Girls' Angle Bulletin

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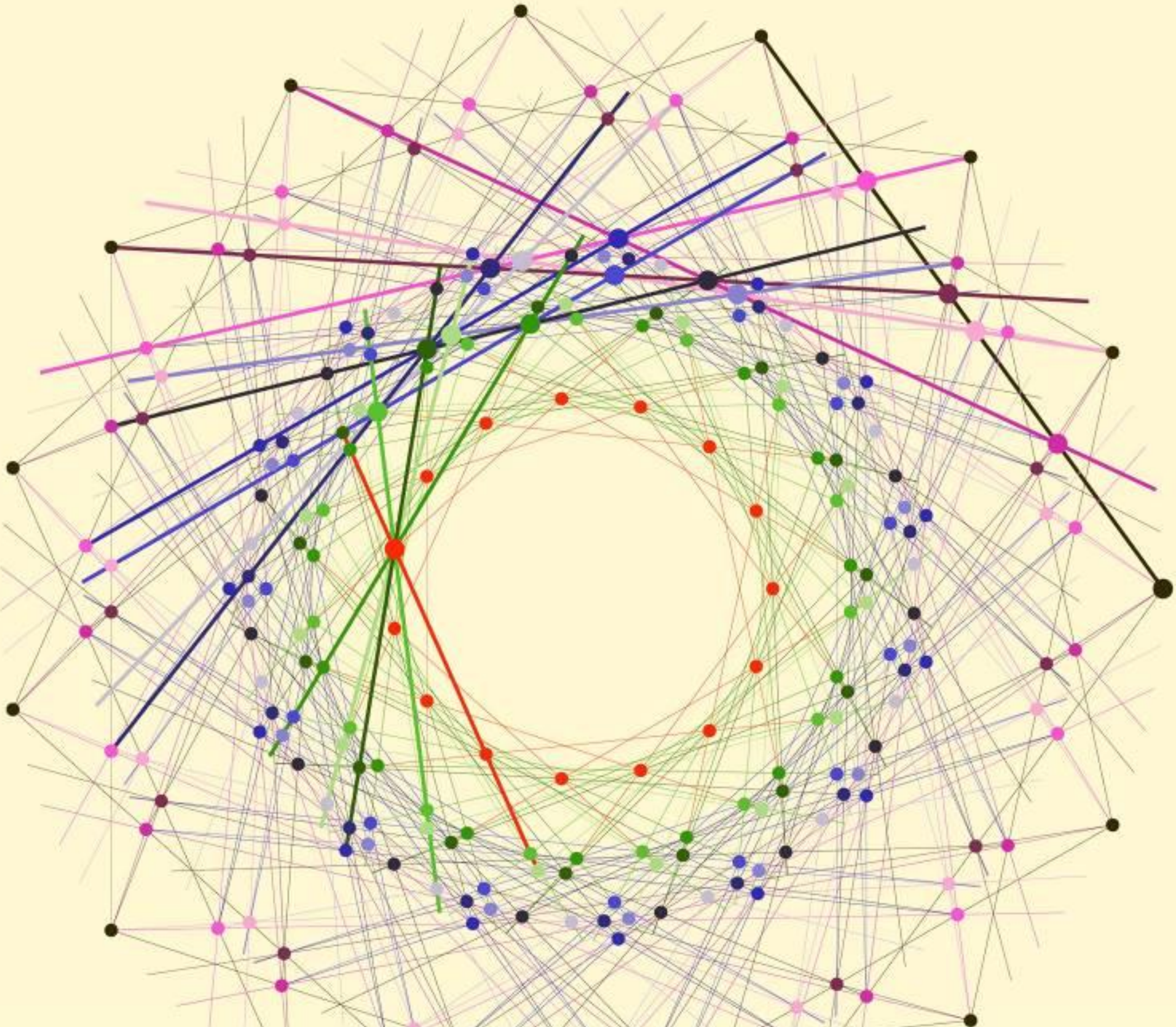
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

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- Ken Fan, President and Founder

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Girls' Angle Bulletin

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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle: A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: A $(240_6, 240_6)$ configuration courtesy of Leah Berman, associate professor of mathematics at the University of Alaska Fairbanks. For more, see *Mathematical Buffet* on page 9.

An Interview with Marie Vitulli, Part 1

Marie Vitulli is professor emerita at the University of Oregon. She received her Ph.D. in mathematics from the University of Pennsylvania under the supervision of Dock S. Rim. She has been at the University of Oregon since 1976. In addition to her mathematical research, she published studies on gender differences in first jobs for new Ph.D.s with Mary E. Flahive in the Notices of the American Mathematical Society.

Part of creating mathematics is asking good questions and the other part is having the capacity to solve the problems by using machinery that already exists or coming up with new machinery.

Ken: You have been a professional mathematician for a long time. To begin, I am hoping to gain some wisdom from you in three areas: learning and studying mathematics, creating mathematics, and leading a life as a mathematician. To that end, would you, in each case, look back over your career and share with us advice and wisdom that you gained that helped you in each area.

So, starting with learning and studying mathematics?

Marie: Mathematics is not a spectator sport. To learn and study it you must be actively engaged. You need to verify what you read and eventually either accept or challenge each statement you come across. There are many branches or types of mathematics. If one branch doesn't hold your interest try another branch before giving up on mathematics.

Ken: And creating mathematics?

Marie: If you start generating questions as you study mathematics that have already been discovered you will have a leg up on creating mathematics on your own. Part of creating mathematics is asking good questions and the other part is having the capacity to solve the problems by using machinery that already exists or coming up with new machinery. I love building new theories that set a framework for asking and answering questions.

Ken: And, finally, leading the life of a mathematician, or, career advice?

Marie: A research mathematician's life is deeply satisfying at times and just as deeply frustrating at other times. When you solve a problem that you have been thinking about for a while you feel like you're on the top of the world. If the problem is untouchable you can get quite frustrated over not being able to make progress on it. The life is challenging in that you have to somehow figure out how to juggle teaching, research, service to your institution and the profession and still have part of you left over for a personal life. On the positive side, you get to do something that you love and you have some flexibility in setting your own schedule. There are always opportunities to travel to attend conferences and workshops and to present your own work. You should realize that a mathematician at a research university works many more hours than someone who has a 9 to 5 job. You frequently work in the evenings and on the weekends. There are many other satisfying careers where knowledge of higher mathematics is beneficial. I went from college to graduate school and then accepted a tenure-track job at the University of

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Vitulli, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
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President and Founder
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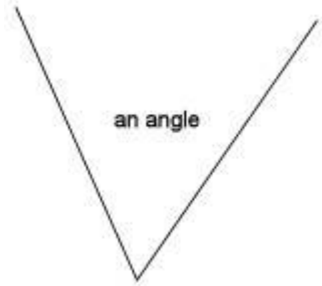
Girls' Angle

Learn by Doing

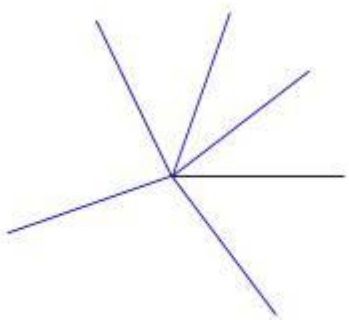
Angle Measure[1]
by Addie Summer | edited by Jennifer Silva



What is an angle?

Angles are formed whenever line segments intersect.

The figure above right shows two line segments that share an endpoint. The space between them is an angle. The line segments that define the angle are the angle's **sides**. The common endpoint of the line segments is the **vertex** of the angle.



Here's another picture with one black line segment and several blue line segments. You're meant to imagine a single blue line segment rotating around. As the blue line segment rotates, it forms several different angles with the black line segment.

Problem 1. Imagine the blue line segment rotating around and around. Do any angles formed stand out to you as being special in some way? Which ones?

At right, I've drawn two angles that strike me as special. Actually, these two angles have stood out to many people and they have special names. The L-shaped angle, which is the angle found in the corner of a piece of copy paper, is called a **right** angle. The other angle, which may hardly seem like an angle at all, is called a **straight** angle.

Angle Measure

Problem 1 shows that angles and circular motion are intimately related. That's why it is very common to measure the size of an angle in terms of circles. We draw a small circle centered at the common endpoint of the two line segments that define the angle, and we ask what fraction of the circle is cut out by the angle. Thus, a right angle is a quarter circle, and a straight angle is a half circle.

People also use **degrees** and **grads** to measure angles. In a full circle, there are 360 degrees and there are 400 grads.

Problem 2. How many degrees are there in a right angle? How many are there in a straight angle?

The symbol "°" stands for degrees. A right angle is 90°, and a straight angle is 180°. Was that your answer for Problem 2? An angle whose measure is between 0° and 90° is called an **acute** angle and an angle whose measure is between 90° and 180° is called an **obtuse** angle.

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[1] This content was supported in part by a grant from MathWorks.

Problem 3. On a clock with an hour and minute hand, what is the angle between the two hands when the time showing is 8 p.m.? What about 2:45 a.m.?

Problem 4. Find all the times when the hour and minute hand of a clock form a 90° angle.

Mathematicians use a third measure for angles: **radians**. To compute radian measure, center a circle at the angle's vertex. The radian measure of the angle is the ratio of the length of the arc that is cut out by the angle divided by the radius of the circle. Because radians are the ratio of two lengths, radian measure is a dimensionless quantity.

Problem 5. Technically, to define radian measure, we need to know that the ratio of the arc length of circle cut out by an angle divided by the circle's radius does not depend on the size of the circle. Why is this true?

Problem 6. Express the radian measures of a right angle and a straight angle in terms of the mathematical constant π. (π is the ratio of a circle's circumference to its diameter.)

Problem 7. Let d be the degree measure of an angle and let r be its radian measure. Write down a conversion equation that relates d and r.

You're walking along when someone shouts at you, "Turn 90 degrees!" Which way do you turn? It's ambiguous! The person who shouted at you should have told you whether to turn left or right. In other words, there's an ambiguity in our definition of angle. When you draw two line segments that share a common endpoint, there are really two angles formed, and you sweep through one or the other depending on whether you rotate the first line segment clockwise or counterclockwise to get to the other. Usually, people are referring to the smaller sector, but not always. If the context does not make it clear which angle is meant, it must be specified.

Problem 8. Let l, m, and n be line segments that share a common endpoint P. Assume that as you rotate l about P in the counterclockwise direction, you pass over m before reaching n. Show that the measure of the angle formed by l and n is equal to the sum of the measures of the angles formed by l and m and m and n, with all angles being measured in the counterclockwise direction.

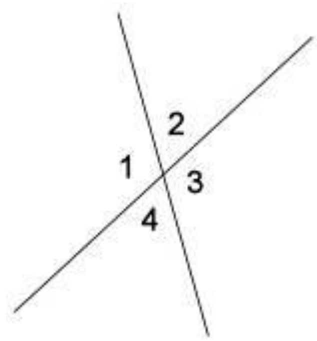
If two angles add up to a straight angle, they are called **supplementary** angles. If two angles add up to a right angle, they are called **complementary** angles.

Problem 9. Imagine walking around the perimeter of a triangle in such a way that the triangle is always on your left. Start your walk in the middle of one of the sides. You approach one of the corners of the triangle. When you go around the corner, you turn counterclockwise through an angle that measures T degrees. The measure of the angle at the corner you just went around is $180 - T$ degrees. When you complete one full circuit and return to your starting point, through how many degrees have you turned in the counterclockwise direction? What does this imply about the sum of the angles of a triangle?

Problem 10. Use the argument hinted at in Problem 9 to show that the sum of the angles in a polygon with n sides is equal to $180(n - 2)$ degrees.

Problem 11. Let's move on to a Cartesian plane with an *x*- and a *y*-axis. Draw a line through the origin. Such a line has equation $y = mx$, where m is the slope of the line. For the moment, let's assume that m is positive, so that the line goes through the first quadrant. Notice that specifying the slope is an effective way of specifying an angle in the first quadrant between the positive *x*-axis and the line. Let $d(m)$ be the degree measure of this angle. What is $d(1)$? What is $d(\sqrt{3})$? As m tends to infinity, what does $d(m)$ approach?

The function *d* defined in Problem 9 is also known as the **arctangent** function. The arctangent function can be regarded as the function that converts the slope of a line into the angle the line makes with the positive horizontal axis.

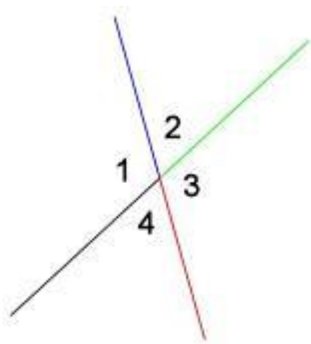


When 2 lines intersect, they form 4 angles (see figure at right).

Problem 12. Which pairs of angles are supplementary to each other?

Angles 1 and 3 are called **vertical angles**. Likewise, angles 2 and 4 are vertical angles.

Problem 13. Show that vertical angles have the same measure.



Another way to think about the equality of the measures of vertical angles is to imagine standing back to back with a friend at the point of intersection of the 2 lines. You and your friend are attached on your backs, so whenever you turn, your friend turns with you. Now face out along the blue line segment (in the figure at left). Your friend will be looking out along the red line segment. If you are asked to turn counterclockwise by the measure of angle 1, you will then be facing down the black line segment. However, your friend will now be facing down the green line segment since the black and green line segments are part of the same line. Having turned through the same amount that you did when you swept through angle 1, it follows that angle 1 and angle 3 have the same measure.

Problem 14. Under what circumstance do all 4 angles created by 2 intersecting lines have the same measure?

People often say that the wall and the ceiling meet at right angles. When people refer to the angle between two planes, they are referring to the **dihedral angle**. Imagine 2 planes that intersect in a line. The intersection of these 2 planes with a plane perpendicular to their line of intersection will be 2 intersecting lines. The 2 intersecting lines form 4 angles. The dihedral angle between the 2 planes can be taken to be any one of these 4 angles.

Problem 15. Take a piece of paper and fold it to form a dihedral angle. Fold a paper contraption that contains a 60° dihedral angle.

Problem 16. What is the dihedral angle between adjacent faces of a cube?

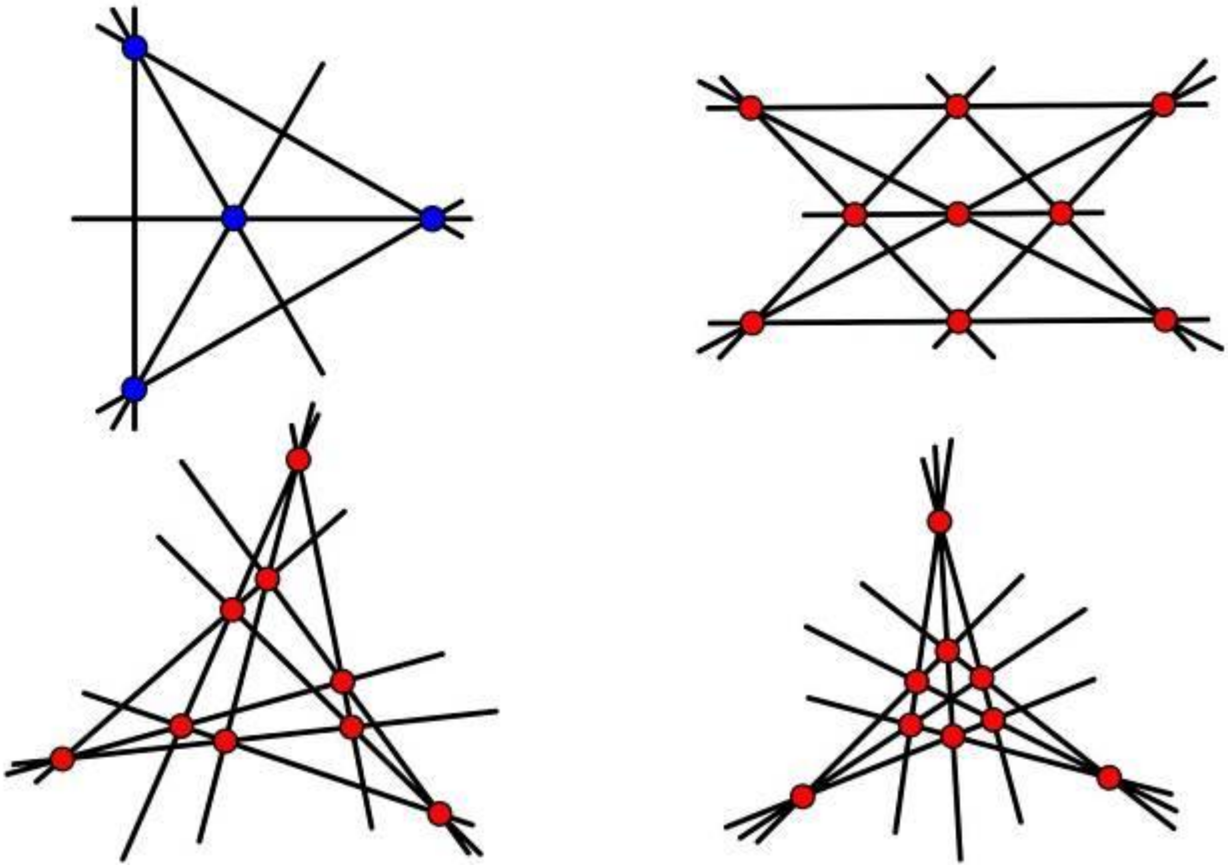
Mathematical Buffet[1]

Planar Configurations by Nadine Alise and Leah Berman

Two points define a line. But if you arbitrarily pick out 3 points, they most likely will not sit on a line. When 3 or more points sit on a line, it's special and there's a word to describe that: **collinear**. In fact, there are theorems whose main point is that 3 points are collinear. (For an example, google "Euler line".)

If you arbitrarily pick out 2 lines, they will intersect in a point, except in the special situation where the lines are parallel. But it is unusual when 3 lines intersect in a point and there is a special word to describe when that happens too: **concurrent**.

A **planar configuration** is a finite set of lines and points in the plane that exhibit a fascinating regularity with respect to collinearity and concurrency. If you have P points and L lines, and if every line has p of the points and every point is on l of the lines, then you have a (P_l, L_p) configuration.

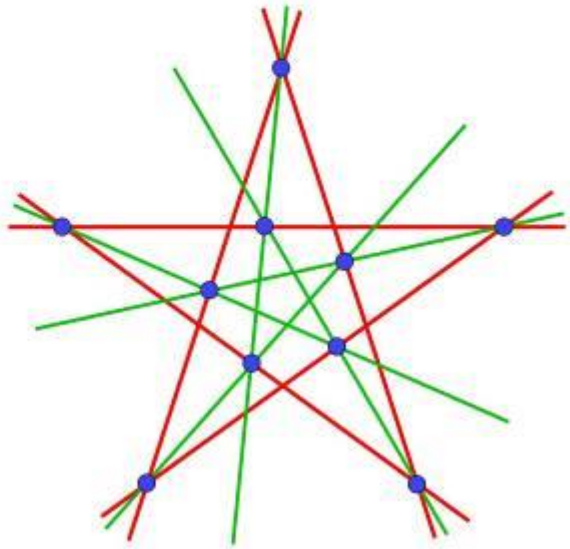
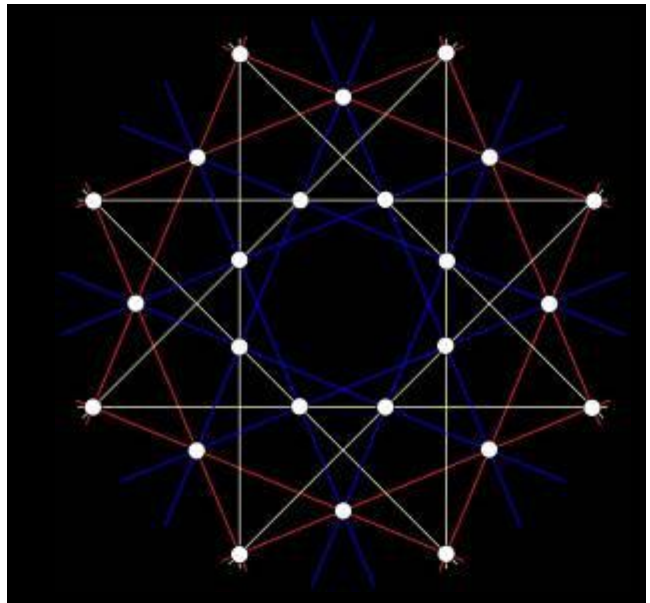
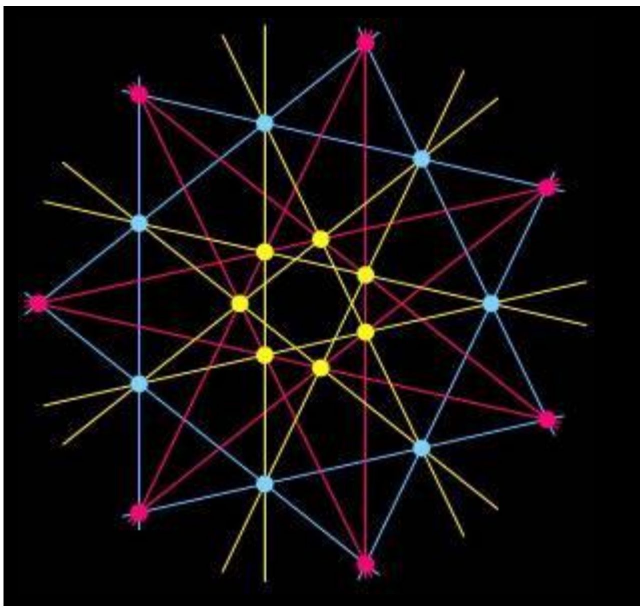
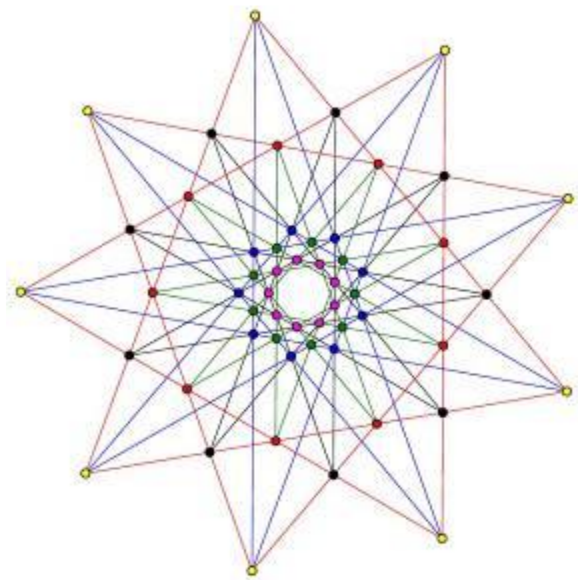
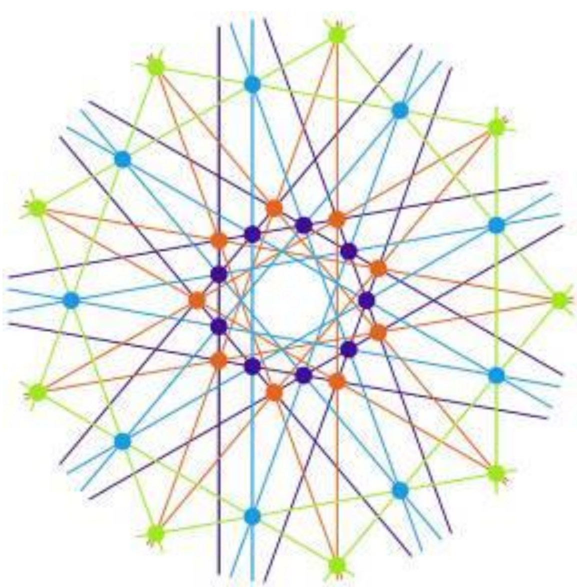
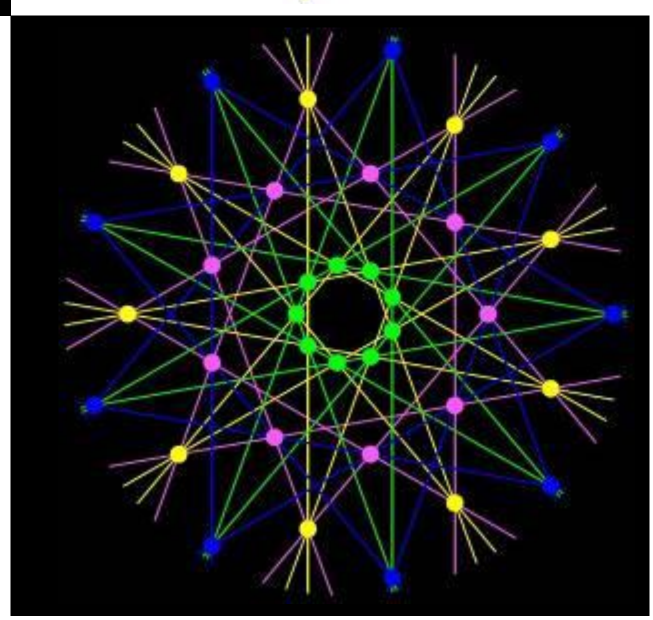


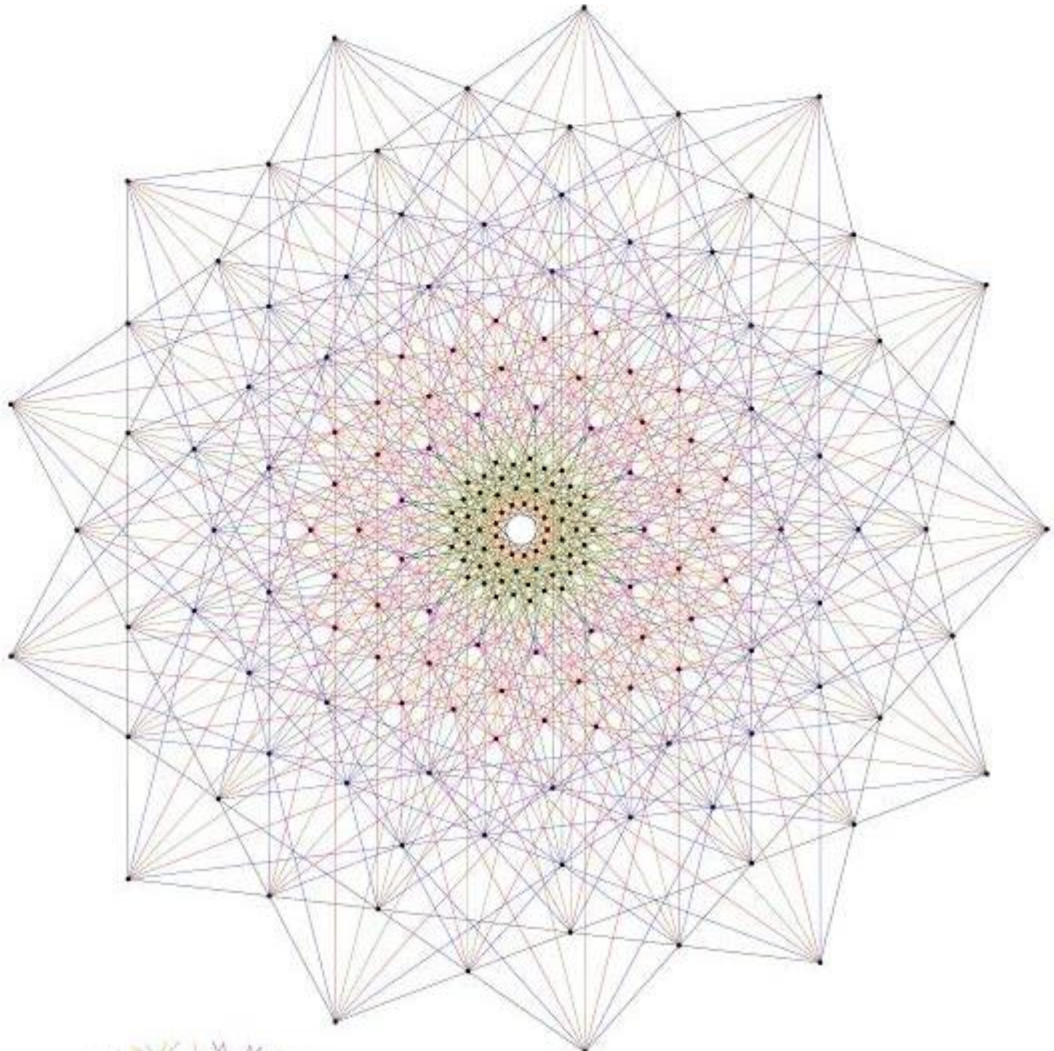
The upper left shows a $(4_3, 6_2)$ configuration: every line has 2 of the 4 blue points and every blue point sits on 3 of the 6 lines. The other three are all $(9_3, 9_3)$ configurations. The vertices of a regular N-gon and the lines that contain its edges form an (N_2, N_2) configuration.

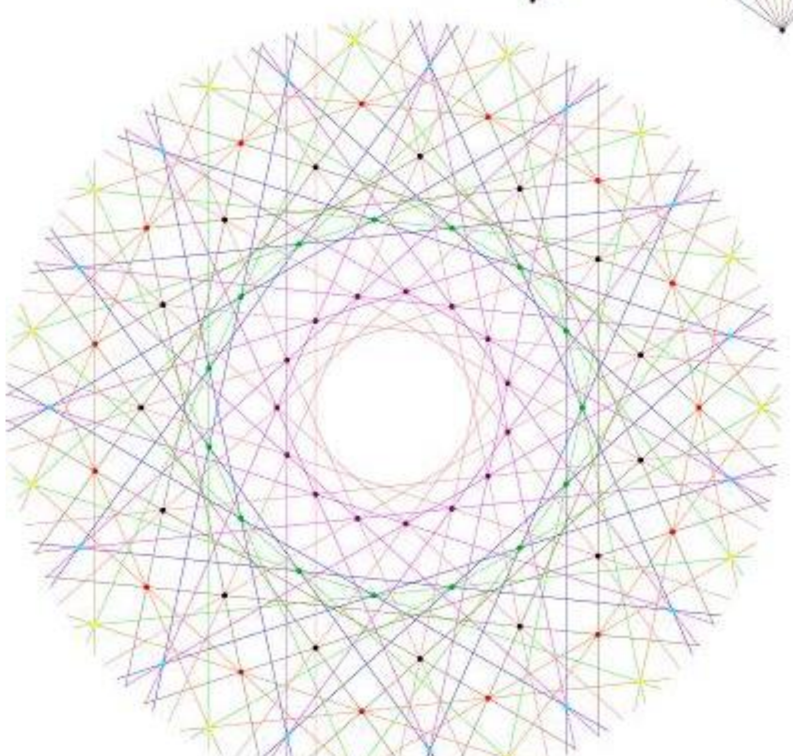
In this *Mathematical Buffet,* we present images of configurations created by Nadine Alise and Leah Berman. How many points and lines do they have? How many of the lines pass through each point and how many of the points sit on each line? Do you see a relationship between P, L, p, and l?

The cover displays a magnificent $(240_6, 240_6)$ configuration.

[1] This content was supported in part by a grant from MathWorks.





Stained Glass Angles

by Lightning Factorial | edited by Jennifer Silva

Emily and Jasmine are designing a stained glass window. Let's listen in.

Emily: There are so many possibilities!

Jasmine: Too many!

Emily: We could depict sea life in a coral reef…

Jasmine: Or we could make an abstract design…

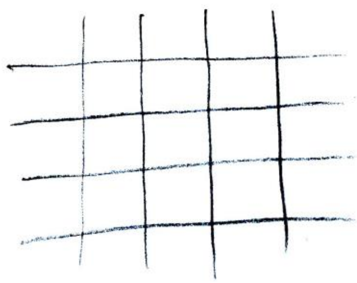
Emily: I suppose we could just make something random. Maybe we can shatter a piece of glass and use that as our design.

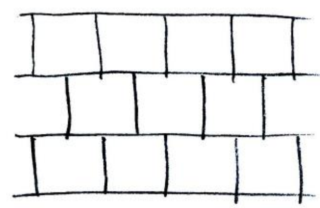
Jasmine: Too dangerous!

Emily: Staring at this blank sheet of paper, I don't know how to begin.

Jasmine: Why don't we start by limiting possibilities? Let's insist that every piece be a square.

Emily: Okay…



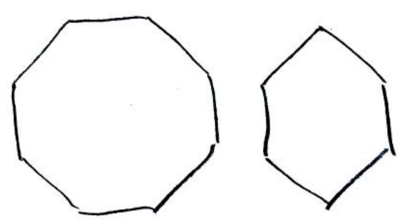


Jasmine: Hmm. That doesn't look very interesting. It's like graph paper.

Emily: Or a brick wall. Though I do like straight edges.

Jasmine: Yes, straight edges will make it easier to build. Let's insist on using straight edges.

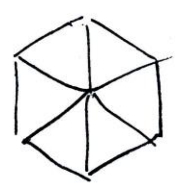
Emily: What if we also insist that all edges have the same length? Would that be too restrictive?



Jasmine: We could make regular polygons. Or maybe they don't have to be regular. They just have to have equal side lengths.

Emily: Empty polygons look like window frames, not stained glass windows.

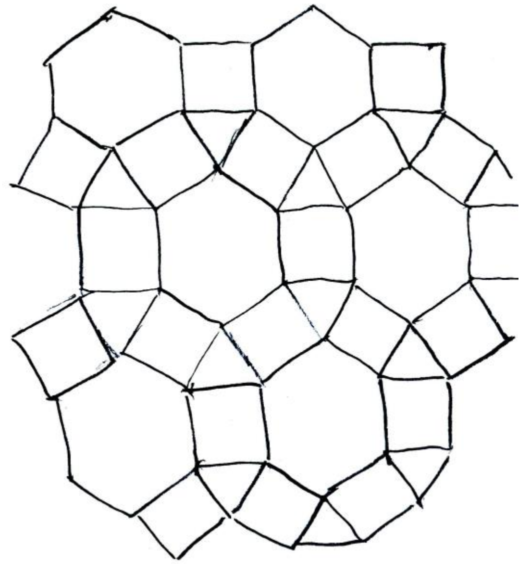
Jasmine: What if we draw in the spokes?



Emily: Okay. But if we stick to edges all the same length, then it means that we have to join the ends of adjacent spokes to form equilateral triangles.

Jasmine: Oh, that's true. That gives us a window in the shape of a regular hexagon.

Emily: I'd like to see something more intricate.



Jasmine: We could make a tiling that uses polygons that all have the same edge length.

Emily: Like the brick pattern?

Jasmine: Yes, only I was thinking of using more than one type of polygon, such as an equilateral triangle, square, and hexagon.

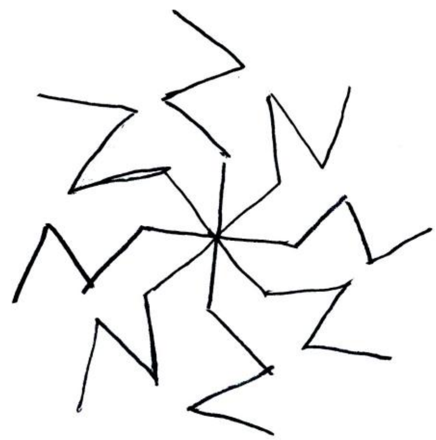
Emily: Oh, I think I've seen a tiling like that. It goes like this…

Jasmine: Yes, that's it!

Emily: It's pretty, but maybe it's more suitable for a bathroom floor.

Jasmine: Maybe. Well, anyway, let's look for other ideas.

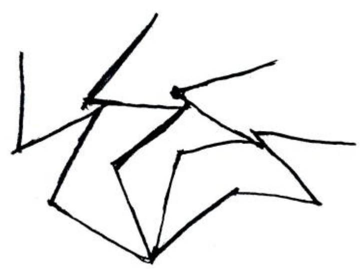
Emily: What if we go back to the spokes idea, except that we make the spokes out of several edges?



Jasmine: How do you mean?

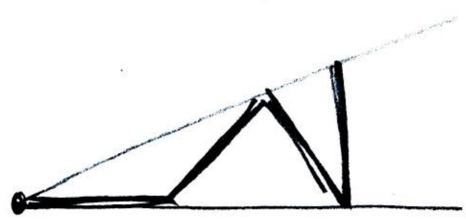
Emily: I'm not sure… I'm just thinking aloud. What if we imagine a long spoke made up of several edges, all the same length, and then rotate copies of it around like this…?

Jasmine: Then you just get one piece of glass with some metal edges running out from the center. I suppose we could try to bunch up the spokes so that adjacent spokes touch each other. That would close off some pieces of glass.



Emily: Hmm. How about this: We draw an angle and then bounce back and forth between the sides of the angle to form the spoke, like this.

Jasmine: That's a curious spoke. I'm intrigued.



Emily: We have to make the angle in such a way that copies of it will fit snugly inside a full circle… like pizza slices.

Jasmine: Right, the angle has to measure 360/N degrees, where N is an integer. Let's see what happens if we start with a right angle, which corresponds to N = 4.

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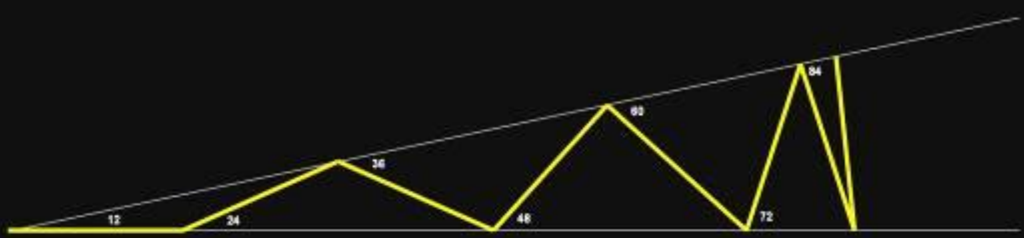


A 12-Degree Spoke

What patterns do you see in Emily and Jasmine’s stained glass window? How many differently shaped rhombi are there? What is the simplest way you can think of to describe the design?

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues her investigation into the equation $x^x = n$.

There's something about the exponentials that I worked on last time that makes me feel like there's more to be said. I want to revisit that. I'll start by summarizing what I found last time.

Last time, I found this identity for $a > 0$. The identity shows how to write numbers between $1/e$ and 1 as x to the power of x in two different ways.

$$\left(a^{\frac{1}{1-a}}\right)^{a^{\frac{1}{1-a}}} = \left(a^{\frac{a}{1-a}}\right)^{a^{\frac{a}{1-a}}} \qquad a > 0$$

I'll define 2 functions f and g to be the 2 values of x such that x to the x are equal.

$$f(a) = a^{\frac{1}{1-a}} \qquad g(a) = a^{\frac{a}{1-a}} = f(a)^a$$

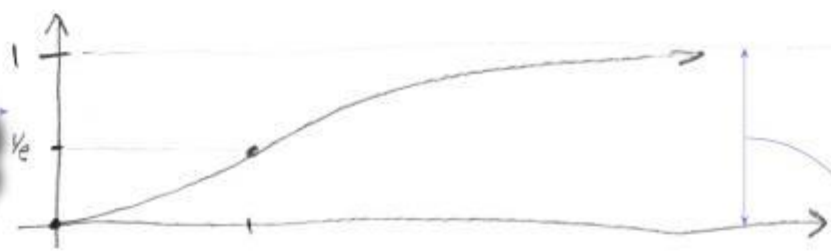
Last time, I also showed that both $f(a)$ and $g(a)$ tend to $1/e$ as a goes to 1.

$$\lim_{a\to 1} f(a) = \lim_{a \to 1} g(a) = \frac{1}{e}.$$

As a tends to infinity, $1 - a$ goes to negative infinity much faster than $\log a$, so the exponent tends to 0. So $f(a)$ tends to 1.

$$f(a) = a^{\frac{1}{1-a}} = e^{\frac{\log a}{1-a}} \to 1 \quad \text{as } a \to \infty.$$

I'll sketch a graph of the function f. At $a = 1$, I'll define it to be $1/e$ since that's it's limit as a tends to 1.



Since the range of f includes all values between 0 and 1, for any a between 0 and 1, there must be a $b > 1$ that satisfies $f(a)$ to the $f(a)$ equals $f(b)$ to the $f(b)$... that is $f(b)$ equals $g(a)$.

Let $0 < a < 1$.

There exists $b > 1$ such that $f(a)^{f(a)} = f(b)^{f(b)}$.

$$f(b) = a^{\frac{a}{1-a}}$$

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

I wonder if I can express this b in terms of a.

$$b^{\frac{1}{1-b}} = a^{\frac{a}{1-a}}$$

I'll try the same approach that worked before. Since I have different bases on the left and right side of the equation, I'll express b as a power of a.

$$b = a^x$$

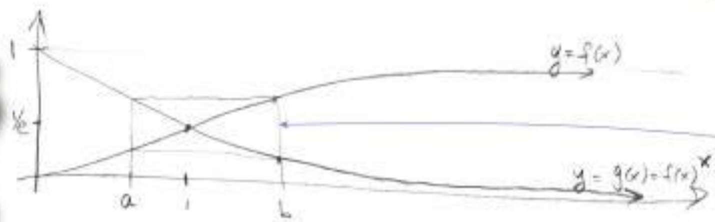
$$b^{\frac{1}{1-b}} = (a^x)^{\frac{1}{1-a^x}} = a^{\frac{x}{1-a^x}} = a^{\frac{a}{1-a}}$$

$$\frac{x}{1-a^x} = \frac{a}{1-a}$$

$$x(1-a) = a(1-a^x)$$

Hm. This doesn't seem to help much. I can't see how to solve this equation for x.

I feel this is a peculiar situation. I think I'll go ahead and graph both functions f and g on the same graph.



Huh...that must mean $f(a)$ is equal to $g(b)$...which means this is a rectangle.

Let's see... b is defined so that $f(b) = g(a)$. Hm. But $f(a)$ is the other value which, when raised to itself, equals $g(a)$ to the $g(a)$ and so is $g(b)$...hum...

$$f(a) = g(b) \qquad g(a) = f(b)$$

$$a^{\frac{1}{1-a}} = b^{\frac{b}{1-b}} \qquad a^{\frac{a}{1-a}} = b^{\frac{1}{1-b}}$$

That is...
But this equation surely means $ab = 1$! Oh, so $b = 1/a$.

$$\left(b^{\frac{b}{1-b}}\right)^a = b^{\frac{1}{1-b}}$$

$$b^{\frac{ab}{1-b}} \Rightarrow \underline{ab = 1}$$

So $g(1/a) = f(a)$? I've got to check that directly...

$$g\left(\tfrac{1}{a}\right) = \left(\tfrac{1}{a}\right)^{\frac{1/a}{1-1/a}} = \left(\tfrac{1}{a}\right)^{\frac{1}{a-1}} = (a^{-1})^{\frac{1}{a-1}} = a^{\frac{1}{1-a}} = f(a)$$

It works!

Equivalently, $g(a) = f(1/a)$.

$$g(a) = f\left(\tfrac{1}{a}\right)$$

$$\boxed{f(a)^{f(a)} = f\left(\tfrac{1}{a}\right)^{f\left(\frac{1}{a}\right)}} \qquad f\left(\tfrac{1}{a}\right) = f(a)^a$$

So f and g are different ways of expressing what is essentially the same function. I can get by with just f... and f satisfies these properties. Neat!

$$x(1-a) = a(1-a^x)$$

$$x = -1 \qquad -(1-a) = a\left(1-\tfrac{1}{a}\right) = a-1 \ ✓$$

Why couldn't I see that $x = -1$ was a solution of my equation relating x and a? That's amusing!

ABB 4.27.14

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

Solid Geometry: Wood Sculptures by Kosticks

by Ken Fan
edited by Jennifer Silva

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Art and geometry combine in the minds of John and Jane Kostick. The result is a collection of wondrous wood sculptures that manifest a wealth of interesting mathematical facts.

The best way to get a feel for this mix of math and art is to play with an actual example, and that's just what we'll do. This article contains all you need to make your own paper model of one of the Kosticks' latest creations: the **Quintetra Assembly**.

Inspiration To appreciate the elegance of the Quintetra Assembly, it helps to think about a few more basic shapes with special focus on the directions of their edges. Let's start with a cube, paying particular attention to its 12 edges. Notice that the edges of a cube point in 1 of 3 different directions, just like the axes of a 3D Cartesian coordinate system.

This observation raises the following question: What solids have all of their edges restricted to the same 3 directions as the edges of a cube? Because this restriction is severe, we can get a very good idea of what these shapes look like with a little bit of experimentation. Any brick shape is possible, and so is any solid built by joining bricks together, provided that all of the bricks are consistently oriented to respect the restriction on edge directions. Of these shapes, only the isolated brick will be convex, and of these bricks, the cube is the most symmetric and is the only convex one with edges all of the same length. (A shape is convex if it contains the line segment joining any two of its points. For example, a circular disk is convex, but an annulus is not.)

Let's make a game of this, now using a different set of allowed directions: the 4 directions specified by the **major diagonals** of a cube. The major diagonals are the line segments that connect opposite vertices. What shapes can you find whose edges are each parallel to one of these 4 directions? Note that if we use only 2 of the 4 directions to travel in a circuit by moving in one direction, then the other, then back in the first direction, then back to the starting point in the second direction, we will trace out a parallelogram. Also, keep in mind that if we wish to stay within a plane, we have to restrict ourselves to using just 2 of the 4 directions. Therefore, such solids, if convex, must have faces that are parallelograms. By analyzing the angles between pairs of directions, we find that these parallelograms involve 2 specific angles, namely $\cos^{-1} 1/3 \approx 70.5°$ and its supplement, $\cos^{-1} -1/3 \approx 109.5°$.

Is there an equilateral convex solid whose edges are each parallel to one of the 4 major diagonals of a cube? If such a shape existed, all of its faces would have to be congruent rhombi.

Think about this before reading further.

Continued on page 23

Pythagorean Triples Challenge

by Tom Moore[1] | edited by Jennifer Silva

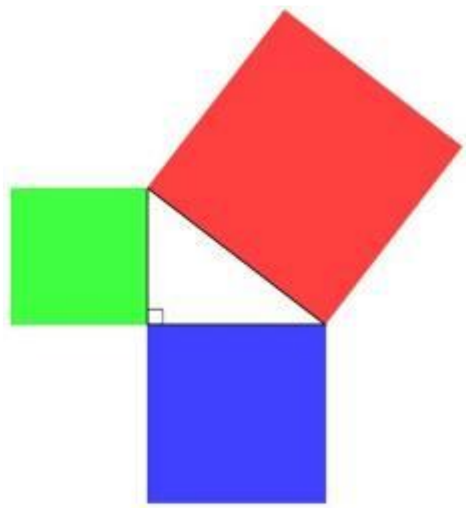
If you know a lot about Pythagorean triples, feel free to skip right to the 5 challenge problems at the end of this article. Otherwise, here is some background information.

A **Pythagorean triple** (a, b, c) is a triple of positive integers that can be used to form the sides of a right triangle with legs of lengths a and b and hypotenuse of length c. According to the Pythagorean theorem, $c^2 = a^2 + b^2$. Conversely, if a, b, and c are positive integers that satisfy the Pythagorean equation $c^2 = a^2 + b^2$, then a, b, and c can be used as the lengths of the sides of a right triangle.

A Pythagorean triple (a, b, c) is called **primitive** if a and b share no common factor other than 1. For example, (3, 4, 5), (5, 12, 13), (8, 15, 17), and (7, 24, 25) are all primitive Pythagorean triples, but (6, 8, 10) is not primitive, even though it is a Pythagorean triple.

Suppose (a, b, c) is a primitive Pythagorean triple. Show that a and b have opposite parity.

A Little History



From the ancient Greek manuscript *Elements*, which was written by Euclid over 2,000 years ago, we learn both the statement and proof of Pythagoras's theorem. In Book I of the *Elements*, we find Proposition 47: *In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle*.[2] That is, the area of the red square is equal to the combined areas of the green and blue squares in the figure at right.

In Book X, Proposition 29, Lemma 1[3], we learn how to generate all primitive Pythagorean triples. Euclid's statement is geometric, but we can put it algebraically, like this: if $m > n$ are positive integers of opposite parity and their highest common factor is 1, then $(2mn, m^2 - n^2, m^2 + n^2)$ is a primitive Pythagorean triple. Furthermore, all primitive Pythagorean triples may be obtained in this manner (switching the leg lengths if necessary so that the even one comes first).

Since every Pythagorean triple can be obtained by scaling a primitive Pythagorean triple by an integer scale factor, the problem of finding all Pythagorean triples is reduced to that of finding all primitive Pythagorean triples. So Euclid's proposition 29 solves the problem of finding Pythagorean triples.

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[1] Thomas Moore is professor emeritus at Bridgewater State University.
[2] See aleph0.clarku.edu/~djoyce/java/elements/bookI/propI47.html.
[3] See aleph0.clarku.edu/~djoyce/java/elements/bookX/propX29.html.

The first few primitive Pythagorean triples derived using Euclid's Proposition 29 are:

m	n	$2mn$	$m^2 - n^2$	$m^2 + n^2$
2	1	4	3	5
3	2	12	5	13
4	1	8	15	17
4	3	24	7	25
5	2	20	21	29
5	4	40	9	41
6	1	12	35	37
6	5	60	11	61

During the years 800 through 200 BCE, books emerged in India that are now grouped under the name **Sulbasutras**. The following primitive Pythagorean triples appear in the Sulbasutras:

$$(3, 4, 5),\quad (5, 12, 13),\quad (8, 15, 17),\quad (7, 24, 25),\quad (12, 35, 37).$$

There are geometric constructions in the Sulbasutras that lead to algebraic formulas that can be used to produce Pythagorean triples. For example, in one construction of a square with a given area t, the algebraic identity[4]

$$t = \left(\frac{t+1}{2}\right)^2 - \left(\frac{t-1}{2}\right)^2$$

can be inferred. If we substitute $t = (2x + 1)^2$ into this identity, we get the identity

$$(2x + 1)^2 = (2x^2 + 2x + 1)^2 - (2x^2 + 2x)^2.$$

This identity yields the Pythagorean triples $(2x^2 + 2x, 2x + 1, 2x^2 + 2x + 1)$, where x is a positive integer. Similarly, substituting $t = x^2$ into the identity yields Pythagorean triples of the form $(2x, x^2 - 1, x^2 + 1)$, where x is any integer greater than 1. Here's a table of some of the Pythagorean triples produced by these formulas:

x	$(2x^2 + 2x, 2x + 1, 2x^2 + 2x + 1)$
1	(4, 3, 5)
2	(12, 5, 13)
3	(24, 7, 25)
4	(40, 9, 41)
5	(60, 11, 61)

x	$(2x, x^2 - 1, x^2 + 1)$
2	(4, 3, 5)
3	(6, 8, 10)
4	(8, 15, 17)
5	(10, 24, 26)
6	(12, 35, 37)

In ancient Babylon, archeologists unearthed thousands of clay tablets with writing on them from a system called **cuneiform**. One of these tablets, known as Plimpton 322, lists some Pythagorean triples, although it is not known whether the Babylonians interpreted these numbers as the sides of a right triangle. The cuneiform system used a base 60 number system. To learn more about this and try your hand at deciphering the contents of Plimpton 322, look up Plimpton 322 on the internet.

[4] See www.math.tifr.res.in/~dani/pyth.pdf

Pythagorean Triples Challenges

I have gathered a few problems that I have published over the years related to this topic. You are hereby challenged to try and solve some of them! The last one is a new problem created especially for Girls' Angle.

Girls' Angle thanks Professor Moore for problem #5!

We welcome you to submit your solutions! Send them to girlsangle@gmail.com.

1. From the *Pi Mu Epsilon Journal*, 1993 (used with permission from Steve Miller):

For $a < b < c$ positive integers, if $\gcd(a, b) = 1$ and $a^2 + b^2 = c^2$, then we say (a, b, c) is a primitive Pythagorean. If both a and c are primes, we call it a prime primitive Pythagorean triple. (i) If (a, b, c) is a prime primitive Pythagorean triple, deduce that $b = c - 1$. (ii) Find all prime primitive Pythagorean triples in which a) a and c are twin primes; b) both are Mersenne primes; c) both are Fermat primes; d) one is Mersenne and the other Fermat.

2. From *The Pentagon*, 2012 (used with permission from Pat Costello):

Prove that there are infinitely many primitive Pythagorean triples (a, b, c), such as (5, 12, 13), with hypotenuse c such that the odd leg is a pentagonal number and the even leg is consecutive with the hypotenuse.

3. Submitted to *The Pentagon*, 2013 (used with permission from Pat Costello):

Prove that there are infinitely many Pythagorean triples (a, b, c) with "legs" a and b, one of which is an abundant number and the other a deficient number.

4. From *MathProblems Journal*, 2013 (used with permission from *MathProblems*):

The examples (3, 4, 5), (5, 12, 13), and (13, 84, 85) show that the same odd number may occur as the "hypotenuse" and as the "odd leg" of primitive Pythagorean triples. Provide explicit constructions of such triples to show that there are infinitely many such odd numbers.

5. For the *Girls' Angle Bulletin*:

Let (a, b, c) be a Pythagorean triple. (i) Prove that the highest power of 2 dividing a cannot equal the highest power of 2 dividing b. (ii) Prove the same for the highest power of 3 dividing a.

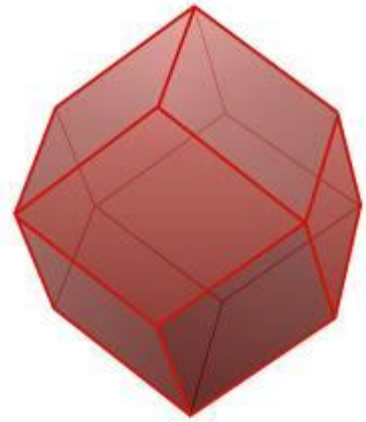
For More

See www.hps.cam.ac.uk/people/robson/neither-sherlock.pdf for more on Plimpton 322.

For a proof of the Pythagorean theorem, check out the Girls' Angle WIM Video featuring Ina Petkova. Also, check out the *Visual Proof of the Pythagorean Theorem* on the Girls' Angle YouTube channel.

There is such a solid, and it is called a **rhombic dodecahedron**.

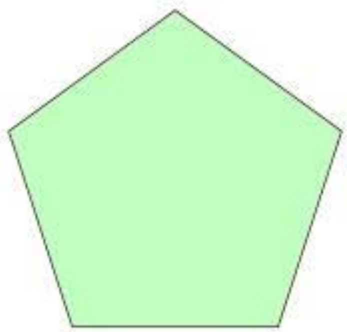
A nonconvex example of a shape whose edges are all parallel to the 4 major diagonals of a cube is the Kosticks' **Tetraxis** puzzle. The name comes from the fact that the edge directions are parallel to the 4 major diagonals of a cube. The video *Tetraxis Geometry* visually explains the geometry of the rhombic dodecahedron and Tetraxis. You can watch it on the Girls' Angle YouTube channel.



A rhombic dodecahedron.

> Restrict yourself to the directions defined by the diagonals of the faces of a fixed cube. Find an equilateral convex solid whose edges each run parallel to one of these 6 directions.

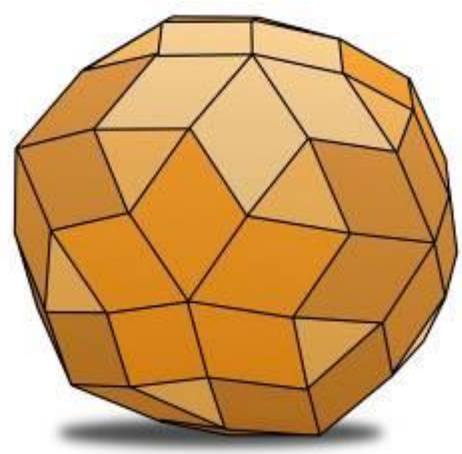
A Leap of Imagination We're ready to explain the Quintetra Assembly. Instead of exploring shapes whose edges are all parallel to a fixed set of 3 or 4 directions, as we have done so far, the Kosticks explored shapes whose edges are parallel to a fixed set of 30 directions!



What 30 directions? Start with a regular dodecahedron. A regular dodecahedron is one of the five **Platonic solids**. It has 12 faces that are congruent regular pentagons, with 20 vertices and 30 edges. Three edges emanate from every vertex. To get a good feeling for the shape, build one! If you make 12 copies of the regular pentagon shown at left, you will find that the dodecahedron practically assembles itself because there is little choice for how to put the faces together. You can also turn to page 11 of Volume 3, Number 4 of this *Bulletin* and find the net of a regular dodecahedron that you can print out and fold.

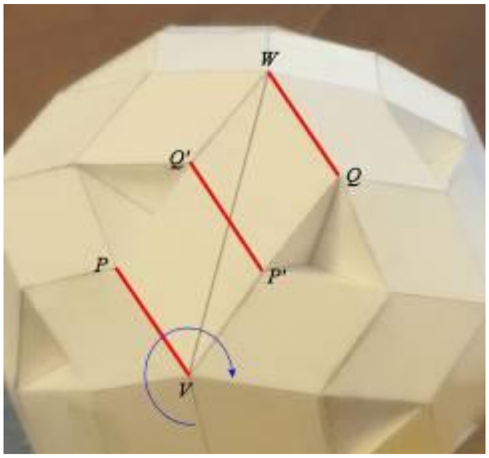
The 20 vertices of the dodecahedron can be grouped into 5 sets of 4 vertices each. In each set, the 4 vertices are the vertices of a regular tetrahedron. If done properly, each of the 5 vertices of any pentagonal face will belong to a different tetrahedron. A tetrahedron has 6 edges, so these 5 tetrahedra collectively have 30 edges. These 30 edges represent the 30 directions to which the Kosticks restricted their explorations.

The Kosticks managed to discern the amazing equilateral convex polyhedron[1] shown at right. By construction, each of its edges runs parallel to one of the 30 directions. The polyhedron consists of 20 equilateral triangular faces and 60 congruent rhombic faces. It has 72 vertices and 150 edges. The centers of the triangular faces form the vertices of a regular dodecahedron, and the rhombi are laid out like a path between the triangles. Most of the vertices are surrounded by 3 rhombi and a triangle, but at 12 of the vertices, 5 rhombi come together to form 5-pointed stars. These 12 special vertices form the vertices of a regular icosahedron.



An equilateral polyhedron with 60 congruent rhombic faces and 20 triangular faces.

[1] According to John Kostick, Zometool is a terrific aid to explore possibilities.



In order to make a model of this polyhedron, the Kosticks had to compute the angles of the rhombic face. One way to find the angles is to determine which of the 30 directions correspond to the adjacent sides of a rhombic face and compute the angle between those 2 directions. I'll sketch another way to find these angles that enables computation of the Cartesian coordinates of all vertices. To follow this approach, you need to be comfortable with trigonometry, vectors, and matrices.

The figure at left shows part of the Quintetra Assembly. Let $\varphi = (1+\sqrt{5})/2$. Vertices V and W are 2 of the 12 vertices where 5 rhombi meet. These 12 vertices form the vertices of an icosahedron. We exploit the fact that the 12 points whose Cartesian coordinates are $(\pm 1, 0, \pm\varphi)$, $(\pm\varphi, \pm 1, 0)$, and $(0, \pm\varphi, \pm 1)$ are the vertices of an icosahedron (where all possible combinations of signs are taken). Without loss of generality, we may assume that $V = (1, 0, \varphi)$ and $W = (-1, 0, \varphi)$.

The 180° rotation about the line that passes through the origin and the midpoint of segment VW interchanges P' and Q'. Therefore, segment $P'Q'$ is parallel to the planes that are perpendicular to the axis of rotation, which include the xy-coordinate plane. That is, P' and Q' have the same z-coordinate. Because $VPQ'P'$ and $WQP'Q'$ are rhombi, we know that PV and WQ are parallel to $P'Q'$. Hence, P, V, Q, and W all have the same z-coordinate, which is φ. Let $P = (x, y, \varphi)$. We seek x and y. By symmetry, we know that $Q = (-x, -y, \varphi)$.

The 72° rotation about the line that passes through the origin and V in the direction indicated by the blue arrow sends P to P'. We use this fact to express the coordinates of P' in terms of the coordinates of P. After some linear algebra, we find

$$P' = \left(\frac{x}{2} + \frac{\varphi}{2} y + \frac{1}{2}, -\frac{\varphi}{2} x + \frac{1}{2\varphi} y + \frac{\varphi}{2}, \frac{1}{2\varphi} x - \frac{y}{2} + \frac{4\varphi+3}{2\varphi+4} \right).$$

Next, we use the fact that $Q'P'$ is parallel to and the same length as PV. This can be expressed by saying that the vector that points from Q' to P' is the same as the vector that points from P to V. When this condition is expressed mathematically and simplified, we arrive at the following system of linear equations in the unknowns x and y:

$$\begin{aligned} 2x + \varphi y &= 0 \\ x - y &= 1 \end{aligned}$$

Solving these for x and y and substituting into our expressions for P and P', we find

$$P = \left(\frac{\varphi}{\varphi+2}, \frac{-2}{\varphi+2}, \varphi \right) \text{ and } P' = \left(\frac{1}{\varphi+2}, \frac{1}{\varphi+2}, \varphi + \frac{2-\varphi}{\varphi+2} \right).$$

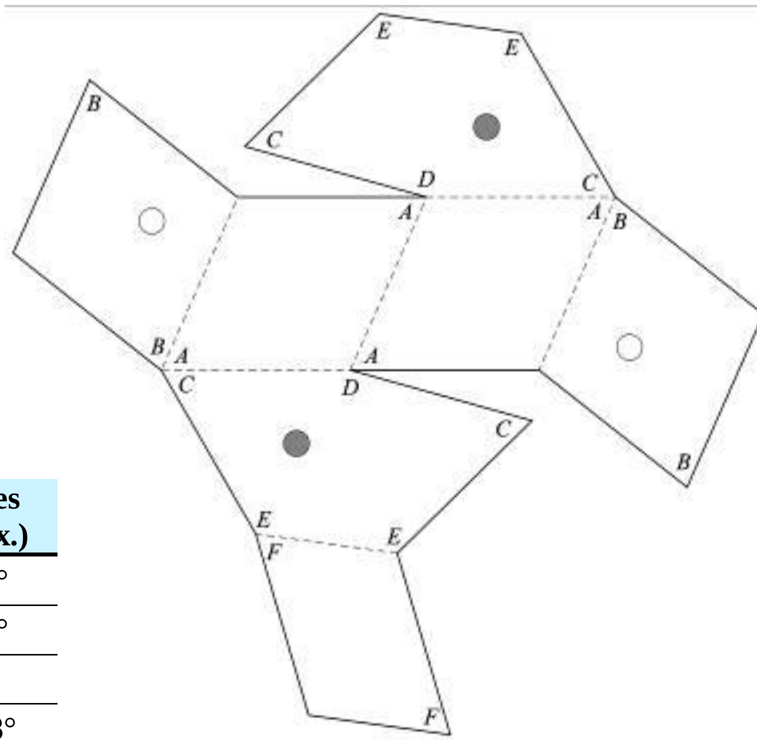
From these, we can compute the angle $P'VP$ (for instance, by using the dot product). We find that angle $P'VP = \cos^{-1}(\varphi/4)$, which is approximately 66.14°.

Jane went beyond understanding the surface of the polyhedron. She designed a unique block, called the **Quintetra block**, from which the polyhedron can be built. The Quintetra block consists of 4 rhombic faces, 2 pentagonal faces, and 1 parallelogram face. It takes 30 Quintetra blocks to build the polyhedron.

Photo courtesy of the Kosticks

The Kosticks' Quintetra Assembly.

The lower left image shows the Kosticks' Quintetra block in 3 different types of wood. The image on the right shows the completed model.



Take It To Your World Make 30 copies of the net shown at right. Cut on the solid lines and fold on the dotted lines. Glue or tape the blocks together so that the dark circles connect to the light circles.

Angles The table below gives the measures of angles in the net. If an angle is unmarked, it is part of a parallelogram with a marked angle.

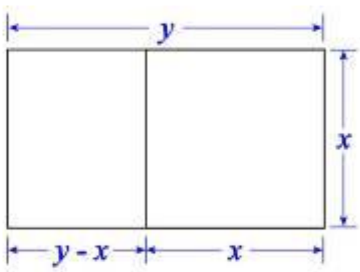
Angle	Exact Measure	Degrees (approx.)
A	$\cos^{-1}(\varphi/4)$	66.14°
B	$\cos^{-1}(1/4)$	75.52°
C	60°	60°
D	$\cos^{-1}((1 - 3\varphi)/4)$	164.48°
E	$210° - D/2$	127.76°
F	$\tan^{-1}\sqrt{5}$	65.91°

More Amazing Facts We urge you to build the model. It will help you to follow the remainder of this article.

There are 20 dimples in the Quintetra Assembly. Recall that all edges of the polyhedron run parallel to the directions determined by the 30 edges of the 5 regular tetrahedra obtained from the 20 vertices of a regular dodecahedron. If you extend a tetrahedron's edges beyond its vertices, you will form "inverted" tetrahedra, and if you extend the edges of these 5 direction-determining tetrahedra, you get the dimples of the Quintetra Assembly!

If you pick out one of the triangular faces T, you can walk from it to 3 other triangular faces on a path of rhombi that each contain edges parallel to the sides of T. These 3 triangular faces, brought together with T (without changing their orientation), will make a regular tetrahedron.

To explain our last observation, we must describe the **rhombic triacontahedron**. A rhombic triacontahedron is a solid with 30 congruent **golden rhombi** for faces. A golden rhombus is formed by connecting the midpoints of the sides of a **golden rectangle**. A golden rectangle is a rectangle with a unique property: if you chop off the largest square possible from one side, the leftover piece will be a rectangle with the same proportions as the original.



Let's walk backwards through these definitions in more detail. Shown at left is an x by y rectangle. A vertical line is drawn inside to mark the left edge of an x by x square. The defining property of a golden rectangle is that the x by $y - x$ rectangle that remains after removing the x by x square is similar to the x by y rectangle. That is, $x : y - x = y : x$. Cross-multiplying, we get $x^2 = y(y - x)$, or $(y/x)^2 - (y/x) - 1 = 0$. This is a quadratic equation in y/x. Since the ratio is positive, we find that $y/x = \varphi$.

Now that we know the exact proportions of a golden rectangle, we can illustrate a fine example, shown at right. To get the golden rhombus, we connect the midpoints of the 4 sides as shown below left. We cut along the lines. The result is the golden rhombus shown at right. Show that the smaller angle in the golden rhombus has a measure of $\tan^{-1} 2$, which is about 63.435°.

To build a rhombic triacontahedron, make 30 of these golden rhombi, all the same size. Join them edge-to-edge to build a 3-dimensional solid. As you join the faces, like angles should meet like angles: 5 golden rhombi meet at each acute-angled vertex, and 3 meet at each obtuse-angled vertex.

What was the point of describing the rhombic triacontahedron? The interior of the Quintetra Assembly is empty. Amazingly, this space will snugly receive a rhombic triacontahedron! So snug is the fit that each of the rhombic triacontahedron's faces will be flush with the inside face of a Quintetra block. For a challenge, prove this fact and find the exact size relationship needed for a snug fit.

Photo courtesy of Jane Kostick

A rhombic triacontahedron built by Jane Kostick.

For more examples of the Kosticks' work and to learn more about the fascinating properties of the Quintetra Assembly, visit their website at www.kosticks.com.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 14 - Meet 5 Mentors: Hannah Larson, Jennifer Matthews, Wangui Mbuguiro
March 6, 2014

Similarity, path counting, segment trisection, the Pythagorean theorem, and variables were on tap at Meet 5.

Session 14 - Meet 6 Mentors: Jennifer Matthews, Wangui Mbuguiro, Liz Simon
March 13, 2014
Visitor: Sarah Spence Adams, Olin College

Prof. Sarah Spence Adams presented on secret codes and error correcting codes. Her first example of an error correcting code was a simple "repeat the code" idea. If you use a code where "1" denotes "yes" and "0" denotes "no," to try to ensure that your message does not get garbled, you could agree to transmit "1111" for "yes" and "0000" for no. Then if you receive "1110," it is likely to have meant "yes." However, if you receive a "1010," then the message is ambiguous. Fortunately, you can tell that an error must have occurred, so you can ask the sender to resend the message. To make the system even more robust, you could increase the number of 1's and 0's. However, such a code would be inefficient.

So instead of repeating the message, Sarah introduced a parity idea. For example suppose North is encoded as "11", South as "00", East as "01", and West as "10". To make the code more robust, one could add a single bit which represents parity, that is, one adds a 0 or a 1 depending on whether the number of prior bits is even or odd, respectively. Thus, North would be sent as "110", South as "000", East as "011", and West as "101". When one receives an encoded compass heading, one can check to see if the parity bit is consistent.

Sarah concluded by explaining the UPC codes found on products in stores. The UPC code is a 12 digit code with the last digit reserved as a checksum. If the first 11 digits are X_1, X_2, X_3, …, X_{11} and the last digit is P, then P is chosen so that

$$3(X_1 + X_3 + X_5 + X_7 + X_9 + X_{11}) + X_2 + X_4 + X_6 + X_8 + X_{10} + P = 0 \pmod{10}.$$

Sarah left us with the following problem: Determine which adjacent transpositions in the UPC are undetectable. In other words, when might two adjacent digits switch places in an undetectable way?

Session 14 - Meet 7 Mentors: Liz Simon
March 20, 2014

We worked on personal math projects, perspective drawing, and building a multiplication sculpture.

Session 14 - Meet 8 Mentors: Inhwa Chi, Hannah Larson, Wangui Mbuguiro, Isabel Vogt
April 3, 2014

Several members have been thinking about perspective drawing. A key to understanding perspective drawing is that perspective drawings are designed to be seen from a specific vantage point. This idea can sometimes be hard to discover because our mind can interpret scenes even if there is some distortion. That's why perspective paintings look fine from several vantage points. Even when we view them from an extreme vantage point, we'll still remember that the painting depicts a Venetian cityscape.

To see that perspective drawings must be constructed for a specific vantage point, look through a window with one eye. If an artist wants to fool you into thinking you were looking at the same scene when you are actually looking at a painting, the artist should paint objects on a canvas of the exact same size as the window and place the objects in the same exact position on the canvas as they appear in the window. If a tree branch enters the scene 5 inches below the upper left corner of the window, then that is where the artist should paint the branch entering the canvas. Now observe that moving your head even the slightest amount in any direction causes the entire image through the window to change. The branch that entered 5 inches below the upper left corner might now enter 4.5 inches below. Objects you could see before may no longer be visible and objects you couldn't see before creep into view. If, when you first gazed through the window, what you were actually looking at was a painting installed in place of the window, nothing would change as you moved your head about. In fact, if you move your head about and nothing in the window changes, that's a telltale sign that you're not actually looking through a window, but at a painting. The fact that the view through an actual window does change as you move about shows that perspective drawings must be designed for a specific vantage point.

Some members studied proofs in Roger Nelsen's book *Proofs Without Words: Exercises in Visual Thinking*. This wonderful collection of visual proofs can also serve as a terrific source of material for those who want to practice articulating mathematics, either orally or in writing.

Session 14 - Meet 9 Mentors: Hannah Larson, Jennifer Matthews, Isabel Vogt
April 10, 2014
Visitor: Anna Frebel, Department of Physics, MIT

Last semester, Prof. Anna Frebel told us how we can learn about a star by looking at its light. This time, she explained how stars produce that light. She explained how nuclear fusion occurs inside of stars and described some of the fusion reactions. She had us computing the energy of the light released from such reactions by comparing the mass of the resulting particles to the reactants and applying Einstein's famous equation $E = mc^2$, which says that the energy is proportional to the mass with constant of proportionality equal to the square of the speed of light.

Session 14 - Meet 10 Mentors: Jordan Downey, Jennifer Matthews, Wangui Mbuguiro
April 17, 2014

Some members completed a multiplication sculpture and pondered its properties. For other examples of multiplication sculptures, google "multiplication sculpture" or take a look at Volume 2, Number 2 of this *Bulletin*.

Calendar

Session 14: (all dates in 2014)

January	30	Start of the fourteenth session!
February	6	
	13	
	20	No meet
	27	
March	6	
	13	Sarah Spence Adams, Olin College
	20	
	27	No meet
April	3	
	10	Anna Frebel, Department of Physics, MIT
	17	
	24	No meet
May	1	Karen Willcox, Dept. of Aeronautics and Astronautics, MIT
	8	

Session 15: (all dates in 2014)

September	11	Start of the fifteenth session!
	18	
	25	No meet
October	2	Emily Pittore, iRobot
	9	
	16	
	23	
	30	
November	6	
	13	
	20	
	27	Thanksgiving - No meet
December	4	
	11	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

All Girls' Angle Members and Subscribers are invited to email math questions, solutions, comments, and suggestions. We will respond!

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- □ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- □ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, assistant professor, UCSF Medical School
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, Tamarkin assistant professor, Brown University
Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.

☐ Enclosed is $216 for one session (12 meets)

☐ I will pay on a per meet basis at $20/meet.

Nonmembers: Please choose one.

☐ I will pay on a per meet basis at $30/meet.

☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ______________________________________