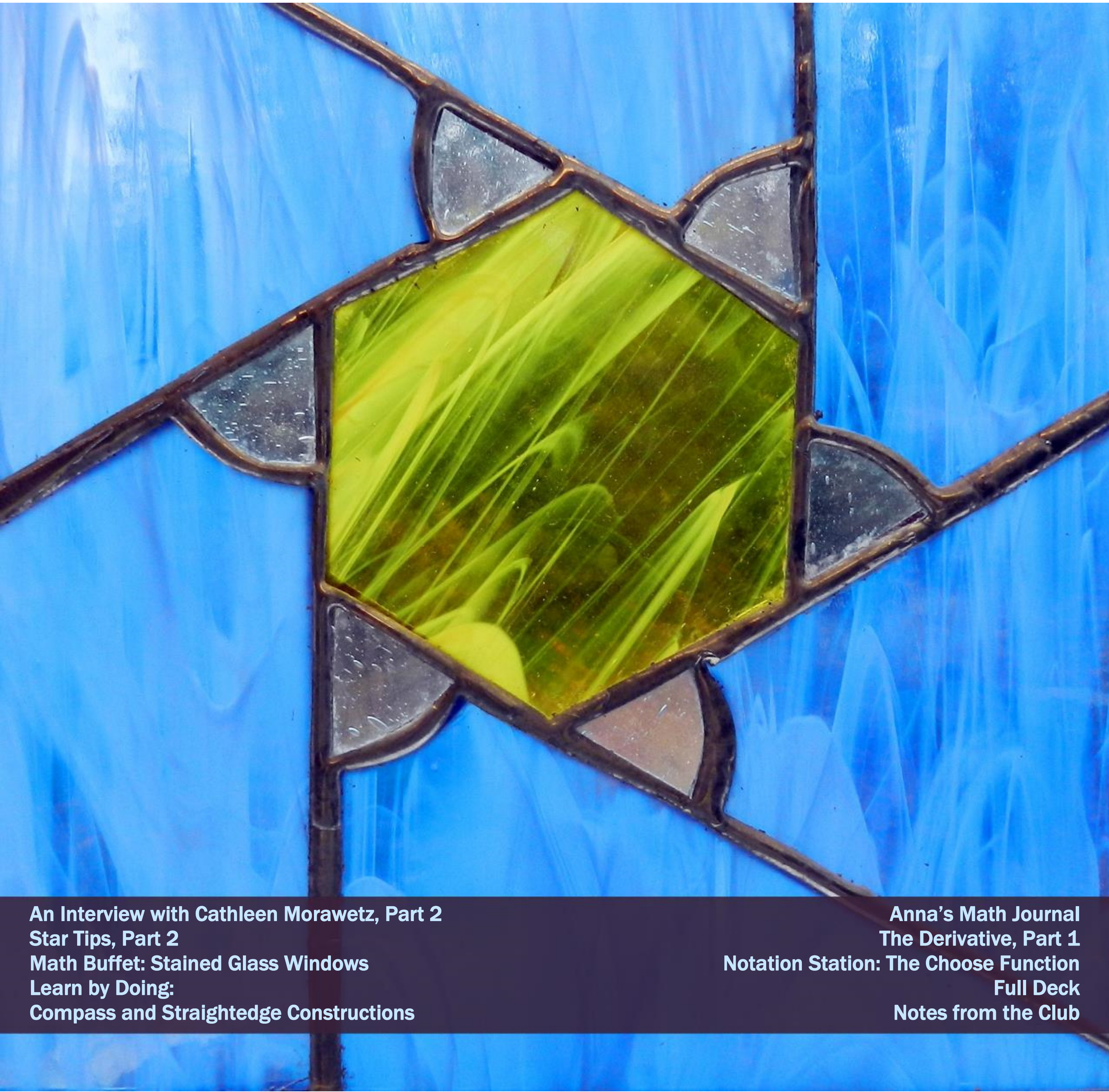
Girls' Angle Bulletin

December 2014 • Volume 8 • Number 2

To Foster and Nurture Girls' Interest in Mathematics



From the Founder

If you get stuck on something in the Bulletin, write to us about it! Subscriptions to the Bulletin come with mathematical support. Don't be shy. To learn math well, you have to do math, and writing us with math questions is a good way to start articulating your mathematical thoughts.

- Ken Fan, President and Founder

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Girls' Angle Bulletin
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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Detail of a stained glass window by Millie Wert. For more math-related stained glass windows, see this issue's *Math Buffet*.

An Interview with Cathleen Morawetz, Part 2

This is the concluding half of Girls' Angle Program Assistant Margo Dawes interview with Courant Institute of Mathematical Sciences Professor Emerita Cathleen Morawetz.

> *I was always much more concerned about whether I* wanted *to do it, not whether I* could *do it.*

Margo: That sounds great. Are you working on anything right now?

Cathleen: I stopped doing mathematics a year ago. Stopped *trying* to do mathematics.

Margo: Do you miss it?

Cathleen: No, I read a lot. And I have taking care of the house and stuff like that to keep me occupied. I have a daughter who lives nearby, who in turn has two daughters, and they come to see me. Right now actually I have a grandson from Nashville who's here visiting with his wife and son.

Margo: Yes! You mentioned having a great-grandson at summer camp before we started the interview.

Cathleen: Yes, my one and only great-grandchild.

Margo: Wonderful. Going back to gender and mathematics, do you think there is still a gender bias in the field today?

Cathleen: There may be. I'm not really familiar with what other places are like. On the other hand, there are also many strong supporters of women. In the early levels, at the school-teaching level, for example, I think there are still predominantly women teachers. In the first stages of mathematics, which is, after all, arithmetic, there are lots of women involved. And some of them go on then and get higher degrees. We have several women on the faculty now, when of course in decades past there was only me. Maybe me and one other person.

Margo: You make a good point, though, about how the divide is relatively equal early on. But when you go up to the undergraduate and graduate level you see the representation drop off sharply.

Cathleen: There may be a big difference between doing arithmetic and doing mathematics.

Margo: There is an explanation put forth by some psychologists that the disparity between men and women in the upper levels of mathematics is not due a gender bias so much as due to a difference in preferences. What do you think of that argument? Do you think it's valid?

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Morawetz, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
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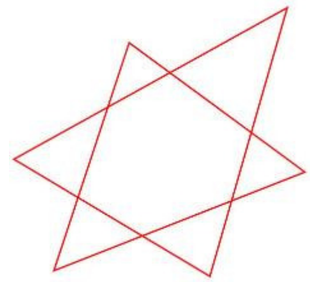
Girls' Angle

Star Tips, Part 2

by Madison Evans and Ken Fan | edited by Jennifer Silva

In Part 1, Emily and Jasmine showed that the sum of the tip angles in any 5-pointed star is 180°.



Emily: Do you think there's a similar result for the sum of the tip angles in a 6-pointed star?

Jasmine: Probably. Let's draw a 6-pointed star. (See figure at right.)

Emily: It reminds me of a Star of David. It's made up of two overlapping triangles.

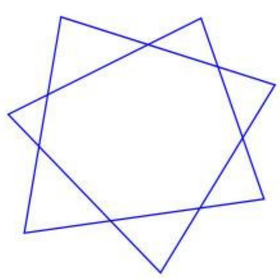
Jasmine: So it is! That means that the sum of its tip angles is just the sum of the angles of two triangles, and that's 360°...

Emily: ... double the amount for a 5-pointed star.

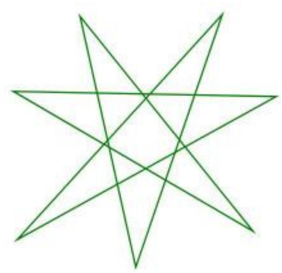
Jasmine: That worked out well. Let's try 7-pointed stars!

Emily: Okay!

Jasmine and Emily both draw 7-pointed stars, then look at each other's drawings.



Emily's 7-pointed star



Jasmine's 7-pointed star

Jasmine: They look quite different.

Emily: Yours splits the plane into more regions, and it looks more star-like than mine.

Jasmine: But yours looks more like the 5- and 6-pointed stars we've studied. All of those looked like polygons with triangles attached to each side, and yours is a 7-sided polygon with triangles on each of its sides.

Emily: How did you construct yours? I like it.

Jasmine: I put 7 dots roughly in a circular arrangement. Then I connected each dot to the dots farthest away around the circle.

Emily: I see. I made my 7-pointed star in a similar way. I also put 7 dots in a circular arrangement, but then I connected each dot to the ones 2 away around the circle.

Jasmine: So the difference between our 7-pointed stars is that in yours, dots are connected to those 2 away around the circle, whereas in mine, dots are connected to those 3 away around the circle?

Emily: I think that's right.

Jasmine: Wait a sec. If there are different kinds of 7-pointed stars, maybe there are different kinds of 5- and 6-pointed stars. Are there?

Emily: Let's see. If we place 5 points roughly around a circle and connect each point to the ones 1 away, we get a pentagon. If we connect each point to the ones 2 away, we get a 5-pointed star like the ones we considered. And if we connect each point to the ones 3 away ...

Jasmine: Since the points are arranged around a circle, that's the same as connecting each point to the ones 2 away. In general, if you have n points in a circle, connecting each point to the ones k away is the same as connecting each one to the ones $n - k$ away.

Emily: Good point. That means there's only one type of 5-pointed star ... at least, of the sort we're making.

Jasmine: Or two types if you think of the pentagon itself as a kind of 5-pointed star.

Emily: Actually, what is a star? Until I saw your star, I had been thinking of an n-pointed star as an n-sided polygon with triangles on each side, where each triangle is formed by extending the sides of the polygon. But I'd prefer a definition of star that includes yours.

Jasmine: How about we define an n-pointed star to be a figure obtained by plotting n points in a circle and connecting each point to the points that are k away, going around the circle? Maybe we can call such a star an (n, k)-star.

Emily: So an $(n, 1)$-star is just an n-gon? And an $(n, 2)$-star would be like the 5- and 6-pointed stars we've already thought about and the 7-pointed star that I just drew. The star you drew would be a (7, 3)-star?

Jasmine: Right.

Emily: Hold on. This definition *doesn't* include the stars we just drew, nor does it cover many of the stars we've seen before, because of the requirement that the vertices sit on a circle.

Jasmine: Oh, you're right. Let's relax the condition that the vertices be on a circle. I only wanted them around a circle so that each vertex had two well-defined neighbors. Perhaps we can just assign n dots an arbitrary order and use that order to determine which dots connect to each other.

Emily: You mean you'd like to assign the numbers 0 through $n - 1$ to the n dots and connect dot x to dot $x + k$, where we add modulo n?
Jasmine: Yes.

Emily and Jasmine pause while they think about this intriguing possibility.

Emily: There's a problem with this idea.

Jasmine: What's that?

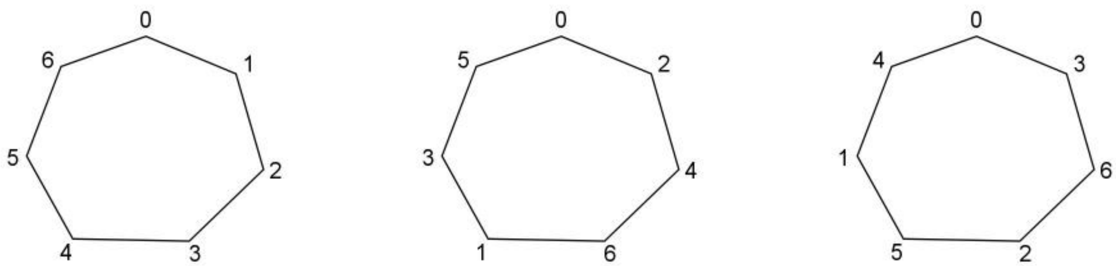
Emily: I think it makes the parameter *k* irrelevant.

Jasmine: How do you mean?

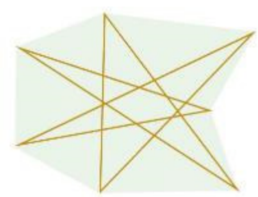
Emily: We want to think of an *n*-gon as an (*n*, 1)-star, right?

Jasmine: Right.

Emily: But if I can label the vertices anyway I wish, I can make an (*n*, *k*)-star an *n*-gon for values of *k* that we don't want to represent *n*-gons. For example, these drawings show how an *n*-gon could be viewed as a (7, 1)-star, a (7, 2)-star, or a (7, 3)-star.



Jasmine: Oh, you're right. That's a big problem! So much for that idea. I guess we do need some kind of geometric structure on the vertices so that skipping around the vertices by 1, 2, or 3 produces different types of stars. What if we simply insist that the *n* dots form the vertices of a *convex n*-gon? That's less restrictive than requiring them to be on a circle.



Emily: I think that definition does capture all of the stars we've seen so far, but what about this one? (See figure at right.)

Jasmine: The definition does exclude that one and several others, but it includes all of the stars we had previously considered, as well as many others. How about we go with this definition for the time being and see if we can show that the sum of the tip angles doesn't depend on the precise location of the vertices. We can call these the "convex stars." We can always refine our definition later. I just like convexity because it helps us understand which edges of the star intersect each other.

Emily: All right. So here's the definition we're going to use: Let *n* be an integer greater than 4. Let *k* be an integer between 0 and *n*/2. A convex (*n*, *k*)-star is a figure obtained by connecting each vertex of the *n* vertices of a convex *n*-gon to the vertex *k* over, going around the *n*-gon.

Jasmine: I'm happy with that, and I'd be even happier if we call the parameter *k* the "spikey factor"!

Emily: The "spikey factor"?

Jasmine: It seems like each time k increases, the star tips become pointier. When k is 1, you get a plain old n-gon. When k is 2, you get triangles on an n-gon. When k is 3, you get something even spikier, hence the "spikey factor"!

Emily: Okay. So is it true that all n-vertex stars with spikey factor k have the same tip angle sum? And if so, what is that sum?

Jasmine: Hmm. Sometimes, an (n, k)-star is really a union of stars with fewer vertices; an example would be the (6, 2)-star, which is a union of 2 triangles. For those stars, we could use induction on n. If an (n, k)-star is a union of T smaller (n', k')-stars, then the sum of the tip angles of the (n, k)-star is T times the sum of the tip angles of an (n', k')-star.

Emily: It looks like we should try to figure out when an (n, k)-star is a union of smaller stars and when it isn't.

Jasmine: Let's call an (n, k)-star "reducible" if it is a union of stars with fewer vertices. Otherwise, let's call it "irreducible."

Emily: Okay. We only need to figure out the sum of the tip angles in irreducible stars to figure out the sum of the tip angles for all stars.

Jasmine: If an (n, k)-star is a union of smaller stars, then the following holds true: if you start at a vertex and connect it to the vertex k over, then connect that to the one k over, and so on, you'll end up back at the starting vertex *before* you've visited all of the vertices.

Emily: This is just like adding in modular arithmetic! If we label the vertices 0, 1, 2, 3, ..., $n - 1$ going clockwise around the n-gon, then we are connecting vertex x to vertex $x + k$, modulo n. That is, modulo n, vertex 0 is connected to vertex k, vertex k is connected to vertex $2k$, vertex $2k$ is connected to vertex $3k$, and so on.

Jasmine: And we return to vertex 0 for the first time with the first positive multiple of k that is divisible by n.

Emily: So we must find the smallest positive integer a such that ak is divisible by n.

Jasmine: If n is relatively prime to k, this happens when a equals n. So an (n, k)-star is irreducible if n and k are relatively prime.

Emily: And if the greatest common factor of n and k is d, then we're looking for the smallest positive integer a such that $a(k/d)$ is divisible by n/d; that happens when $a = n/d$, because k/d and n/d are relatively prime. So an (n, k)-star is irreducible *if and only if* n and k are relatively prime.

Jasmine: In fact, if we look at the $(n/d, k/d)$-star that contains vertex 0, it consists of n/d evenly-spaced vertices. If we rotate from these vertices 1 over in the same direction, we will have all the vertices of another $(n/d, k/d)$-star component of our (n, k)-star. We can keep rotating like this to find all of the sub-stars of our (n, k)-star. This means that an (n, k)-star is a union of exactly d smaller $(n/d, k/d)$-stars.

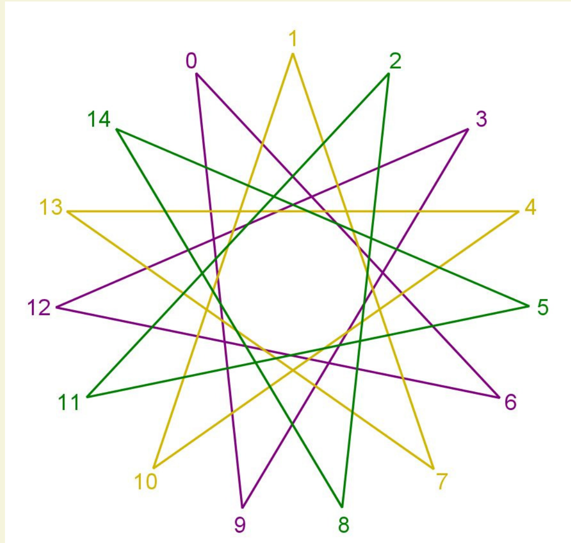
Emily: Yes, each (n/d, k/d)-star will consist, modulo n, of the vertices

$$m, m + k, m + 2k, ..., m + (n/d - 1)k,$$

for a fixed m between 0 and $d - 1$, inclusive. For each such value of m, we get a different (n/d, k/d)-star inside the (n, k)-star.

Jasmine: These are all basic facts about modular arithmetic.

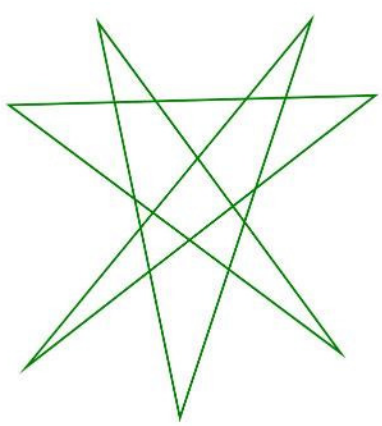
Emily: It's not too surprising that modular arithmetic pops up here since we're systematically connecting points around a loop.



A (15, 6)-star is the union of 3 (5, 2)-stars.

Jasmine: So we've reduced to the case of the convex (n, k)-star where n and k are relatively prime. If we can show that the sum of its tip angles is independent of the vertex locations, then we'll have proven that the sum of the tip angles is constant for all (n, k)-stars, regardless of whether n and k are relatively prime.

Emily: But how are we going to do that? Your (7, 3)-star looks so complicated. I bet (n, k)-stars can get even more complicated for larger values of n and k. The way we computed the sum of the tip angles in a (5, 2)-star relied on it being a pentagon with triangles built on each side. In general, I don't even think that there will necessarily be an n-gon in the center of (n, k)-stars.



Jasmine: You're right. Take a look at this star. (See figure at left.) It's a (7, 3)-star with no central 7-gon. What can we do with something like this?

Emily and Jasmine think quietly.

Emily: Do you remember that invariance property you noticed last time?

Jasmine: You mean, that the sum of the tip angles doesn't change when we move one vertex while keeping the others fixed? What about it?

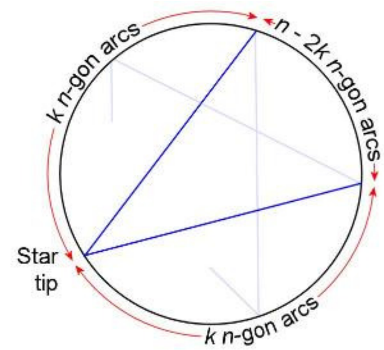
Emily: If that's true in general, perhaps we can move the vertices around one by one until we get a regular star, where the vertices are from those of a regular n-gon. That would prove that the sum of tip angles only depends on the number of vertices and the spikey factor. We could then calculate the value of the sum by computing it for the regular star.

Jasmine: It sounds like a plausible approach, though there are restrictions on the movement of a vertex. The vertices have to preserve their circular order throughout the transformation as well.

Emily: Plus, we can only move one vertex at a time. This is pretty daunting.

Jasmine: It does seem difficult, but I like the idea. It strongly suggests that the sum of the tip angles will be independent of the locations of the vertices in general; we can definitely morph some (n, k)-stars to other (n, k)-stars using the technique. We just don't know if we can morph *any* (n, k)-star to a regular (n, k)-star. Also, I think we should be able to compute the sum of the tip angles for a regular (n, k)-star, because we can use the formula that relates the measure of an angle inscribed in a circle with that of the subtended arc.

Emily: That's a good point! Let's figure out the sum of the tip angles of a regular (n, k)-star. In a regular (n, k)-star, all of the tip angles are congruent. So we only have to compute one of them, then multiply that by n.



Jasmine: Funny that we didn't use this approach when we computed the tip angles in the regular 5-pointed star!

Emily: The vertices of a regular n-gon divide the perimeter of the circle into equal arcs, each with a central angle of $360/n$ degrees.

Jasmine: But in an (n, k)-star, each vertex is connected to the vertices that are k vertices away around the circle. So the central angle of the arc cut off by a tip angle would be (n – 2k) times $360/n$ degrees.

Emily: Since the inscribed angle is half the central angle, each tip angle measures (n – 2k) times $180/n$ degrees. When we multiply that by n, we get that the sum of the tip angles of a regular (n, k)-star is $180(n – 2k)$ degrees!

Jasmine: Neat! I want to check this against what we found for the (5, 2)- and (6, 2)-stars. When n is 5 and k is 2, the formula gives us 180(5 – 2(2)) or 180 degrees.

Emily: Check! And when n is 6 and k is 2, we get 360 degrees. I bet that's the general formula. We just have to figure out how to morph each (n, k)-star to a regular one to prove it.

Jasmine: Hmm.

Emily: Jasmine, suppose this formula is correct for all convex (n, k)-stars. Do you find it curious that the sum of the tip angles always comes out to be a multiple of 180°?

Jasmine: That *is* curious. In fact, that's got to be a clue!

Emily: You mean a clue as to how to morph convex (n, k)-stars to regular ones?

Jasmine: No, I mean that that observation has to be trying to tell us something … Wait a sec! I think I know!

What do you think Jasmine knows? Tune in next time to find out!

Math Buffet

Stained Glass Windows
Layout by Toshia McCabe

Marion Walter suggested the theme for this Math Buffet: mathematically inspired stained glass windows. After reading *Stained Glass Angles* (Vol. 7, No. 4 of this Bulletin), she came upon John Rose's creation (see page 15) at the Eugene Public Library and contacted him to contribute a photo.

Created by Jim Sawyer

Mr. Sawyer displayed these polyhedral stain glass works at the 2000 Math/Chem/Comp Conference which was held in Dubrovnik, Croatia.

How are the two large polyhedra related?

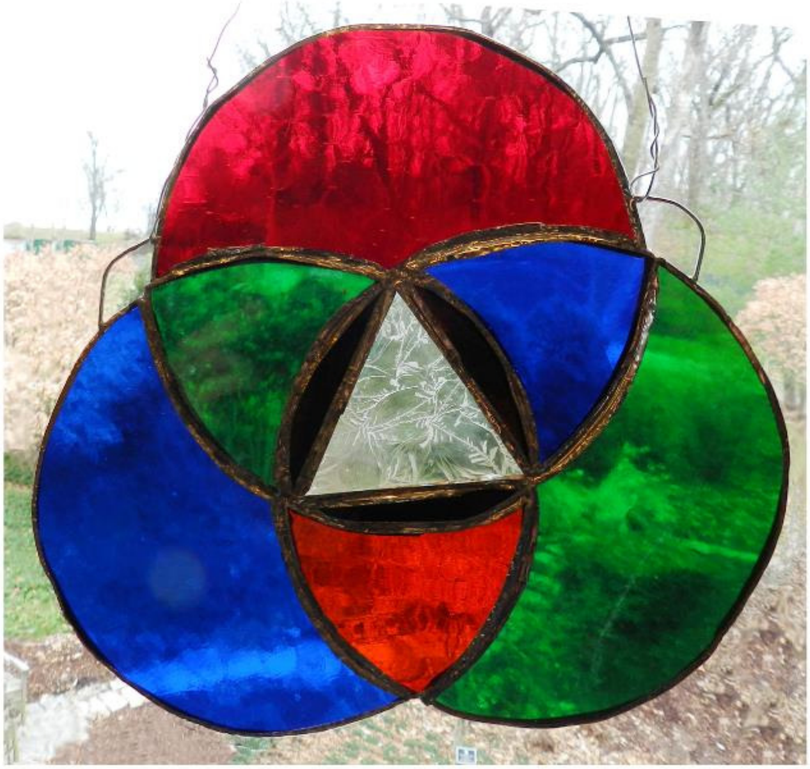
If you extend the edges where two hexagonal faces meet, you will obtain a special polyhedron. What is it?

Created by Thaddeus Wert

Mr. Wert teaches math at Harpeth Hall in Nashville, Tennessee.

Some of his student's works are featured in this *Math Buffet*.

In this work, Mr. Wert builds an entire stain glass window out of circles. All the ways two circles can relate to each other in a plane are depicted.



Created by
Sydney Webber

Ms. Webber is a Junior at Harpeth Hall.

Here, Ms. Webber creates a stained glass window design based on Borromean rings. Borromean rings are 3 hoops interlinked in such a way that no 2 are linked. Based on Ms. Webber's design, can you construct a triple of Borromean rings?

How does this design compare and contrast with Mr. Wert's design?

Created by
Shelby Nutter

Ms. Nutter is a junior at Harpeth Hall.

Ms. Nutter's window design depicts a projection of the 4D hypercube, which is also known as a "tesseract."

In the actual tesseract 4 edges meet at each corner, and any pair of these 4 edges meet at a right angle.

Notice how Ms. Nutter used textures and color to reinforce the mathematical structure. It makes the window look 3 dimensional.

Created by
Claire Trabue

Ms. Trabue is a Sophomore at Harpeth Hall.

Ms. Trabue's design illustrates Johnson's theorem: When 3 congruent circles intersect in a common point, the circle through the other 3 pairwise intersections is congruent to the original 3 circles.

Can you prove the theorem?

Notice that you can regard any 3 of the 4 circles in the window as the given circles.

Ms. Trabue's design also draws attention to a special triangle. How does this triangle relate to the circles?

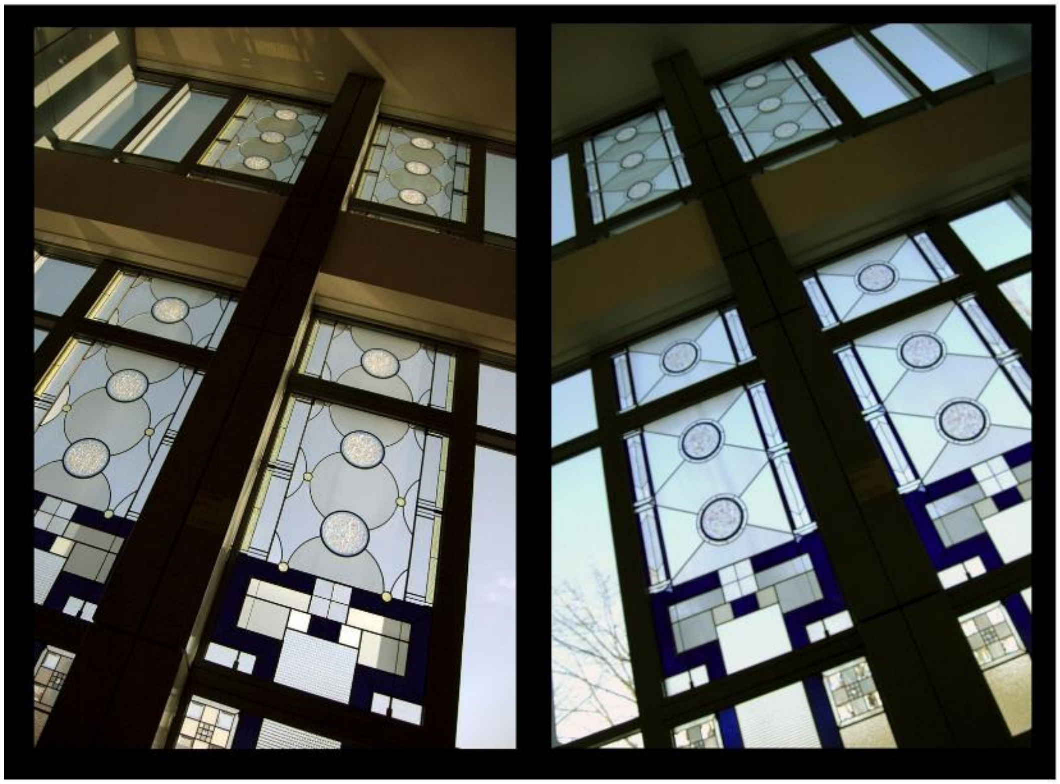
Created by
Millie Wert

Ms. Wert is Harpeth Hall class of 2013.

A detail of Ms. Wert's design can be found on this issue's cover.

Although it may look like a flower, the design is actually inspired by the theorem that says that the sum of the exterior angles of a polygon is a full circle.

Can you determine all of the angle measurements in Ms. Wert's design?



Created by
John Rose

Mr. Rose created these windows for the Eugene Public Library in Eugene, Oregon.

Imagine studying at a desk awash in the light passing through these windows, like the people in the background below. Every now and then you could look up from your studies and ponder upon the beautiful designs.

Special Thanks to all the contributors and to Marion Walter for suggesting the idea and helping to make it happen.

Learn by Doing

Compass and Straightedge Constructions
by Girls' Angle Staff

What figures can you draw if the only tools you have are a compass and a straightedge?

We'll explore the answer to this question in this installment of Learn by Doing. Before we begin, let's address exactly what we mean by a compass and straightedge.

A **compass** is a tool for making circles and circular arcs. A typical compass has two legs joined by a hinge. At the end of one leg, there is a sharp point. At the end of the other leg, there is a pencil tip. To use the compass, one anchors the sharp point and swings the pencil tip around to trace out the desired arc. For our purposes, we will assume that the compass does not collapse when lifted from the paper. This means that you can use the compass to transfer lengths.

A **straightedge** is like a ruler, except that it doesn't have any markings. You cannot measure distance with a straightedge. You can only use it to draw straight lines.

This problem set is a lot more fun if you actually have a compass and straightedge to experiment with. But if you don't, you can still do the problems in a theoretical way. Whenever we use the word "construct," we mean to construct using your compass and straightedge.

Let's get started!

Problem 1. Draw some line segments and circles using your compass and straightedge.

Problem 2. Draw a long line segment. Using the compass, mark off several consecutive segments of equal length along this line segment.

Problem 3. Draw a line segment. You might be able to eyeball the midpoint of the line segment quite accurately. However, with compass and straightedge, you don't have to guess. Figure out a way to locate the exact midpoint of the line segment using your compass and straightedge. (The last line segment or circle you draw should be one that intersects the give line segment exactly through its midpoint.)

Problem 4. Draw a line segment. Construct a line segment that not only passes right through the midpoint of the given segment, but does so at right angles. In other words, construct the perpendicular bisector of the given segment.

Problem 5. Draw a line segment. Let L be the line that contains the line segment. Mark some arbitrary point P not on L. Construct a line segment that passes through P and is perpendicular to L.

Problem 6. Draw a line segment. Let L be the line that contains the line segment. Mark some arbitrary point P not on L. Construct a line segment that passes through P and is parallel to L.

Problem 7. Problem 2 is about marking off equally spaced points along a line segment. For that problem, the exact distances between points does not matter. Draw a line segment. Explain how to divide the line segment into n equal pieces for any positive integer n using compass and straightedge. In other words, construct $n - 1$ points on the line segment, which, together with the endpoints, form $n + 1$ equally spaced points.

Problem 8. Draw a circle. Construct 3 equally spaced points around the circle, or, what is essentially the same thing, construct an equilateral triangle inscribed in the circle.

Problem 9. Draw a circle. Construct 4 equally spaced points around the circle, or, what is essentially the same thing, construct a square inscribed in the circle.

Problem 10. Given two line segments, construct a line segment whose length is equal to the sum of the lengths of the given segments.

Problem 11. Given an arbitrary triangle and line segment, construct a triangle that uses the line segment as one of its sides and is similar to the given triangle.

For the remaining problems, assume that there is drawn a line segment of unit length.

Problem 12. Draw two line segments of arbitrary length. Let x and y be their lengths. Construct a line segment of length xy. (Hint: use the concept of similarity.) Why is it necessary to be provided with a line segment of unit length?

Problem 13. Draw a line segment of arbitrary length x. Construct a line segment of length $1/x$.

Problem 14. Explain how to construct a line segment whose length is any positive rational number. (A rational number is a number that can be expressed as the ratio of two integers.)

Problem 15. Draw two circles of the same radius side by side so that they are touching. Tangent to both of these circles and centered at the point where they touch, draw a larger circle. Construct a circle that is internally tangent to the larger circle and externally tangent to the two smaller circles.

Problem 16. Let's take a small break from compass and straightedge constructions and solve a geometry problem. Consider a circle with diameter $\overline{AB}$. Let P be a point on $\overline{AB}$ such that $AP = 1$ and $PB = x$. Let C be the chord perpendicular to $\overline{AB}$ that passes through P. What is the length of C in terms of x?

Problem 17. Draw a line segment of arbitrary length x. Construct a line segment of length $\sqrt{x}$.

Problem 18. Draw an arbitrary quadrilateral. Explain how to construct a square of the same area.

Problem 19. Draw 3 line segments and suppose that they have lengths a, b, and c, respectively. Let $s + ti$ be a root of $ax^2 + bx + c = 0$, where $i^2 = -1$ and s and t are both real numbers. Construct two line segments, one of length $|\, s \,|$ and the other of length $|\, t \,|$.

Problem 20. Draw an arbitrary angle. Let T be the measure of the angle. Construct line segments with lengths sin T and cos T.

Problem 21. Draw a line segment and suppose it has length $x \leq 1$. Construct an angle whose cosine is x.

Problem 22. Construct a regular pentagon. Hint: cos 72° = $\left(\sqrt{5}-1\right)/4$

Problem 23. Construct a regular hexagon.

If you've done Problems 8, 9, 22, and 23, you've constructed an equilateral triangle, square, regular pentagon, and regular hexagon. A natural question to ask is: Which regular polygons can be constructed with compass and straightedge?

In 1796, Carl Friedrich Gauss showed that an n-gon is constructible if and only if n is a product of **Fermat primes**, which are prime numbers of the form $2^{2^n}+1$, and a power of 2. Since 7 is not a Fermat prime, the regular heptagon cannot be constructed with compass and straightedge.

The known Fermat primes are:

$$2^{2^0}+1=3,\ 2^{2^1}+1=5,\ 2^{2^2}+1=17,\ 2^{2^3}+1=257,\text{ and } 2^{2^4}+1=65537.$$

It is not known if there are any more.

Problem 24. Draw an arbitrary triangle. Construct its circumscribed and inscribed circles.

Problem 25. Draw an arbitrary triangle ABC. Construct point X on $\overline{AB}$ and Y on $\overline{CB}$ so that $AX = XY = YC$.

Problem 26. Draw 3 points A, B, and C. A collapsing compass is a compass that doesn't hold its shape if it is lifted from the paper. Show that even with a collapsing compass and straightedge, one can still construct a circle of radius AB centered at C. In other words, although you cannot transfer lengths directly with a collapsing compass, you can still do so via a construction. Therefore, anything that you can construct with compass and straightedge can also be done with a collapsing compass and straightedge.

Problem 27. A rusty (or "fixed") compass is a rigid compass whose angle cannot be changed. Explain how to construct equilateral triangles with sides of length any rational number using a rusty compass and straightedge.

Problem 28. Incredibly, any construction that can be carried out with compass and straightedge can be carried out with *only a compass*, where we consider a line to be constructed if two points on that line have been constructed (each as the intersection of two circular arcs). This result was proven by George Mohr in 1672 and independently discovered by Mascheroni in 1797. Can you prove it? For a detailed proof, see Essay Fifteen in Ross Honsberger's book *Ingenuity in Mathematics*. There you'll also find a proof of Steiner's result that anything that can be constructed with compass and straightedge can be constructed with *only a straightedge* provided that a circle and its center are given.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna turns her attention to cross sections of the surface $z = xy$.

Investigating the paraboloid of revolution was fun. I think I'll study cross sections of the surface defined by $z = xy$ next.

I'll start with vertical planes.

I'm orienting things so that the z-axis points straight up.

$z = xy$

Cross sections with vertical planes.

$x = c$, c constant.

Planes parallel to the yz coordinate plane intersect the surface in lines...

$z = cy \Rightarrow$ straight line

$y = mx + b$

$z = x(mx+b) = mx^2 + bx$

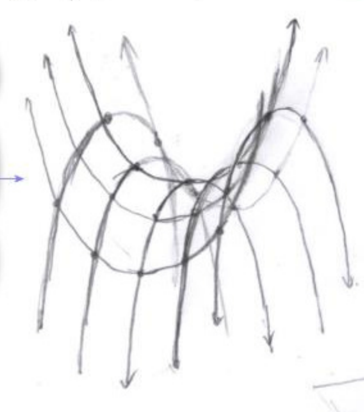
Other planes intersect the surface in parabolas.

$\Rightarrow$ parabola with roots located where plane intersects x and y axes.

The parabolas always intersect the x and y axes!

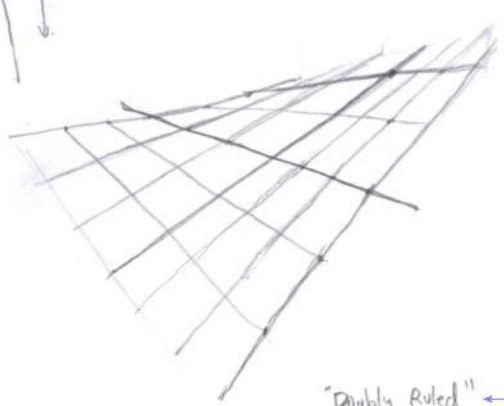
Actually, when the cross sectioning plane is parallel to the xz coordinate plane, the cross section is a line. Algebraically, that's because $m = 0$.

$m = 1, -1$



This means that cross sections with planes parallel to the plane $x = y$ are all parabolas congruent to each other, and similarly for cross sections with planes parallel to the plane $x = -y$. Just like the paraboloid of revolution, it's a parabolic rack of parabolas! Only, this time we hang upside down parabolas on a right side up parabola.

Hey, wait a sec... cross sections with planes parallel to the xz and yz coordinate planes are all lines. I should be able to sketch this surface using only straight lines!



"Doubly Ruled"

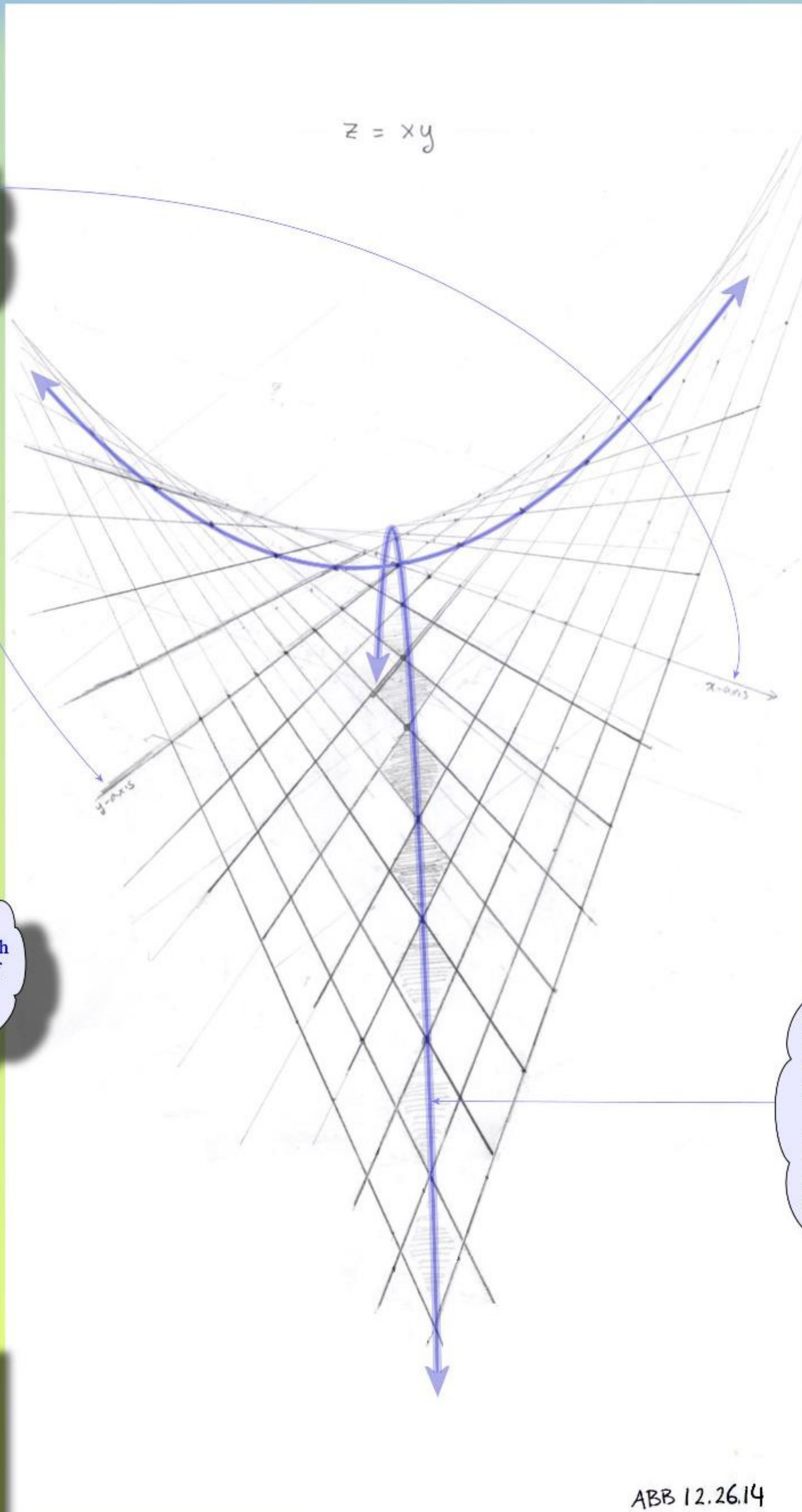
This is what people must mean when they say that a surface is "doubly ruled."

Key:
- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

I should use a ruler!

I'll draw in the x-axis first, then the y-axis.

I'll try to be bolder and capture a greater range of slopes.



Oriented this way, the part of the surface over the fourth quadrant blocks the view of the part of the surface over the second quadrant.

Neat! I can see the "opposing" parabolas that sit above $y = x$ and $y = -x$.

Knowing that everything drawn here is a straight line makes it much easier to sketch the surface.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

The Derivative, Part 1

by Ken Fan | edited by Jennifer Silva

Calculus is a subject well covered in textbooks. Instead of another textbook treatment of the subject, we aim to provide an illuminating, though non-rigorous, explanation.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Morawetz, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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Notation

by Lightning Factorial

Through the years, certain mathematical symbols have become widely adopted. Learning the notational conventions is important for communication. The best way to learn new notation is to use it. It's no different from what each of us did to learn the alphabet: repetition.

The Choose Function

There are 30 students in Mr. Eldredge's class. Four of them must be selected to represent the class in the March for Peace. How many possibilities are there for choosing the representatives?

The counting question that asks for the number of ways to choose r objects from a collection of n appears frequently in many branches of math:

> If you flip 30 coins, in how many ways can exactly 4 heads come up?
> How many 4 element subsets are there of a set with 30 elements?
> What is the coefficient of x^4 in the expansion of $(x + 1)^{30}$?

All four questions stir up the same math and have the same answer.

Because this basic counting question is fundamental, mathematicians created special notation that represents the concept: the **choose** notation. It looks like this:

The top number is the total number of objects one can choose from. In our first example, it is the number of students in Mr. Eldredge's class.

$$\binom{n}{r}$$

The bottom number is the number of objects we want to choose. In our first example, it is the number of students that must be sent to the March for Peace.

It is spoken, "n choose r." People also write ${}_nC_r$ for $\binom{n}{r}$. A nice expression to compute $\binom{n}{r}$ is:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

You might be wondering, "Why invent a notation for this concept? Why not simply write the answer down or just use the formula?" One reason is that there are times when it is more important to emphasize the meaning of something than to give the actual value. For example, it happens that the answer to all four questions above is 27,405. But if you simply wrote down this number, few would realize that it represents the number of ways to select the 4 representatives for the March of Peace from Mr. Eldrige's class of 30. But if you write $\binom{30}{4}$, people will know what it means. If someone needs the exact value, they can always use the formula.

Make a table of values of ${}_nC_r$. Do you recognize it?

Full Deck

by Girls' Angle Staff

Full Deck is a card game designed for practicing single digit multiplication.

Materials

> A standard deck of cards (with no jokers)
> Scratch paper and pencils for all players

Number of Players

> Theoretically, up to 52, but it's better to use 1 – 6. It's also more effective if players have about the same level of skill at decimal multiplication.

How to Play

Gather players around a table. Shuffle the deck and place it face down in the center of the table.

All players must keep track of a number called the "current number." The current number begins at 1.

Some player is selected to begin and play proceeds clockwise around the table.

When a player's turn comes up, the player flips over the top card on the remaining stack in the center and places it onto a growing pile of overturned cards.

If the card is...

...an Ace, 2 through 10, or a Jack,	the player changes the current number by multiplying the current number by the face value of the card. Aces are 1 and Jacks are 11.
...a Queen,	the player resets the current number to 1.
...a King,	the player gathers up the flipped over cards and places them aside in a safe pile. The current number is reset to 1.

If a player makes a computation error, all cards not in the safe pile are gathered up, reshuffled, and placed face down in the center of the table. The current number is reset to 1. Play resumes with the player who made the error.

Play continues until all cards in the deck are successfully flipped.

Can players make it through a full deck?

Anybody who makes it through a full deck playing solo is known as a "decker."

Purpose

Facility with single digit decimal multiplication is the crucial technical skill required to master the multiplication and division of decimal numbers.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

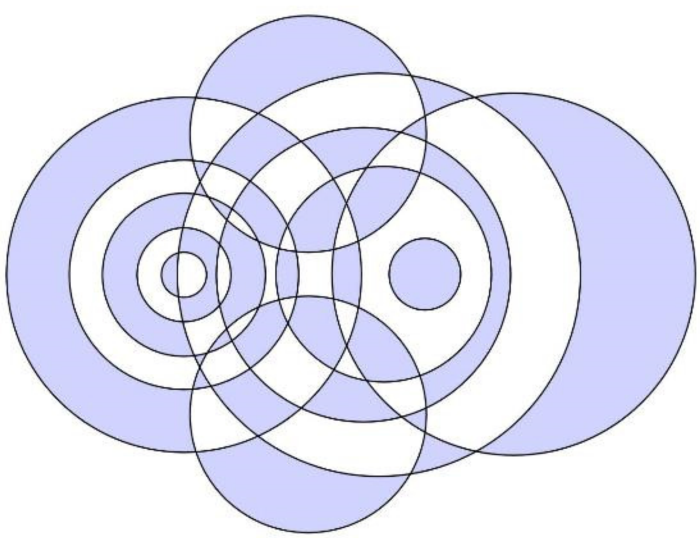
Session 15 - Meet 8 November 6, 2014	Mentors:	Rediet Abebe , Bridget Bassi, Anthea Chung, Karia Dibert, Lydia Goldberg, Jennifer Matthews, Isabel Vogt, Jane Wang

Normally, we highlight one or two items per meet. However, to give a sense of the variety of math at any given meet, here's a list of the main math questions and topics that member's pondered: If you could use only 7 yes/no questions to determine a number between 1 and 100, and you had to determine all 7 questions *in advance*, what would your questions be? What happens when you cut a bagel along a Möbius strip? Hilbert's Hotel Infinity. How do you convert fractions to decimals? Which fractions are terminating decimals? Signs of permutations. Zolotarev's proof of Quadratic Reciprocity. Arithmetic modulo prime powers. Proof by contradiction and by induction. Folding an origami tetrahedron. What is the largest number of acute angles that a convex hexagon can have? Combinations and permutations.

Session 15 - Meet 9 November 13, 2014	Mentors:	Bridget Bassi, Karia Dibert, Alexandra Fehnel, Lydia Goldberg, Anu Vajapeyajula, Jane Wang, Sibo Wang
	Visitor:	Cornelia Van Cott, University of San Francisco

Cornelia Van Cott is an Associate Professor of Mathematics at the University of San Francisco. She was raised in Indiana and double majored in math and music. She went to the University of Indiana for graduate work in topology. She gave us a presentation on the mathematics of coloring and doodling.

To start, she had each member pick a number between 5 and 8, then draw that many overlapping circles on a piece of paper. Each design split the paper into several regions. She asked members to try to color in their designs using two colors while adhering to two properties. First, each region had to be filled with a single color. Second, no two regions that shared a borderline could be the same color. She asked, "Is it always possible to do this?" Also, "Is it possible to create a design that requires more than two colors to fulfill both properties?"



Cornelia explained that if you place a number in each region that tells how many circles contain that region, then because the numbers change by +1 or -1 every time you cross a borderline, if you color the even numbered regions one color and the odd numbered regions another, you will get a solution.

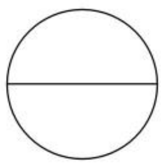
She then asked, "Would this still be possible if, instead of circles, one used overlapping rectangles to create the design?" Can you construct a design using rectangles that requires 3 different colors if you want regions that share a borderline to be of different colors?

In general, a design that can be colored with n colors so that no two regions sharing a borderline are the same color is called ***n*-colorable**.

"Is there a way to characterize all doodles that are 2-colorable?"

She explained that the key is to look at the intersections. The degree of an intersection is the number of borders coming out of it. A design is 2-colarable if and only if the degree of every intersection is even.

She then posed the following challenge:



Consider the shape obtained by taking a circle and drawing in a diameter. Create a design out of such shapes in such a way that no two drawn diameters are collinear. Prove that the resulting design is 3-colorable (that is, the regions can each be colored 1 of 3 different colors in such a way that no two regions that share a borderline are the same color).

Cornelia concluded with a discussion of the 4-color theorem.

Session 15 – Meet 10 November 20, 2014	Mentors:	Rediet, Abebe, Bridget Bassi, Alexandra Fehnel, Lydia Goldberg, Jennifer Matthews, Anu Vajapeyajula, Isabel Vogt, Jane Wang, Sibo Wang

We have several members who are mastering fundamental arithmetic operations with decimal numbers. Because of the distributive law and the nature of decimal representations, all four of the basic arithmetic operations can essentially be broken down to operations that involve only the digits zero through nine.

To motivate doing lots of exercises involving multiplication by a digit, we created a number of games, such as a card game we call "Full Deck." For rules, see page 25.

Other members explored how permutations can be realized as matrices of 1s and 0s where each row and each column has exactly one 1. For one thing, they showed that the sign of a permutation is the determinant of the corresponding matrix representation.

Session 15 - Meet 11 December 4, 2014	Mentors:	Bridget Bassi, Karia Dibert, Alexandra Fehnel, Lydia Goldberg, Anu Vajapeyajula, Isabel Vogt, Jane Wang, Sibo Wang

One question that arose at this meet was: Solve for x: $| x – 1 | = | x – 2 |$. I'm bringing this question up here to illustrate the value of a conceptual approach to mathematics.

For this problem, a common way to handle the presence of absolute values is to eliminate them by considering cases. Here, for each of the three cases $x \leq 1$, $1 < x < 2$, and $2 \leq x$, the equation can be expressed without absolute values. For example, when $1 < x < 2$, the equation becomes $x – 1 = -(x – 2)$. For each case, the equation becomes a linear equation in one variable and can be solved using basic one-variable linear algebra.

However, a more conceptual way to proceed is to interpret $| x - y |$ as the distance between x and y on the number line. Using this interpretation, the equation asks, "What number is the same distance from 1 as it is from 2?" Under this interpretation, does the answer jump out at you?

Try to solve the following problem conceptually: For what value of x is the expression

$$| x - 1 | + | x - 2^2 | + | x - 3^2 | + \ldots + | x - 2015^2 |$$

minimized? Find a quiet spot and think, paper unnecessary. (If you're having trouble, email us!)

Another question some members pondered was the following classic frog jumping problem: There are N green frogs and N red frogs arranged in a row consisting of $2N + 1$ spaces. The green frogs occupy the N leftmost spaces and the red frogs occupy the N rightmost spaces. The goal is to interchange the positions of the red and green frogs. Frogs of the same color are considered indistinguishable from one another. A frog can move one space over or, if the adjacent space is occupied, leapfrog two spaces over, but only if the destination space is empty. Each space can be occupied by at most one frog.

How can the red and green frogs exchange places and what is the minimum number of moves required to accomplish this, as a function of N?

Session 15 - Meet 12	Mentors:	Bridget Bassi, Karia Dibert,
December 11, 2014		Jennifer Matthews, Isabel Vogt

We held our traditional end-of-session Math Collaboration!

Here are a few problems from the event:

A group of 84 students form a 6 by 14 rectangle. The leftmost person in the front row has a bean bag. The person with the beanbag passes it to the person directly to the right and behind. The people in the front row are considered to be directly behind the people in the back row and the people in the left column are considered to be directly to the right of people in the right column. The bean bag is passed on until the leftmost person in the front row gets it back. How many students handle the bean bag?

In what row does the number 1,287 first appear in Pascal's triangle?

How many ways can 6 identical pencils be distributed among 3 people?

If $1 / \mathrm{N} = 0.41\overline{6}$, what is N?

In the country Noetheria, there are coins that are worth 5 idems and there are coins worth 9 idems. (The unit of currency in Noetheria is the idem.) Using only these two coin denominations, there are some amounts of idems that are impossible to have. How many impossible (positive integer) amounts of idems are there?

Calendar

Session 15: (all dates in 2014)

September	11	Start of the fifteenth session!
	18	
	25	No meet
October	2	Emily Pittore, iRobot
	9	
	16	
	23	
	30	
November	6	
	13	Cornelia A. Van Cott, University of San Francisco
	20	
	27	Thanksgiving - No meet
December	4	
	11	

Session 16: (all dates in 2015)

January	29	Start of the sixteenth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	Bathsheba Grossman, Artist
	19	
	26	No meet
April	2	
	9	
	16	
	30	
May	7	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls’ Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant’s Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don’t like math, what don’t you like? If you love math, what do you love? What would you like to get out of a Girls’ Angle Membership?

____

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls’ Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls’ Angle**. Mail to: Girls’ Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is \$216 for one session (12 meets)	☐ I will pay on a per meet basis at \$30/meet.
☐ I will pay on a per meet basis at \$20/meet.	☐ I'm including \$36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____