don’t match up. The same is true for the x’s. Therefore, neither the 3s nor the x’s can be canceled in this situation.

To see why addition must be canceled by subtraction whereas multiplication must be canceled by division, we need to examine what cancellation really means and how it works. First, however, we must consider two very special numbers: 0 and 1. Zero is called the *additive identity* because when 0 is added to any number, our original number remains unchanged. In other words, for every number n,

$$n + 0 = n.$$

On the other hand, 1 is called the *multiplicative identity* because multiplying a number by 1 also doesn’t change the original number. That is, for every number n,

$$n \cdot 1 = n.$$

Cancellation within an expression can occur only when there are terms that add to 0 or factors that multiply to 1.

Consider an example of correct additive cancellation:

$$x+\cancel{3}-\cancel{3}=x\,.$$

The cancellation notation above is really shorthand for saying the following:

$$x + 3 - 3 = x + (3 - 3) = x + (3 + (-3)) = x + 0 = x.$$

Subtraction of a number is actually addition of its negative. The numbers 3 and -3 are called *additive inverses* of each other because they add up to the additive identity, 0. We can therefore think of them as being exact opposites of each other. Exact opposites cancel.

Here is an example of correct multiplicative cancellation:

$$\frac{\cancel{3}x}{\cancel{3}y} = \frac{x}{y}.$$

This is an abbreviated way of saying

$$\frac{3x}{3y} = \frac{3}{3}\cdot\frac{x}{y} = 3\cdot\frac{1}{3}\cdot\frac{x}{y} = 3\cdot 3^{-1}\cdot\frac{x}{y} = 1\cdot\frac{x}{y} = \frac{x}{y}.$$

So we can see that cancellation notation is very handy indeed; it saves us from writing down all of these steps! Recall that $3^{-1} = 1/3$, a fact discussed in the previous issue. We say that 3 and 3^{-1} are *multiplicative inverses* of each other because they multiply to make the multiplicative identity, 1. The numbers 3 and 3^{-1} are thus exact opposites of each other so they cancel when multiplied together.

Be careful to keep additive and multiplicative inverses straight, however. Neither of the following attempted cancellations work.

$$x + \cancel{3} + \cancel{3^{-1}} = x$$

is wrong because, when we want to cancel terms that are being added together, they must be *additive* inverses of each other, not multiplicative inverses or anything else. We probably wouldn't be tempted to make the analogous mistake

$$x \cdot \cancel{3} \cdot (\cancel{-3}) = x,$$

but that's exactly what we would be doing if we reversed the roles of addition and multiplication in the previous incorrect equation. Meanwhile

$$\frac{x + \cancel{3}}{x - \cancel{3}} = \frac{x}{x}$$

doesn't work either, as we can't cancel additive inverses across division, just as we wouldn't cancel multiplicative inverses across subtraction. That is,

$$x + \cancel{3} - \cancel{3^{-1}} = x$$

is incorrect. For some reason, I've never come across an example of the similar mistake

$$\frac{x}{\cancel{3}} - \frac{\cancel{3}}{y} = \frac{x}{y},$$

but this is wrong for basically the same reasons as the example above it.

Tempted as we may be to do the following, cancellations such as

$$\frac{x + \cancel{3}}{y + \cancel{3}} = \frac{x}{y}$$

are not proper either, because there is no way to factor out the 3s. In some sense, the 3s are glued by the plus signs to the *x* and to the *y*, respectively. There are two implicit pairs of parentheses in the expression on the left, which we really read as

$$\frac{x+3}{y+3} = \frac{(x+3)}{(y+3)}.$$

Because the order of operations says that we must perform all operations within parentheses first, there is no way to break apart the parentheses and factor out the 3s. Similarly, we wouldn't cancel them out in the situation

$$\frac{5+\cancel{3}}{1+\cancel{3}}=\frac{5}{1}$$

because we know that the correct simplification of this expression is

$$\frac{5+3}{1+3}=\frac{8}{4}=2$$

(note that both additions are done before the division). In fact, a good way to tell that we can't cancel in certain situations is to plug in actual numbers for any variables; then we can readily see why we wouldn't cancel within the new expressions. So indeed

$$\frac{x+\cancel{3}}{\cancel{3}x}=\frac{x}{3}$$

is wrong because we can see that, if we substitute in a 2 for the *x*,

$$\frac{2+\cancel{3}}{\cancel{3}(2)}=\frac{2}{2}$$

does not hold true. Rather,

$$\frac{2+3}{3(2)}=\frac{5}{6}\neq\frac{2}{2}.$$

We've talked about canceling within an expression, but we can also perform cancellation on equations. To be canceled across an equation, two factors or terms must play the exact *same* roles on both sides of the equation. For instance, in the equation $x + 3 = y + 3$,

$$x+\cancel{3}=y+\cancel{3}$$

works; so does

$$\cancel{3}(x+1)=\cancel{3}y$$

in the equation $3(x + 1) = 3y$. But, in the equation $x + 3 = 3y$, we wouldn't want to cancel like this:

$$x+\cancel{3}=\cancel{3}y,$$

because the two 3s are definitely not playing the same roles.

For practice, try simplifying the expressions in the following exercises. Determine whether each cancellation is valid or not. The answers can be found on page 27.

1. $\frac{x+\cancel{5}}{y+\cancel{5}}$

2. $\frac{\cancel{x}(x+5)}{5\cancel{x}}$

3. $x+\cancel{5}+5y-\cancel{5}$

4. $\frac{\cancel{5}x}{x+\cancel{5}}$

5. $\frac{5\cancel{x}}{x^2-5\cancel{x}}$

6. $\frac{5\cancel{x}}{\cancel{x}(x+5)}$

7. $-\cancel{x^2}+5x+1+\cancel{x^2}$

COACH BARB'S CORNER

by Barbara Remmers

OWNING IT

A wonderful aspect of math is that deep understanding can replace memorization. Instead of trying to remember multiple disconnected facts, the knowledge becomes part of you.

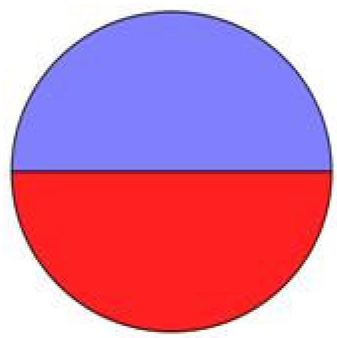
Today we consider the circle. If the formulas for its circumference and area seem like unrelated bizarre facts, read on.

First let's think about circles, not as a shape that you've known forever, but as if you had discovered them. How might that have happened? Perhaps you noticed the shape left by your goat after it grazed in a grassy field while tethered to a stake.

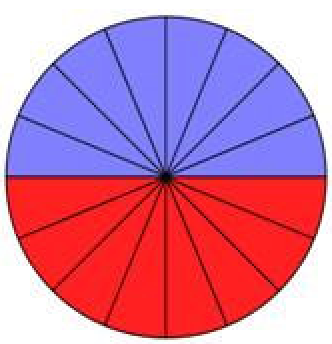
After deciding to name the new shape 'circle', you might start thinking about how to make more circles and different circles. Bigger circles and smaller circles. You'd consider what determined the size of the circle, and conclude that it was the length of the tether. So you'd give that a name, 'radius,' or 'r' for short.

Now about π. Should you accept it as a mysterious number that goes on forever and is about equal to 3.14? No. It is a property of a circle. Imagine that after discovering your round shape, you started studying it. You kept measuring all the way around the circle and comparing it to the distance across it, straight through the center. Again and again you found the ratio of the circumference to the diameter was π. So you remembered π. It was very useful. You could use it to compare the cost of using a tether for your goat to the cost of building a fence to contain the animal.

At this point the circle circumference formula, $2\pi r$, makes a certain amount of sense, but what about the area formula, πr^2? How does the π we know as a ratio of the circle's circumference to its diameter connect to the area? Consider the following.

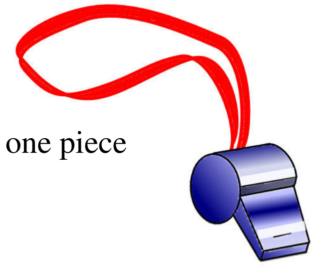


Forget the goat. Imagine a pizza, a circular pizza. A pizza that is half red and half blue. Its radius is r. So its circumference is $2\pi r$. You can see it at right.

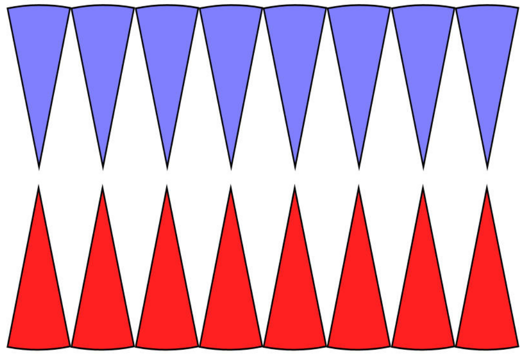


Now imagine taking a pizza cutter and cutting it straight through the middle so that you have two half-circle pizzas, one red and one blue. The circumference of each half-circle pizza has a straight part and a rounded part. The straight part is $2r$. The rounded part is half the uncut pizza's circumference, so its πr. Fine.

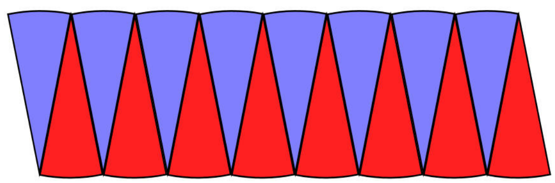
Now, let's cut the pizza. Cut it seven more times, in the usual way, so that all the pizza slices are the same size (at left).



Now, take the red half-pizza and bend the curved crust into a straight line. Keep it in one piece though! Do the same for the blue side, and here's what your pizza looks like.

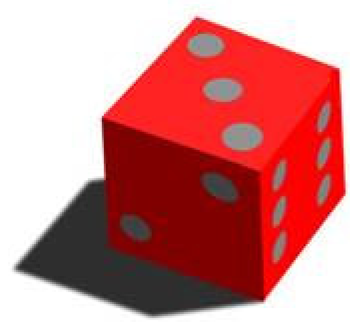


Now, move your red and blue half-pizzas together so that the zig-zag parts fit together. Expect to get a little messy. Step back and look at it.



Now it's not exactly a rectangle (In math, anything that is not exactly true is completely false so we can just say it's not a rectangle) but it reminds me of a rectangle. The rectangle it reminds me of has a short side of r and a long side of πr. So its area is ... πr^2!

MATH SHOCKER



Do you know that the number of dots on opposite sides of a die add up to seven? You can use this fact to convince some people you have super powers.

Take a die. Casually mention that you have X-ray vision.

When doubt is expressed, calmly insist that you can see the face-down side of a die. Roll the die a few times. By subtracting the face-up number from seven, you can "see" the face-down number.

You can also "see" the sum of the hidden numbers on the tops and bottoms of a stack of dice. You know that every die's top and bottom numbers add up to seven, so that all the face-up and face-down sides of the stack add up to seven times the number of dice in the stack. So you can subtract off the number on the top face of the top die from the multiple of seven you computed. Then you know the sum of all the hidden numbers that you "see."

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 8 – Meet 5 – March 3, 2011

Mentors: Jennifer Balakrishnan, Keren Gu, Samantha Hagerman,
Ryan Heffrin, Jennifer Melot, Liz Simon

Among the topics worked on this meet: coordinate geometry, Pick's theorem, intersections of diagonals in polygons, and a nice round of the Define This Game (see page 8).

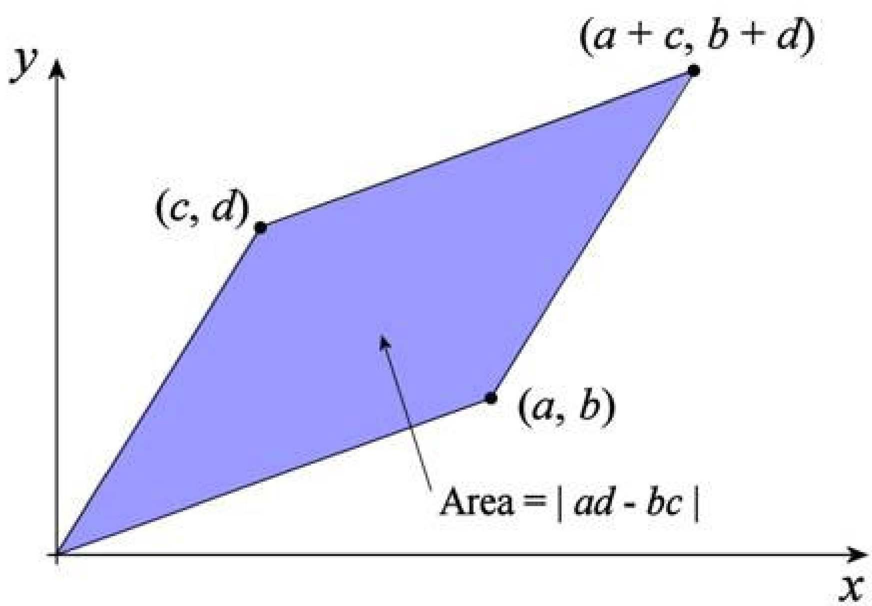
Session 8 – Meet 6 – March 10, 2011

Mentors: Jennifer Balakrishnan, Eli Grigsby, Keren Gu, Ryan Heffrin,
Natasha Jensen, Kate Rudolph, Rediet Tesfaye

Special Guests: Meera Krishnan and Eva Nazarewicz, Harvard Business School

A very nice formula deduced at this meet is the area of a parallelogram in the coordinate plane whose vertices are (0, 0), (a, b), (c, d), and ($a + c$, $b + d$). It turns out that the area is given by $|ad - bc|$. The expression inside the absolute value, $ad - bc$, is a beautiful formula with myriad applications. It is sometimes referred to as the **signed area** of the parallelogram. It is also the **determinant** of the matrix:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$



Can you figure out how to tell from the picture of the parallelogram whether the signed area is going to be positive, negative, or zero?

Eva and Meera talked about the mathematics involved in picking the price point for selling an item. They explained the factors that affect production, showed how to compute the cost of production, and showed a way to estimate profit as a function of the selling price. Next, they split the girls into two groups. Each group worked on pricing a picture frame with the aim of maximizing profit.

Session 8 – Meet 7 – March 17, 2011

Mentors: Jennifer Balakrishnan, Samantha Hagerman, Jennifer Melot,
Lucia Mocz, Kate Rudolph, Liz Simon

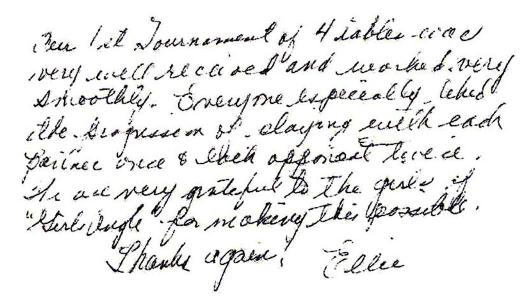
Members who have been designing a dollhouse have had to make many computations involving fractions. See page 6 for more on fractions.

Session 8 – Meet 8 – March 31, 2011

Mentors: Jennifer Balakrishnan, Keren Gu,
Samantha Hagerman, Ariana Mann,
Lucia Mocz, Liz Simon, Rediet Tesfaye,
Bianca Viray

Special Guest: Susan Barry, Mt. Holyoke College

Girls' Angle received an interesting Community Outreach problem from Ellie Hawthorne of Rochester, New York. With Samantha mentoring, the girls, especially **Canis Lupus**, solved it! See page 10 for details.


Our 1st Tournament of 4 tables was very well received and worked very smoothly. Everyone especially liked the progression of playing with each partner once & each opponent twice. We are very grateful to the girls of "Girls Angle" for making this possible. Thanks again! Ellie

A message from Ellie to the girls: "Our 1st Tournament of 4 tables was very well received and worked very smoothly. Everyone especially liked the progression of playing with each partner once & each opponent twice. We are very grateful to the girls of 'Girls' Angle' for making this possible. Thanks again, Ellie."

Susan gave a presentation on how we see in 3D. She was born incapable of seeing the world in 3D. Her visual system was unable to interpret the two views of an object that come from the two eyes as two views of the same object. When young, she endured 3 surgeries to lessen her cross-eyed appearance, but these procedures did not enable her to see in 3D. In fact, she was informed that she would never perceive a 3D world. Yet, through a combination of insight, hard work, and special training, she was able to relearn how to see. When she turned 48, for the first time, she opened her eyes to a 3D vision of the world!

Interesting facts about binocular vision

About 1 in 20 people are born without the ability to see in 3D.

A curious application that exploits our binocular vision is to the detection of counterfeit money.

Many animals have binocular vision, including birds of prey and praying manti.

To give us a sense of what it is like to see only in 2D, she had the girls thread a straw with a narrow stick and catch tossed balls, all with one eye closed. To develop the understanding of how binocular vision works, she gave us 3D red/blue glasses and showed us anaglyphs. Then she had us hold a tube over one eye and place an open hand adjacent to the tube at the far end. With both eyes open, it would appear as if there were a "hole in the hand." She had them look at a bead on a string (called a "Brock string") to see "two strings" crossing at the bead.

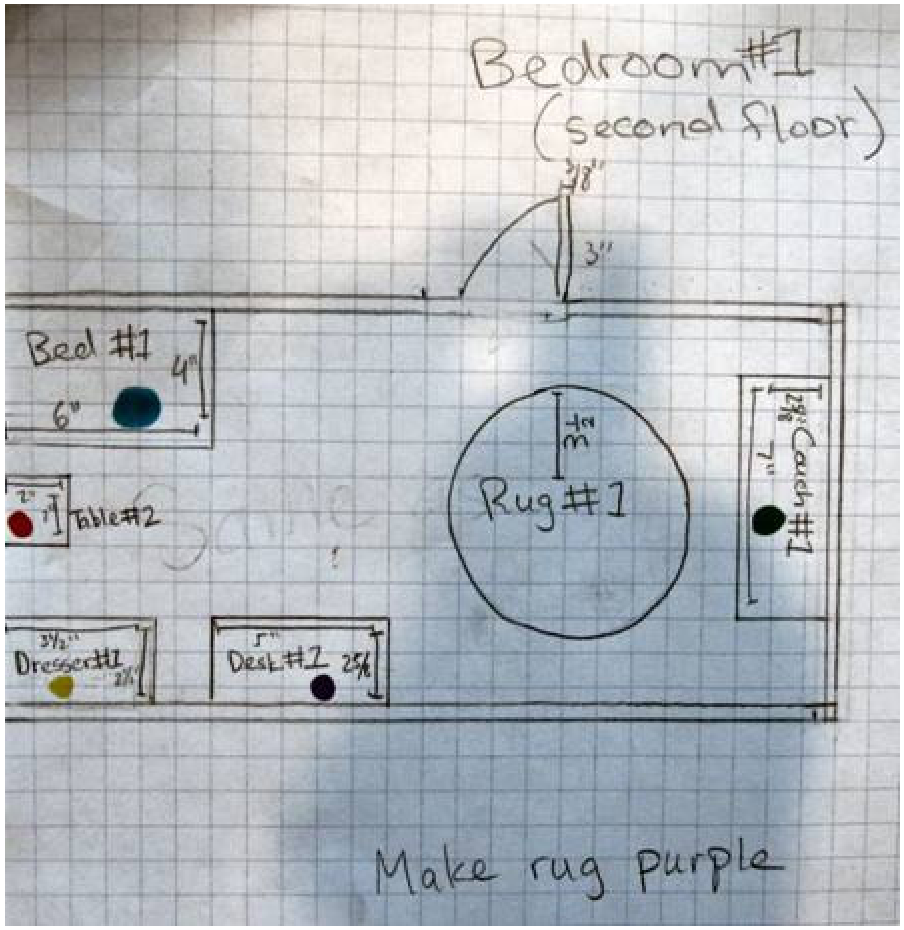
When Susan gained the ability to see in 3D, suddenly the world expanded. For the first time, she fully grasped the meaning of Lewis Carroll's famous book title, "Through the Looking Glass." She also was able to follow, for the first time, fast moving balls in baseball and tennis. You can read more about her fascinating story in her recent book, *Fixing My Gaze: A Scientist's Journey Into Seeing in Three Dimensions*.

<u>Session 8 – Meet 9 – April 7, 2011</u>

Mentors: Jennifer Balakrishnan, Jennifer Melot, Liz Simon

Special Guest: Jane Kostick, woodworker

Jane visited to check up on the dollhouse design blueprints.



A room from the dollhouse blueprints.

Some girls looked for integer solutions to the equation $x^2 - 2y^2 = 1$. Integral solutions to this equation are strongly connected to the content of David Speyer's articles in Volume 4, Numbers 2 and 3 of this Bulletin. Here's why: Suppose you have a solution to the equation $x^2 - 2y^2 = 1$ with both x and y positive. If we rearrange terms, this is equivalent to

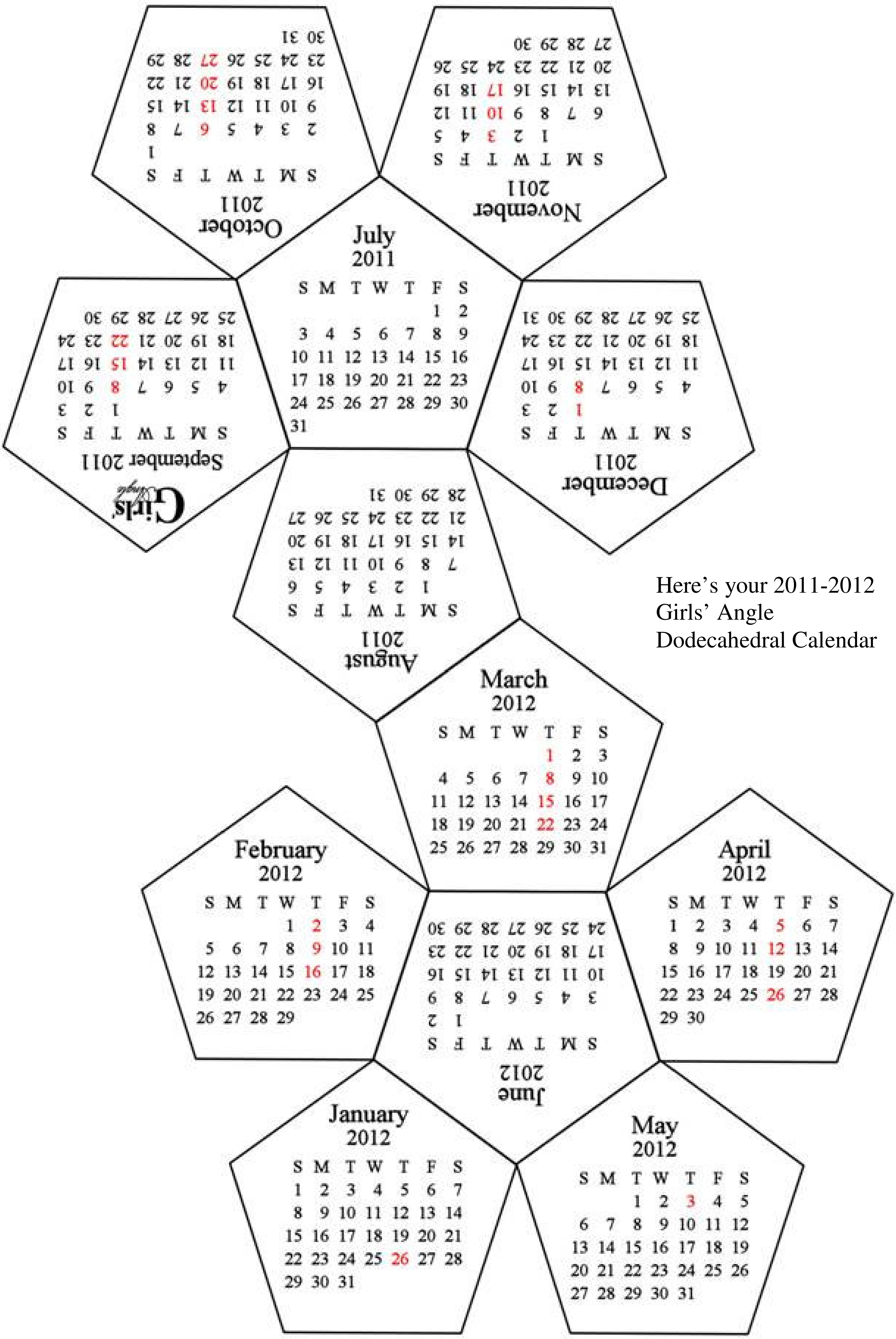
$$\frac{x}{y} = \sqrt{2 + \frac{1}{y^2}}\,.$$

When y is a large number, the right hand side of this equation is very close to the square root of 2. In other words, a solution to $x^2 - 2y^2 = 1$ in positive integers x and y produces a good rational approximation, x/y, to the square root of 2, and the larger y is, the better this approximation will be. Recall that in David's articles, he sought good rational approximations to the number π. Even though here we get good rational approximations to the square root of 2 instead of π, it is reasonable to expect to find content in his articles applicable to these rational approximations as well.

<u>Session 8 – Meet 10 – April 14, 2011</u>

Mentors: Samantha Hagerman, Jennifer Melot, Liz Simon, Rediet Tesfaye, Bianca Viray

Several members made geometric origami models, both single sheet and modular. Everyone was working on folding a Fujimoto Hydrangea at the end.



Here's your 2011-2012 Girls' Angle Dodecahedral Calendar

Calendar

Session 8: (all dates in 2011)

January	27	Start of eighth session!
February	3	
	10	
	17	Felice Frankel, photographer and scientist
	24	No meet
March	3	
	10	
	17	
	24	No meet
	31	Susan Barry, Mount Holyoke
April	7	Jane Kostick, woodworker
	14	
	21	No meet
	28	Jennifer Che, tiny urban kitchen
May	5	

Session 9: (all dates in 2011)

September	8	Start of the ninth session!
	15	
	22	
	29	Start of Rosh Hashanah – No meet
October	6	
	13	
	20	
	27	
November	3	
	10	
	17	
	24	Thanksgiving - No meet
December	1	
	8	

Here are answers to the *Errorbusters!* problems on page 20.

1. no
2. yes
3. yes
4. no
5. no
6. yes
7. yes

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) comes with membership. We are working hard to lower the cost of the Bulletin. Until we do, however, nonmembers can receive the printed version by becoming a Bulletin Sponsor. Please contact us if interested.

The Bulletin targets girls roughly the age of current members. Each issue is likely to contain some material that feels very challenging or difficult to understand. If you are a member or Bulletin Sponsor and have any questions about the material, feel free to ask us about it!

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

How do I join? Membership is granted per session. Members have access to the club and receive a printed copy of the Girls' Angle Bulletin for the duration of the membership. You can also pay per meet, but it is slightly more expensive. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply. If you cannot attend the club, you can purchase a **Remote Membership** which comes with a year-long subscription to the Bulletin and a 25% discount for any club meet attended. Remote members may email us math questions (although we won't do people's homework!).

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, Tamarkin assistant professor, Brown University
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For (please circle): Membership Remote Membership

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

For **membership**, please fill out the information in this box. **Bulletin Sponsors** may skip this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a one session Membership
  - ☐ $108 for a one year Remote Membership
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ______________________________________Date: ______________

Print name of applicant/parent: ______________________________________

Print name(s) of child(ren) in program: ________________________________

A Math Club for Girls