Girls' Angle Bulletin

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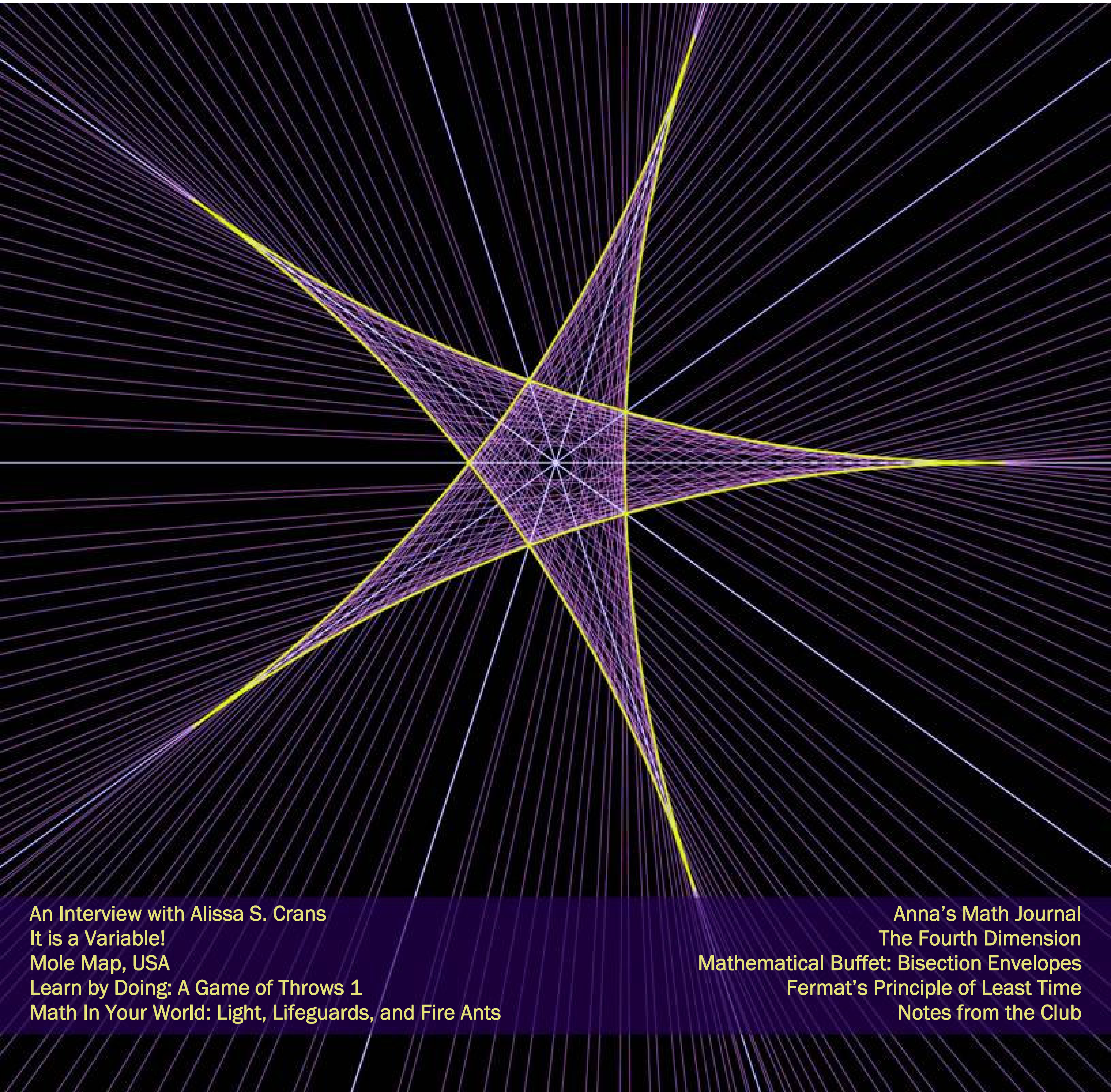
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Mathematics presents a never-ending series of conceptual hurdles to overcome. Grasping negative numbers is one of the earliest. Another hurdle is the mastery of variables. Yet another is developing comfort with spaces of high dimension. Imaginary numbers is another and the concept of infinity is a big one. In this issue, we separately address variables and the fourth dimension. To learn new concepts, play with them. Read articles that involve them, talk about them, use them, and don't give up!

- Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: The bisection envelope of a regular pentagon. For more on bisection envelopes and an explanation, see the *Mathematical Buffet* on page 22.

An Interview with Alissa S. Crans

Don't be shy about asking lots of questions!

Dr. Alissa S. Crans is an Associate Professor of Mathematics at Loyola Marymount University. She is also the Associate Director of Diversity and Education at the Mathematical Sciences Research Institute.

Ken: How did you become interested in mathematics?

Alissa: I always liked math, so I took as much of it as I could and learned as much as I could. I was very lucky to have teachers that inspired and challenged me. In the 5th grade, Mr. Boots took the time to teach me how to cross multiply (or cross cancel) when multiplying fractions before he taught the rest of the class. In the 6th grade, Mr. Peschman would put problems on our homework and tests specifically to challenge me. Also, my dad was a math major in college, so all throughout elementary and middle school, I would sit down with him after dinner and explain my math homework to him. If I had made a mistake on any problem, he'd ask me to show him how I did it, and then I would see my error and correct it. So for me, mathematics was a very social, fun activity.

In college I starting learning more abstract math and it was even more exciting and fun than the mathematics I had learned previously! I remember when I started learning about infinity and just feeling like my mind was being blown. For example, we were asked whether we thought there are more counting numbers {1, 2, 3, ...} or even counting numbers {2, 4, 6, ...}. Of course we all said counting numbers because there are clearly twice as many of those than the even ones. But we were wrong! These collections of numbers are the same size! You can see this by matching them up with each other: We match 1 with 2, 2 with 4, 3 with 6, 4 with 8, and so on. It was just incredible and, at the time, shocking!

Ken: On your website, you quote the mathematician Sonia Kovalevskaya where she says that a mathematician must have a spirit like that of a poet. Can you please give an example of something in math that illustrates this claim? Where is the poetry in mathematics?

Alissa: I don't think of math as being exactly like poetry, but I like this quote because it likens mathematics to an art. Mathematicians find beauty in their work and are extremely creative when posing and answering questions. We all love what we do and often appreciate mathematics as it is, regardless of whether it has a practical or real-life application. I often ask my students what the practical application is in a painting or sculpture or novel or piece of music. Mathematics is no different – to many people it is beautiful, interesting, and fun on its own and can be enjoyed and appreciated much in the same way that we appreciate and enjoy visiting museums, reading poetry, or attending concerts.

To many people, thinking of math as an art may seem strange because I think that when many people talk about mathematics, they are thinking about arithmetic computation and various formulas and rules they were told to memorize such as the quadratic formula, SOH-CAH-TOA, the Pythagorean theorem and the like. But when I talk about mathematics, I'm thinking about patterns and symmetries – like understanding not just how to use the Pythagorean theorem to compute something, but why it is so and what its implications are. For instance, in my own mathematical research, I rarely perform numerical computations. In fact, I can't remember the last time I wrote down a number while working on my own research!

This brings up another aspect of mathematics. Math is a huge subject! So if you don't particularly like the math you're doing in school, please don't conclude that math just isn't your thing. For example, while many mathematicians love studying the patterns in numbers, there are also lots of mathematical fields that have nothing to do with numbers, like topology, and in particular knot theory, which is one of my main research interests.

Ken: In your research statement, you describe your desire to find definitions for certain concepts, such as "higher-dimensional cocycles." I think the idea of finding the definition of something is new to many K12 students because in K12 math, definitions are generally supplied by a textbook or teacher. Could you please explain this process of defining something? How do you know if you have succeeded?

Alissa: This is an excellent question!

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the rest of this interview with Dr. Crans, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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It is a Variable!

by Timothy Chow[1]

Have you ever played the game, “Twenty Questions”? Your friend secretly thinks of a person, place, or thing, and you have to guess what your friend is thinking of by asking at most twenty questions. Your friend has to answer your questions truthfully. Also, you must ask questions to which the answer is either “yes” or “no”; otherwise you could just cheat by asking your friend, “What are you thinking of?” and your friend would have to tell you!

Here’s how a game might go. Your friend says, “I’m thinking of a thing.”

“Is it bigger or smaller than a person?”

“Yes.”

“Oops...I mean, is the thing you’re thinking of bigger than a person?”

“No.”

“Is it something you own?”

“Yes.”

“Is it an iPad?”

“No.”

“Is it bigger or smaller than...I mean, is it smaller than an iPad?”

“Yes.”

“Is it electronic?”

“No.”

“Is it something you can eat?”

(Laughing) “No.”

“Is it something you use at school?”

“Yes.”

“Is it a book?”

“No.”

“Is it a pencil?”

“Yes! Lucky guess!”

Twenty Questions is a fun game, but what I want to focus on is not the game itself, but a certain word that was used over and over during the course of the game. It is a very common word that

[1] This work supported in part by a grant from MathWorks.

we use all the time without even thinking about it. Which word? Well, I just said it! And I just said it again. The word I want to focus on is "it."

Notice that in every question, you used the word "it." Let's think for a moment why that is. After all, you didn't *have* to use the word "it." In each question, you could have said "the thing you're thinking about" instead of "it." But of course, "it" is much shorter and faster to say, so why say five words when one word will do?

The word "it" is what English teachers call a **pronoun**. A word for a specific thing, such as "pencil," "book," "iPad," and so on, is called a **noun**. But the word "it" is different. "It" can refer to anything. A pencil is not a book, so you cannot use the word "pencil" to refer to a book. But "it" can refer to a pencil or to a book, or to anything at all, as long as both the speaker and the listener have a common understanding of what "it" is. In other words, what "it" refers to **varies** with the context. This variability is what distinguishes pronouns such as "it," "he," and "she" from nouns.

There is something else to notice about pronouns. Usually, a pronoun is simply used as shorthand for a noun, and you could just as well use the noun instead. For example, suppose you say, "My aunt is a teacher. She teaches science." You could instead say, "My aunt is a teacher. My aunt teaches science." The meaning would be the same, and using the pronoun "she" just shortens the second sentence. But let's think about "Twenty Questions" again. In our example, "it" turned out to be a pencil. In our questions, could we have used "pencil" instead of "it"? Well, we could have, *if* we had known that "it" was a pencil, but of course the whole point of the game was that we didn't know what "it" was, and had to keep asking questions. Pronouns have the nice feature that they can be used even when the thing being referred to is **unknown**, or only partially known.

By now you may have guessed what all this discussion of "Twenty Questions" and "it" has to do with mathematics. I would like to suggest that you think of variables (such as the variables "x" and "y" that appear all the time in algebra) as pronouns. For example, suppose we are faced with two equations like this:

$$\begin{aligned} 2x + 5y &= 22 \\ 6x - y &= 2 \end{aligned}$$

What does this mean? It means that there are two secret numbers, about which we have some information, and our job is to discover what they are. To state the information about the secret numbers, we use pronouns, because the numbers are unknown. The pronoun "it" would work just fine, except for one thing: Since there are *two* unknown numbers, which one would we mean by "it"? We need two different pronouns. For example, we could refer to the first number as "her" and the second number as "him." Then the information we have could be stated this way:

"2 times her plus 5 times him equals 22."

"6 times her minus him equals 2."

These two sentences are exactly equivalent to the equations "$2x + 5y = 22$" and "$6x - y = 2$." So if they are equivalent, why do mathematicians use "x" and "y" (which they call **variables**) instead of “her” and “him”? After all, it is easier to read sentences with familiar English words like “her” and “him” than it is to read an equation with funny symbols like “x” and “y” in them.

There are a couple of reasons for using variables. The most important one is that mathematicians often deal with complicated problems for which many pronouns are needed. In English, there are only a few pronouns, and there just aren’t enough to handle all the problems that mathematicians need to solve.

A second reason is that using variables allows us to express the information we have very concisely. For simple mathematical problems with only one or two variables, it may be possible to keep all the information in your head without using variables. But for more complicated mathematical problems, keeping track of all the information becomes hard, and variables allow you to write down the information neatly and concisely. This is very important for a complicated problem for which the process of **solving** (that is, determining what the secret numbers are) cannot be done in your head, but requires several steps that must be carried out systematically with 100% accuracy.

Speaking of solving, one question that many people have about variables is, when I am solving equations that involve variables, what rules am I allowed to apply? For example, when I see two x’s, can I cancel them? Since I don’t know what x is, how do I know what I can or can’t do with x?

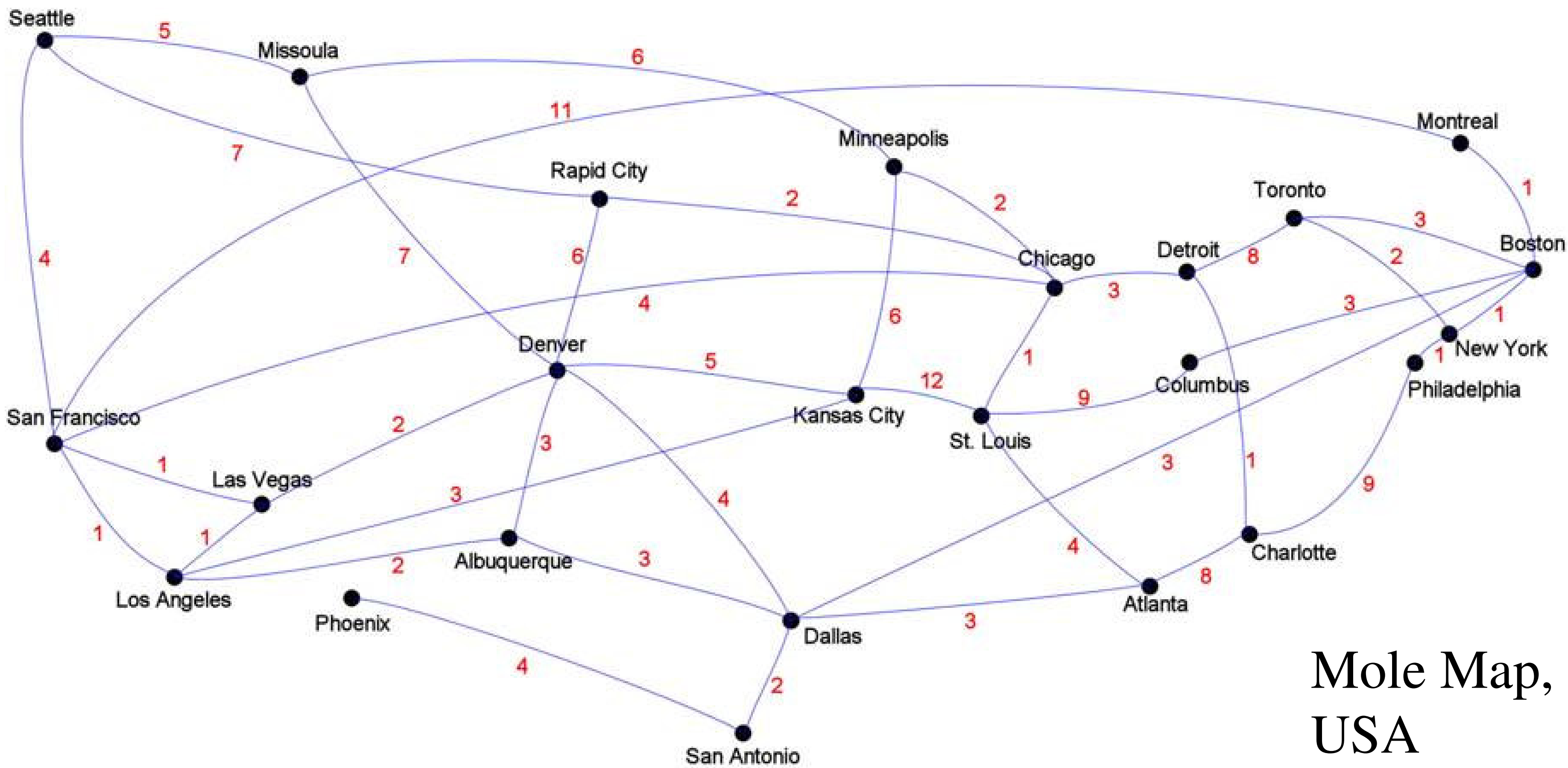
The answer is, you can apply a rule to an equation involving x if the rule would apply *no matter what number x is*. But if there is even one exception – if there is even one possible number that x could be where the rule would not apply – then we cannot apply the rule.

Think back to “Twenty Questions.” After you learn that “it” is smaller than an iPad, would you bother to ask, “Is it smaller than a TV?” Not if you’re smart! Even though you don’t know what “it” is, you do know that “it” is smaller than an iPad, and since an iPad is smaller than a TV, “it” must be smaller than a TV, no matter what “it” is. Asking if it is smaller than a TV would be a waste of one of your twenty questions. With variables, we would say it this way: If we are given that $x < y$, and we are given that $y < z$, then we can conclude that $x < z$, because we know that x must be less than z no matter what numbers x, y, and z are.

As another example, if we have the equation $x + y = x + 3$, can we subtract x from both sides of the equation to conclude that $y = 3$? The answer is yes, because no matter what number x is, we can subtract it from both sides of the equation.

Here’s a trickier example. Suppose we have the equation $3y = xy$. Can we cancel the y from both sides of the equation and conclude that $x = 3$? At first sight we might think so, because if $y = 1$ or $y = 2$ or $y = 3$ for example, we could divide both sides of the equation by y to conclude that $x = 3$. But wait! Is there any exception? Yes! What if $y = 0$? We are not allowed to divide by 0. Unless we happen to have some additional information that y is not zero, we cannot cancel y. We can cancel y only if we can cancel it no matter what y is, and if there is even one exception, then we cannot cancel.

I hope that this discussion has helped you understand variables better.



Mole Map, USA

The Map above shows various cities in the United States. Every blue edge has a red number. The number is the number of weeks it takes for Fred, the mole, to burrow between the two cities connected by the edge. The only paths that Fred can take between cities are given by the blue edges. For example, from Phoenix, Fred can only travel to San Antonio, and it will take him 4 weeks to dig his way there.

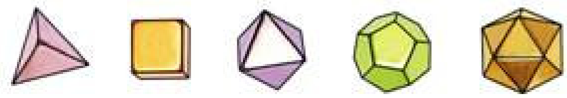
Help Fred find the quickest path from Boston to Kansas City.

After you've tried, look up Dijkstra's algorithm. Dijkstra's algorithm is a systematic way to find optimal paths. Perform this algorithm to find the quickest paths between Boston and each of the other cities. Was the original path you found to Kansas City the quickest?

Learn by Doing

A Game of Throws 1[1]

by Robert Donley



It is easy to overlook computing tools of time's past, for example, the abacus, slide rule, Napier's bones, and the ancient random number generator, the die. Everyone knows the six-sided die, but table-top gamers work with other types:

Name	Faces	Face Type	Edges	Vertices
Tetrahedron	4	triangle	6	4
Cube	6	square	12	8
Octahedron	8	triangle	12	6
Dodecahedron	12	pentagon	20	30
Icosahedron	20	triangle	30	12

Table 1. A collection of 5 different die types that we'll call a **gaming set**.

In this table, each shape is taken to be regular: all vertices look the same and all faces are congruent regular polygons of the indicated type. By using these **Platonic solids**, each die will be fair in the sense that each face comes up with the same probability as any other face.

Our motivating problem: if we lose all of our 12-sided dice, how can we compensate by using the other die types? Specifically, can we simulate the uniform, random generation of 12 numbers using just a single roll of one of the other dice in the gaming set? This is a real problem; since the 12- and 20-sided dice are close to spherical, they tend to get lost at a faster rate than the other types. We will also consider how to simulate "dice" with a number of faces not listed in Table 1 by using only one roll of a die in the gaming set.

From here on, all dice referred to in this article will be considered to be fair dice.

Problem 1. Verify the counts in Table 1. Rather than check by brute force, count as follows: for the cube, each face has 4 edges, so we find at most $4 \cdot 6 = 24$ edges. Since each edge meets exactly 2 faces, we have double counted, so there are 12 edges. A similar argument works for vertices: each face of the cube has 4 vertices, so there are at most $4 \cdot 6 = 24$ vertices. Since each vertex meets 3 faces, we have triple-counted, so there are 8 vertices.

We can simulate a 2-sided die (a.k.a. a coin) with a cubical die by declaring that odd numbers correspond to 1 and even numbers correspond to 2. The next problem explores such groupings.

Problem 2. By grouping sides together to represent a single die outcome, for what N can we simulate a fair N-sided die by using a single roll of one of the dice in the gaming set? What is the smallest N that is unobtainable? What does this problem have to do with divisibility?

Unfortunately, this method doesn't enable us to simulate a 12-sided die without using the Dodecahedral one. Also, this method will never allow us to use a die to simulate one with more sides. However, believe or not, we can turn a 4-sided die into a 12-sided die using just a single roll! (By the way, normally, when we roll a 4-sided die, we check the down face.)

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[1] This content was supported in part by a grant from MathWorks.

Problem 3. Take a 4-sided (tetrahedral) die. When we roll this die, one of its sides will be face down. Also, there will be 3 vertices on the floor, and 1 vertex pointing up. Let us agree to rotate the die about a vertical axis in the clockwise direction (when looking down upon it) until the first moment when one of the vertices on the floor points straight at the die roller. How many ways can the die be oriented after we perform this rotation? For example, one way might be with the 1 face down, the 2 facing away from the die roller, and the 3 and 4 on the other exposed faces in clockwise order from the 2. Another way might be with the 2 face down, the 1 facing away from the die roller, and the 4 and 3 on the other exposed faces in clockwise order from the 1.

(Spoiler Alert! Please try to solve Problem 3 before reading further!)

With a 4-sided die, there are 4 possibilities for the down face, and once chosen, there are 3 possibilities for the vertex that is pointing straight at the die roller. This gives 12 equally likely orientations for a given roll. So that's how to simulate a 12-sided die using a 4-sided die! We take the outcome of a roll to be the orientation that comes up after aligning as in Problem 3.

Problem 4. For each die in the gaming set, determine the total number of different orientations that the die can come up (after applying the alignment procedure described in Problem 3, which works for all the different dice in the gaming set). Combining this result and the idea in Problem 2, now what are all the possible number of sides that can be simulated using a single roll of a single die from our gaming set?

Let's focus on how the tetrahedron yields a 12-sided die. Let's figure out a way to uniquely identify each of the different orientations that can arise in Problem 3. First, we'll fix a reference orientation to be the one where 1 is on the facedown side, 2 is on the side facing away from the die roller, and 3 and 4 follow clockwise from 2. We will then specify each of the other orientations as a permutation that tells what each number in the reference orientation has become. For example, if, after we roll the die and align it, 2 is face down, 1 is facing away from the die roller, and 4 and 3 follow clockwise from 1, then we will denote this as the permutation

$$1 \to 2,\ 2 \to 1,\ 3 \to 4,\ 4 \to 3.$$

Problem 5. Write down all the permutations that can be obtained from the different orientations that can appear after rolling a 4-sided die and aligning it as in Problem 3.

Problem 6. One of the permutations you wrote down in Problem 5 corresponds to the reference orientation. Which permutations correspond to rotating the die in the reference orientation around an axis that passes through a vertex? Which permutations correspond to rotating the die in the reference orientation around an axis that passes through the midpoint of an edge?

Problem 7. How many permutations of the numbers 1 through 4 are there in total?

Problem 7 shows us that the permutations obtained from all the different orientations of the 4-sided die from Problem 3 do not account for all permutations.

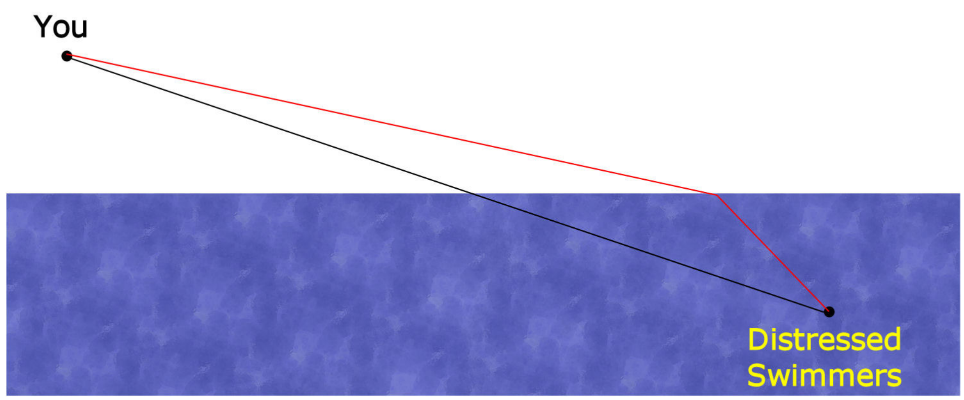
We see a powerful idea here: we have taken a potentially complicated set of symmetries, reduced them to both tidy qualitative and quantitative descriptions, and used mathematics to guarantee that our bookkeeping is complete. In the next part, we consider the symmetries of the cube – all our work with the tetrahedron will come into play.

Light, Lifeguards, and Fire Ants

by Lightning Factorial
edited by Jennifer Silva

You're a lifeguard. The sun is shining, and the beach is alive with activity. Out of the corner of your eye, you notice a couple of kids drifting a bit far out into the ocean. Something doesn't look right. You look through your binoculars. One of them looks exhausted, and the other seems to be trying to help. Now they're both struggling. They're not calling out for help, but they are definitely having trouble keeping their heads above water. You've got to get to them as fast as you can.

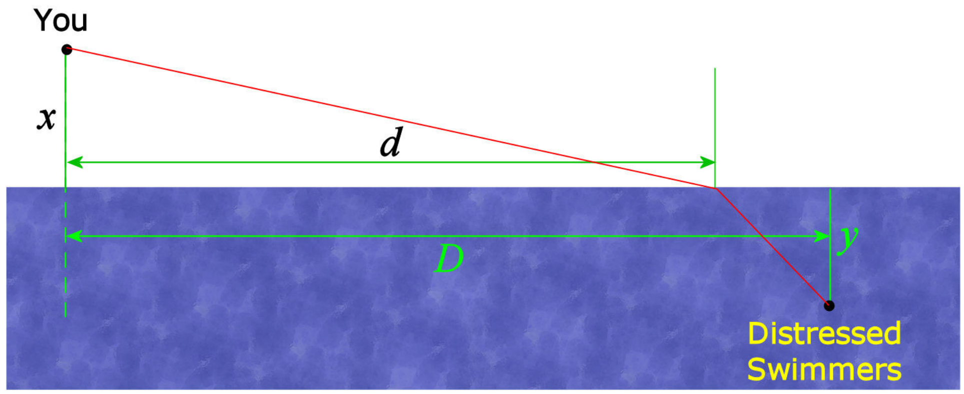
What path should you take?



You can run much faster than you can swim. This suggests that going along the straight line shortest distance path (in black in the figure) won't be the fastest since you spend more time swimming that way. Intuitively, it would be better to stay on the beach longer so you can move faster, as in the red path in the figure. But where exactly should you enter the water? As a lifeguard, this is a critical decision, because every second matters.

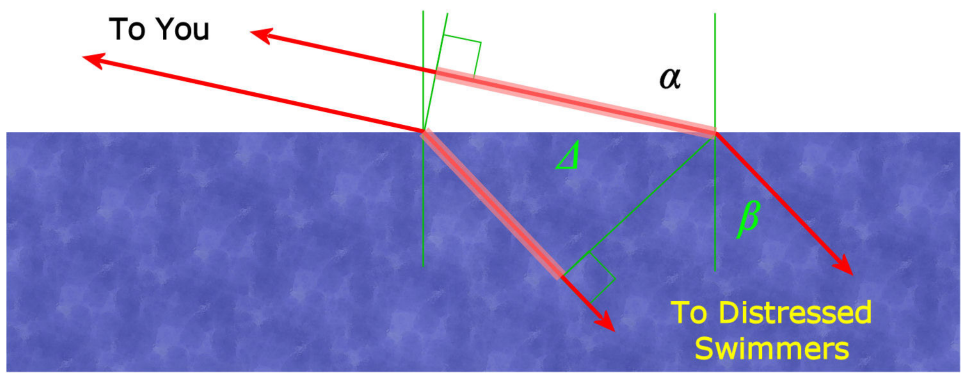
Let's analyze the situation mathematically. We'll introduce variables to denote some key lengths in the figure. Let x be the distance that you are from the shoreline. Let y be the distance that the distressed swimmers are from shore. Let D be the horizontal separation between you and the distressed swimmers, and let d be the horizontal distance between you and your point of entry into the water. We'll denote your running speed by v_r and your swimming speed by v_s. See the figure at the top of the next page.

Let $T(d)$ denote the time it takes for you to reach the distressed swimmers. We've expressed this time as a function of d, since it depends on where you enter the water. We can apply the Pythagorean theorem to find the length of the two legs of the red path. Then, we can use the formula *distance equals rate times time* to compute $T(d)$.



We find that $T(d) = \dfrac{\sqrt{x^2+d^2}}{v_r} + \dfrac{\sqrt{y^2+(D-d)^2}}{v_s}$. We are interested in minimizing $T(d)$.

Let's roughly examine how $T(d)$ changes if we alter d by a very small amount, which we will denote by Δ. To do this, we will zoom in on the point of your entry into the water. When Δ is very small, the angles that the red paths make with the shoreline will be almost identical. (Similarly, when you look at the moon, you don't have to change the direction you look even if you move about; the underlying reason is the same in both instances. Try this the next time you are driving along in a car and can see the moon.) Up close, the situation looks like this:



The thickened portions of the two paths show the distance by which the running and swimming legs change between these two paths. The running portion changes by approximately $\Delta \sin \alpha$. The swimming portion changes by approximately $\Delta \sin \beta$. Hence, the difference in time it takes to run the red path on the left versus the red path on the right will be approximately $\Delta \sin \alpha / v_r - \Delta \sin \beta / v_s$, or $\Delta(\sin \alpha / v_r - \sin \beta / v_s)$.

Notice that this time difference is proportional to Δ. If the constant of proportionality is not equal to zero, it means that moving your entry point over slightly will alter the time of the path; depending on the sign of the constant of proportionality, your time will improve if you move to the left or the right. What this means is that the optimal entry point must occur when the constant of proportionality is equal to 0.

We conclude that the ideal entry point is located where

$$\frac{\sin \alpha}{v_r} = \frac{\sin \beta}{v_s}.$$

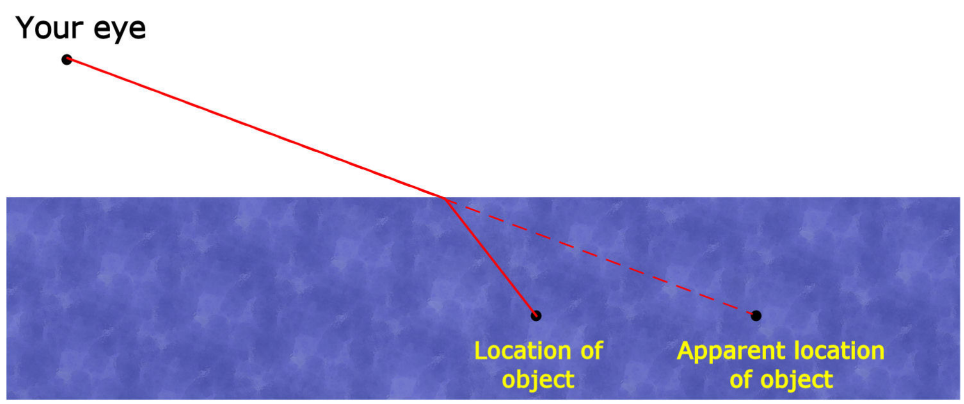
This fact is known as **Snell's law**. The angle α is called the **angle of incidence**, and the angle β is called the **angle of refraction**.

Notice that if $v_r = v_s$, then the angle of incidence would equal the angle of refraction; this makes sense because if you can swim as fast as you can run, the straight line shortest distance path would also be the fastest route to take.

The argument we used to deduce Snell's law can be made rigorous using calculus. If you find where the derivative of $T(d)$ with respect to d is equal to 0, you will recover Snell's law. Although, note that in our derivation, we did not actually have to use the formula for $T(d)$!

Light

Pierre Fermat observed that light travels along the path that minimizes travel time. This is known as **Fermat's Principle of Least Time**. This means that the lifeguard computation is the same computation that applies to the behavior of light when it passes from one medium to another, such as from air into water. You can see the effects of this kinked light path by looking down through the water at a submerged object. If you reach out with your hand to pick up the object, it will not be where it seems! That's because the light kinks according to Snell's law.



Fire Ants

Earlier this year, Jan Oettler and his colleagues found that fire ants also follow Fermat's Principle of Least Time to reach a source of food. How do you think these ants succeed in solving Snell's law?

A trail of fire ants following Snell's law. Photograph by Simon Tragust. Used under the terms of the Creative Commons Attribution License. For the source, please see the reference below.

Take It To Your World

Fermat's Principle of Least Time is a powerful tool you can use in everyday situations. Get a mirror. Have two friends stand some distance apart in a hallway. Position them so that they are not the same distance from the walls of the hallway. Use the Principle of Least Time to determine where you should place the mirror on the wall so that each friend can see the other in the mirror. For more, turn to page 26.

Further Reading

Feynman, R. P., *The Feynman Lectures on Physics*, Volume 1, Chapter 26. (This is highly recommended to obtain a deeper understanding and modern statement of the Principle of Least Time.)

J. Oettler, V. S. Schmid, N. Zankl, O. Rey, A. Dress, J. Heinze. (2013), *Fermat's Principle of Least Time Predicts Refraction of Ant Trails at Substrate Borders*. PLoS ONE 8(3): e59739. doi: 10.1371/journal.pone.0059739.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues to investigate whether products of consecutive numbers can be perfect squares.

I got so confused toward the end of last time. I took a long break from the problem and feel ready to tackle it again. I was trying to show that the product of 6 consecutive integers cannot be a perfect square.

$n_1, n_2, n_3, n_4, n_5, n_6$ – 6 consecutive numbers.

Can $\prod_{k=1}^{6} n_k$ be a perfect square?

Write $n_k = a_k b_k^2$ where a_k is "square-free"
i.e. prime factors appear with exponent 0 or 1.

My editor informed me that numbers that are not divisible by the square of any prime are called "square free."

For primes > 5, only one of the n_k can be divisible by that prime, so if the product is a perfect square, then such a prime must appear with even exponent in any n_k that it divides. Hence, the only prime factors of a_k are 2, 3, 5.

This is where I started to get confused last time. So this time, I'm going to avoid the table I tried to make.

If a number is divisible by 5, then all the numbers 4 below it to 4 above it, except for the number itself, will not be divisible by 5. So if one of the middle of my consecutive numbers is divisible by 5, that would be the only one.

If $5 \mid n_k$, $2 \le k \le 5$, then n_k is the only one of the 6 divisible by 5 and so 5 must appear with even exponent.
That is, $5 \mid b_k$ but $5 \nmid a_k$.
In this case, only 2, 3 are possible prime factors of the a_k.

$\Rightarrow a_k = 1, 2, 3,$ or 6.

$\Rightarrow a_k = a_j$ for some $1 \le k < j \le 6$.

There's that pigeonhole principle again!

$$\Rightarrow \left.\begin{array}{l} n_j = a_j b_j^2 = a_k b_j^2 \\ n_k = a_k b_k^2 \end{array}\right\} \quad \underbrace{n_j - n_k}_{j-k} = a_k(b_j^2 - b_k^2)$$

$\Rightarrow b_j^2 - b_k^2 = 1, 2, 3, 4, 5$
$\Rightarrow b_j^2 = 9, b_k^2 = 4$ or $b_j^2 = 4, b_k^2 = 1$.

In this case (where 5 divides only 1 of the 6 consecutive numbers), things seem to proceed much like the case of 5 consecutive numbers.

If $b_j^2 = 9, b_k^2 = 4$, then $j - k = 5a_k$ so $a_k = 1$
$k = 1, j = 6 \quad n_1 = 4 \quad n_6 = 9$
but $4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \ne \square$.

Recall that last time I used a square to denote a perfect square. I decided to continue using this nonstandard notation in my notes.

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

When 5 divides only 1 of the 6 numbers, this line of reasoning works out rather nicely!

If $b_j^2 = 4$, $b_k^2 = 1$ then $j - k = a_k \cdot 3$

$\Rightarrow a_k = 1$, $j = k + 3$ so $n_1 = 1$

but $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \neq \square$.

Let's see about the case where 5 divides both n_1 and n_6.

If $\prod_{k=1}^{6} n_k = \square$, then $5 | n_1$ and n_6.

Must have $5 | a_1$ and a_6 too, otherwise back to previous analysis.

$5 \nmid a_k$, $k = 2, 3, 4, 5$.

Whenever 2 of the numbers have equal square free parts, I've been able to reduce to checking a few specific cases which, so far, have turned out not to yield perfect squares. I have a hunch that this will be true in general.

$\boxed{\text{If } a_k = a_j \text{ for some } 1 \le k < j \le 6 \text{ then not } \square. \ ?}$

Hey! How convenient that all the square free numbers that can be made from 2 and 3 yield a perfect square when all multiplied together!

$\{a_2, a_3, a_4, a_5\} = \{1, 2, 3, 6\}$

$\Rightarrow n_2 n_3 n_4 n_5 = \square$

$\Rightarrow n_1 n_6 = \square$.

Actually, that's not a coincidence! If I write all the square free numbers made from some finite set of prime numbers, each prime will occur 2 to the (number of primes minus 1) times.

I haven't proven this yet. Perhaps I should say only that it's likely not to be a perfect square. Certainly it reduces the problem to checking a small number of cases.

$a_1 = 5a_1'$, $a_6 = 5a_6'$

For $n_1 n_6 = \square$, must have $a_1' = a_6' \in \{1, 2, 3, 6\}$.

$\Rightarrow a_1 = a_6 \Rightarrow$ so done.

If $a_1 = a_6$, then n_1/a_1 and n_6/a_6 are positive perfect squares that differ by at most 1...impossible.

---

6 numbers wasn't so bad... I'll go for 7 consecutive numbers. Can their product be a square?

Case: 7 consecutive numbers. $n_k = a_k b_k^2$, a_k square-free.

Only 2, 3, 5 can be prime factors of a_k.

At most 2 of the a_k are divisible by 5 leaving 5 only divisible by 2, 3 $\Rightarrow$ $a_k = a_j$ for some $1 \le k < j \le 7$.

$$b_j^2 - b_k^2 = \frac{j-k}{a_k}$$

This case yields to this line of reasoning quite nicely!

Cases $b_j^2 = 9$, $b_k^2 = 4$

$\Rightarrow a_k = 1$

So either 3, 4, 5, 6, 7, 8, 9 or 4, 5, 6, 7, 8, 9, 10

Both have only 1 seven.

$b_j^2 = 4$ $b_k^2 = 1$

$\Rightarrow a_k = 1$ or $a_k = 2$

If $a_k = 1$, then 1, 2, 3, 4, 5, 6, 7

If $a_k = 2$, then $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$

Again, only 1 seven.

Let's see... with 8 numbers, I have to worry about the primes 2, 3, 5, and 7 now, because 7 could divide 2 of the numbers. If it just divided 1 of the 8 numbers, then the previous case's reasoning would essentially apply here too...I'd get 1 or 2 numbers divisible by 5, leaving 6 numbers with square free parts divisible by only 2 or 3, and that would lead to at least 2 equal square free parts. Then I could apply the general principle to reduce to checking a few small cases. So let's assume 7 divides 2 of these numbers...it would have to be the first and last. Hm. Then, for similar reasons, I'd need 5 to divide at least 2 of the remaining 6 numbers. Otherwise, I'd again find equal square free parts and boil down to a small number of cases to check. That means 5 divides the second and seventh square free parts. And for the remaining 4 numbers to have all different square free parts, all 4 possible square free values 1, 2, 3, and 6 must occur, so their product would be a perfect square. Hm.

Seems like finding j, k s.t. $a_k = a_j$ is very useful.

Reduces to checking a finite number of cases.

(and each case was easily eliminated because there was a prime that appeared just once)

I might as well see if I can show that the product of 8 consecutive numbers cannot be a perfect square.

Case: 8 numbers

$$\underline{7} \quad \underline{5} \quad \underbrace{\_\_ \ \_\_ \ \_\_ \ \_\_}_{\text{all possibilities } 1,2,3,6} \quad \underline{5} \quad \underline{7}$$

product is $\square$

Oh dear...I'm stuck again! I can't see how to push this line of reasoning all the way through for 8 numbers.

Key:
- Anna's thoughts
- Anna's afterthoughts
- Editor's comments

ABB 12.26.13

Can you help Anna? Can the product of 8 consecutive numbers be a perfect square?

The Fourth Dimension

by Ken Fan | edited by Jennifer Silva

It often seems as though we are confined to a world with 3 spatial dimensions. We can move left and right, forward and backward, and up and down. Because all of our experience takes place in this world, it takes imagination and practice to think about a fourth spatial dimension.

Here, I'll explain one natural way in which the 4^{th} and higher dimensions arise. I'll start with 1 dimension, then add another, and another, and yet another, to obtain a model for the 4^{th} dimension. I'll add these dimensions in a uniform way each time. By the time you finish this article, I hope you'll agree that high-dimensional spaces are common and useful and that you'll feel comfortable working in spaces with any number of dimensions.

1 Dimension

Let's think about horses.

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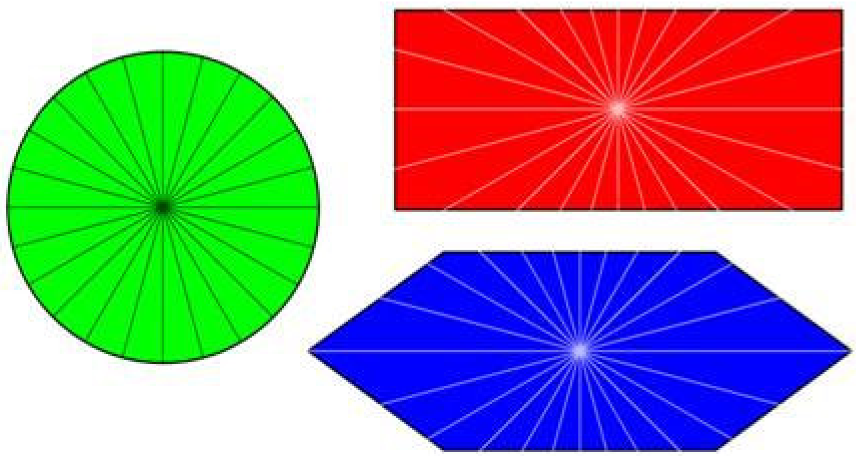
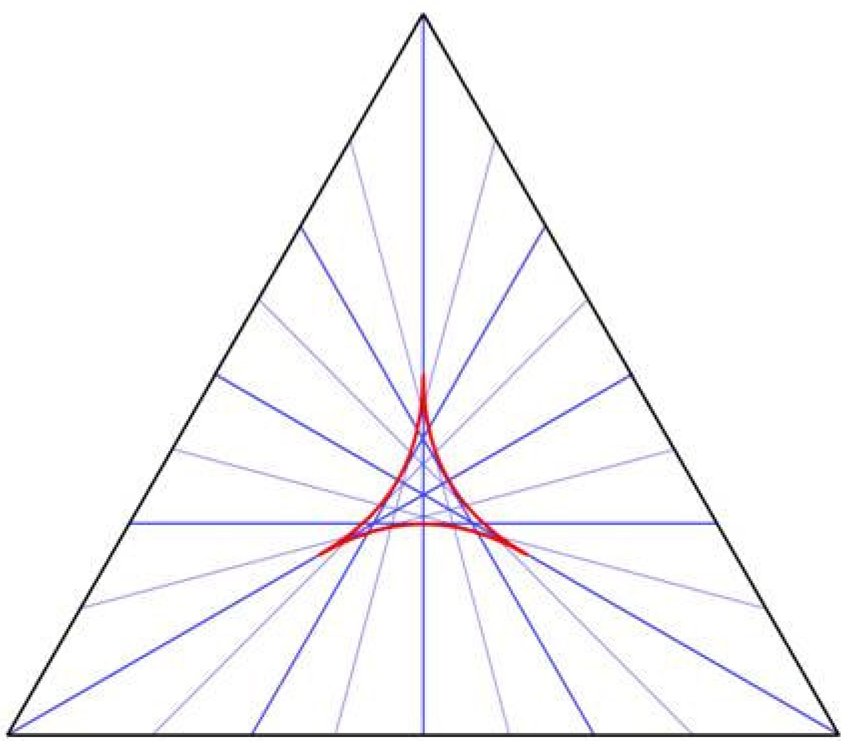
Mathematical Buffet

Bisection Envelopes

Consider a (bounded) planar shape, like a circle, triangle, or trapezoid. For each direction, there is a unique line in that direction that bisects the shape's area. To see this, imagine starting with a line in the given direction, and with the shape sitting entirely to one side of it. As we sweep the line across the shape (while maintaining its direction), more and more of the shape will transfer continuously from one side of the line to the other. Eventually, all of the area will have slipped across the line. Somewhere in between, exactly half the area will be on each side of the line.

For shapes with 180º rotational symmetry about a point (like those at right), all of these bisecting lines will intersect at the point of symmetry. However, for other shapes, the situation becomes more complex. The figure below shows several bisecting lines in an equilateral triangle. Assuming the shape is convex, the midpoints of all the bisecting line segments trace out a curve, which Noah Fechtor-Pradines has dubbed the **bisection envelope** in his paper by the same name which will soon appear in the journal *Involve*. Noah unearthed a number of interesting properties of bisection envelopes. Just a few of these properties are illustrated here.



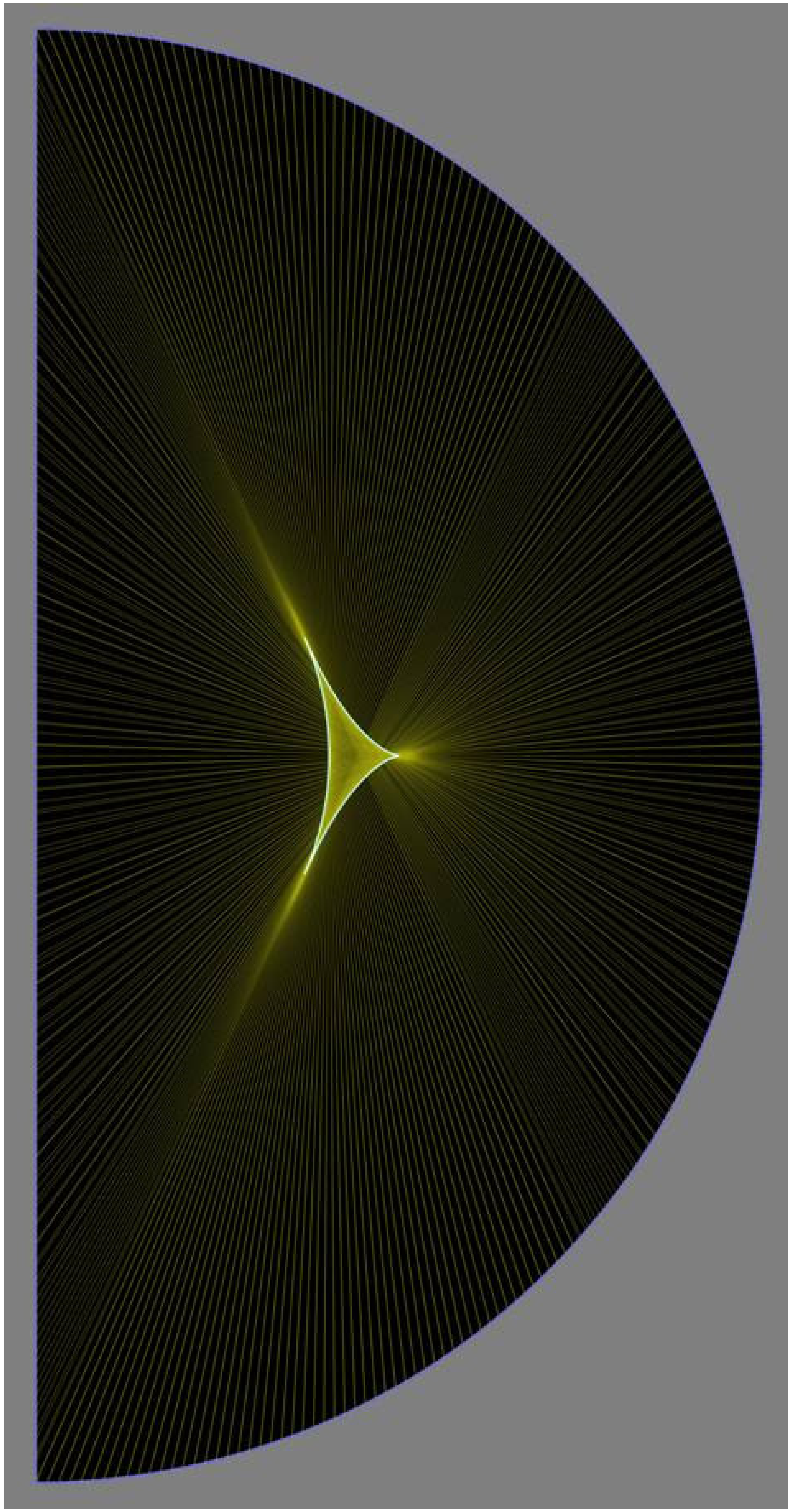


In the equilateral triangle, notice that all of the bisecting lines are tangent to the bisection envelope (shown in red). This is a general fact. Also, for polygons, the bisection envelope consists of arcs of hyperbolas.

Further Reading

Fechtor-Pradines, N., *Bisection Envelopes*, to appear in the journal Involve.

A Semicircle

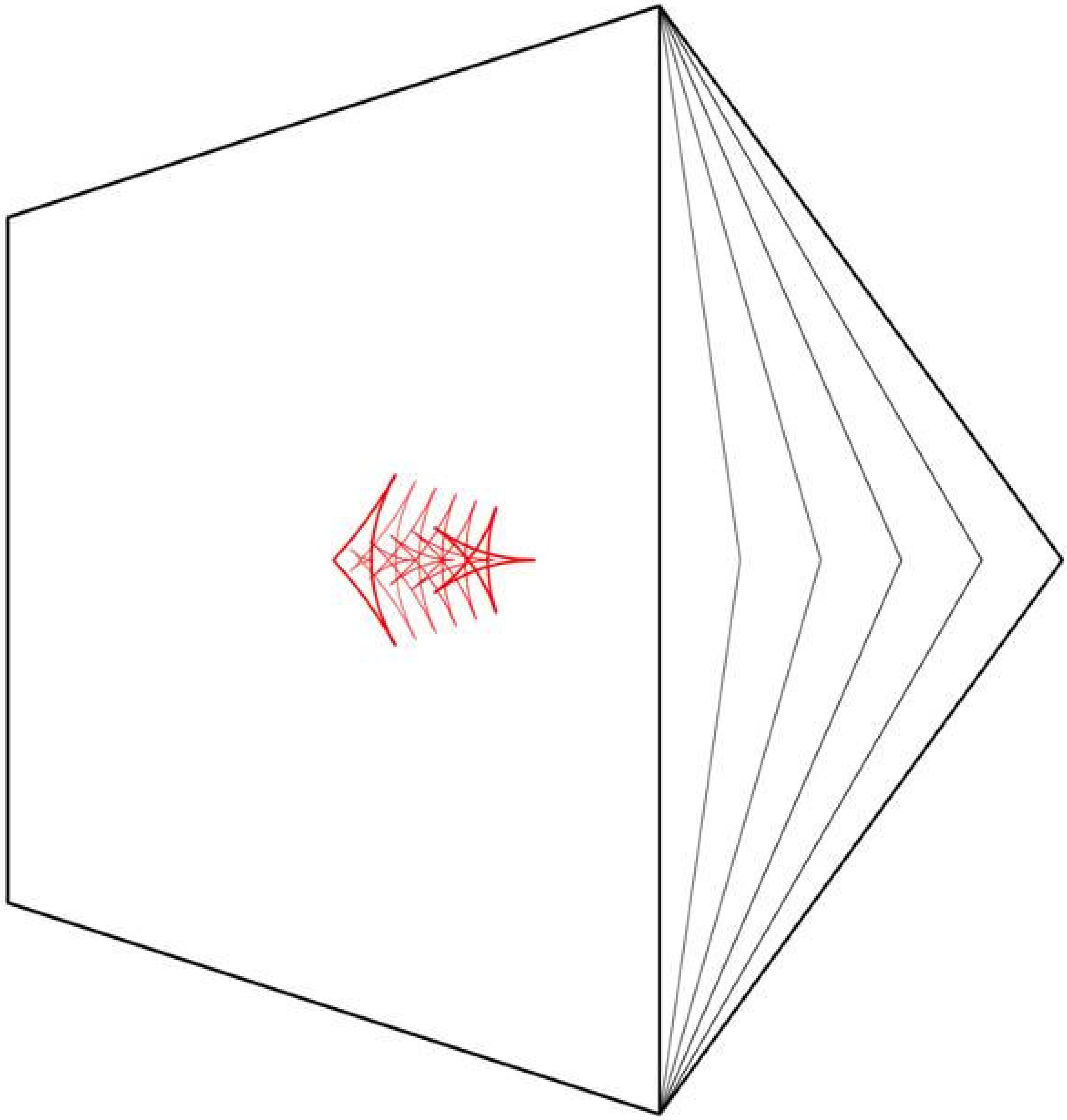


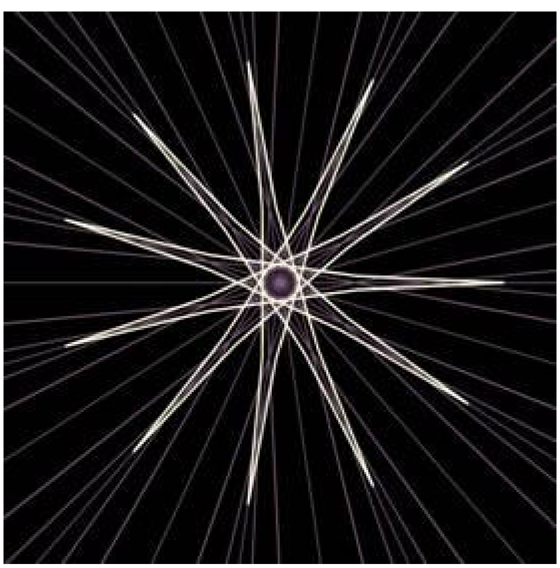
Bisection envelopes always have an odd number of cusps (places where the curve makes a sharp point). The bisection envelope of a semicircle consists of 3 smooth arcs joined at 3 cusps.

What kind of curve is the longest of the 3 arcs?

A beautiful fact that Noah proved is that the points that sit on exactly one bisecting line are precisely the set of points outside the bisection envelope. For a point inside the bisecting envelope, the number of bisecting lines that pass through it depends on the winding number of the bisection envelope about that point. (For more on winding numbers, see Soren Galatius' article in Volume 5, Numbers 1 and 2 of this *Bulletin*.)

Notice that the bisection envelope enjoys the same symmetries as the original shape.



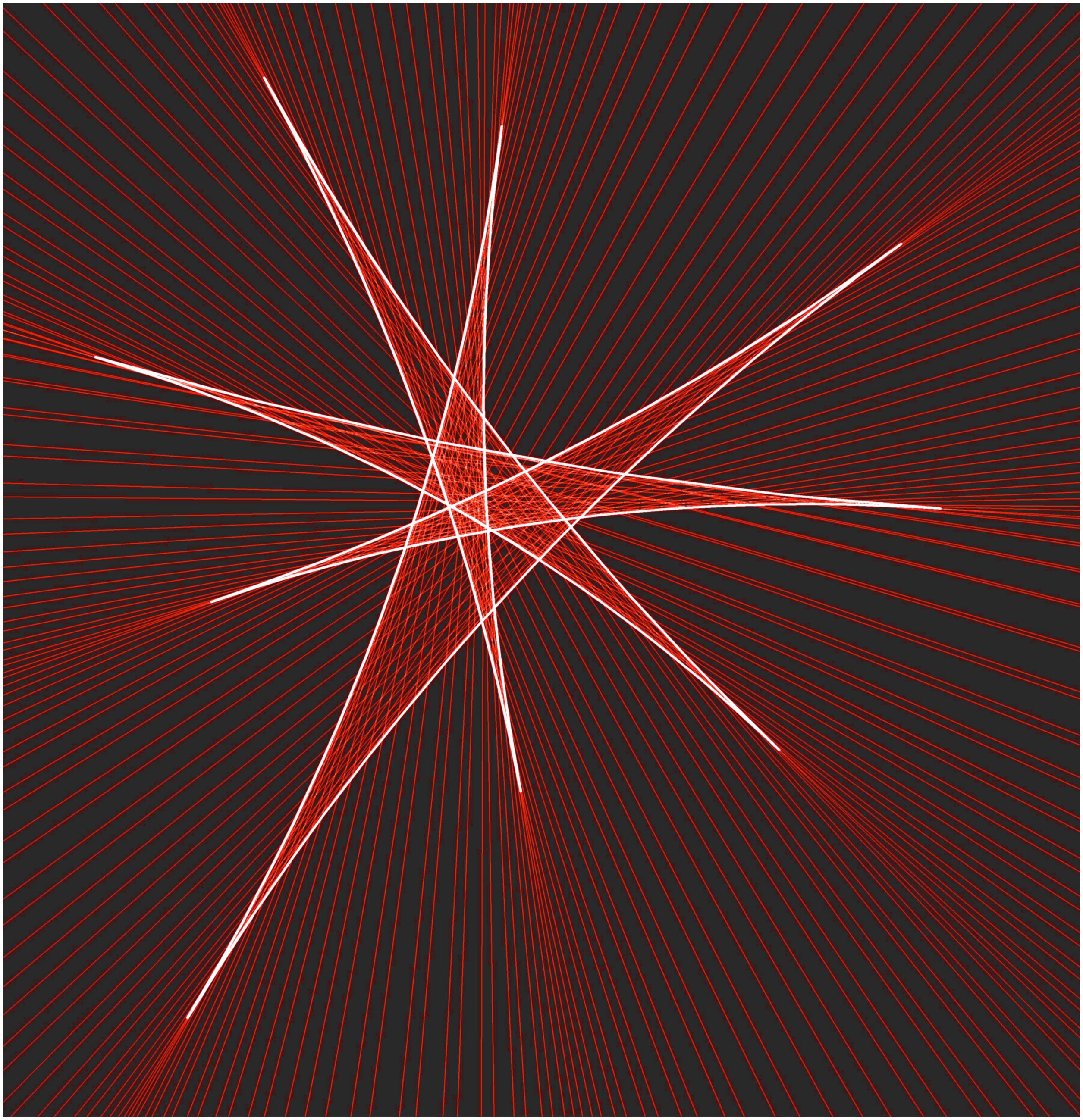


Trapezoid to Regular Pentagon

The figure above shows how the bisection envelope of a trapezoid morphs into that of a regular pentagon. The leftmost bisection envelope (which has 3 cusps) is that of the trapezoid. Going from left to right, 2 new cusps immediately appear, and the bisection envelope morphs into a symmetric 5-pointed star. For a close-up of the 5-pointed star, see this issue's cover.

Another Regular Polygon

At left is shown a close-up image of the bisection envelope of another regular polygon. How many sides does the regular polygon have?



A Random Nonagon

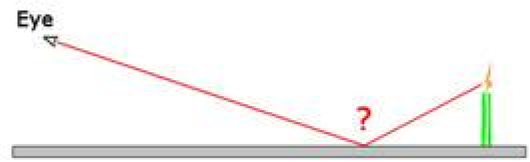
The bisection envelope shown above is that of the irregular nonagon at right. It was constructed by slightly perturbing each vertex of a regular nonagon in a random way.

For more about bisection envelopes, check out Noah's paper!

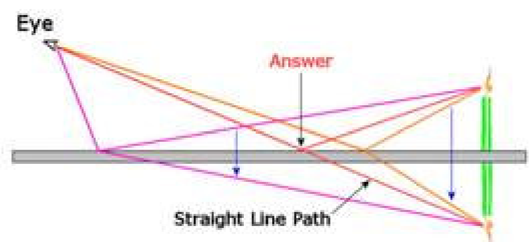
Fermat's Principle of Least Time

If you haven't read *Math In Your World* (page 13) yet, please do, and think about the problem posed at the end of it before reading on. I'm about to give a solution to it.

Here's an illustration of how to use Fermat's Principle of Least Time. Imagine a candle placed on a mirrored surface. How can we find the location where the light from the candle flame will reflect off the mirror and into the observer's eye?



To solve this, we'll assume that the shortest distance between two points is a straight line. According to Fermat's Principle of Least Time, to find where the candlelight reflects off the mirror, we have to locate the point on the mirror where the sum of the distances from that point to the eye and the candle flame is as small as possible. Imagine reflecting the part of the light path between the mirror and the candle flame over the plane of the mirror. Each candidate light path now becomes a two-legged journey from the eye, across the mirror, to the reflection of the candle flame. The kink is located at the surface of the mirror. Each candidate light path has the same length as its reflected light path because reflections preserve distance. Of these reflected light paths, the one that has the shortest length is the one that isn't kinked at all. So to find the location where the light bounces off the mirror, draw a straight line from the eye to the reflection of the candle flame and observe where this line intersects the surface of the mirror. Problem solved!



Try your hand at solving the following problems using Fermat's Principle of Least Time.

1. Take a cylindrical drinking glass or glass jar and fill it up halfway with water. Look at various objects through the bottom half and top half of the glass. What do you see? Explain what you see using the Principle of Least Time and the fact that light travels about 3/4 as fast through water and about 2/3 as fast through glass than through the air.

2. Light travels a little bit slower through the atmosphere than through the vacuum of outer space. Use the Principle of Least Time to explain why daylight can last longer than 12 hours. For example, in Malabo, the capital of Equatorial Guinea, on March 21, 2014, sunrise will occur at 6:29 a.m. and sunset will follow more than 12 hours later at 6:36 p.m. Hint: At sunset, where is the actual location of the sun?

3. The setup below consists of 2 mirrors and a candle. Where will the candlelight bounce off the first mirror and into the observer's eye? Where will the candlelight bounce off the second mirror and into the observer's eye? Where will the candlelight bounce off both mirrors before entering the observer's eye?

Note: A more accurate statement of the Principal of Least Time is that light will travel along a path where small changes to the path do not change its travel time, up to first order. (After all, light will not only bounce off the mirror; it will also travel straight to the eye.)

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 13 - Meet 8 Mentors: Asra Ali, Wangui Mbuguiro, Emmy Murphy, Isabel Vogt
November 7, 2013

Two pressing questions that arose at the club:

1. Is there an algebraic identity of the form $(x^2 + y^2)(m^2 + n^2) = (?)^2 + (?)^2$?

2. If you're making a perspective drawing of two flat roads that intersect at right angles, how do you determine the locations of the two vanishing points defined by them?

For the first problem, there's a spoiler in the next meet. For the second, we'd love to hear your solution, so please send them to us at girlsangle@gmail.com.

We also studied equivalent definitions. Two definitions are equivalent if the objects defined by one definition are the same as those defined by the other. Equivalent definitions give us two ways to look at the same thing. Can you show that the following two statements are equivalent?

Let X, Y, and Z be non-collinear points in the Cartesian plane.

Definition 1. A triangle is the convex hull of the points X, Y, and Z.

Definition 2: A triangle is the set of points in the plane of the form $xX + yY + zZ$, where x, y, and z, are nonnegative real numbers that sum to 1.

Session 13 - Meet 9 Mentors: Asra Ali, Jordan Downey, Wangui Mbuguiro
November 14, 2013
Visitor: Kate Jenkins, Akamai Technologies

Kate Jenkins is a software architect at Akamai Technologies. Akamai produces software that makes the internet faster, more reliable, and more secure. About ¼ to ⅓ of all internet traffic flows over the Akamai platform which is a network of servers built on top of the internet. The math involved in writing such software includes combinatorics, linear algebra, and probability. At Akamai, Kate works to solve problems that affect many people. She describes working at Akamai as fun, collaborative, and rewarding.

Throughout her childhood, Kate always liked math. In middle school, she participated in Mathcounts. In College, she was a math major, though she took it on faith that taking math courses would keep many doors open. She also worked at the Geometry Center at the University of Minnesota because she liked the marriage of math and art. After college, she combined math, art, and computer programming by working as a graphic programmer for a computer game company in Cambridge. After 3 years as a graphic programmer, she decided to go to graduate school in applied math at Cornell, where she studied graph algorithms. Following her graduate work, she joined Akamai and has been with Akamai for a dozen years.

At the club, Kate explained Dijkstra's algorithm. See page 10.

A member found the identity $(x^2 + y^2)(m^2 + n^2) = (xm - yn)^2 + (xn + ym)^2$. In terms of complex numbers, this tells us that $|zw| = |z||w|$. (Take $z = x + yi$ and $w = m + ni$.)

Session 13 - Meet 10 Mentors: Jennifer Matthews, Wangui Mbuguiro, Isabel Vogt
November 21, 2013

Some members became detectives as they sought the following numbers:

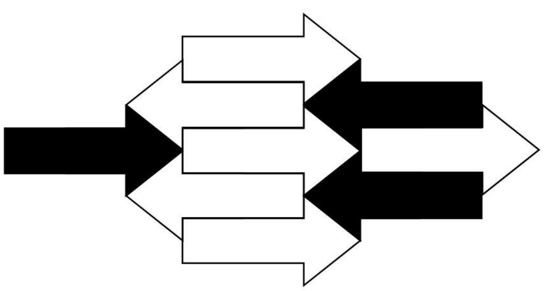
The fifth root of 537,824.
The seventh root of 17,249,876,309.
The seventeenth root of 18,654,364,505,825,863,257,034,516,199,263,461,660,466,879,247.

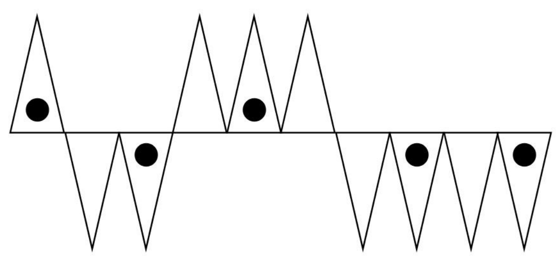
The only hint members were given was that all the answers were integers. Can you find these numbers too?

Some members worked on gaining facility with manipulating expressions that contain variables. If you're having trouble with variables, see Timothy Chow's article on page 7.

Session 13 - Meet 11 Mentors: Jennifer Matthews, Isabel Vogt
December 5, 2013

Some members played an advanced game of Math Pictionary. In this game, a member would try to get another member to replicate an image (up to similarity) using verbal cues only. The explainer would be allowed to study the image for 1 minute, before the image was taken away. The explainer had to sit with hands down (to avoid gesturing information) and facing away from the other girls so that she could not see how their drawings were progressing. To give a sense of the complexity of the drawings they had to describe, here are two of the images we used:





Member's soon learned that in order to play the game well, they had to break down images into discrete pieces of information, such as counting the exact numbers of certain shapes or finding distinctive patterns. How would you describe these images?

Session 13 - Meet 12 Mentors: Jordan Downey, Wangui Mbuguiro
December 12, 2013

We held our traditional end-of-session Math Collaboration. The girls broke into the treasure chest with 6 minutes to spare. Great work, All!

Calendar

Session 13: (all dates in 2014)

September	12	Start of the thirteenth session!
	19	No meet
	26	
October	3	
	10	
	17	Anna Frebel, Department of Physics, MIT
	24	
	31	
November	7	
	14	Kate Jenkins, Akamai Technologies
	21	
	28	Thanksgiving - No meet
December	5	
	12	

Session 14: (all dates in 2014)

January	30	Start of the fourteenth session!
February	6	
	13	
	20	No meet
	27	
March	6	
	13	
	20	
	27	No meet
April	3	
	10	
	17	
	24	No meet
May	1	
	8	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

All Girls' Angle Members and Subscribers are invited to email math questions, solutions, comments, and suggestions. We will respond!

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- □ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- □ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, assistant professor, UCSF Medical School
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, Tamarkin assistant professor, Brown University
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________