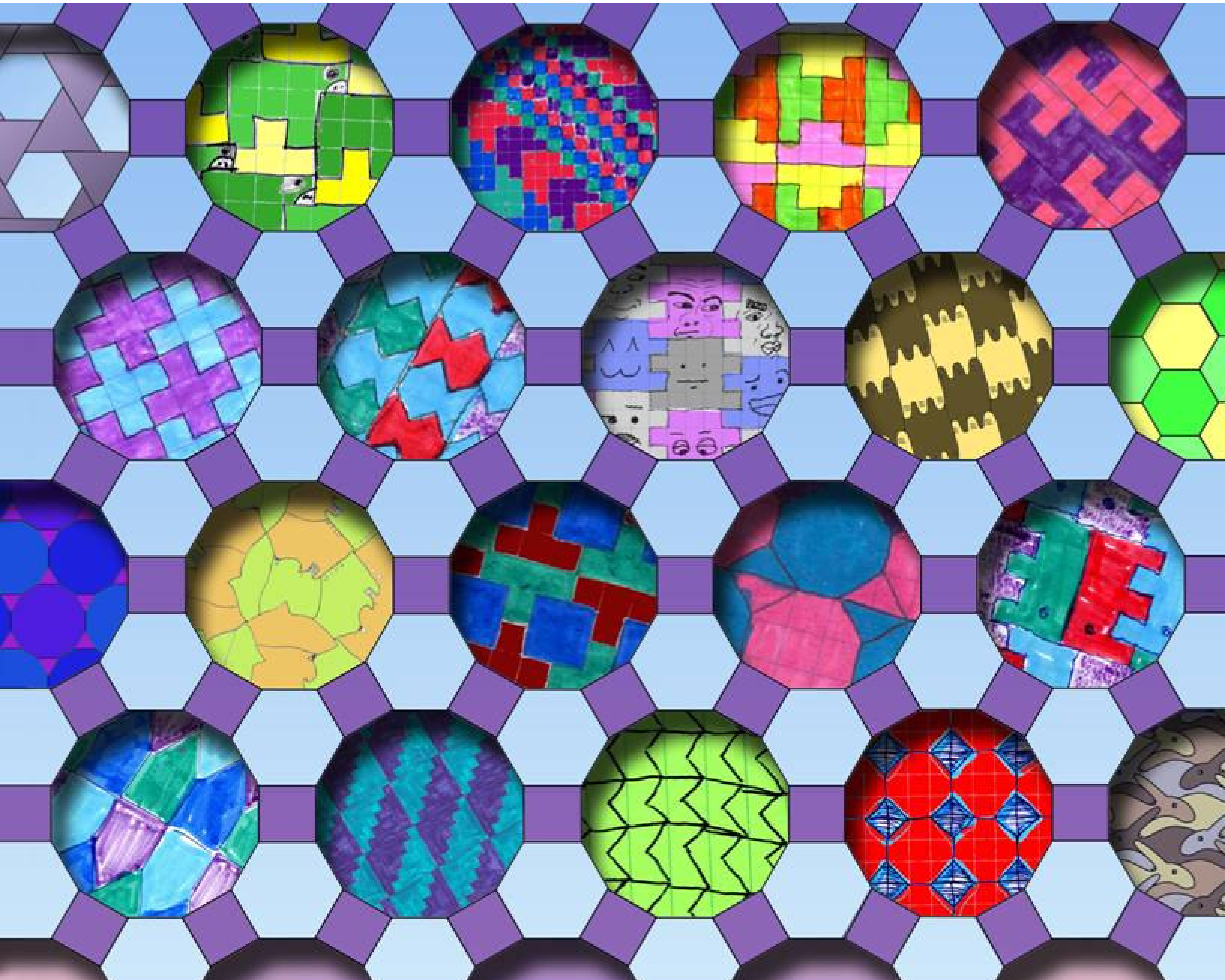
Girls' Angle Bulletin

April 2012 • Volume 5 • Number 4

To Foster and Nurture Girls' Interest in Mathematics



From the Founder

First, a very special thanks to Prof. Jean Pedersen who gave us a marvelous 3-part interview filled with mathematics and history. We hope her interview inspires you to get active with the math!

Inspired by Dr. Sarah Spence Adams' presentation on secret codes, there's a secret message challenge for you on page 15. Decipher the messages, determine their associated numbers, and send in your solution for a chance to win a prize. Everything you need to know to decipher the messages can be found in this issue, but feel free to seek additional help or work with others.

Last March, Rediet Abebe, Keren Gu, Samantha Hagerman, and Julia Zimmerman did a wonderful job hosting our booth at the 2012 FIRST Robotics Career Expo in New York City. If you missed us, we hope to see you there next year!

- Ken Fan, President and Founder

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Girls' Angle Bulletin
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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: A Tessellation of Tessellations, showing a number of tessellations made by members this session. For identities, please see this issue's Mathematical Buffet.

An Interview with Jean Pedersen, Part 1

Photography by Chris Pedersen. Illustrations by Sylvie Donmoyer.

Jean Pedersen

Jean Pedersen is a professor in the Mathematics and Computer Science Department at Santa Clara University. In the past she has held positions as a Governor of the Northern California Section of the Mathematical Association of America (MAA), as a member of the Editorial Board for *Mathematics Magazine*, and (twice) as a visiting Erskine Fellow at Christchurch, New Zealand. She has lectured about mathematics in Europe, South Africa, Canada, Mexico, New Zealand and Australia. She is currently serving on the Board of Editors for the MAA series, *Spectrum*. She is also the Director of the Individual Studies Program at Santa Clara University.

Ken: How did you become interested in mathematics? Do you remember the mathematical idea or result that first attracted your attention?

Jean: I was always reasonably good at arithmetic as a child and my father, who was a medical doctor, encouraged me. He taught me about negative numbers and infinity when I was in third grade and I believe I caused the teacher — who didn't know much about either idea — a great deal of trouble when I didn't say the problem was impossible, and gave the appropriate answer that she didn't understand. I am told that numerous parent-teacher conferences took place during that year. Like most young children I enjoyed the attention and so I became rather adept at doing multiplication and long division problems.

But I didn't really learn to love mathematics until after I had obtained my Master's Degree (from the University of Utah) and began teaching. Then, in an effort to make mathematics more interesting for my students than my own education had been, I began reading books about how to motivate students. My favorite authors at that time were George Pólya [9], Martin Gardner [4], W. W. Rouse Ball [1], and Eric Temple Bell [3]. Pólya's *Mathematical Discovery* is a real treasure for teachers who want ideas about motivating students. Gardner's first article in *Scientific American* was my introduction to hexaflexagons. Ball's book, *Mathematical Recreations and Essays*, was a fascinating collection of mathematical ideas I'd never met. Bell's book covered accounts about well-known mathematicians.

I became intrigued with hexaflexagons and have used them ever since in discussions with students when we talk about symmetry or group theory.

In the late 1960's I was asked by Jerry Alexanderson — the chairman of the Department of Mathematics at Santa Clara University, where I was teaching — to give a talk to some high school students. When I asked for a suggestion about possible topics, he suggested I should do "something on geometry." Naturally the first thing that came to mind was the hexaflexagon. In preparing for this event I constructed from a strip of adding machine tape a pattern piece with 10 equilateral triangles on it. I was using this to cut pieces from the adding machine tape that I could fold so that each student could be given the stack of triangles and the talk would begin by having each student construct a hexaflexagon. My son, Chris, who was about 6 years old at the time, offered to help cut the pieces. After explaining that he had to do it very carefully, I gave him scissors and he went to work. But, as he cut each piece he trimmed off just a tiny bit of the pattern piece.

The next day when I sat down to watch a football game with my husband, Kent, I put a breadboard on my lap and began to fold the strips that Chris had cut. It was then that I noticed the first fold didn't produce the expected equilateral triangle — because the initial angle was less than 60°. However, because I was distracted by the football game I continued to fold and when I next looked at the piece of paper I noticed that the triangles had miraculously turned into what appeared to be equilateral triangles. I thought about this for a moment and then pointed out what had happened to my husband who, accustomed to hearing me quote from Pólya's books, said, "Can you prove it?" I went to the study and produced the following proof (where $\pi = 180°$).

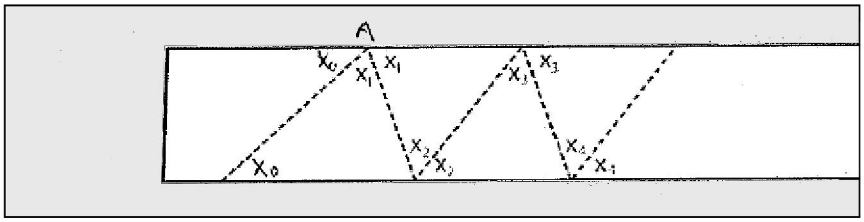

Figure 1 Tape that has been bisected once at each edge.

Assume that the initial angle $x_0 = \frac{\pi}{3} + \varepsilon$ (and ε can be either positive or negative!). Then at vertex *A* we have $2x_1 + \frac{\pi}{3} + \varepsilon = \pi$. Solving for x_1 we get

$$x_1 = \frac{\pi}{3} - \frac{\varepsilon}{2}.$$

This means that every time you make a correct fold, bisecting the obtuse angle between the last fold line and an edge of the tape, you cut the previous error in half and change the sign. Thus we see that, in general

$$x_n = \frac{\pi}{3} + (-1)^n \frac{\varepsilon}{2^n},$$

so that the smallest angle on the tape must approach $\frac{\pi}{3}$ (= 60°) as *n* gets large — that is, as you continue to fold!

Encouraged by this I returned to the family room and told Kent that I had a proof. The football game wasn't over, so gently pushing me aside to see the last replay, he said, "That's great! Does the idea generalize?"

I returned to the study and began a systematic folding where I bisected the obtuse angle between the last fold line and the edge of the tape *twice*. I soon realized that it was converging to something because the spaces between similar folds were becoming more regular. Playing with this piece of folded tape I discovered that if you folded it on successive short lines you obtained what Peter Hilton and I eventually called the short-line 5-gon shown in Figure 2. From this it was clear, from plane geometry considerations, that the smallest angle on this tape was approaching an angle of $\frac{\pi}{5}$.

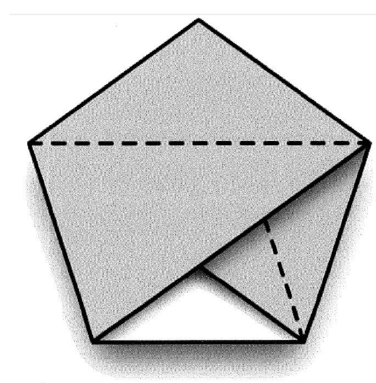

Figure 2 A short-line 5-gon.

An error-correcting proof similar to the one for the tape that had been folded once at each edge of the tape goes as follows.

In Figure 3 assume $x_0 = \frac{\pi}{5} + \varepsilon$, than at vertex *A* we have $x_0 + 4x_1 = \pi$, or

$$\frac{\pi}{5} + \varepsilon + 4x_1 = \pi .$$

Solving for x_1, we see that

$$x_1 = \frac{\pi}{5} - \frac{\varepsilon}{2^2} .$$

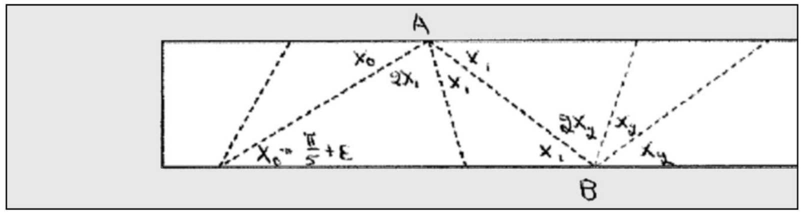

Figure 3 Tape that has been folded twice at each edge.

Then, at vertex *B* we would have $x_1 + 4x_2 = \pi$, or

$$\frac{\pi}{5} - \frac{\varepsilon}{2^2} + 4x_2 = \pi .$$

Solving for x_2 we have

$$x_2 = \frac{\pi}{5} + \frac{\varepsilon}{2^4} .$$

In general, $x_n = \frac{\pi}{5} + (-1)^n \frac{\varepsilon}{2^{2n}}$, from which it follows that the smallest angle on this tape approaches $\frac{\pi}{5}$ as you continue to fold twice on each edge of the tape.

Later my daughter, Jennifer, found some of these folded strips in my study and asked if she could play with them. After a bit she came back to me saying, “Look mommy, if you fold on this long line, and then this long line, . . . isn’t that pretty?” What resulted is the figure shown in Figure 4. I had to agree that it was, indeed, very pretty. Intrigued with its symmetry I made 12 of these pentagons and glued them together with scotch tape along their edges to form a dodecahedron. When it was finished I noticed that the folded strips seemed to go around a “great circle” of the model and it looked as if 6 strips might be braided together to make what I later named a *golden dodecahedron*, because the ratio of the long line to the short line on this folded tape is the golden ratio.

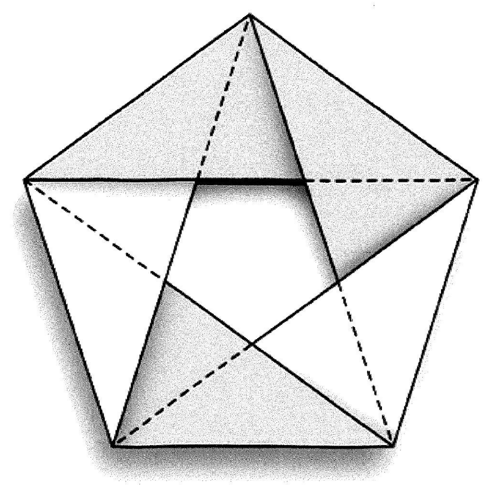

Figure 4 A long-line 5-gon

To see if it would be possible to construct such a braided model I made six strips from gummed tape that had been folded repeatedly twice on each edge, glued them to colored paper, cut out the strips, refolded on the long lines and actually braided them together using paper clips to hold the pieces in place; these were attached at the holes of the faces as indicated by the arrows in Figure 5. Much to my surprise and delight, when the model was finished all of the paper clips could be removed and it remained stable! (See Figure 5.) More complete instructions for the construction of this beautiful model can be found in [5, 7, 8].

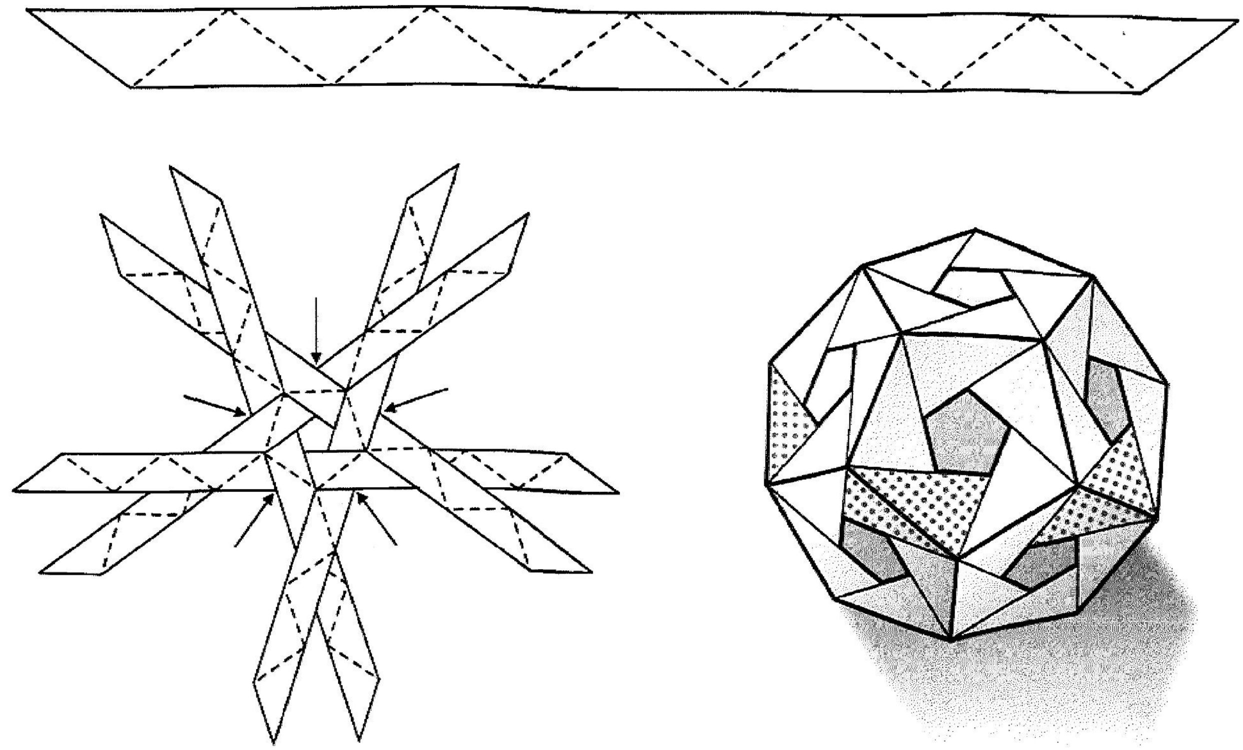

Figure 5 Top is a typical strip from the paper folded twice at each edge with just the long fold lines shown. Bottom left is the beginning layout of 5 strips around the "north pole" of the Golden Dodecahedron (the arrows indicate where paper clips may be attached to help hold the pieces together). When the sixth strip is braided about the "equator" and the original 5 strips come together about the "south pole" the model on the bottom right is obtained.

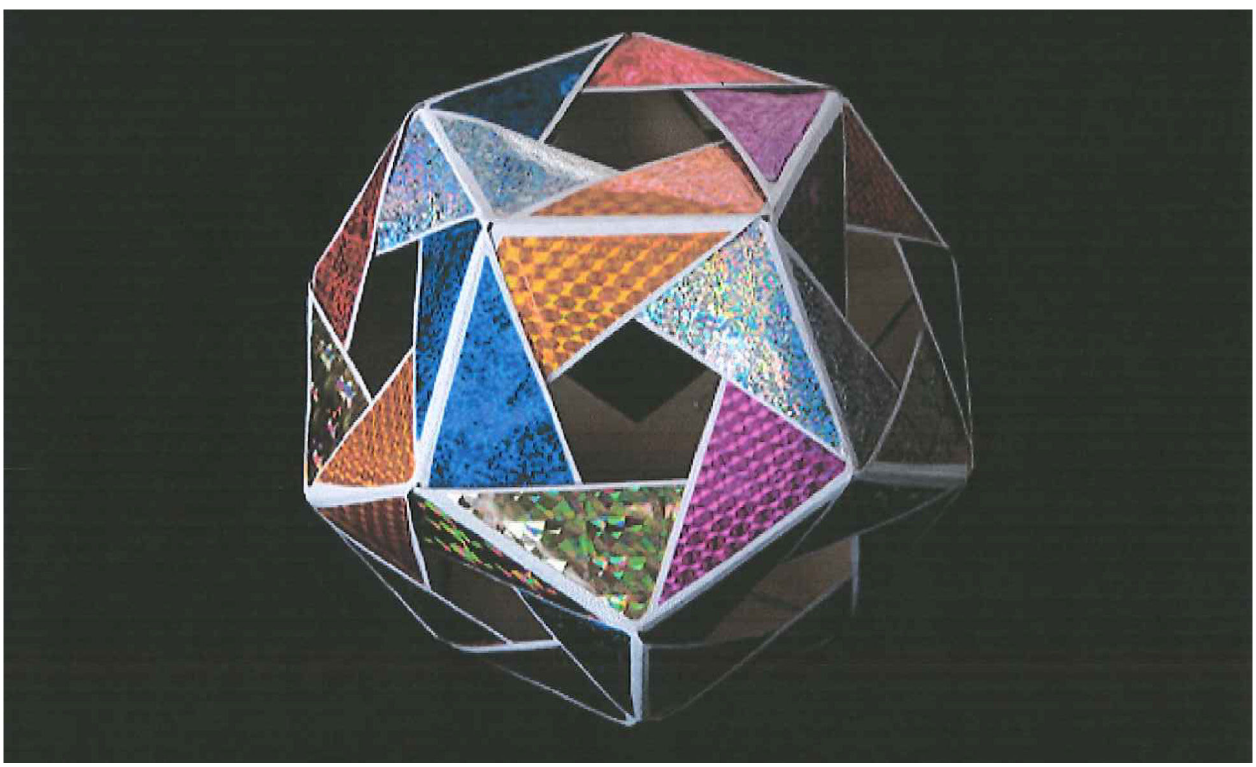

Figure 6 The Golden Dodecahedron.

This was the beginning of a long study, first of geometry in the plane, and then of polyhedra. Eventually I discovered how to use the straight strips of paper to braid together each of the Platonic Solids (see [5, 8]). The reader may notice a certain pattern connected with the number of strips used in each case.

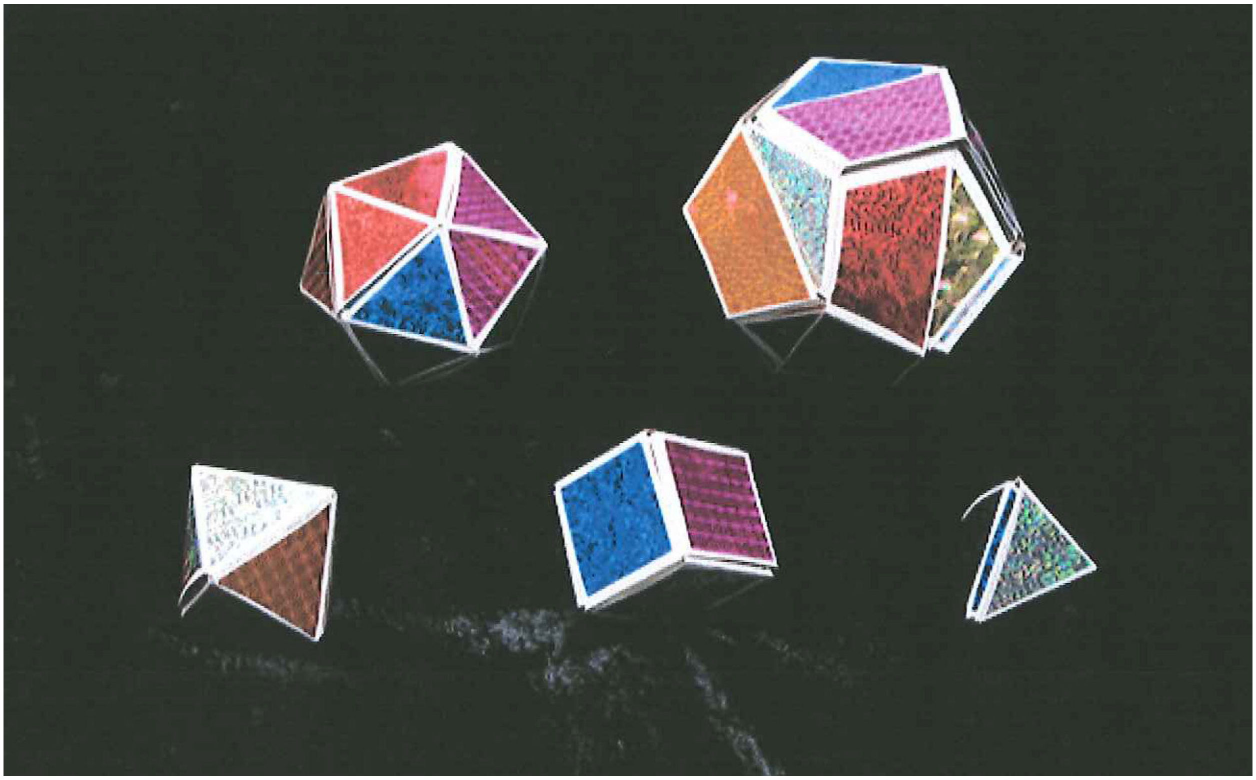

Figure 7 Braided Platonic Solids (the numbers in parentheses indicate the number of straight strips used in the construction). Top: Icosahedron (5), Dodecahedron (6). Bottom: Octahedron (4), Cube (3), Tetrahedron (2).

Next came discoveries with non-regular and non-convex polyhedra. Figure 8 shows a pentagonal dipyramid made from one strip (how many other convex polyhedra can be made from one strip and have all the faces and edges, except the beginning-end faces, covered the same number of times by a straight strip of equilateral triangles?), a woven tetrahedron, and a braided ring of tetrahedra. Instructions for building these can be found in [5, 8].

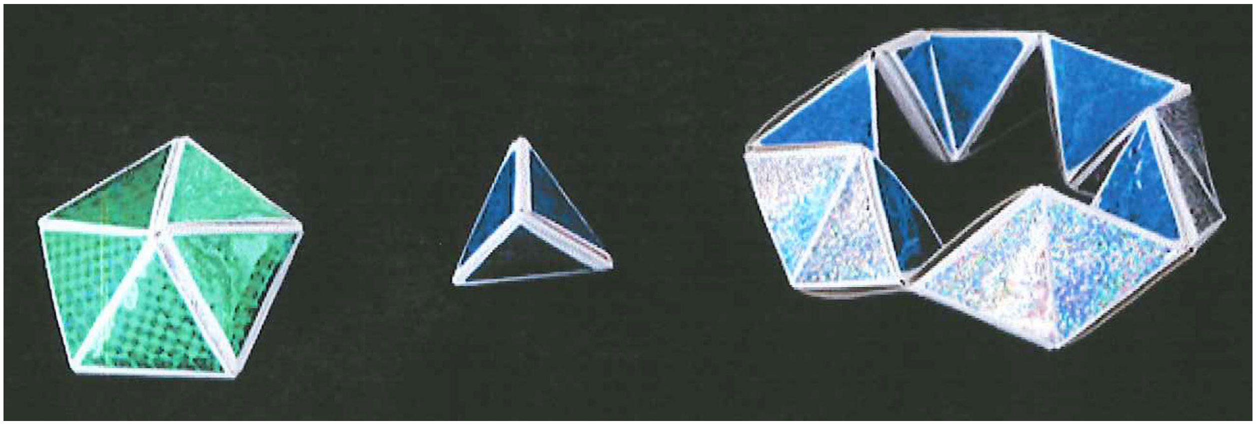

Figure 8 A pentagonal dipyramid made from one strip of 31 equilateral triangles, a tetrahedron braided from two strips of 5 triangles each, and a rotating ring of 10 tetrahedra braided from 2 strips with 42 triangles each.

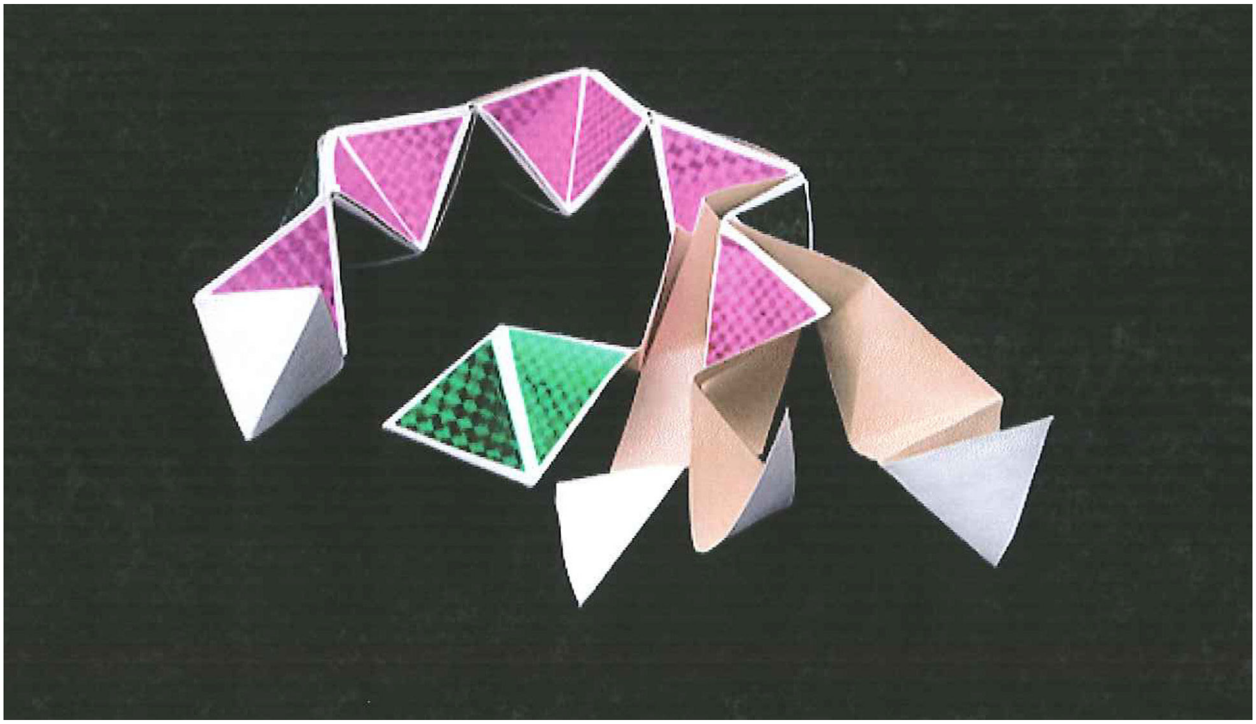

Figure 9 The rotating ring coming apart.

Ken: When did you realize that you wanted to be a mathematician? Was it difficult to become a mathematician?

To be continued...

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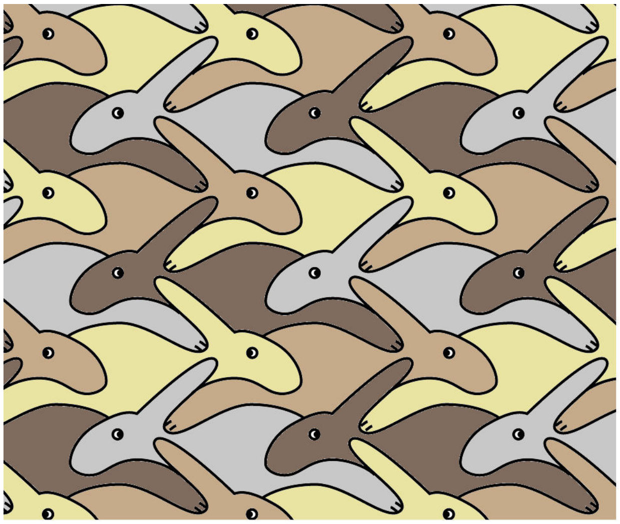
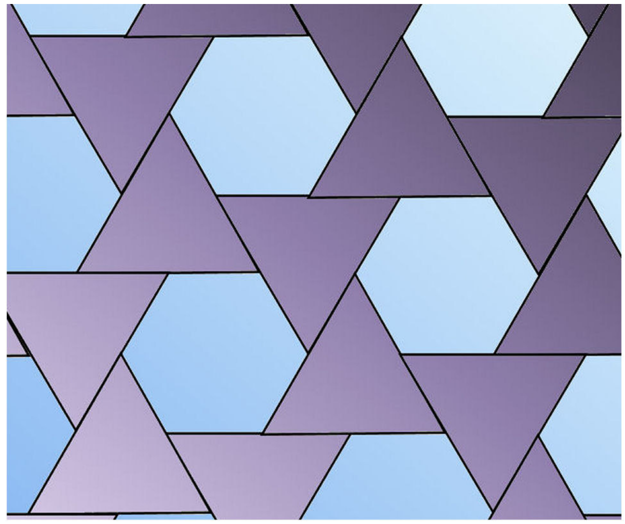
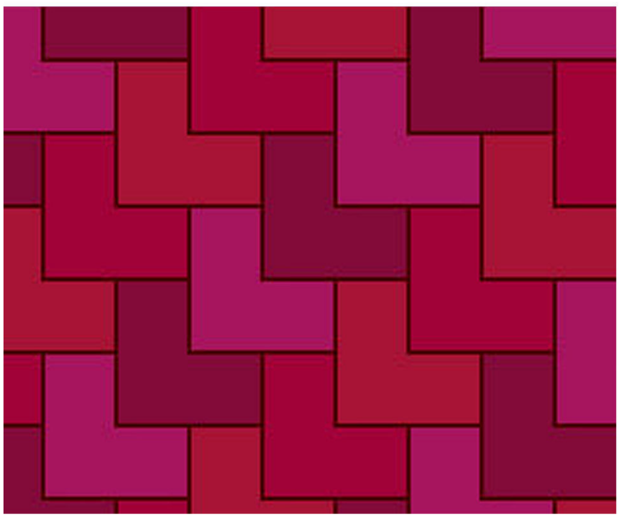
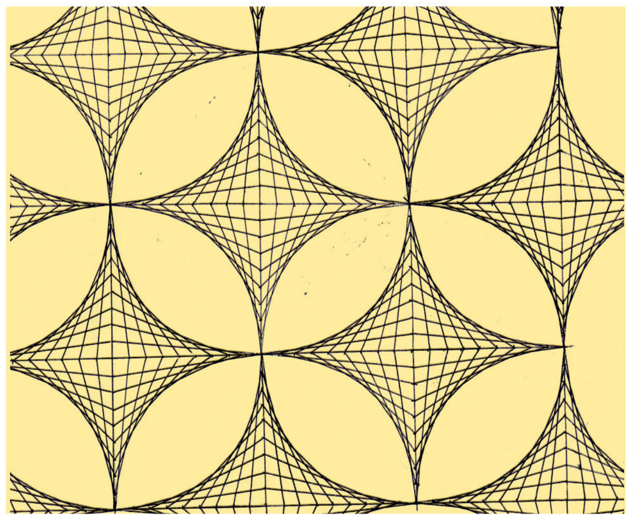
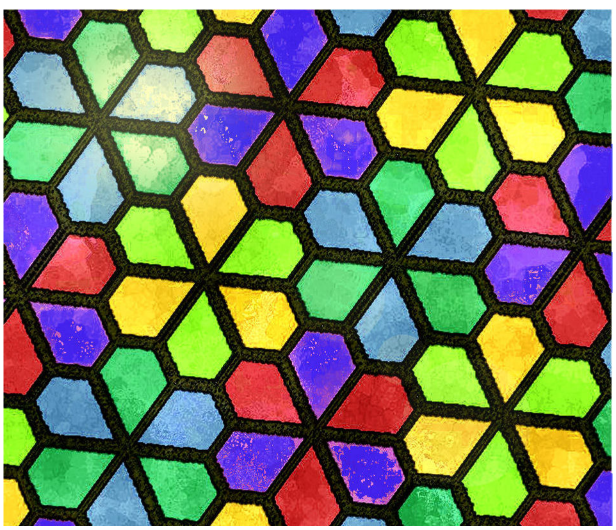
Mathematical Buffet

Tessellations


Left to right, top to bottom: Bunny tessellation by Toshia McCabe, equilateral triangle/hexagon, gnomon, grid-based tessellation by Connie Liu, floret-pentagonal, heart based tessellation by Rediet Abebe.

From left to right, top to bottom, the tessellations are by: **Pixie**, **Coolio566** and **Super Monkey Cow**, **Pixie**, **Ninja Cow**, **Channah**, **Ninja Cow**, **Horse**, **Bad Poker Face**, **Molly**, **Lily**, **Pixie**, **Lily**, **Molly**, **Fiz**, **Horse**, **Ninja Cow**, **Tigers**, **Ninja Cow**, **Pixie**, **Lily**, **Pixie**.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues her exploration of sums of the first *n* *k*th powers.

If you don't understand what Anna is doing, please read the last few installments of Anna's Math Journal. This is a continuation of her investigation into sums of kth powers.

I'll let $S_k(n)$ be the sum of the first n kth powers.

Let $S_k(n) = 1^k + 2^k + 3^k + \cdots + n^k$

I'll make a table of the results I know so far.

k	$S_k(n)$	Coefficients 1	n	n^2	n^3	n^4	n^5	n^6	n^7	n^8
0	n	0	1	0	0	0	0	0		
1	$\frac{1}{2}n(n+1)$	0	1/2	1/2	0	0	0	0		
2	$\frac{1}{6}n(n+1)(2n+1)$	0	1/6	1/2	1/3	0	0	0		
3	$\frac{1}{4}n^2(n+1)^2$	0	0	1/4	1/2	1/4	0	0		
4	$\frac{1}{30}(6n^5+15n^4+10n^3-n)$	0	-1/30	0	1/3	1/2	1/5	0		
5		0			0	5/12	1/2	1/6		
6		0				0	1/2	1/2	1/7	
7		0					0	7/12	1/2	1/8

The only pattern I can see is that the lead coefficient looks like it is $1/(k+1)$.

I added these entries later as I computed the formulas for the higher order coefficients.

Instead of trying to do the case $k = 5$, I'm going to try to do the general case and see if I can pick out a pattern.

$$n^k = S_k(n) - S_n(n-1)$$

$$S_k(n) = a_{k,0} + a_{k,1}n + a_{k,2}n^2 + \cdots + a_{k,k+1}n^{k+1}$$

$$n^k = (a_{k,0} + a_{k,1}n + a_{k,2}n^2 + \cdots + a_{k,k+1}n^{k+1})$$
$$- (a_{k,0} + a_{k,1}(n-1) + a_{k,2}(n-1)^2 + \cdots + a_{k,k+1}(n-1)^{k+1})$$

I'll carefully start comparing the coefficients.

Coefficients

n^{k+1}: 0

n^k: $a_{k,k} - a_{k,k}$

$+ (k+1)a_{k,k+1}$ ~~$a_{k,k+1}$~~

$= (k+1)a_{k,k+1} = 1 \Rightarrow \boxed{a_{k,k+1} = \frac{1}{k+1}}$

Use binomial theorem:

$$(n-1)^p = n^p - \binom{p}{1}n^{p-1} + \binom{p}{2}n^{p-2} - \cdots (-1)^{p-1}\binom{p}{p-1}n + (-1)^p$$

To perform these computations, I made frequent use of the binomial theorem, which gives the coefficients in the expansion of $(n - 1)^p$ in terms of the binomial coefficients "n choose m."

Neat, this verifies my earlier observation about the lead coefficient.

n^{k-1}: $a_{k,k-1} - \left(a_{k,k-1} - k a_{k,k} + \binom{k+1}{2} a_{k,k+1}\right)$

$= k a_{k,k} - \frac{k(k+1)}{2} a_{k,k+1} = 0$ ~~$\Rightarrow$~~

$k a_{k,k} - \frac{k}{2} = 0 \Rightarrow \boxed{a_{k,k} = 1/2}$

Curious...the second highest order coefficient is always one half...why's that?

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

I guess I'll keep on going...

n^{k-2}: $a_{k,k-2} - \left(a_{k,k-2} - (k-1)a_{k,k-1} + \binom{k}{2}a_{k,k} - \binom{k+1}{3}a_{k,k+1}\right)$

$$= (k-1)a_{k,k-1} - \frac{k(k-1)}{2}\cdot\frac{1}{2} + \frac{(k+1)(k)(k-1)}{6}\cdot\frac{1}{k+1}$$

$$= (k-1)a_{k,k-1} - \frac{k(k-1)}{4} + \frac{k(k-1)}{6} = 0$$

$$a_{k,k-1} = \frac{k}{4} - \frac{k}{6} = \frac{k}{12} \qquad \boxed{a_{k,k-1} = \frac{k}{12},\ k \geq 2}$$

Don't see anything, so I'll keep on going!

n^{k-3}: $(k-2)a_{k,k-2} - \binom{k-1}{2}a_{k,k-1} + \binom{k}{3}a_{k,k} - \binom{k+1}{4}a_{k,k+1}$

$$= (k-2)a_{k,k-2} - \frac{(k-1)(k-2)}{2}\cdot\frac{k}{12} + \frac{k(k-1)(k-2)}{6}\cdot\frac{1}{2} - \frac{(k+1)k(k-1)(k-2)}{24}\cdot\frac{1}{k+1}$$

$$= (k-2)\left(a_{k,k-2} - \frac{k(k-1)}{24} + \frac{k(k-1)}{12} - \frac{k(k-1)}{24}\right) = 0$$

$$a_{k,k-2} = k(k-1)\left(\frac{1}{24} - \frac{1}{12} + \frac{1}{24}\right) = 0 \Rightarrow \boxed{a_{k,k-2} = 0 \quad k \geq 3}$$

...one more coefficient! I sure hope I don't make a mistake!

n^{k-4}: $(k-3)a_{k,k-3} - \binom{k-2}{2}a_{k,k-2} + \binom{k-1}{3}a_{k,k-1} - \binom{k}{4}a_{k,k} + \binom{k+1}{5}a_{k,k+1}$

$$= (k-3)a_{k,k-3} + \frac{(k-1)(k-2)(k-3)}{6}\cdot\frac{k}{12} - \frac{k(k-1)(k-2)(k-3)}{24}\cdot\frac{1}{2} + \frac{(k+1)k(k-1)(k-2)(k-3)}{120}\cdot\frac{1}{k+1}$$

$$0 = a_{k,k-3} + \frac{k(k-1)(k-2)}{72} - \frac{k(k-1)(k-2)}{48} + \frac{k(k-1)(k-2)}{120}$$

$$a_{k,k-3} = k(k-1)(k-2)\left(-\frac{1}{72} + \frac{1}{48} - \frac{1}{120}\right)$$

$$= k(k-1)(k-2)\left(-\frac{10}{720} + \frac{15}{720} - \frac{6}{720}\right)$$

$$= -\frac{k(k-1)(k-2)}{720} \Rightarrow \boxed{a_{k,k-3} = -\frac{k(k-1)(k-2)}{720} = -\frac{1}{120}\binom{k}{3},\ k \geq 4}$$

Hmmm...in a way it's amazing that each time I ended up with a sum of different constants times the same polynomial...

It does seem that the coefficient $a_{k,k-p}$ is equal to some constant that only depends on p multiplied by a binomial coefficient.

Seems like $a_{k,k-p} = (\text{some constant independent of } k)\binom{k}{p} = C_p\binom{k}{p}$ $\quad(\star)$

$n^{k-(p+1)}$: $(k-p)a_{k,k-p} - \binom{k-p+1}{2}a_{k,k-p+1} + \binom{k-p+2}{3}a_{k,k-p+2}$

$$- \cdots + (-1)^{p+1}\binom{k+1}{p+2}a_{k,k+1}$$

Typical term: $(-1)^m\binom{k-p+m}{m+1}a_{k,k-p+m} \qquad 0 \leq m \leq p+1$

Let's see what happens if one of these coefficients is computed on this assumption...doing so should confirm whether or not the observation is in fact true.

If $\star$ is true: $(-1)^m\binom{k-p+m}{m+1}C_{p-m}\binom{k}{p-m}$

$$= (-1)^m C_{p-m}\frac{(k-p+m)!}{(m+1)!(k-p-1)!}\cdot\frac{k!}{(p-m)!(k-p+m)!}$$

$$= (-1)^m(k-p)C_{p-m}\frac{k!}{(m+1)!(p-m)!(k-p)!}$$

$$= (-1)^m(k-p)C_{p-m}\frac{p!}{(m+1)!(p-m)!}\binom{k}{p}$$

Interesting! It does work out! The expression is complex, but since the factor of $k - p$ cancels throughout, this computation does show that $a_{k,k-p}$ will be a constant independent of k times the binomial coefficient k choose p.

I'll focus my attention on computing these c_p's...next time!

ABB 4.27.12

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

Chocolate Lover's Index

Written by Katherine Sanden

This past week I was reminded of how math can be found in (almost) everything – including candy. My friend introduced me to Mini Charleston Chews (I'll refer to them as "Minis" throughout this article). Each Charleston Chew is a block of taffy coated in chocolate.

As I was eating the Minis with my friend, and reflecting on my memories of Original Charleston Chews (I'll refer to them as "Originals" throughout this article), I realized I was tasting a different candy than I was used to – the ratio of chocolate to taffy had changed drastically, and this actually made for a noticeable difference in taste.

If a Mini is just a tiny version of an Original, why should the ratio of chocolate to taffy change at all? To investigate this question, let's assume that the chocolate coatings of both the Mini and the Original are the same and relatively thin. This assumption allows us to model the amount of taffy and chocolate in Charleston Chews with the mathematical notions of **volume** and **surface area**.

I estimate that the dimensions of an Original are about 21 cm by 2.5 cm by 2 cm and those of the Mini are about 1 cm by 1 cm by 4 cm. Using these dimensions, we can compute the ratio of surface area (i.e. chocolate coating) to volume (i.e. taffy) in both candies, which will give us an idea of the average proportion of chocolate in each bite.

Mini Charleston Chews[1]


Original Charleston Chew[1]

Surface area of Mini:

$$2(1 \times 1) + 2(4 \times 1) + 2(4 \times 1) = \mathbf{18\ cm^2}$$

Volume of Mini: $1 \times 1 \times 4 = \mathbf{4\ cm^3}$

Surface area of Original:

$$2(21 \times 2.5) + 2(21 \times 2) + 2(2.5 \times 2) = \mathbf{199\ cm^2}$$

Volume of Original: $21 \times 2.5 \times 2 = \mathbf{105\ cm^3}$

Surface area to volume ratio of Mini: $18 / 4 = \mathbf{4.5\ cm^{-1}}$

Surface area to volume ratio of Original: $199 / 105 \approx \mathbf{1.9\ cm^{-1}}$

We can think of these ratios as a "Chocolate Lover's Index." The higher the ratio, the higher the average proportion of chocolate per bite. This Index can be used to compare chocolate coated candies whenever they have thin coatings of about the same thickness.

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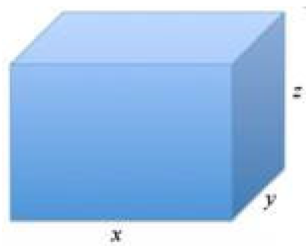
[1] Mini Charleston Chews photo by Windell H. Oskay, www.evilmadscientist.com, for licensing info: creativecommons.org/licenses/by/2.0/. Original Charleston Chew photo by Evan-Amos courtesy of en.wikipedia.org/wiki/File:Charleston-Chew-Split.jpg.

Take it to Your World

Suppose you could remake the Charleston Chew bars according to your own choice of dimensions, as long as you maintain the shape of a rectangular box. What dimensions would you choose to increase or decrease the Chocolate Lover's Index? Experiment with different shapes and sizes.



In this case you'd be working with the ratio $\frac{2(xy+yz+zx)}{xyz}$.

From this expression, we can see that the ratio will shrink if you enlarge the rectangle by the same factor in all dimensions, so if you love chocolate and you're choosing between a big and small chocolate coated apple, go for the smaller one!

What if you could totally redesign the shape of a Charleston Chew – subject to the constraint that the volume remains 4 cm^3? What shape would you choose to make a big Chocolate Lover's Index? In this case, I would probably choose something very thin – a wafer-like shape. The sky is the limit in how high we can increase the Chocolate Lover's Index – we could make it as long and thin as we wanted, until it no longer became practical as a candy to hold in your hand. Can you show mathematically that the Chocolate Lover's Index is large when the shape is like a thin wafer?

But suppose you're not much of a chocolate lover. You'd want to minimize the ratio of surface area to volume. What shape should you use (keeping to blocks of volume 4 cm^3)? In fact, it turns out that the shape to make would be a perfect cube. We can justify this mathematically using the **arithmetic-geometric mean inequality**, which tells us that the arithmetic mean of a collection of positive numbers is always greater than or equal to their geometric mean, with equality if and only if all the numbers are equal. To get our candy result, we'll compare the arithmetic and geometric means of the three numbers *xy*, *yz*, and *zx*:

$$\frac{xy+yz+zx}{3} \ge \sqrt[3]{(xy)(yz)(zx)} = \sqrt[3]{(xyz)^2} = \sqrt[3]{16}\ \text{cm}^2,$$

with equality if and only if $xy = yz = zx$. Notice that the left side of this inequality is one-sixth of the surface area. Because the volume is being held constant, we minimize the surface area to volume ratio by minimizing the surface area. The arithmetic-geometric mean inequality tells us that this minimum occurs exactly when $xy = yz = zx$, which means the block is a cube.

What would you do if you free yourself of the block constraint? Now you can make whatever shape you wish, so long as the volume is 4 cm^3. What shape would you make to minimize the Chocolate Lover's Index? It can be shown that, given a fixed volume, a sphere is the shape that will minimize the surface area. You probably intuitively understand why already. Suppose you were sitting in a cold room trying to keep warm. Would you curl up into a ball or sprawl out lying down? By curling up into a ball, you are making your body as sphere-like as possible, minimizing the surface area of your body that is exposed to the cold air around you. Can you think of other examples in your life where the ratio of surface area to volume is at play?

Our Chocolate Lover's Index gives a reasonable idea of how much chocolate you'd get in a typical bite only under a number of assumptions. The chocolate coating has to be relatively thin and applied uniformly over the entire candy. You might feel that the Chocolate Lover's Index isn't really accurate for your purposes. Perhaps you're interested in comparing nut-filled chocolate bars where there really isn't a coating at all. What Chocolate Lover's Indexes would you invent to suit your needs?

Secret Message Challenge

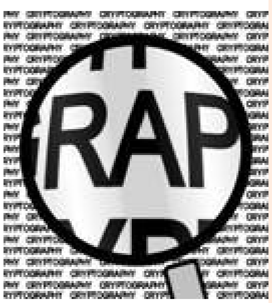
Instructions: There are four secret messages for you to decode. Each message gives a clue to a specific number. Figure out the four numbers and add them together to get a single master number. Send this master number along with your contact information to girlsanglepuzzler@gmail.com for a chance to win a prize. Entries are due by June 15, 2012.

For information on how to decipher the first 3 messages, read about Sarah Spence Adams' visit to Girls' Angle on page 23.

For more details, visit the Girls' Angle blog at girlsangle.wordpress.com

Message #1: A Caesar cipher

DOHA PZ AOL IPNNLZA ADV KPNPA WYPTL UBTILY?



Message #2: Another Caesar cipher

This one's harder than the first because all of the spaces have been removed!

BKQNBWENZEYAWNANKHHAZWJZPDAENOQIEO
NAYKNZAZ.PDEOEOZKJANALAWPAZHUKRANWJ
ZKRAN.KBWHHPDALKOOEXHAOQIO,SDEYDOQIS
EHH,EJPDAKNU,KYYQNIKNAPDWJWJUKPDAN?

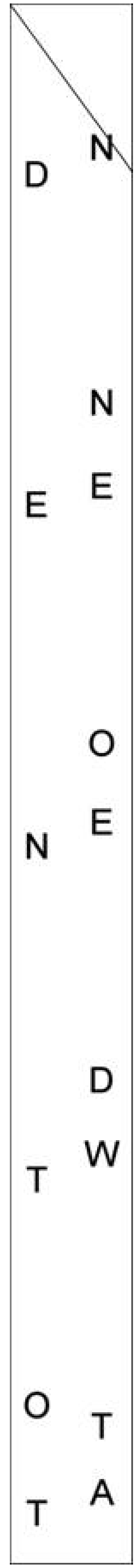
Message #3: A Vigenère cipher

Hint: The cipher key used is a four letter word that is very popular at Girls' Angle and starts with the letter M!

UN MOQ GBYXS' TUSLX CUDXV RETAGRBUS EEPEST
VLAGUQ, A VHNLX JMR TWBETYE IG VZE LJQNX.
DTAM PE TAL ZUFIQR HU FHTA OAUSQ CTY?

Message #4: A variant on the scytale cipher.

To decipher this message, cut out the strip at right and sort out the letters by grouping them according to the vertices of a "short-side pentagon" (see page 4). The first crease you need to make is indicated for you. Make all your creases mountain folds. You will find five words, one for each vertex, that explain how to get the number you need.



Errorbusters!

by Cammie Smith Barnes / edited by Jennifer Silva

In this issue, we will discuss finding **inverse functions** (when they exist). Suppose $f(x) = 3x + 5$. An error that I sometimes see students make is to say that the inverse function to f is the function $f^{-1}(x) = \frac{x}{3} - 5$, which is not true. Rather, $f^{-1}(x) = \frac{x-5}{3}$.

This error is related to the error that we discussed previously when we addressed the distributive law (in Volume 5, Number 2). The new context of this error, however, gives us an opportunity to show a valid way to find inverse functions and to come to grips with when inverse functions do and do not exist.

What is an inverse function? Recall that last time we focused on function composition. We noted that for functions $f(x)$ and $g(x)$, where the domain of f contains the image of g, the **composition** $f \circ g$ is given by $f \circ g\,(x) = f(g(x))$. The inverse f^{-1} of a function f is the function such that $f \circ f^{-1}(x) = x$ and $f^{-1} \circ f(x) = x$. In other words, if you compose a function with its inverse, the composition that you get is just the **identity function** (the function that sends every value to itself). You can think of f^{-1} as the function that "undoes" f. For example, if $f(x) = x + 2$ (the function that returns 2 more than its input), then the inverse function $f^{-1}(x) = x - 2$ (the function that returns 2 less than its input). Addition by 2 and subtraction by 2 "undo" each other: $f \circ f^{-1}(x) = (x - 2) + 2 = x - 2 + 2 = x$ and $f^{-1} \circ f(x) = (x + 2) - 2 = x + 2 - 2 = x$. (Remember that since the order of composition matters (that is, in general $f \circ g \neq g \circ f$), when we show that a function is the inverse of another, we have to check the composition in both orders.)

Similarly, if $g(x) = x/7$, then the inverse $g^{-1}(x) = 7x$. (Multiplication by 7 "undoes" division by 7.) So how do we find the inverse of a slightly more complicated function, such as $f(x) = 3x + 5$? One method is to solve the equation $f(x) = 3x + 5$ for x in terms of $f(x)$, as follows:

$$\begin{aligned} f(x) &= 3x + 5 \\ f(x) - 5 &= 3x \\ \frac{f(x) - 5}{3} &= x \end{aligned}$$

Then we replace $f(x)$ with x and x with $f^{-1}(x)$ to get $f^{-1}(x) = \frac{x-5}{3}$, being careful to keep the 5 in the numerator of the fraction. The final substitutions of x for $f(x)$ and $f^{-1}(x)$ for x reflect the fact that, for the inverse function, the roles of input (x) and output ($f(x)$) are reversed.

If the final substitutions of x for $f(x)$ and $f^{-1}(x)$ for x seem confusing, however, then try a slightly different algorithm that yields the same result. First replace $f(x)$ with y to get $y = 3x + 5$. Then switch y and x to get $x = 3y + 5$. Next solve for y in terms of x to get $y = \frac{x-5}{3}$. Finally, replace y with $f^{-1}(x)$, yielding $f^{-1}(x) = \frac{x-5}{3}$ once again.

Why does $f(x) = 3x + 5$ even have an inverse function? Let us examine this function further to answer this. First, note that f is **injective** (also known as **one-to-one**): for every output of f, there is exactly one input value. In other words, whenever we change the input value, the output value changes as well, no matter what new input we choose. This is true because if $3x_1 + 5 = 3x_2 + 5$ (that is, if the output values are the same), then we can solve this equation to get $x_1 = x_2$ (which means that the input values must also be the same). Note further that f is **surjective**: for every real number y, there exists a real number x such that $y = 3x + 5$ (this is because we can solve the equation $y = 3x + 5$ for x to get $x = \frac{y-5}{3}$). When a function is both injective and surjective (we call such a function **bijective**), then there is a one-to-one correspondence between input and output values. Hence inputs and outputs can switch roles in such a way that the new "function," which sends an output of the original function back to the input it came from, is indeed a bona fide function. Recall that the definition of function requires that, for each input, there is exactly one output. If for each output, there is also exactly one input, then the roles can safely be interchanged.

To give better insight into what injective and surjective mean, consider the function $h(x) = x^2$. It maps from the set of real numbers to the set of real numbers, but it is neither injective nor surjective. Note that both $h(2) = 4$ and $h(-2) = 4$, so that h is **not** injective. Note furthermore that, for the real number -1, there does **not** exist a real number x such that $x^2 = -1$ (so h is **not** surjective). In fact, $h(x)$ is always greater than or equal to 0. Thus, h does **not** have an inverse function.

However, if we restrict its domain, we can obtain a function that has an inverse. For example, if we restrict the domain of h to the nonnegative real numbers, then $h^{-1}(x) = \sqrt{x}$. This is because h is bijective on this smaller domain. Moreover, to solve $x = y^2$ for y in terms of x we know that we must take the square root of both sides, and we can only do this when $x \geq 0$.

Let's walk through the process of finding the inverse function for another nonlinear polynomial, $f(x) = 2x^3 + 16$. First, let's set $y = f(x)$, then switch x and y to get $x = 2y^3 + 16$. Now, let's solve for y:

$$\begin{aligned} x &= 2y^3 + 16 \\ x - 16 &= 2y^3 \\ \frac{x-16}{2} &= y^3 \\ \frac{x}{2} - 8 &= y^3 \\ \sqrt[3]{\frac{x}{2} - 8} &= y \end{aligned}$$

Hence $f^{-1}(x) = \sqrt[3]{\frac{x}{2} - 8}$. (Notice that in the penultimate step of solving for y, we were careful to distribute the denominator of 2 to both the x and the 16.)

For practice, try finding the inverses of the following functions and indicate whether or not you have to restrict the domain for the inverse function to exist. In every case, the domain and range of the function are both equal to the set of real numbers except for the last problem where both the domain and range are equal to the set of real numbers excluding zero. The answers can be found on page 28.

1. $f(x) = 4x$
2. $f(x) = 2x - 6$
3. $f(x) = |x|$
4. $g(x) = 8x - 3$
5. $g(x) = 5x^3$
6. $g(x) = \frac{x+3}{8}$
7. $h(x) = x^2 + 2x + 1$
8. $h(x) = 6x^3 - 18$
9. $h(x) = \frac{1}{x}, x \neq 0$

COACH BARB'S CORNER

by Barbara Remmers | edited by Jennifer Silva

Owning it: Fraction Satisfaction, Part 4

Your younger sister *said* she wanted to understand fractions, and your mom offered you $10 for the job. When you started to explain, however, your sister started fuming the first time she was the least bit confused. One thing led to another, and now not only is the $10 looking unlikely, but Sis is also threatening to tell what you have hidden underneath your socks.

Things may not be as bleak as they seem, though, because here comes your devoted friend, 3/7.

$\frac{3}{7}$: Hi dearies. Why all the sour faces?

Sis: She's supposed to be making me understand fractions. Instead she's confusing me worse than ever.

$\frac{3}{7}$: Well missy, I'm sure this can all be fixed. I do want to point out, though, that no one can make someone else understand something. You, too, must make an effort.

Sis: I *am* making an effort. I listen, but she makes no sense!

You: I do too make sense! She just doesn't listen!

$\frac{3}{7}$: Oh dear. I sense the dialogue has broken down. Let's see if I might assist. First we need to abide by a saying I picked up in my youth. It's as applicable to life as it is to doing math. Keep Calm and Carry On.

Sis: I get the Keep Calm part, but how can you Carry On when nothing makes any sense?

$\frac{3}{7}$: Excellent question, sweetie. You Carry On by asking questions. Calmly, of course. Now let's try again. Why don't you ask a question about your biggest point of confusion?

Sis: Huh?

You: The thing that most confused you … ask me nicely.

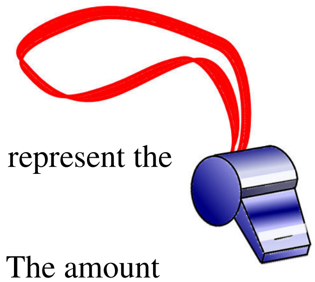
Sis: Well, just when I understood what 2/3 meant – that you divide 1 into 3 equal pieces and take 2 of them – you said it can also have another name, which makes no sense at all!

$\frac{3}{7}$: Don't attack the messenger, darling. I suspect your elder sister can handle questions much better than a character assassination. Go ahead, dearie.

Sis: How can 2/3 have another name?

You: It helps if you think about 2/3 as an amount.

$\frac{3}{7}$: Lovely. I like to think of the amount in terms of special chocolate bars that each represent the amount 1, and that are easy to break into equal-sized pieces.



You: Okay. Imagine one of those chocolate bars divided into 3 equal-sized pieces. The amount we care about, 2/3 of a chocolate bar, is 2 of those 3 pieces.

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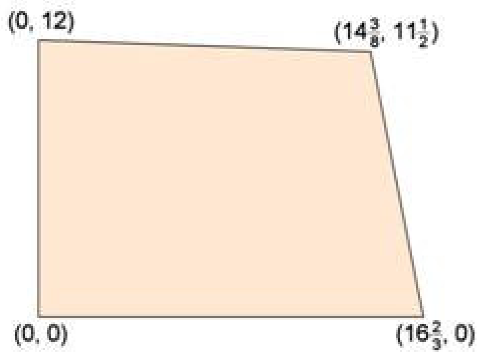
Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 10 – Meet 5 – March 1, 2012

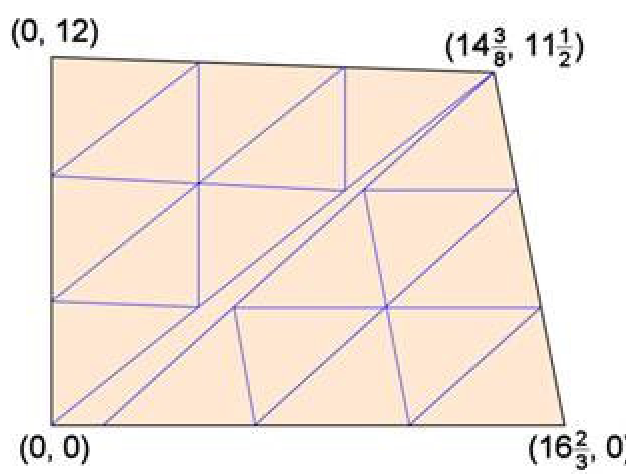
Mentors: Jennifer Balakrishnan, Samantha Hagerman, Ariana Mann, Bensey Schnip, Fan Wei



Special thanks to Petsi Pies bakery for donating a delectable brownie baked to exquisite perfection for our brownie cutting challenge. The order of the day: Figure out how to cut the unusually shaped brownie into 19 equal pieces. To describe the shape, it's useful to plot the brownie on a coordinate grid with each unit representing one inch (see figure at right). The corners could be placed onto the points with coordinates (0, 0), (0, 12), (14 3/8, 11 1/2), and (16 2/3, 0).

Several members computed the total area of the brownie and then divided by 19 to figure out that each piece should be 9 7/12 square inches. Then things started to get really difficult because the shape was so unusual. Little progress was made during the first half of the meet. During the break, we asked the girls to dream. In your wildest dreams, what do you wish were possible? Dream something definite and then check to see if the dream can actually be realized.

One member dreamt that the brownie could be cut along one of its diagonals, reducing the problem to the dissection of two triangles. To see if this dream is just a fantasy, the thing to check is whether the area of one of the resulting triangles is an integral multiple of 9 7/12. If you slice along the diagonal that connects (0, 12) with (16 2/3, 0), then the lower triangle will have area ½ (12)(16 2/3) = 100 square inches, but 100 is not an integral multiple of 9 7/12. However, if you slice along the other diagonal, then the upper triangle will have area ½ (12)(14 3/8) = 86 ¼ square inches, and, very fortunately, 86 ¼ = 9 (9 7/12). Thus, cutting along the diagonal from (0, 0) to (14 3/8, 11 1/2) reduces the problem to splitting one triangle into 9 equal pieces and the other into 10. The figure shows the cutting scheme we ended up using. Can you dream up another way?



Session 10 – Meet 6 – March 8, 2012

Mentors: Rediet Abebe, Jennifer Balakrishnan

Special Guest: Julie Yoo, Kyruus

Julie Yoo explained how start-up companies are financed. She described the basic ideas behind the process of share allocation, budget determination, and fundraising, drawing on her own experience working for various start-ups, such as Endeca Technologies, and as the co-founder of Kyruus. We hope her visit will inspire some of our members to create and lead companies of their own someday. Julie also generously invited our members to visit her at work, something that one of our members actually took her up on. Thank you for that, Julie!

<u>Session 10 – Meet 7 – March 15, 2012</u>

Mentors: Jennifer Balakrishnan, Samantha Hagerman, Ariana Mann, Charmaine Sia

We began a multi-meet math treasure hunt! Also, some girls began working on tessellations for this issue's *Mathematical Buffet* (see page 9).

<u>Session 10 – Meet 8 – March 22, 2012</u>

Mentors: Samantha Hagerman

More treasure hunt and tessellations!

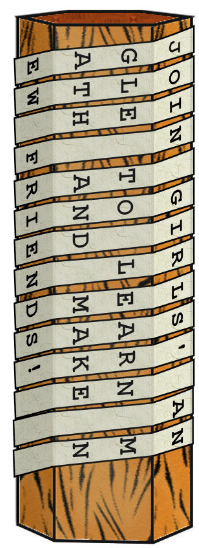
<u>Session 10 – Meet 9 – April 5, 2012</u>

Mentors: Rediet Abebe, Jennifer Balakrishnan, Connie Liu, Jennifer Melot

More treasure hunt and tessellations!

<u>Session 10 – Meet 10 – April 12, 2012</u>

Mentors: Jennifer Balakrishnan, Samantha Hagerman, Jennifer Melot, Fan Wei



A scytale.

Special Guest: Sarah Spence Adams, Olin College

Sarah began with a brief history of secret codes. The earliest secret messages were made secret simply by somehow hiding the message. One method was to shave a messenger's head, write the message on the shaved scalp, and let the hair grow back to cover up and hide the message. To retrieve, the recipient would shave the messenger's head and read the messenger's scalp! Scrambling or altering of messages came later. An early method for scrambling a message is the **scytale cipher** (see figure above): take a tape and wrap it around a cylinder. Then write the code on the tape. When the tape is unraveled, it will appear to contain random letters. The intended recipient would need a cylinder of the same diameter to decode the message.

Next, Sarah discussed **Caesar ciphers**. Named after the Roman emperor Julius Caesar, the Caesar cipher cyclically shifts letters around the alphabet a certain number of places. Supposedly, Caesar himself used a 3 shift which Sarah demonstrated by enciphering "PIE" to "SLH." She gave the girls green and red index cards with Caesar enciphered messages to decode. Green cards had messages that included spaces between words. Knowing word lengths is a big clue. To remove this clue, the red cards had messages with all spaces deleted.

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Some members deciphered their secret message by making a table that showed every shift of a group of letters. They'd look for the row that had a snippet of text that made sense. To combat that technique, Sarah introduced the **Vigenère** cipher. The Vigenère cipher is a based on the Caesar cipher, but is slightly more sophisticated. In the Vigenère cipher, letters are shifted according to a set pattern. Sarah illustrated by shifting to a pattern of repeated "CAT"s, with the letter A being regarded as having value zero. Thus, the "C" represents a shift by 2, the "A" by zero (which is the same as not shifting at all), and the "T" by 19. To encipher "I AM AWESOME" using this Vigenère cipher, the first letter, "I" is shifted by "C" or 2 to become "K", the next letter, "A" (in "AM") is shifted by the "A" in "CAT," so remains "A." Next, the "M" in "AM" is shifted by "T" or 19 to become "F", and so on. Notice that the next "A" that begins "AWESOME" is shifted by the "C" in "CAT," so becomes "C." This shows how the Vigenère cipher can encode the same letter differently. It is this property that renders the technique the members developed to decode their Caesar ciphers unusable. The word "CAT" is called the **cipher key**. To decode, the recipient needs to know the cipher key.

When there's a cipher key, there's a problem: How do you communicate the cipher key to the recipient? You end up having to communicate two secret codes, the message and the key! But if you're going to secretly deliver the key, why not just secretly deliver the message? This issue was resolved with the invention of **Public Key Cryptography**, which was invented in the 70's.

Test your deciphering skills on page 15. If you can decipher the messages and find the numbers the messages refer to by June 15, 2012, enter our free raffle for a chance to win a prize! For more details, please visit our blog at girlsangle.wordpress.com.

Session 10 – Meet 11 – April 26, 2012

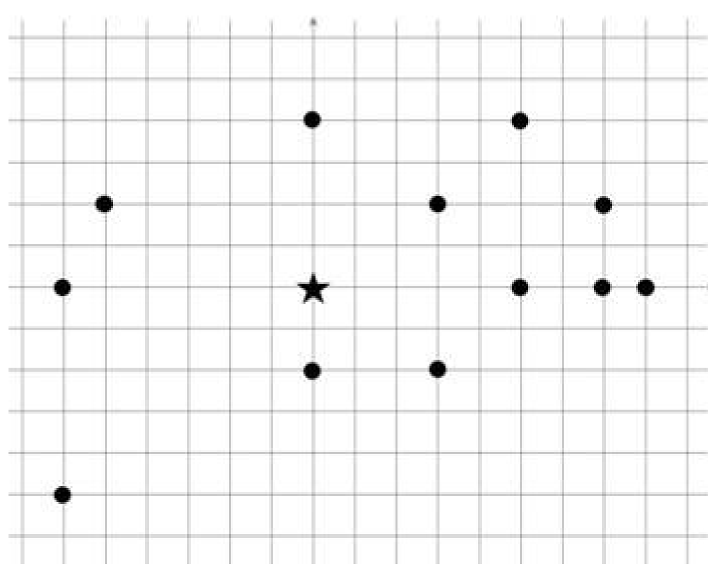
Mentors: Rediet Abebe, Jennifer Balakrishnan, Samantha Hagerman, Connie Liu, Jennifer Melot

Members finished up the multi-meet treasure hunt! Congratulations!

The 2012 New York/New Jersey FIRST Robotics Career Expo – March 18, 2012

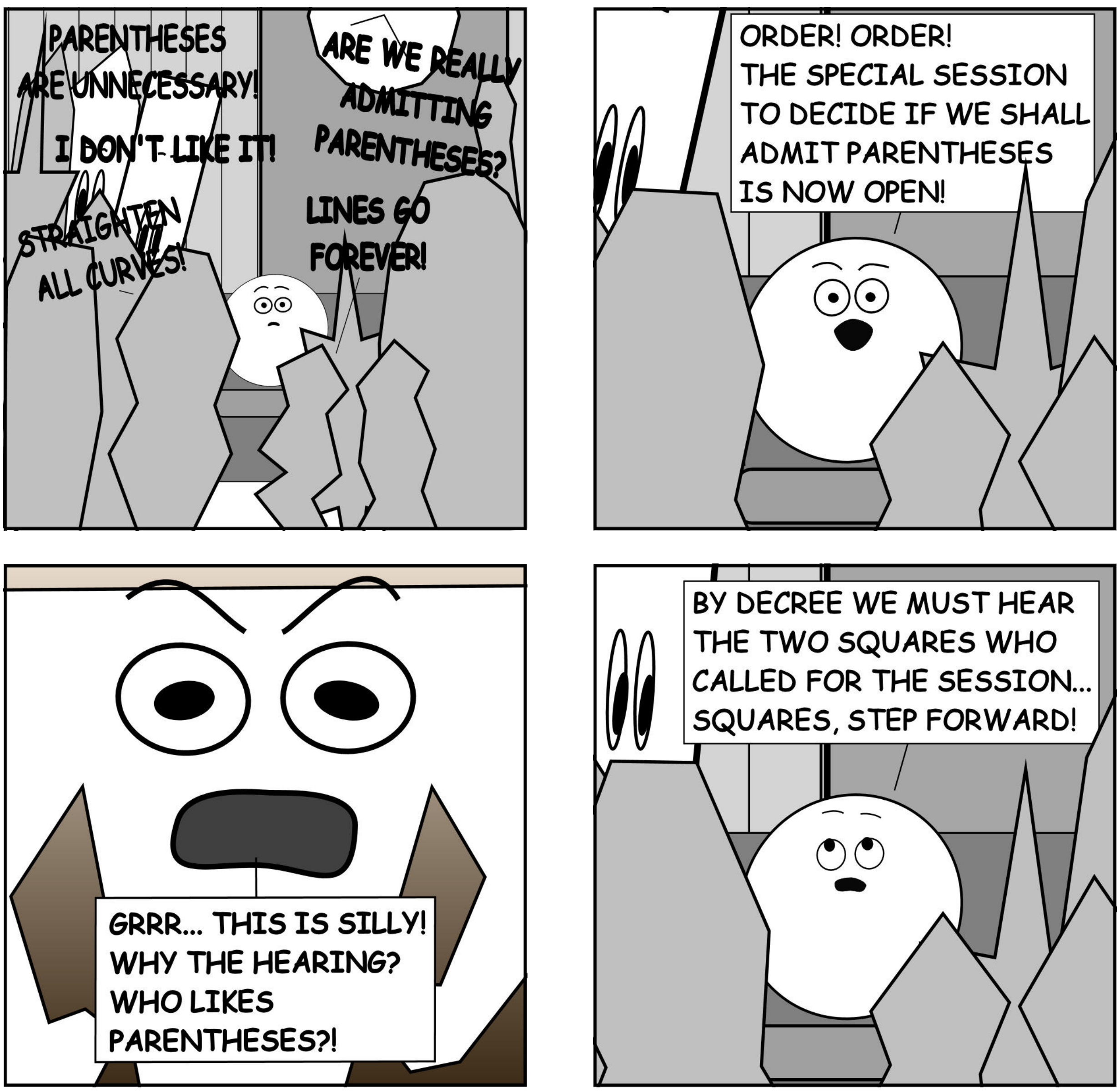
Here's a raffle problem from our booth at the FIRST Robotics Career Expo in New York City:

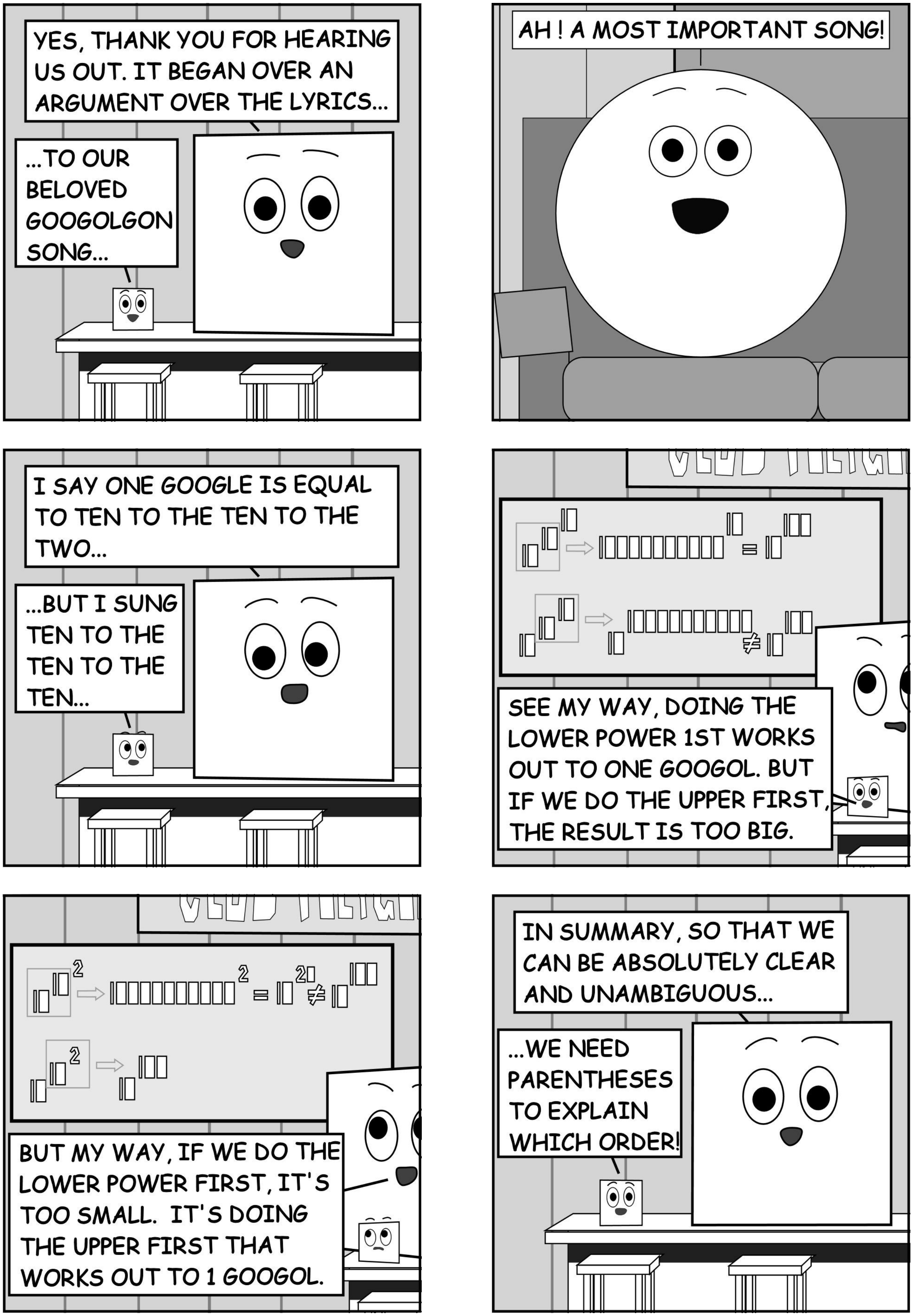
The dots in the graph mark the locations of various places that Marion must visit to run a bunch of errands. She begins at the star and travels up, down, left, or right only. She picks a direction and walks until she reaches the first unvisited place in that direction. She then picks another direction and walks until she reaches the first unvisited place in the new direction. She continues in this manner and manages to visit every single place. Can you reconstruct her path?

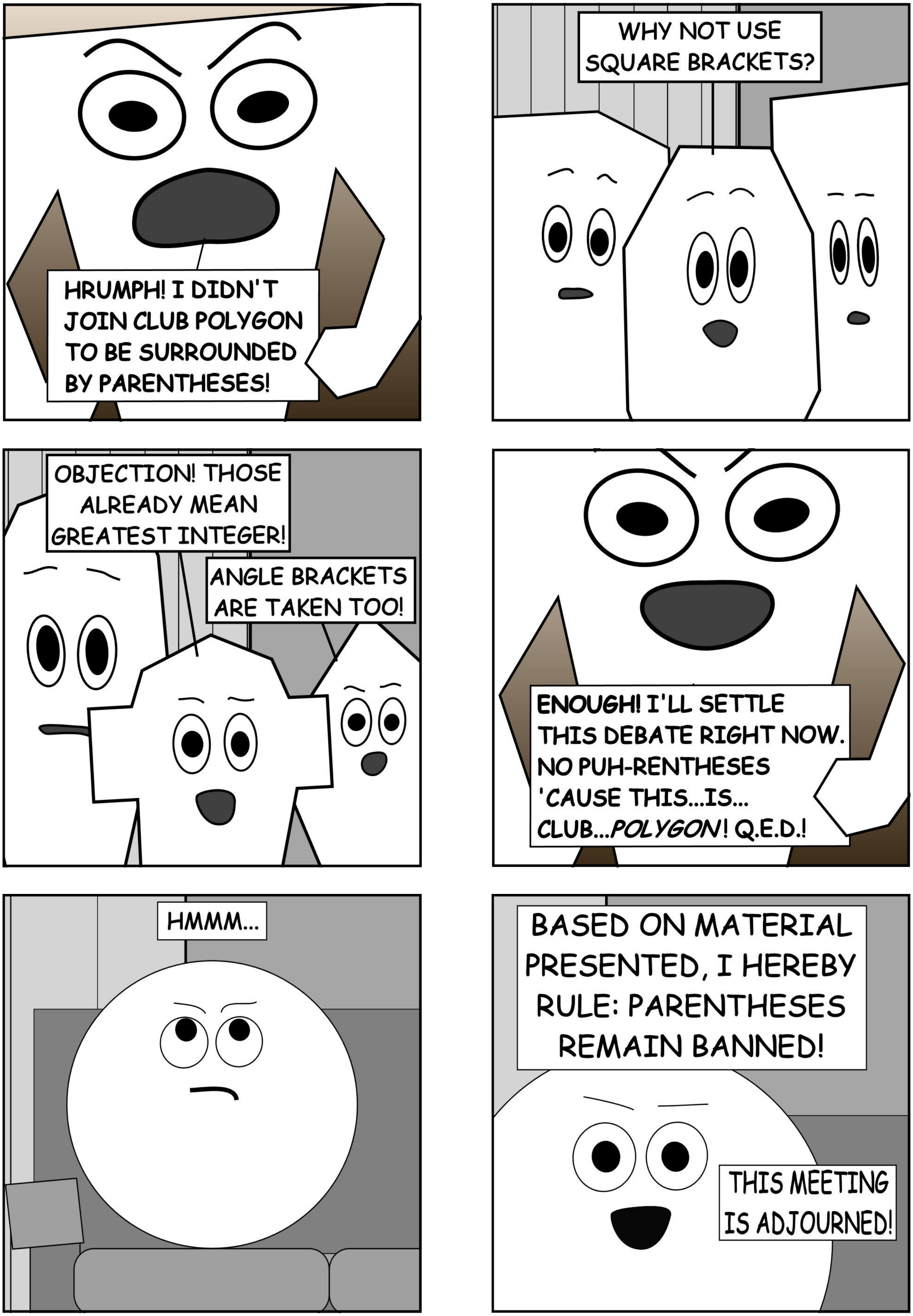


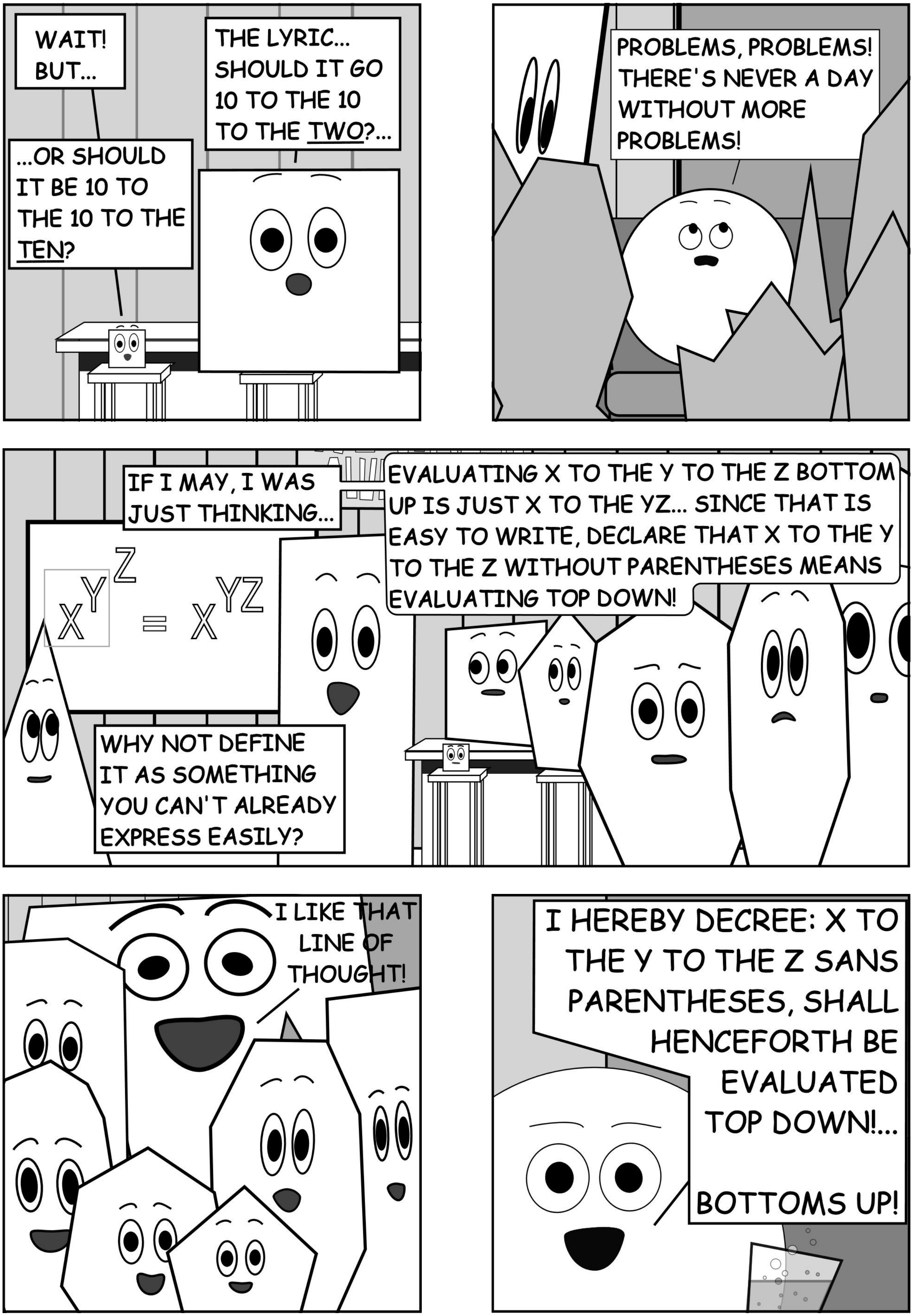
It Figures!

by CKFan




YES, THANK YOU FOR HEARING US OUT. IT BEGAN OVER AN ARGUMENT OVER THE LYRICS...
...TO OUR BELOVED GOOGOLGON SONG...
AH ! A MOST IMPORTANT SONG!
I SAY ONE GOOGLE IS EQUAL TO TEN TO THE TEN TO THE TWO...
...BUT I SUNG TEN TO THE TEN TO THE TEN...
SEE MY WAY, DOING THE LOWER POWER 1ST WORKS OUT TO ONE GOOGOL. BUT IF WE DO THE UPPER FIRST, THE RESULT IS TOO BIG.
BUT MY WAY, IF WE DO THE LOWER POWER FIRST, IT'S TOO SMALL. IT'S DOING THE UPPER FIRST THAT WORKS OUT TO 1 GOOGOL.
IN SUMMARY, SO THAT WE CAN BE ABSOLUTELY CLEAR AND UNAMBIGUOUS...
...WE NEED PARENTHESES TO EXPLAIN WHICH ORDER!


HRUMPH! I DIDN'T JOIN CLUB POLYGON TO BE SURROUNDED BY PARENTHESES!
WHY NOT USE SQUARE BRACKETS?
OBJECTION! THOSE ALREADY MEAN GREATEST INTEGER!
ANGLE BRACKETS ARE TAKEN TOO!
ENOUGH! I'LL SETTLE THIS DEBATE RIGHT NOW. NO PUH-RENTHESES 'CAUSE THIS...IS... CLUB...POLYGON! Q.E.D.!
HMMM...
BASED ON MATERIAL PRESENTED, I HEREBY RULE: PARENTHESES REMAIN BANNED!
THIS MEETING IS ADJOURNED!


WAIT! BUT...
...OR SHOULD IT BE 10 TO THE 10 TO THE TEN?
THE LYRIC... SHOULD IT GO 10 TO THE 10 TO THE TWO?...
PROBLEMS, PROBLEMS! THERE'S NEVER A DAY WITHOUT MORE PROBLEMS!
IF I MAY, I WAS JUST THINKING...
EVALUATING X TO THE Y TO THE Z BOTTOM UP IS JUST X TO THE YZ... SINCE THAT IS EASY TO WRITE, DECLARE THAT X TO THE Y TO THE Z WITHOUT PARENTHESES MEANS EVALUATING TOP DOWN!
$X^{Y^Z} = X^{YZ}$
WHY NOT DEFINE IT AS SOMETHING YOU CAN'T ALREADY EXPRESS EASILY?
I LIKE THAT LINE OF THOUGHT!
I HEREBY DECREE: X TO THE Y TO THE Z SANS PARENTHESES, SHALL HENCEFORTH BE EVALUATED TOP DOWN!...
BOTTOMS UP!

Calendar

Session 10: (all dates in 2012)

January	26	Start of the tenth session!
February	2	
	9	Meike Akveld, ETH Zürich
	16	
	23	No meet
March	1	
	8	Julie Yoo, Kyruus
	15	
	22	
	29	No meet
April	5	
	12	Sarah Spence Adams, Olin College
	19	No meet
	26	
May	3	

Session 11: (all dates in 2012)

September	13	Start of the eleventh session!
	20	
	27	
October	4	
	11	
	18	
	25	
November	1	
	8	
	15	
	22	Thanksgiving - No meet
	29	
December	6	

Here are answers to the *Errorbusters!* problems on page 17.

1. $f^{-1}(x) = x/4$
2. $f^{-1}(x) = \dfrac{x+6}{2}$
3. No inverse
4. $g^{-1}(x) = \dfrac{x+3}{8}$
5. $g^{-1}(x) = \sqrt[3]{\dfrac{x}{5}}$
6. $g^{-1}(x) = 8x - 3$
7. No inverse.
8. $h^{-1}(x) = \sqrt[3]{\dfrac{x+18}{6}}$
9. $h^{-1}(x) = \dfrac{1}{x},\ x \neq 0$

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) comes with membership. We are working hard to lower the cost of the Bulletin. Until we do, however, nonmembers can receive the printed version by becoming a Bulletin Sponsor. Please contact us if interested.

The Bulletin targets girls roughly the age of current members. Each issue is likely to contain some material that feels very challenging or difficult to understand. If you are a member or Bulletin Sponsor and have any questions about the material, feel free to ask us about it!

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

How do I join? Membership is granted per session. Members have access to the club and receive a printed copy of the Girls' Angle Bulletin for the duration of the membership. You can also pay per meet, but it is slightly more expensive. We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply. If you cannot attend the club, you can purchase a **Remote Membership** which comes with a year-long subscription to the Bulletin and a 25% discount for any club meet attended. Remote members may email us math questions (although we won't do people's homework!).

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, assistant professor, UCSF Medical School
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, Tamarkin assistant professor, Brown University
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For (please circle): Membership Remote Membership

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

For **membership**, please fill out the information in this box. **Bulletin Sponsors** may skip this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a one session Membership
  - ☐ $108 for a one year Remote Membership
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at $20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

Girls' Angle

A Math Club for Girls