Girls' Angle Bulletin

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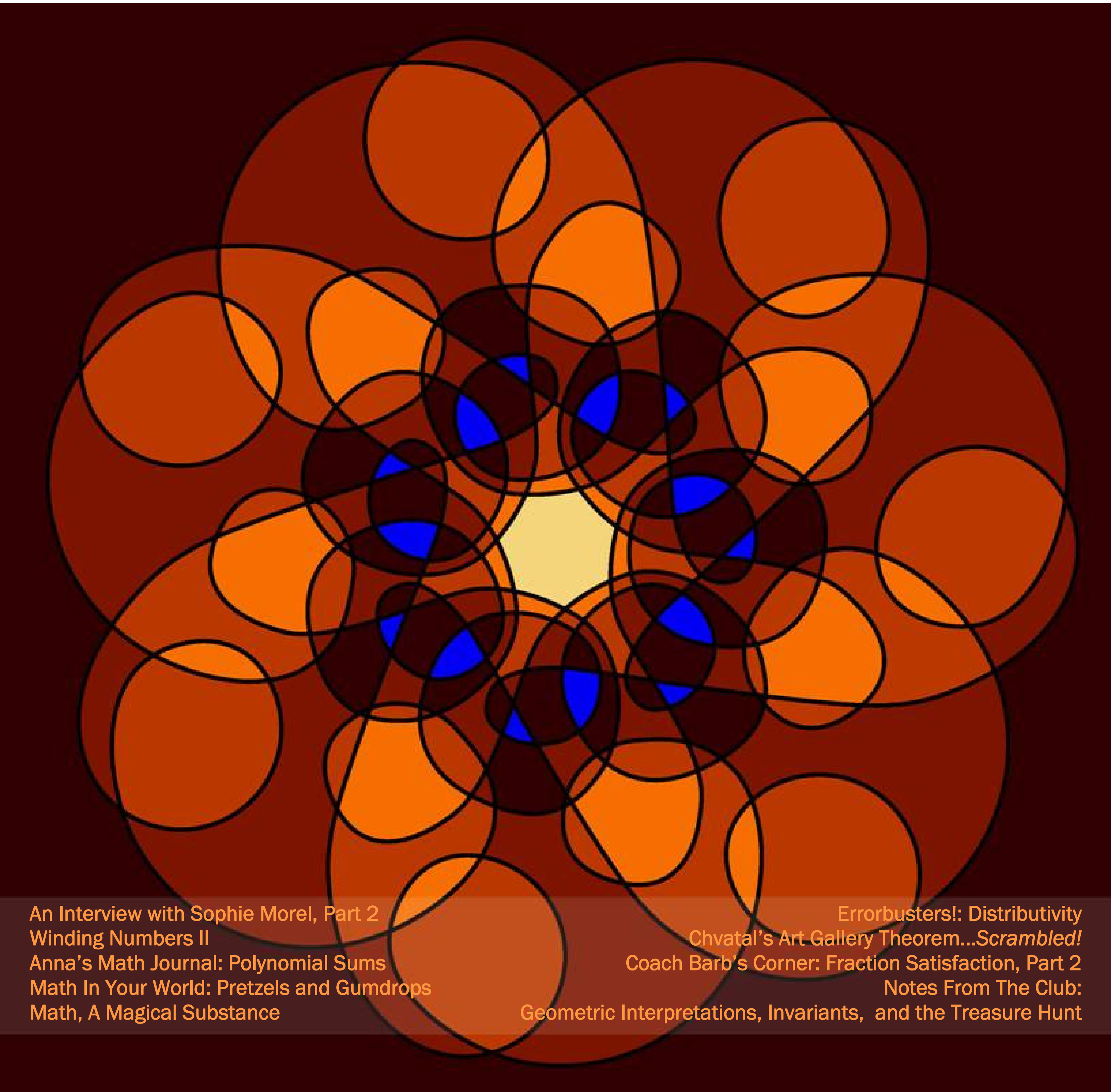
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

We're thrilled to announce that spots for SUMiT 2012 filled up in just a few days. For those of you participating, get ready to think yourselves through a wild math adventure! We wish we had the resources to accommodate more participants, but if you didn't get a spot, keep an eye out for SUMiT 2013.

Registration is open for both Math Contest Prep and the Girls' Angle Club, which start up again on January 22 and 26, respectively. We've already lined up some excellent visitors.

Mathematics is a subject that rewards effort and perseverance. The more you put in to learning math, the more you will get out, and what you get out is something that is valuable for many more things than just math: the ability to think more deeply, more subtly, and more effectively.

- Ken Fan, President and Founder

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Girls' Angle Bulletin

The official magazine of Girls' Angle: A Math Club for girls
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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Winding Number Colors* by Ken Fan. For a hint about the color patterning, read *Winding Numbers II*.

An Interview with Sophie Morel, Part 2

This is the second half of our interview with Dr. Sophie Morel, Professor of Mathematics at Harvard University and a Research Fellow at the Clay Mathematics Institute.

Ken: How do you think about mathematics? Do you think geometrically? Algebraically?

Sophie: Algebraically, most of the time. (But that's just me. Lots of people think geometrically.)

Ken: I'd like to talk about a very specific mathematical concept: the polynomial. How do you understand this concept? Is it something that you learned once, understood, and moved on? Or does your understanding of this concept change over time? If it changes over time, how does it change? Do you think of polynomials as tools? as objects? as a device? When you think of polynomials, do you think pictorially? Do you think in terms of properties?

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

We will make the rest of this interview available here at some time in the future. But what we hope is that you consider the value of interviews with mathematicians such as Harvard Professor Sophie Morel and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider sponsoring the Bulletin. Currently, such sponsorships cost $50/year, and as the number of sponsors increases, this cost will drop. As a sponsor, you will receive a printed copy of this Bulletin with complete contents and no advertisements.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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The print version is ad free.

Winding Numbers II

written and illustrated by Søren Galatius

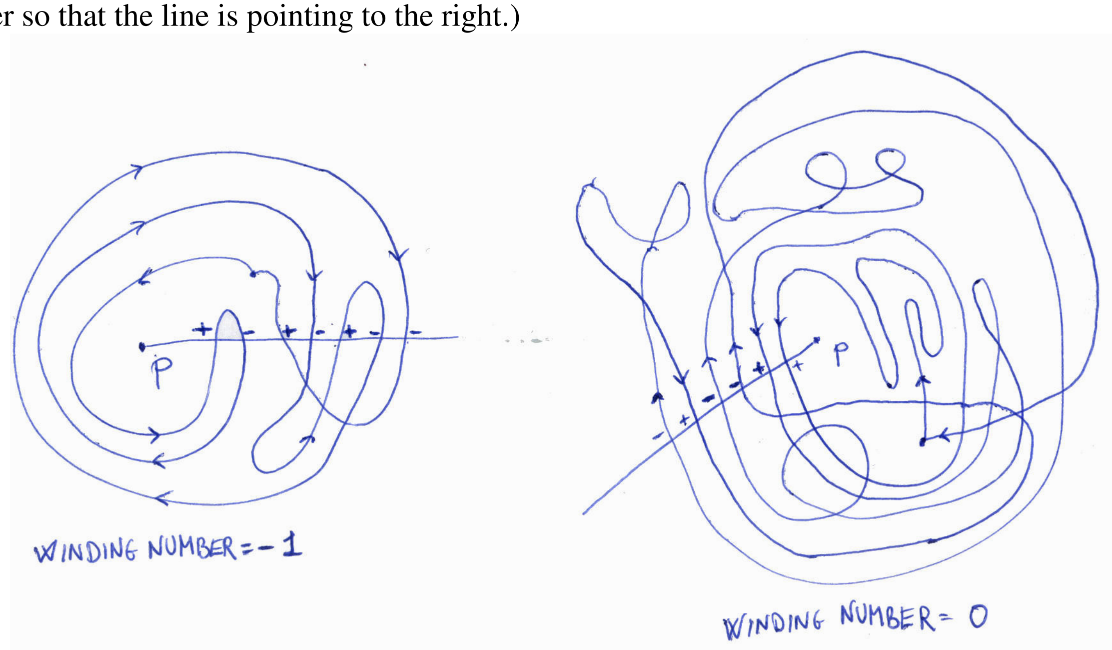
More about the winding number

Remember how I defined the winding number of a closed curve *C* around a point *P* (which must not be on the curve) using a formula in the previous issue of this *Bulletin*? What we did was that for each point on the plane we imagined a line from that point to *P*, and one from *P* going straight to the right; these two lines formed an angle at *P*, and we watched what happened to that angle as the point moved around the curve. Then the winding number was defined as

$$w(C,P) = \frac{\text{end angle} - \text{begin angle}}{360^\circ} \qquad (1)$$

.

There is a *completely different* way to calculate the same number, which doesn't involve angles at all! Let me explain how. Suppose we want to calculate the winding number of a closed curve *C* around a point *P*, drawn on a piece of paper. On the same piece of paper, draw a straight line in any direction, starting at *P* (you can use a ruler if you want it to be completely straight, but that's actually not important). Draw it long enough, for example to the edge of the paper, and rotate the paper so that the straight line is pointing to the right. Now look closer at the points where the curve crosses the straight line—for some of them the curve *C* will be moving upwards (from below the straight line to above), and for some it will be moving downwards. Then mark the crossing points where *C* moves upwards with a plus and the others with a minus. Here are two examples. (If you're confused about the second, remember that you first have to turn the paper so that the line is pointing to the right.)

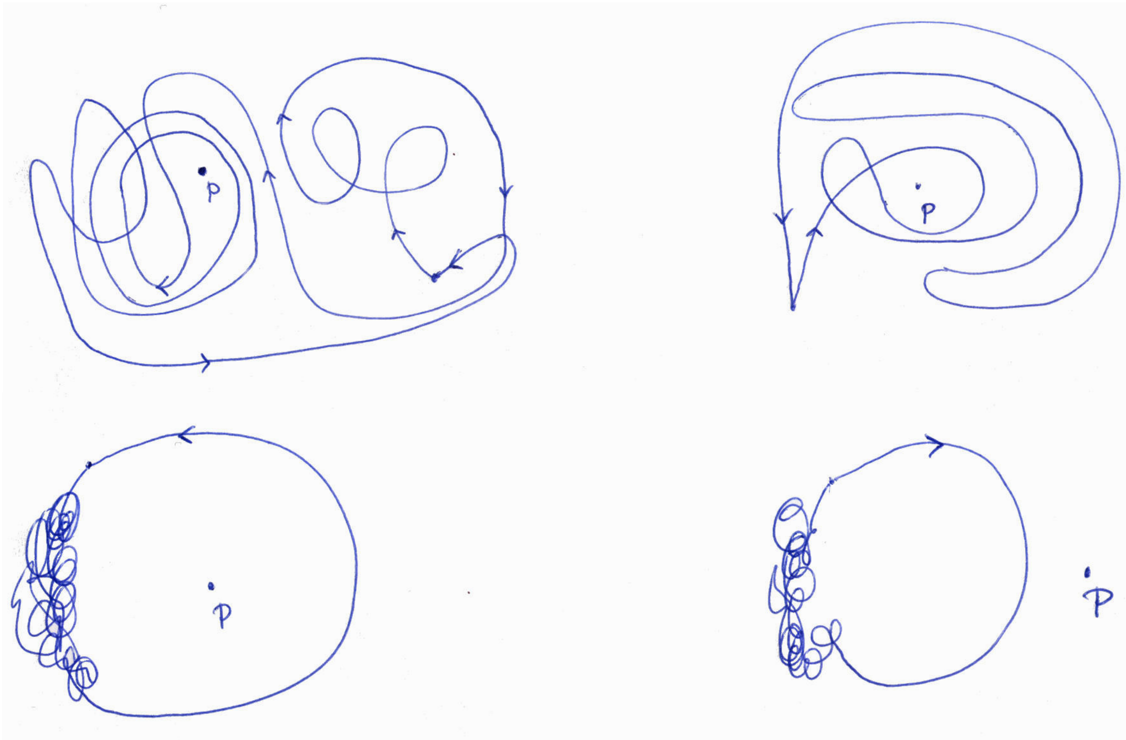


I promised to explain how to calculate the winding number without using any angles. Here it is: Count the number of plusses and minuses, then the winding number is the number of plusses minus the number of minuses! For example, in the first picture above there are 3 plusses and 4 minuses, so we calculate the winding number (without ever using any angles!) as 3 – 4 = –1. In the second it is 3 – 3 = 0.

The new method for calculating the winding number can be written as a formula:

$$w(C, P) = \text{number of plusses} - \text{number of minuses}. \qquad (2)$$

Exercise: Use the new method to find the winding number of these curves:



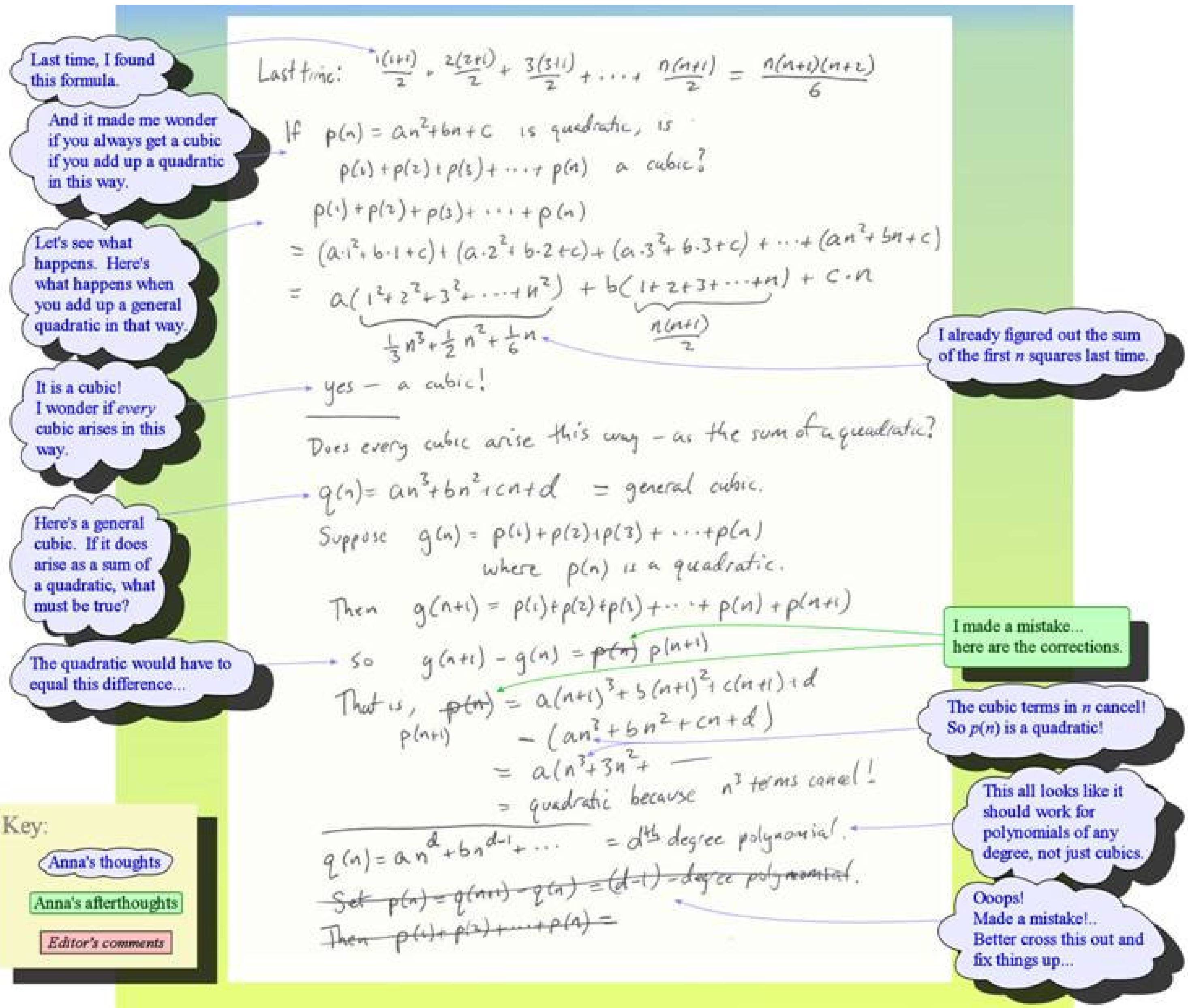
Why do these two methods (formula (1) and (2)) give the same number, no matter what curve C and what point P (which must not be on the curve) you draw? Well, I'll let you think about that on your own, but I think it's an *amazing* fact. Let's go back to our story about Chris and Hanna. Remember that Hanna was standing on the soccer field and Chris was bicycling around her, while she was watching him. She kept track of what direction she was looking in a very precise way, namely the angle between east and the direction she was looking. If he passed to the east of her, she did not jump from 360° to 0°, but kept counting beyond 360. He started and ended at the same place, and when he stopped, she calculated the difference between the angle he stopped at and the angle he started at, and divided by 360°. What she's done is of course to calculate Chris' winding number using formula (1), the angle method. Hanna is of course an avid reader of the Girl's Angle *Bulletin*, so she knows that there's a *completely different* way to calculate the same number. After Chris is done bicycling, she walks from the place she's been standing to the edge of the soccer field. While she walks, she is looking down to see how many times she passes his tracks. The soccer field is a little bit muddy, so she can see his tracks very clearly. In fact, the tracks are so clear that each time she passes them, she can tell whether he was going to the left or to the right. She counts the number of times he was going left and subtracts the number of times he was going right. If she doesn't make mistakes in the counting or the calculations, then *this number is the same as the number she calculated from the angles*!

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna furthers her work from last time and finds a formula for the sum of the first *n* cubes.



Now that I've corrected the error, I'll start again with the general degree d situation.

Set $p(n) = q(n) - q(n-1) = (d-1)$-degree polynomial.

I set $p(n)$ equal to the difference between consecutive values of $q(n)$...

Then $p(1) + p(2) + p(3) + \cdots + p(n)$

$$= \big(q(1) - q(0)\big) + \big(q(2) - q(1)\big) + \big(q(3) - q(2)\big) + \cdots + \big(q(n) - q(n-1)\big)$$

$$= q(n) - q(0)$$

So $q(n) = q(0) + p(1) + p(2) + p(3) + \cdots + p(n)$

So it turns out that $q(n)$ is the sum of the first n values of $p(n)$, plus $q(0)$.

I feel like summarizing these findings...

Conclusion: If $p(n)$ is a degree $d-1$ polynomial, then $p(1) + p(2) + p(3) + \cdots + p(n)$ is a degree d polynomial, call it $q(n)$, and $p(n) = q(n) - q(n-1)$. Conversely, every d-degree polynomial $q(n)$ can be written as $q(0) + p(1) + p(2) + p(3) + \cdots + p(n)$ for a $(d-1)$-degree polynomial $p(n)$, and, in fact $p(n) = q(n) - q(n-1)$. If $q(0) = 0$, then

$$q(n) = p(1) + p(2) + \cdots + p(n).$$

Neat relationship. So if I start with a degree d polynomial, the sum of its first n values is given by a degree $d+1$ polynomial, and, up to a constant, every degree $d+1$ polynomial is the sum of the first n values of a degree d polynomial.

I'll make a table of polynomials that I already know are related in this way here.

$p(n)$	$q(n) = p(1) + p(2) + p(3) + \cdots + p(n)$
1	n
n	$\frac{n(n+1)}{2}$
n^2	$\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$
$\frac{n(n+1)}{2}$	$\frac{n(n+1)(n+2)}{6}$
$\frac{n(n+1)(n+2)}{6}$	$\frac{n(n+1)(n+2)(n+3)}{4 \cdot 3 \cdot 2 \cdot 1}$?
n^3	?

Well, these last two I'm not sure about yet...

$q(n) = an^4 + bn^3 + cn^2 + dn$

$n^3 = q(n) - q(n-1)$

I think I'll determine the sum of the first n perfect cubes first. I set $q(n)$ equal to a general 4th degree polynomial...except I know I don't need the constant term.

$$= an^4 + bn^3 + cn^2 + dn - a(n-1)^4 - b(n-1)^3 - c(n-1)^2 - d(n-1)$$

Since I have a polynomial equation, I'll set the coefficients on both sides equal to each other.

n^3: $1 = b + 4a - b = 4a \Rightarrow a = 1/4$

n^2: $0 = c - 6a + 3b - c = -6\left(\frac{1}{4}\right) + 3b \Rightarrow b = \frac{1}{2}$

n: $0 = d + 4a - 3b + 2c - d = 1 - \frac{3}{2} + 2c \Rightarrow c = \frac{1}{4}$

n^0: $0 = -a + b - c + d = -\frac{1}{4} + \frac{1}{2} - \frac{1}{4} + d \Rightarrow d = 0$

$$\Rightarrow q(n) = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 = \frac{n^2}{4}(n^2 + 2n + 1) = \frac{n^2(n+1)^2}{4}$$

So $$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Hey, how neat...the sum of the first n perfect cubes is always a perfect square!

ABB 12.27.11

Can you verify the remaining entry in Anna's table? What about other relationships not in the table, such as p(n) = n⁴

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

Mathematics and Gingerbread Houses

Written and Illustrated by Katherine Sanden
Edited by Jennifer Silva

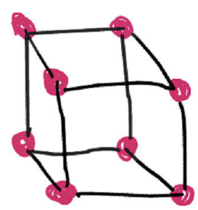
MATH IN YOUR WORLD

Logo Design by Hana Kitasei

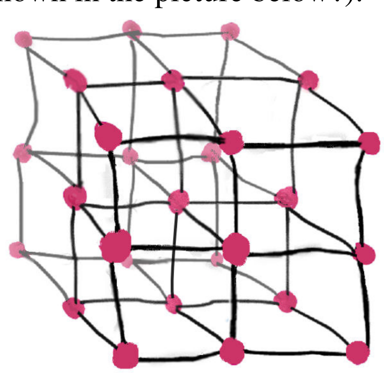
Last week I participated in a gingerbread house-making contest. Each team was given a pack of graham crackers, a bag of icing, and bowls of various types of candy. As my team and I got going, we found ourselves gravitating toward two bowls: the pretzel sticks and the gumdrops. It turns out there are quite a few shapes you can build with just pretzels and gumdrops!

In this article we'll explore a few of those shapes, and hopefully give you a leg up if you decide to enter a gingerbread house-making contest anytime soon. As you read, you might enjoy following along with your own pretzels and gumdrops (toothpicks and gumdrops work well, too).

My team and I started by making a unit cube (black lines represent pretzel sticks and pink circles represent gumdrops):



We then tried making bigger cubes by connecting unit cubes (how many unit cubes were connected to form the structure shown in the picture below?):



As our cubes got bigger, we started to run out of supplies and had to barter with other teams to get more.[1] We wanted to barter efficiently, so we began thinking about exactly how many more pieces of each candy we'd need.

Suppose we wanted to expand the cube above, so that its dimensions were 3 unit cubes across instead of 2. What is the total number of pretzels we'd need? What is the total number of gumdrops?

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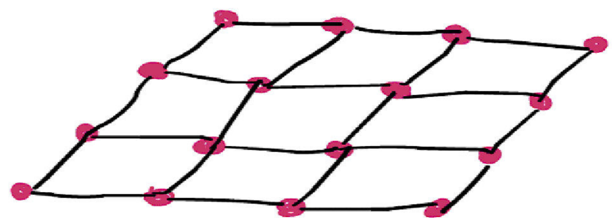
[1] Assuming we began with equal quantities of gumdrops and pretzels, which do you think we ran out of first?

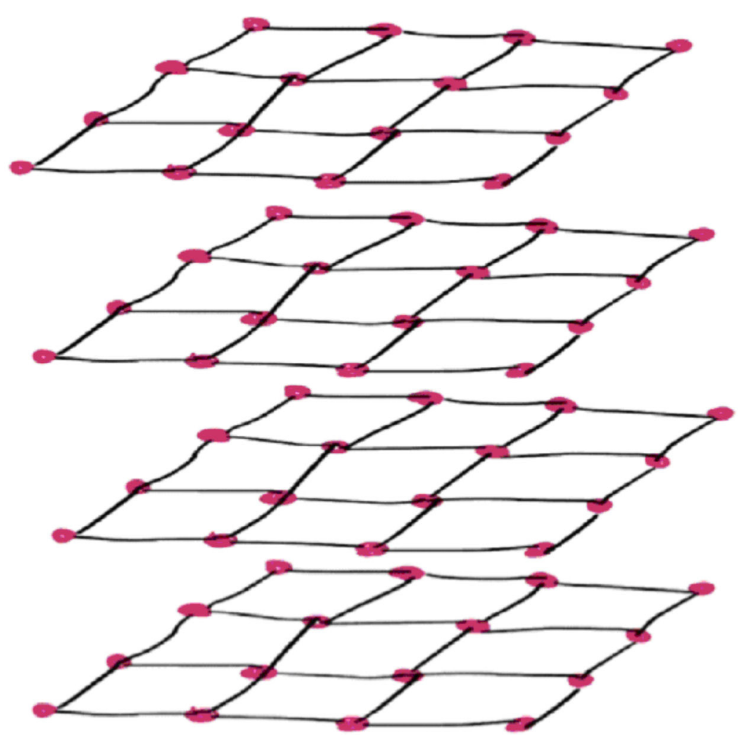
To figure that out, let's take the cube apart and look at the base layer first:



The base layer is a 4-by-4 grid of gumdrops, yielding a total of 16 gumdrops. To count pretzel sticks, I chose to look first at all pretzel sticks facing left to right. I noticed that there are 4 rows of 3 pretzel sticks each, yielding a total of 12 pretzel sticks facing left to right. Similarly, there are 12 pretzel sticks facing front to back, for a grand total of 24 pretzel sticks.

Note: This is just one technique for counting pretzel sticks. It's the technique that came most naturally to me. You might have another technique that is more intuitive to you, such as counting the number of pretzels that stick out of each gumdrop, or grouping the pretzels in squares instead of lines. If you have another technique, please share it! Each technique sheds light on a different aspect of the shape, so coming up with multiple techniques can lead to all sorts of new observations.

Now we know that there are 16 gumdrops and 24 pretzel sticks in the base layer of the cube. Since there are 4 such layers, we have now racked up 64 gumdrops and 96 pretzel sticks.



We can tell from the picture that we have successfully counted every gumdrop. The only pretzel sticks missing are the vertical ones, of which there are 16 pillars each 3 pretzel sticks high, yielding 48 extra pretzel sticks. So we found that for a 3-by-3 cube, we need 64 gumdrops and 144 pretzel sticks.

We can go back and count the numbers of gumdrops and pretzel sticks for the smaller cubes, then organize our findings in the following chart:

Dimensions of cube	Number of Gumdrops	Number of Pretzel Sticks
1-by-1-by-1	8	12
2-by-2-by-2	27	54
3-by-3-by-3	64	144
4-by-4-by-4	$5^3 = 125$	300
5-by-5-by-5	$6^3 = 216$	?
n-by-n-by-n	?	?

Can you replace the question marks in this chart with the correct numbers? What is the general formula? That is, how many gumdrops and pretzel sticks would be needed for a cube whose dimensions are n-by-n-by-n? Let's answer these questions together, and then I'll leave you with another challenge.

The number of gumdrops is more straightforward. The base of an n-by-n-by-n cube will have $(n + 1)^2$ gumdrops, and there will be $n + 1$ horizontal layers, so:

Total number of gumdrops = $(n + 1)^3$.

To write a general formula for the number of pretzel sticks, let's take another walk through the technique I used to compute this number. I start by looking at the base layer and counting the number of pretzel sticks facing left to right. I find $n + 1$ lines of pretzel sticks with n sticks in each line (draw the picture for $n = 5$ or $n = 6$ to convince yourself of this). So we've got $(n + 1)n$ pretzels. I then multiply this number by 2 to account for the pretzels on the base layer that face front to back. Our pretzel count is now $2(n + 1)n$.

Since there are $n + 1$ layers identical to the base layer, I multiply our count by $n + 1$, bringing our total to: $2(n + 1)n(n + 1) = 2n(n + 1)^2$.

My final step is to include the vertical pretzel sticks. Since there are $(n + 1)^2$ gumdrops on each horizontal layer, there are $(n + 1)^2$ "pillars" of vertical pretzel sticks, each pillar containing n pretzels, for a total of $n(n + 1)^2$ vertical pretzel sticks. Add this to our previous total to obtain:

Total number of pretzel sticks = $2n(n + 1)^2 + n(n + 1)^2 = 3n(n + 1)^2$.

Note: As mentioned before, there are many ways to think about this. Do you have a different method of counting pretzel sticks? Does it produce the same general formula?

Take it to your world

- Can you think of other scalable shapes that might be fun to build with pretzels and gumdrops? How about an equilateral triangular pyramid?[2] Or a geodesic dome?[3]
- Suppose you wanted to know exactly how many pretzels and gumdrops you would need for one of these shapes. Could you construct a table similar to the one we made for cubes? I recommend trying this for triangular pyramids. Make a real one out of gumdrops and pretzel sticks if you need help visualizing it.

Feel free to send us your formulas and pictures of the structures you make. Have fun!

---

[2] See en.wikipedia.org/wiki/Tetrahedron
[3] See en.wikipedia.org/wiki/Geodesic_dome

Math, A Magical Substance

by Ken Fan / edited by Jennifer Silva

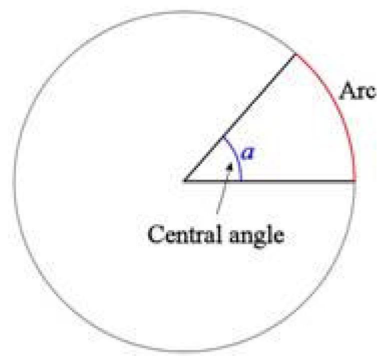
Start pouring juice into a glass. As you pour, the glass begins to fill. And as everybody knows, if you stubbornly keep on pouring, the juice will eventually overflow.

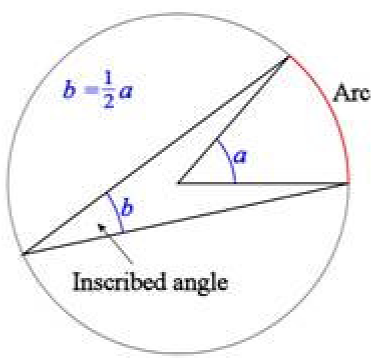
That's the way most things are. They fill up space until there is no more room: if you keep trying to add things, something will burst, spill, or be crushed.

But math is different. The more math you try to put into your mind, the more math it can hold. It's a kind of magic! We'll do a simple illustration of this magical quality of mathematics from the topic of angles and circles.

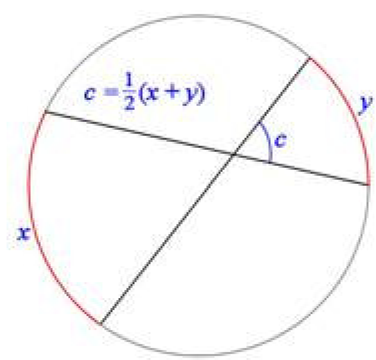
When you first encounter this topic in geometry, it may seem as though there are so many facts to learn. Let's review these facts.

First, there's the notion of the **central angle** of an arc of a circle. The central angle of an arc is the measure of the size of the angle formed when you draw radii connecting the center of the circle to the endpoints of the arc, as illustrated in the figure at right. Here, I'll refer to this central angle as the "arc angle."

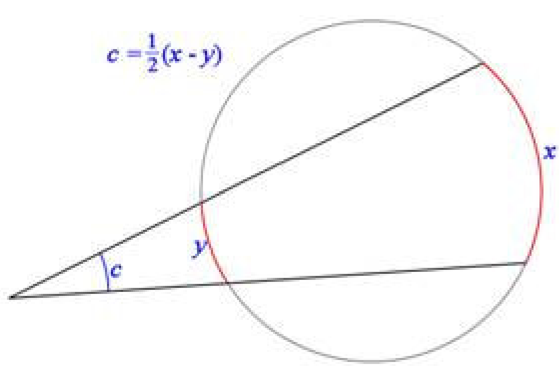




Next, comes the concept of the **inscribed angle**. An inscribed angle is what you get if you join the endpoints of the arc to some other point on the circle (instead of the center). Typically, the first result discussed from our topic is the connection between the measure of the inscribed angle and the arc angle. For now, we'll simply state this result and not prove it: The measure of the inscribed angle is equal to half of the arc angle (see figure at left).

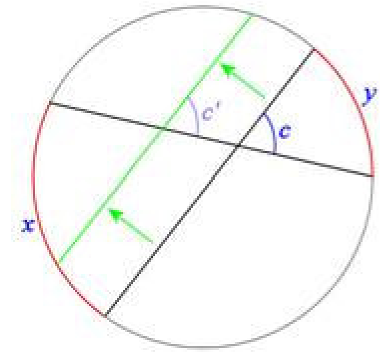


Then comes a second result about the measure of an angle formed by two lines that intersect inside the circle (see figure at right). Again, we'll skip the proof and just state that the measure of such an angle is equal to half of the sum of the arc angles labeled x and y in the figure.

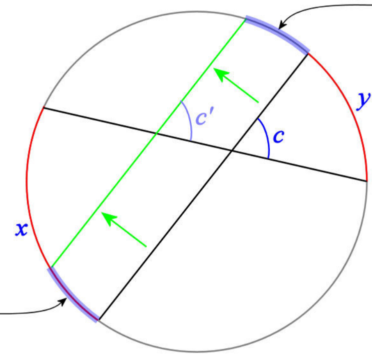


Finally, there's a third result concerning the measure of an angle formed by two lines that intersect outside of the circle, but where the lines still cut through the circle, as shown at left. The measure of this kind of angle is equal to half of the difference between the arc angles, as the equation indicates. Sometimes, particular attention is given to this last result in the special case where one of the lines is tangent to the circle.

When it becomes your turn to master these facts, what are you going to do? Are you going to commit them to memory by repeating them over and over? Although rote memorization isn't ideal, it's not necessarily a bad thing to do. But let's try instead to put *even more* of that magical substance that is mathematics into our minds and see what happens.

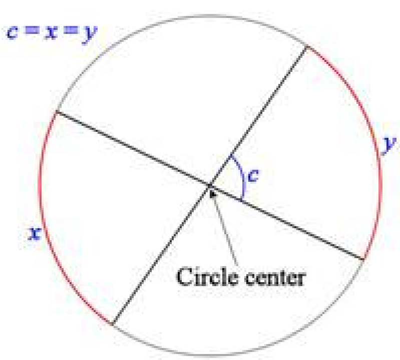


Let's look closely at the situation where two lines intersect inside of a circle. The figure at right illustrates this scenario. We are going to imagine what happens when we slide one of the two lines as indicated. As we slide it, we carefully maintain the line's orientation. The angle between the fixed line and the line we moved is preserved by this parallel transport, so $c' = c$. However, the arc angles of the arcs cut out by the lines change, as shown in the figure below. The arc labeled y gets bigger and the arc labeled x shrinks.



But look closely: the amount that y grows is the same as the amount that x shrinks! (If you reflect the circle in the diameter that is perpendicular to the line that moves, the arc by which y increases is flipped on top of the arc that x loses, and vice versa.) This means that the *sum* of the opposite arc angles cut out by the lines is preserved.

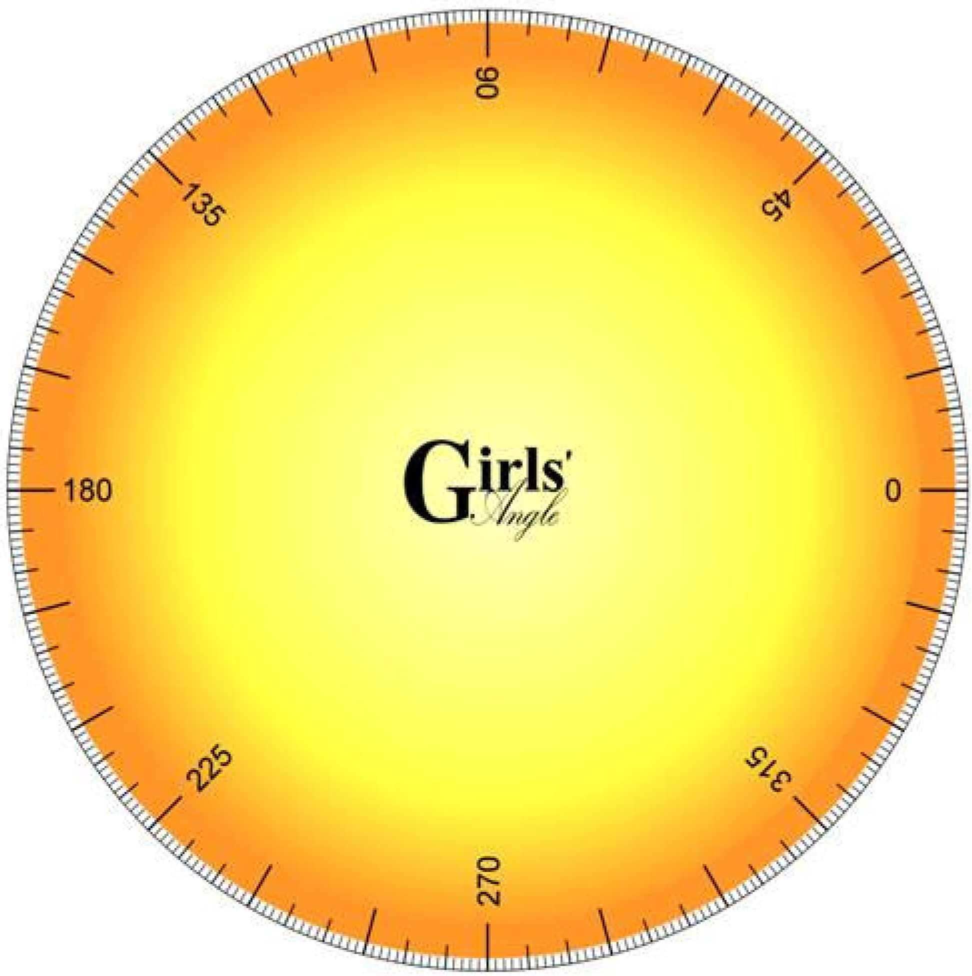
Now observe that moving one of the lines has the same effect as shifting the point of intersection to a new location. Indeed, we can move this point of intersection anywhere we wish inside the circle by sliding the lines about in the manner just described. When we combine this fact with rotations, we gain tremendous freedom. Rotations won't affect the measures of any of the angles or arcs because rotations are rigid motions. In other words, think of the two intersecting lines as a kind of fixed "X" shape, perhaps made out of metal. We can move this "X" around with its intersection inside the circle, slipping it this way and that, and even rotating it. But the angle of the "X" as well as the sum of opposite arc angles cut out by the "X" will never change; they are **invariants** of such motion. When we slide the point of intersection so that it falls directly atop the center of the circle (see figure at left), the angle of the "X" becomes the arc angle of the arcs. This enables us to calibrate the constant angle of the "X" with the constant sum of opposite arc angles: the angle of the "X" is half of this sum.



What has happened? We have, in effect, collapsed the first two results described on the first page into the understanding of a single fact: that the angle of the "X" and the sum of opposite arc angles cut out by the "X" are invariants of motion. We'll leave the details to you, but the third result can also be collapsed into this single understanding. You just have to attach a negative sign to arc angles that are "behind" their corresponding angle in the "X."

By studying more math, you not only gain more insight into the geometry, but you also roll three facts into one invariance principle. Instead of memorizing three facts, you can just concentrate on grasping the one invariance principle. (And if you forget the exact relationship between the angle of the "X" and the sum of the opposite arc angles, just calibrate them by seeking the most symmetric situation: when the intersection of the "X" coincides with the center of the circle.)

Notice that in discovering this invariance principle, we have also *proven* the results we initially stated without proof. It is truly magical how, in mathematics, deeper understandings that pack our knowledge into efficient nooks and crannies of the mind are possible, simplifying proofs along the way. (For another example of this miracle, read the interview with Prof. Morel in this issue.)



To further solidify your understanding of this principle, let's apply it to the measuring of angles formed by intersecting lines using a circular protractor. When people use circular protractors to measure angles, they typically start by carefully aligning the center of the protractor with the vertex of the angle. But our considerations show that this alignment is totally superfluous! You can just plop the protractor down carelessly over the "X" and measure the angle by taking the average of the opposite arc angles cut out by the "X." Because aligning the intersection with the center of the protractor isn't necessary, one need not be able to see through the protractor to make the measurement. You can effectively use an opaque protractor like the one above to measure angles formed by two lines. Try it and see!

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Errorbusters!

by Cammie Smith Barnes / edited by Jennifer Silva

This time of year is always very hectic for both teachers and students. Final exams are stressful for all involved. I recently spent days wading through stacks of final exams and other end-of-term assignments. One of the errors that I found again and again on my precalculus finals happened when I asked students to simplify the expression

$$(x + 3)^2 + 3(x + 3),$$

which came from the composition of the functions $f(x) = x^2 + 3x$ and $g(x) = x + 3$. (For more about function composition, see the next issue.) Too many of my students simplified the expression in the following way, or some version thereof:

$$(x + 3)^2 + 3(x + 3) = x^2 + 9 + 3x + 3 = x^2 + 3x + 12.$$

This simplification is wrong for a couple of related reasons, both involving the distributive law. Can you spot the errors? The first one I call “inadequate squaring,” and the second one is “forgetting to distribute.” Let’s focus on the second error first, as both errors really boil down to the same misapplication of the distributive law.

It is not true that

$$3(x + 3) = 3x + 3.$$

The 3 on the left must be distributed to both terms inside the parentheses, like this:

$$3(x + 3) = 3 \cdot x + 3 \cdot 3 = 3x + 9.$$

For instance, if we had a number instead of the variable x, we could see that

$$3(5 + 3) = 3 \cdot 5 + 3 \cdot 3 = 15 + 9 = 24$$

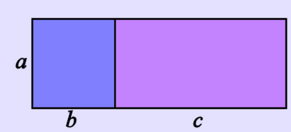
because, indeed, $5 + 3 = 8$ and $3 \cdot 8 = 24$. For some reason, it’s tempting to distribute only on the left and to forget the other term(s) inside the parentheses.

Distributive Law

The distributive law says that if a, b, and c are numbers, then

$$a(b + c) = ab + ac.$$

A geometric interpretation of the law is that the two sides represent two different ways to calculate the area of the same rectangle: the area of a rectangle with width a and length $b + c$ is both equal to $a(b + c)$ and the sum of the areas of two rectangles, one a by b and the other a by c, which yields $ab + ac$.



Now let’s consider the first, more common error:

$$(x + 3)^2 = x^2 + 9.$$

Here the student has forgotten to distribute three times. We’ll write out the square and multiply carefully to see the result that she should have gotten. Here are the first two steps:

$$(x + 3)^2 = (x + 3)(x + 3) = x(x + 3) + 3(x + 3).$$

Note that we have distributed the second ($x + 3$) to both of the terms in the first ($x + 3$). If we were to multiply out the product $(a + b)(c + d)$, we would do the same thing:

$$(a + b)(c + d) = a(c + d) + b(c + d);$$

it just seems a little more confusing when both factors are the same. Next we distribute the first x to the terms of the ($x + 3$) beside it:

$$x(x + 3) + 3(x + 3) = x^2 + 3x + 3(x + 3).$$

Similarly, for the product $(a + b)(c + d)$, we would get

$$a(c + d) + b(c + d) = ac + ad + b(c + d).$$

Finally, we distribute the terms of the remaining ($x + 3$) to the 3 on its left and then combine like terms, giving

$$x^2 + 3x + 3(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9.$$

Doing the same for the product $(a + b)(c + d)$, we get:

$$ac + ad + b(c + d) = ac + ad + bc + bd,$$

which we can see is the sum of the product of the **f**irst terms (a and c) with the product of the **o**uter terms (a and d), the product of the **i**nner terms (b and c), and the product of the **l**ast terms (b and d). This sum of products that you get after applying the distributive law three times when expanding products of binomials (sums of two terms) is often memorized by using the mnemonic "FOIL" which is an acronym for the adjectives **F**irst, **O**uter, **I**nner, **L**ast. (Notice, however, that it doesn't really matter in what order you add up these products; you could do **l**ast, **i**nner**,** **f**irst, and **o**uter, though "LIFO" doesn't sound as good as "FOIL." That is, "FOIL" is merely a memory aid and does not, itself, represent a mathematical law. It is just a convenient way to remember what happens when you properly apply the distributive law to the problem of multiplying two binomials.)

Notice that the expansion of $(x + 3)^2$ has a middle term, $6x$, which is often forgotten. This $6x$ is the sum of the outer and inner products ($3x$ and $3x$), which are just as important to remember as the first and last products (x^2 and 9, respectively). A shortcut for "FOILing" the square of a binomial comes from recognizing that the outer and inner products will always be the same. For instance, if we multiply out $(x + y)^2$, we see that the outer and inner products are xy and yx, respectively, but we know that $xy = yx$ because multiplication is commutative (that is, the order of multiplication doesn't matter). So the terms xy and yx add up to $2xy$ (twice the first term of ($x + y$) times the second term), just as the terms $3x$ and $3x$ add up to $6x$. Hence the square of ($x + y$) is

$$(x + y)^2 = \mathrm{x}^2 + 2xy + y^2.$$

Substituting 3 for the y, we see that the square of ($x + 3$) is

$$(x + 3)^2 = x^2 + 2x(3) + 3^2 = x^2 + 6x + 9,$$

which is the first term of $(x + 3)$ squared, plus twice the first term times the second term plus the second term squared. Hence we see that simplifying $(x + 3)^2 + 3(x + 3)$ gives

$$(x + 3)^2 + 3(x + 3) = x^2 + 6x + 9 + 3x + 9 = x^2 + 9x + 18,$$

which is the correct answer to the exam question missed by numerous students.

Now that we know how to square binomials, let us try cubing one. Note that
because we can "FOIL" the first two factors of $(x + y)$, getting $(x^2 + 2xy + y^2)$ on the left and leaving a factor of $(x + y)$ on the right. Next, we have to multiply a trinomial $(x^2 + 2xy + y^2)$ with a binomial $(x + y)$. To do that, we again use the distributive law. So we distribute the remaining $(x + y)$ to each term of $(x^2 + 2xy + y^2)$ to get

$$(x^2 + 2xy + y^2)(x + y) = x^2(x + y) + 2xy(x + y) + y^2(x + y).$$

> **More Distributive Law**
>
> When we have to expand $a(b + c + d)$, the a distributes to all three terms in parentheses. You can deduce this from the distributive law in steps:
>
> $$\begin{aligned} a(b + c + d) &= a((b + c) + d) \\ &= a(b + c) + ad \quad \text{(distributive law)} \\ &= ab + ac + ad \quad \text{(distributive law)} \end{aligned}$$
>
> You can also interpret this geometrically as computing the area of a rectangle split into three rectangles instead of the two used to illustrate the distributive law.
>
> This pattern generalizes to a sum of any number of terms:
>
> $$a(b_1 + b_2 + b_3 + \ldots + b_n) = ab_1 + ab_2 + ab_3 + \ldots ab_n.$$

Now we finish multiplying out and then gather together like terms:

$$x^2(x + y) + 2xy(x + y) + y^2(x + y) = x^3 + x^2y + 2xyx + 2xy^2 + y^2x + y^3 = x^3 + 3x^2y + 3xy^2 + y^3.$$

For practice, try multiplying out the following expressions. Answers can be found on page 31.

1. $(x + 1)(x + 3)$
2. $(x + 2)^2$
3. $(x + 5)^2$
4. $(x + 4)(x + 6)$
5. $(x - 3)^2$
6. $(x - 2)(x - 4)$
7. $(x + 3)(x - 3)$
8. $(x + 1)^3$
9. $(x + 1)(x + 2)(x + 3)$
10. $(x + 1)^4$
11. $(x + 1)^5$
12. $(x + 2)^4$

You might enjoy making a triangular table of the coefficients you get when you expand $(x + y)^n$. We've already computed the rows corresponding to $n = 0, 1, 2,$ and 3 and they go like this:

1
1 1
1 2 1
1 3 3 1

Add more rows to this growing triangle. What patterns can you find?

Chvatal's Art Gallery Theorem…*Scrambled!*

Chvatal's Art Gallery Theorem is about the number of guards required to keep an eye on every part of an art gallery, assuming that the gallery floor plan is a polygon. We say that a museum is fully guarded if an unobstructed straight line segment can be drawn from any point in the museum to some guard. Cut out these statements and rearrange them into a coherent proof.

This scrambled proof is adapted from the one presented in *Proofs from THE BOOK* by Aigner and Ziegler, who attribute the proof to Steve Fisk.

—

Proof of Lemma.

It does not matter which triangulation we choose, any one will do.

For $n > 3$ pick any two vertices u and v which are connected by a diagonal.

Claim: This graph is 3-colorable.

If not, the triangle ABC contains other vertices.

This diagonal will split the graph into two smaller triangulated graphs both containing the edge uv.

Now look at the two neighboring vertices B and C of A.

Let us draw $n - 3$ non-crossing diagonals between corners of the walls until the interior is triangulated.

Now AZ is within P, and we have a diagonal.

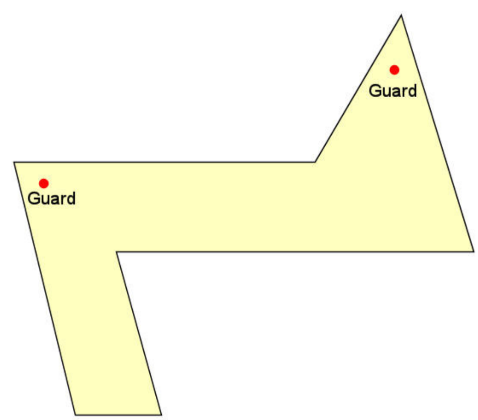
Pasting the colorings together yields a 3-coloring of the whole graph.

Since there are n vertices, at least one of the color classes, say the vertices colored 1, contains at most [n/3] vertices, and this is where we place the guards.

3-colorability

The proof uses the notion of a **3-colorable** graph. A graph is 3-colorable if each of its vertices can be colored one of three colors in such a way that no edge has endpoints that are the same color.

Don't confuse "graph" here with the graph of a function. Here, graph is used in the combinatorial sense. For an explanation, read *Math In Your World* in Volume 4, Number 5 of this *Bulletin*.



A museum "fully guarded" by two guards. If the guard at left were to walk down to the end of the west wing, the museum would no longer be fully guarded.

If the segment *BC* lies entirely in *P*, then this is our diagonal.

By induction, we may color each part with 3 colors where we may choose color 1 for *u* and color 2 for *v* in each coloring.

Call a vertex *A convex* if the interior angle at the vertex is less than 180°.

Since every triangle contains a vertex of color 1 we infer that every triangle is guarded, and hence so is the whole museum.

We proceed by induction on *n*.

Slide *BC* towards *A* until it hits the last vertex *Z* in *ABC*.

Think of the new figure as a plane graph with the corners as vertices and the walls and diagonals as edges.

For $n = 3$ there is nothing to prove.

To use induction, all we have to produce is one diagonal which will split the polygon *P* into two smaller parts, such that a triangulation of the polygon can be pasted together from triangulations of the parts.

For $n = 3$ the polygon is a triangle and there is nothing to prove.

Lemma. Given a polygon with *n* vertices, it is possible to draw in $n - 3$ non-crossing diagonals between vertices so that the polygon's interior is triangulated.

Suppose $n > 3$.

Theorem: A museum with *n* walls, can be fully guarded by [n/3] guards.

Since the sum of the interior angles of *P* is $(n - 2)180°$, there must be a convex vertex *A*.

Proof.

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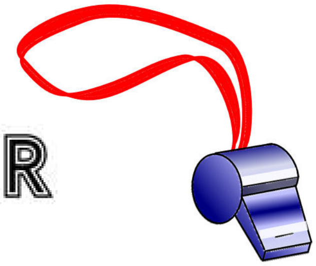


COACH BARB'S CORNER

by Barbara Remmers | edited by Jennifer Silva

Owning it: Fraction Satisfaction, Part 2

As promised last time, we are now going to add fractions. Our previous acquaintance, 3/7, is going to join us. Actually Coach Barb remains a bit daunted by the dame, so she is going to send you, brave girl, to chat with her. Just go up and knock on the door of that dear little gingerbread cottage covered with candy and baked goods. No, no, it cannot be the same cottage that proved unfortunate for Hansel and Gretel – notice that all the treats are number-shaped. What could be more welcoming? Yes, yes, there are loud bangs, sloshing noises, and surprising exclamations emanating from the dwelling, but just go ahead and knock. Coach Barb will climb up the lollipop tree and wait for you.

$\frac{3}{7}$: Who are you? Oh, never mind, you'll do just fine. Here take this bushel of money and help me pin them up out on the drying line.

Q: How did all these dollar bills get wet?

$\frac{3}{7}$: The usual way, dearie, the usual way.

Q: What is the usual way?

$\frac{3}{7}$: Oh, yes, I must keep in mind how little life experience you youngsters have. Well, I was running after my pet beaver who had once again chewed through her wooden cage, and I neglected to take into account my new banana peel carpeting. As I slid and crashed and fell, my billfold flew in one direction and my coin purse in another. Unfortunately my billfold landed in the stew pot. Fortunately I fished it out before it overly seasoned my road kill stew, but then I had to launder my paper money.

Q: I see. I'll hang the bills up, grouped by denomination.

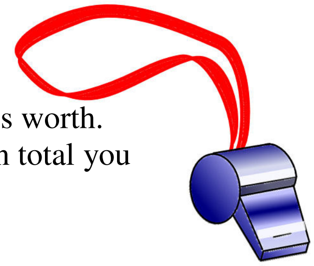
$\frac{3}{7}$: Thank you, sweetheart. They do look nice, with each of the types of bill with their own kind. I do believe I have the same number of each denomination of bill.

Q: Yes. Two twenties, two tens, two fives, and two ones.

$\frac{3}{7}$: Wonderful! So I have eight bills. My, how valuable!

Q: Well, one actually cannot determine the value by merely counting the bills, because they do not all have the same value.

$\frac{3}{7}$: Oh, yes! I am afraid I am a bit muddled in my thinking after that fall. Would you please humor me, darling, and remind me how it's done?



Q: Sure. The important point is that you have to know how many dollars each bill is worth. Then you add up the number of dollars in each bill to figure out how many dollars in total you have.

$\frac{3}{7}$: Ah, yes! It's all starting to come back.

Q: In your case, your bills are worth 20 + 20 + 10 + 10 + 5 + 5 + 1 + 1 dollars. In other words, or other numbers, you have 72 dollars worth of paper money.

$\frac{3}{7}$: Excellent! I have always felt rich and that just confirms it. Whoopsie! What just landed on my head? I do believe it is a lollipop wrapper. Oh, no! I absolutely cannot bear the thought of yet another infestation of pesky orangutans in my lollipop orchard. I must brace myself and try to muster up the courage to confirm what I fear may be another tedious ordeal.

Q: Oh, I am sure you don't have orangutans in your trees, ma'am. How about I gather your coins now?

$\frac{3}{7}$: Oh, splendid idea, dumpling. May I interest you in a bowl of stew?

Q: Oh, no. You just rest your head and have some yourself while I gather the coins.

$\frac{3}{7}$: You are kindness personified, I do declare. I will do just that.

Q: Hey, look! You also have two of each denomination of coin: two quarters, two dimes, two nickels, and two pennies.

$\frac{3}{7}$: Hay is for horses, but otherwise splendid! Please do carry on, snookums, and remind me what the value is of eight coins.

Q: Well you see, it's not quite that simple. The value depends on what types the eight coins are.

$\frac{3}{7}$: Oh, dear. It's that denomination issue raising its head again, right?

Q: Yes, ma'am. You're remembering. Keep at it, and in no time, it'll be automatic.

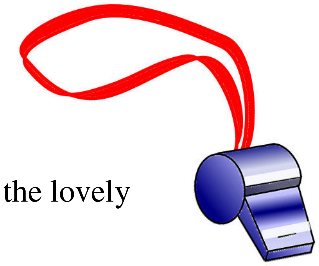
$\frac{3}{7}$: Thank you, toots. Please refresh my memory with this example.

Q: No problem. For coins, we convert them all to cents and then add up the number of cents in each coin to find the total value of all of your coins.

$\frac{3}{7}$: This all sounds familiar ... chin up and carry on, my dear.

Q: So your coins are worth 25 + 25 + 10 + 10 + 5 + 5 + 1 + 1 cents. That is, 82 cents.

$\frac{3}{7}$: Splendid! Marvelous! Terrific! Awesome!



Q: Really?

$\mathbf{\frac{3}{7}}$: Yes! You have just revived my memory. I am healed. It's all reminding me of the lovely process of adding fractions.

Q: Really?

$\mathbf{\frac{3}{7}}$: Yes! The main point is to make sure that you are adding the same type, or denomination, of fraction when you add them.

Q: And we tell the type of fraction by its bottom number, right?

$\mathbf{\frac{3}{7}}$: There's nothing wrong with your memory, sweets. I bet you even remember that fancy folk like to call that number the denominator.

Q: I do. So *when the bottom numbers match, then we are safe to add*. We can add sevenths to sevenths, sixths to sixths, and so on. When we add two things of the same type, we are going to end up with the same type – at least initially, before we simplify. So we know the bottom number of the answer before we've done any addition.

$\mathbf{\frac{3}{7}}$: Yes, dearie. I always write down the bottom number and the fraction bar before I figure out the top number.

Q: Then since the top number tells us how many sevenths, or whatever other thing the bottom number specifies, we can just add the two top numbers whenever the bottom numbers match.

$\mathbf{\frac{3}{7}}$: You got it girly! I'd just caution you that "thing" is a rather inelegant pronoun. "Unit fraction" is the melodic phrase to use. Sevenths, fifths, thirds, etc. are unit fractions. Indeed, since the top number tells you how many unit fractions a fraction contains, you can just add up the two top numbers when the same unit fraction is involved.

$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$$

$$\frac{17}{101}+\frac{83}{101}=\frac{100}{101}$$

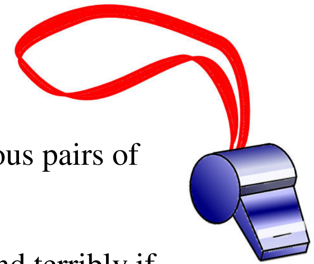
$$\frac{5}{3896}+\frac{85}{3896}=\frac{90}{3896}$$

Q: Okay. Everything's clear when the bottom numbers match: use the same bottom number and the top number is the sum of the two original top numbers. But what about when the bottom numbers do not match? What about adding a third and a quarter? 1/3 + 1/4 = ?

$\mathbf{\frac{3}{7}}$: There's no need to start sentences with "but," darling. Let's be civilized here. Speaking of civilization, I smell that my brownies are done. Let's get them.

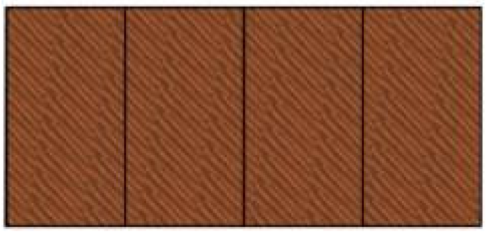
Q: Oooh, you made two pans. Yum!

$\mathbf{\frac{3}{7}}$: Yes, the swamp mud is so deliciously firm this time of year that I doubled the recipe.



Q: Oh. Um. Well. Hey, doubling the recipe must have involved adding up numerous pairs of identical fractions, which would naturally have the same bottom number.

$\frac{3}{7}$: Oh, dearie, I do admire your enthusiasm for all things fractional. Would you mind terribly if we paused a bit while you cut the first pan into thirds and the second into quarters?



Q: No, I don't mind.

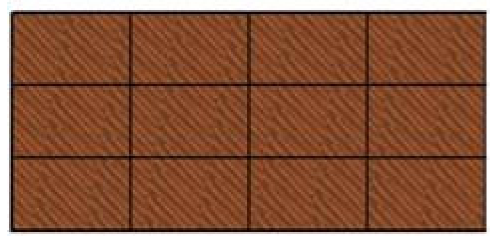
Hey! This illustrates the problem. I can take one row of the first pan and one column of the second and know that I have 1/3 + 1/4 of a pan of brownies, but I want a single fraction for the answer, not 1/3 + 1/4.

$\frac{3}{7}$: Yes, dear. There, there. I know you're worked up over this but PLEASE spare me the "hey." Let's just give it a rest and focus on the lovely brownies, darling. Would you please cut each of the first pan's rows into quarters, and each of the second pan's columns into thirds?

Q: Hey! … I mean ... My oh my! Now the pans look the same. They are both cut into twelfths. Hey! ... My oh my! I can call the one row of the first pan 4/12 and one column of the second pan 3/12, and then I'm in the easy peasy situation of dealing with the same type of fraction. I can say 7/12 instead of 1/3 + 1/4. My oh my oh my oh my!

And I can even add 2/3 and 3/4 by translating them to 8/12 and 9/12 and adding the tops to get 17/12.

I can see why multiplying both bottom numbers together will always work as a new bottom number – one can be the rows and the other can be the columns of the "whole" brownie pan!



$\frac{3}{7}$: Yes, darling. Now, is there any need to discuss subtraction?

Q: Nah... I mean no. As long as I keep in mind what the top and bottom numbers mean and the important issue of working with the same type of unit fractions, I'm fine. Understanding what is going on is better than memorizing a bunch of rules.

$\frac{3}{7}$: I couldn't have said it better myself, dear. Well, it certainly has been most pleasant, not to mention edifying, visiting with you. Might you have any additional bits of wisdom to share while you have a brownie?

Q: Oh, no thanks on the brownie, but you may want to reconsider the use of banana peels for a carpet and wood for a beaver cage.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

<u>Session 9 – Meet 8 – November 3, 2011</u>

Mentors: Jennifer Balakrishnan

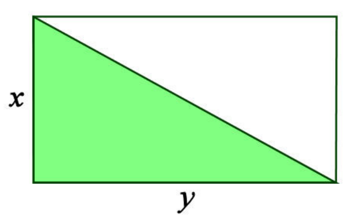
The main topics of the day were the Klein bottle, logic problems, and studying a proof of Chvatal's Art Gallery Theorem. If you enjoyed the scrambled proof challenge in the last issue, there's one of Chvatal's Art Gallery Theorem on page 20.

<u>Session 9 – Meet 9 – November 10, 2011</u>

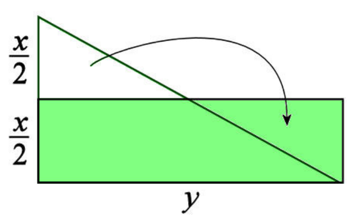
Mentors: Samantha Hagerman, Liz Simon, Rediet Tesfaye

A group of girls have been working steadily through Raymond Smullyan's *The Lady or the Tiger & Other Logic Puzzles*. Smullyan's books are a treasure trove of excellent logic puzzles.

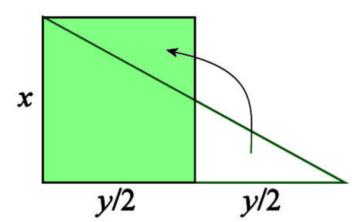
Rediet reports that **billy-bob-joe-bob-jim** illustrated the fact that $(xy)/2 = (x/2)y = x(y/2)$ by showing that all three correspond to the area of a triangle with base x and height y:



The area of the green triangle is half of the area of the rectangle. The area of the rectangle is xy, so the area of the triangle is $(xy)/2$.



…But the area of the triangle can also be seen to be the area of a rectangle with dimensions $x/2$ by y. So the area is also $(x/2)y$.



…Still more, the area of the same triangle is equal to that of a rectangle with dimensions x by $y/2$. Hence the area is also $x(y/2)$…

Hence, the expressions $(xy)/2$, $(x/2)y$, and $x(y/2)$ are all equal to each other.

In a similar spirit, can you make a geometric illustration of the fact that $(x + 1)^2 = x^2 + 2x + 1$? How about $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$? (If you have trouble verifying these identities algebraically, see this issue's *Errorbusters!*)

Some girls proved that $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$ by using induction. If you would like to see another proof of this formula using an elaborate geometric derivation in the spirit of **billy-bob-joe-bob-jim**'s illustrations above, take a look at pages 24 and 25 of volume 4, number 3 of this *Bulletin*.

Session 9 – Meet 10 – November 17, 2011

Mentors: Samantha Hagerman, Jennifer Melot, Liz Simon

One of the many important concepts explored at this meet was that of an **invariant**. An invariant of some action is something that stays the same even when the action is performed. For example, if you take a polygon, cut it into some pieces, and rearrange these pieces to form another polygon (using all the pieces without overlap), then the area of the resulting polygon will be the same as that of the original. In other words, the area is an invariant of the cutting and rearranging action. Invariants are used all the time outside of mathematics too. For example, the amount of money in your wallet is invariant under a wide variety of actions. Only certain acts will cause that amount to change, such as when you use cash to purchase something. Consequently, if you open your wallet and find a different amount of cash in it from what you remembered, you can conclude that something unusual must have happened! For another example of an invariant, check out *Math, A Magical Substance* on page 13.

Session 9 – Meet 11 – December 1, 2011

Mentors: Jennifer Balakrishnan, Jennifer Melot, Liz Simon, Rediet Tesfaye

Special Guest: Alison Malcolm, Department of Earth and Planetary Sciences, MIT

Prof. Malcolm talked about earthquakes and how math can be used to locate their epicenters. She explained that there are 3 basic types of wave that are generated by an earthquake: P, S, and surface waves. The P wave is usually the first to arrive and travels through the earth oscillating along the direction of propagation. The S wave, which also travels through the earth, follows the P wave and oscillates perpendicularly to the direction of propagation. Finally, the surface wave arrives. The surface wave travels along the surface of the Earth and it is this wave that is associated with all the tremors that people feel.

Then, she explained that for nearby earthquakes, one can estimate the distance, in kilometers, of the epicenter by using the formula $10(t_s - t_p)$ where t_s is the time of arrival of the S wave and t_p is the time of arrival of the P wave. The formula breaks down at larger distances. However, by using this formula on data collected by seismometers at several nearby locations, one can get a rough idea of the location of the epicenter.

On August 23, 2011, an earthquake in Virginia caused tremors in more than a dozen states as well as in Canada. She gave members data from 4 seismic recording stations and had members apply the above formula to locate the epicenter.

Session 9 – Meet 12 – December 8, 2011

Mentors: Jennifer Melot

Congratulations to all members for their valiant effort at our traditional end-of-session math treasure hunt! This session's hunt was definitely one of the more challenging hunts. If you would like to try your hand at some of the problems the girls tackled, just proceed to the next page.

4 Problems from the Session 9 Treasure Hunt

1. Construct two truly different 4 by 4 magic squares. That is, make two arrangements of the numbers from 1 to 16 in a 4 by 4 grid so that every row, column, and diagonal adds up to the same number. One magic square should not be a rotation or reflection of the other.

2. Figure out how to partition a square into 6 identical shapes, that are NOT rectangular.

3. The number 1 is a perfect square, a perfect cube, a perfect fourth power, a perfect fifth power, a perfect sixth power, AND a perfect seventh power! The next such number that is a perfect square, cube, fourth, fifth, sixth, and seventh power is N. Determine the remainder when you divide N by 100.

4. Rearrange the following statements to make a coherent proof of the Sylvester-Gallai Theorem.

Since L_0 contains more than two of the given points, it must contain two, say R and S, which are on the same side of Q (with one of them possibly equal to Q).

Consider pairs (P, L) where L is a line that passes through at least two of the given points and P is one of the given points that does not lie on L.

This contradicts our choice of P_0 and L_0.

Suppose, on the contrary, that L_0 passes through more than two of the given points.

Claim: L_0 passes through exactly two of the given points.

Proof.

Let L_1 be the line that passes through P_0 and S.

Let Q be the point on L_0 that is closest to P_0.

Theorem. In any configuration of n points in the plane, not all on a line, there is a line which contains *exactly* two of the points.

Therefore, the theorem must be true.

Among all such pairs (P, L), choose a pair (P_0, L_0) such that P_0 has the smallest distance to L_0.

The distance of R to the line L_1 is smaller than the distance of P_0 to L_0.

Without loss of generality, we can assume that R is between Q and S, where R possibly coincides with Q.

(This proof is adapted from the one in *Proofs from THE BOOK*, by Aigner and Ziegler, who attribute the proof to R. M. Kelly.)

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Girls' Angle is now selling advertising space in the electronic version of the Bulletin.

The print version is ad free.

Calendar

Session 9: (all dates in 2011)

September	8	Start of the ninth session!
	15	
	22	Ally Hartzell, Pixtronix
	29	Start of Rosh Hashanah – No meet
October	6	
	13	Diana Hubbard, Boston College
	20	
	27	
November	3	
	10	
	17	
	24	Thanksgiving - No meet
December	1	Alison Malcolm, MIT
	8	

Session 10: (all dates in 2012)

January	26	Start of the tenth session!
February	2	
	9	Meike Akveld, ETH Zürich
	16	
	23	No meet
March	1	
	8	
	15	
	22	
	29	No meet
April	5	Beth Kanell, Author
	12	Sarah Spence Adams, Olin College
	19	No meet
	26	
May	3	

Special Announcement: If you like solving math contest problems, sign up for Girls' Angle's Math Contest Prep course. Check our website www.girlsangle.org for details.

Here are answers to the *Errorbusters!* problems on page 19.

1. $x^2 + 4x + 3$
2. $x^2 + 4x + 4$
3. $x^2 + 10x + 25$
4. $x^2 + 10x + 24$
5. $x^2 - 6x + 9$
6. $x^2 - 6x + 8$
7. $x^2 - 9$
8. $x^3 + 3x^2 + 3x + 1$
9. $x^3 + 6x^2 + 11x + 6$
10. $x^4 + 4x^3 + 6x^2 + 4x + 1$
11. $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$
12. $x^4 + 8x^3 + 24x^2 + 32x + 16$

Girls’ Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls’ Angle? Girls’ Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls’ interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls’ Angle mentors, the Girls’ Angle Support Network, the Girls’ Angle Bulletin and Community Outreach.

Who are the Girls’ Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls’ Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls’ Angle Bulletin? The Girls’ Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) comes with membership. We are working hard to lower the cost of the Bulletin. Until we do, however, nonmembers can receive the printed version by becoming a Bulletin Sponsor. Please contact us if interested.

The Bulletin targets girls roughly the age of current members. Each issue is likely to contain some material that feels very challenging or difficult to understand. If you are a member or Bulletin Sponsor and have any questions about the material, feel free to ask us about it!

What is Community Outreach? Girls’ Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members’ efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

How do I join? Membership is granted per session. Members have access to the club and receive a printed copy of the Girls’ Angle Bulletin for the duration of the membership. You can also pay per meet, but it is slightly more expensive. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of “catching up with the group” doesn’t apply. If you cannot attend the club, you can purchase a **Remote Membership** which comes with a year-long subscription to the Bulletin and a 25% discount for any club meet attended. Remote members may email us math questions (although we won’t do people’s homework!).

Where is Girls’ Angle located? Girls’ Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, assistant professor, UCSF Medical School
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, Tamarkin assistant professor, Brown University
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For (please circle): Membership Remote Membership

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

For **membership**, please fill out the information in this box. **Bulletin Sponsors** may skip this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a one session Membership
  - ☐ $108 for a one year Remote Membership
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ______________________________________Date: ______________

Print name of applicant/parent: ______________________________________

Print name(s) of child(ren) in program: ________________________________

Girls' Angle

A Math Club for Girls