Girls' Angle Bulletin

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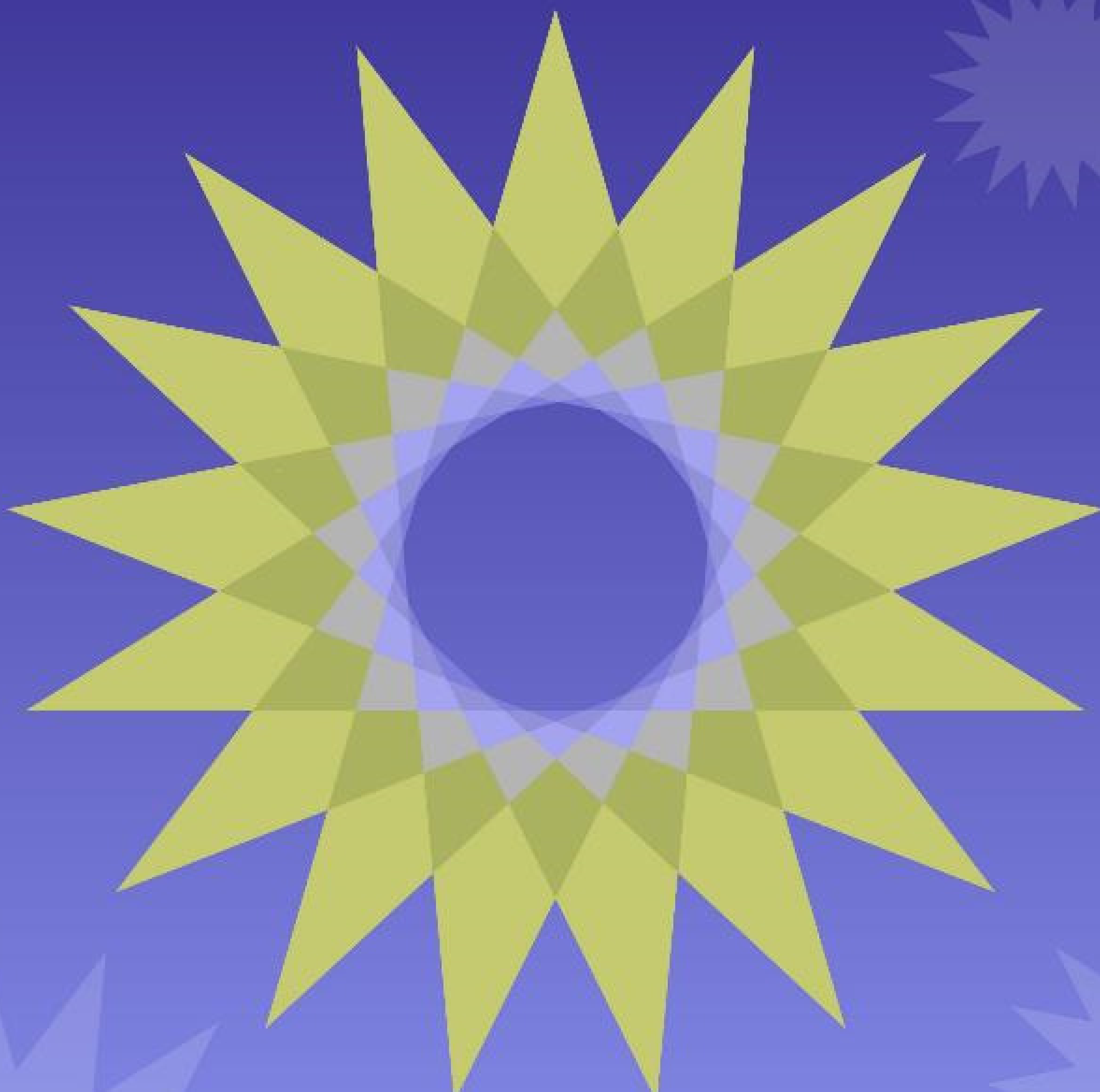
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Mathematics is a kind of exuberance. To experience it, do math. Ponder a math question. Look for patterns. Try to understand them. It's a great feeling to wash away confusion with newfound clarity.

- Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Seventeen Stars by Toshia McCabe. What is the sum of the tip angles? For more, see *Star Tips, Part 3* on page 7.

An Interview with Courtney Gibbons, Part 1

Courtney Gibbons is an Assistant Professor of Mathematics at Hamilton College. She received her Ph.D. in 2013 from the University of Nebraska-Lincoln. She also writes code for an open-source algebra software package called Macaulay 2.

The best way to learn math is to do lots of problems: after all, that's what math is!

Ken: When did you realize that you wanted to become a mathematician? What turned you on to mathematics?

Courtney: Although I loved math when I was very young, I struggled with middle school math and only ended up taking calculus my senior year to prove that I could do it. I remember liking it, but I didn't really know that there was such a thing as a "mathematician" or that I could (or might want to) choose to become one. I actually went to college thinking I would major in French and minor in something like Economics so that I could work for the UN or the World Bank. Luckily, I took multivariable calculus and fell in love. I pretty quickly decided to major in math, and I loved my professors. Once I realized that I could be a professor and do math all the time, I don't think I ever even considered doing anything else!

Ken: What about multivariable calculus, specifically, did you fall in love with?

Courtney: When I walked into my multivariable calculus class my first year of college, I was excited, but I was also nervous because I'd heard it was a hard class. But within five minutes, the professor described how you could use parametric equations to describe how a leaf spiraled through the air as it fell off a tree, and I was hooked! Learning multivariable calculus also showed me the parts of calculus I thought I understood the first time around (but really didn't). I liked seeing that there was more to learn; it turns out there's always more to learn!

Ken: What motivates you to do mathematical research today?

Courtney: I feel very lucky to have a job where I'm encouraged (and expected) to do research, because it's like being paid to play with puzzles. I get excited about trying to figure out why certain patterns emerge. Although I haven't begun working in applied algebra (yet!), there are some very interesting applications of my research area, commutative algebra, to problems in genetics, phylogenic trees, and neuroscience. Knowing that one day scientists might find my research useful is a big motivator for me to get to work every morning.

Ken: What are your goals as a mathematician?

Courtney: I remember having to answer this question a lot when I was applying for jobs, and it was hard to give a good answer that wasn't too specific. Instead of laying out my goals for my mathematical career in terms of the problems I want to solve, let me start by saying that I want to be wholly present in my work, and I want to achieve high quality in all that I do, from teaching to research to serving my college community and the mathematical community. As far as research goes, one of my goals is to include students in my research. There are lots of problems in math that are more tedious than hard, and having students to help work through the details is great for me and for them!

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Gibbons, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
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Star Tips, Part 3

by Ken Fan | edited by Jennifer Silva

Emily: What are you thinking?

Jasmine: Imagine ...

Emily: Wait! Don't tell me. I want to try to figure it out.

Jasmine: Okay.

Emily: Our conjecture is that the sum of the tip angles in a convex (n, k)-star is $180(n - 2k)$ degrees.

Jasmine: Right.

Emily: The fact that this formula always yields a multiple of 180° suggests something important to you.

Jasmine: Yes, it does!

Emily: Hmm. What could it be? Can you give me a hint?

Jasmine: Um. Well, what do multiples of 180° mean to you?

Emily thinks for a moment, then recalls the common phrase "do a 180."

Emily: Well, turning by 180° is doing a uey or an about-face.

Jasmine: Exactly!

Emily: So if you rotate by a multiple of 180°, you'll end up either looking in the same direction or doing an about-face. But that would seem to be a consequence of the conjecture, not a way to prove it. Am I on the right track?

Jasmine bursts out laughing.

Jasmine: Ha ha! Oh, sorry. I'm not laughing at you, but what you just said fits in so very well with what I'm thinking, it's uncanny!

Emily: What did I just say? You mean that it's a consequence, not a way to prove it?

Jasmine: No ...

Emily: That it's like looking in the same direction or doing an about-face?

Jasmine: No ...

Emily: What?

Jasmine: That you're on the *right track*!

Emily: That I'm on the right track?

Jasmine: Yeah!

Emily: Well, am I?

Jasmine: Yes, you are ... but think about that even more – being on the *right track*.

Emily: On the right track? Hmm...

Jasmine: What does that mean literally?

Emily: There's a track?

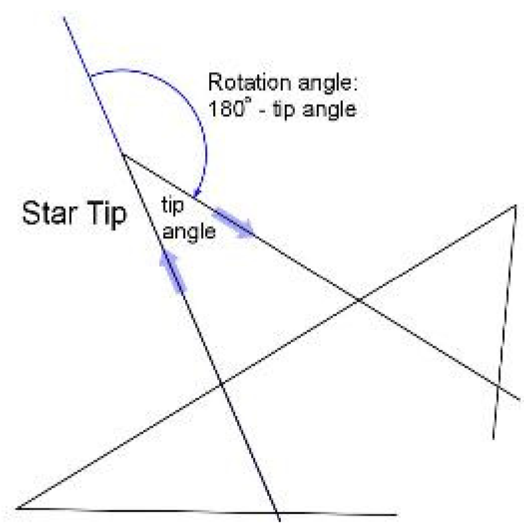
Emily thinks hard while Jasmine patiently waits.

Emily: Oh! Ha ha! Hilarious! You're thinking of the star itself *as a track*!

Jasmine: You got it!

Emily: We pretend we're walking around the edges of the star, from tip to tip to tip. We start somewhere in between two tips and keep going until we return, always facing straight forward in the direction we're walking. We walk the path that we'd trace out with a pencil if we were to draw the star. When we complete a full lap around the star track and return to the starting place, we'll be facing the same direction we faced when we began. That means that the total of all the angle measures we turn through on our walk must be a multiple of 360◦!

Jasmine: Right! As you walk along the star track, at each tip, you turn through an angle of 180° minus the tip angle. If we're on an irreducible star track, this implies that the total number of degrees through which we turn by the time we return to the starting point is $180n - S$, where S is equal to the sum of the tip angles.



Emily: And since we end up facing the same direction we started in, we must turn through some whole multiple of 360°. That is, $180n - S = 360T$, where T is the total number of full turns we rotate through in one full lap of the star track.

Jasmine: All we have to do now is to figure out how many full turns are made in one full lap.

Emily: Let's focus on irreducible (n, k)-stars and worry about reducible ones later.

Jasmine: I think that's a good plan. This way, we will meet every tip on our star-walk.

Recall that Emily and Jasmine define a convex (n, k)-star as follows: Start with a convex n-gon. Label its vertices 0, 1, 2, ..., $n - 1$ in the clockwise direction. Then form the (n, k)-star by connecting vertex x to vertex $x + k$ (modulo n).

Emily: And for definiteness, let's begin our walk somewhere along the segment that connects tip 0 to tip k, heading in the direction from tip 0 to tip k. We'll end our walk on this same segment facing in the same direction. Let's call this the "initial heading."

Jasmine: Since rotating the whole shape doesn't do any harm, we may as well assume, without loss of generality, that the initial heading is due north.

Emily: Okay.

Jasmine: As we walk along a segment, we're walking straight and not turning at all. All the turning takes place at the star tips. So what we have to do is count how many times we find ourselves turning to or through due north at each star tip.

Emily: In the middle of our walk, let's say we arrive at tip x from tip $x - k$ and are about to turn to face toward tip $x + k$, all modulo n, of course. So we're facing in the direction that points from tip $x - k$ to tip x, and we're about to turn right to face in the direction to tip $x + k$ from tip x. For which x will we turn through the heading due north?

Jasmine: Emily? Is it possible that as we rotate at one of the star tips, we rotate through due north twice or more? If that happens, we can't just count the number of such tips to figure out the total number of full revolutions we make. For each tip, we'd have to compute the number of times we rotate through due north and add up these numbers for all the tips.

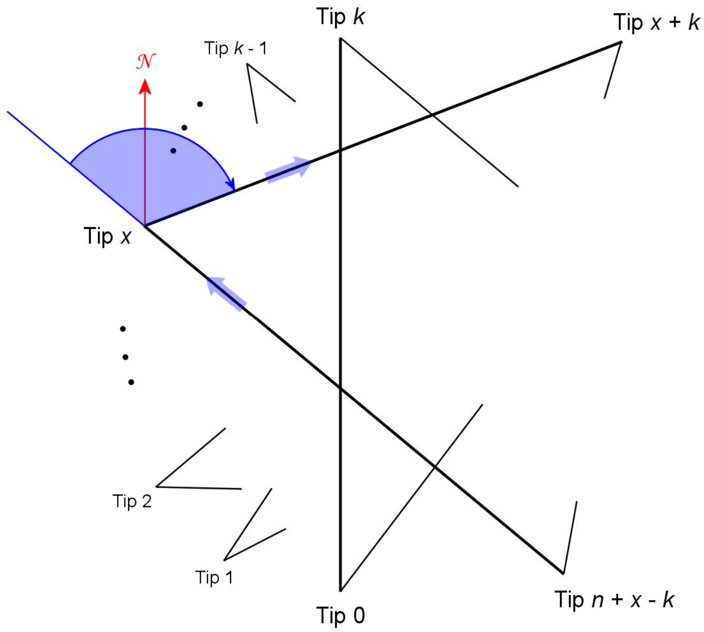
Emily: You're right, but I don't think we have to worry about that because we're assuming that $k < n/2$, and this means that we always turn less than 180° at each tip. If you draw the line through tips $x - k$ and x, the tips $x - k + 1$, $x - k + 2$, $x - k + 3$, through $x - 1$ will all be on the left side of this line (as you walk in the direction of tip x from tip $x - k$), the tips $x - k$ and x will be on the line, and all the other tips will be on the right side of the line. Since $k < n/2$, tip $x + k$ will be among the tips on the right. So by the time we're facing in the direction of tip $x + k$, we'll have turned right through less than 180°.

Jasmine: Good point. That's a relief!

Emily: So as we rotate through each tip, we either look due north once, or not at all.

Jasmine: Yes. Let's see. If $x = 0$, we start off facing some direction, and end up looking due north, toward tip k. So tip 0 contributes to the count.

Emily: Yes. In fact, if we start at any tip x where $0 < x < k$, we will rotate through due north. That's because $n - k < n - k + x < n$ for such x; this means that as we walk from tip $n + x - k$ to tip x, we will move across the vertical line through tips 0 and k from east to west. But since $k < x + k < n$, as we walk from tip x to tip $x + k$, we will cross back west to east over the vertical line through tips 0 and k. The only way that can happen is if we turn through due north.



Turning through due north at tip x, when $0 < x < k$.

Jasmine: I agree. So the tips from tip 0 to tip $k - 1$ each contribute to our full-turns count.

Emily: I think those are the only tips that contribute. At tip k, we start off facing due north, and then rotate through some angle less than 180°. So we do see due north at tip k, but this is where our full-turn count starts at 0.

Jasmine: Well, we still had better check the tips x where $k < x < n$.

Emily: Okay. Suppose we're walking from tip $x - k$ to tip x, then on to tip $x + k$, where $k < x < n$. First of all, tip x is on the east side of the vertical line through tips 0 and k. We want to make sure that we do not turn through due north as we make our right turn at tip x. Could it happen that the direction from tip $x - k$ to tip x *is* due north? It doesn't seem possible.

Jasmine: If that direction were due north, then tips $x - k$ and x would be vertically aligned and both would have to be east of the vertical through tip 0. That would imply that $x - k \geq k$, since otherwise $0 < x - k < k$, but the tips y with $0 < y < k$ are on the west side of the vertical through tip 0. And if $x - k = k$, then tips 0, $x - k = k$, and x would all be on the same vertical; that isn't possible, so $x - k > k$. So in clockwise order, the tips would go 0, k, $x - k$, x.

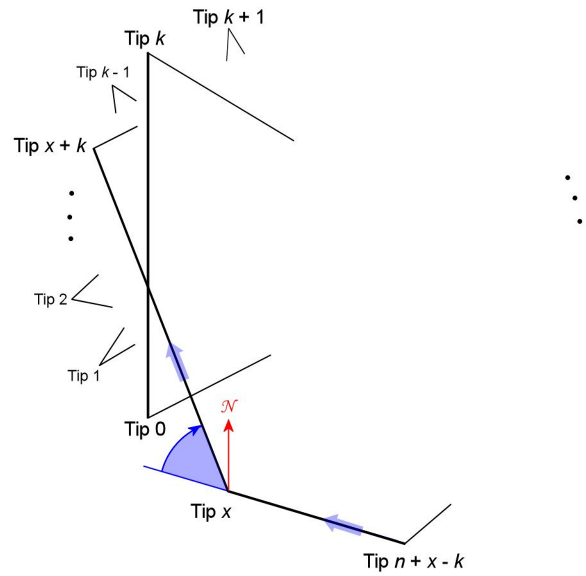
Emily: Right. So if we focus on these four tips, they would form the vertices of a trapezoid. And since the vertices of a convex polygon formed by a subset of the tips inherits their circular order, the direction from tip $x - k$ to tip x must actually be due south! The upshot of all of this is that if $k < x < n$, then we never approach tip x from the south.

Jasmine: Gosh, that's quite an argument! I wonder if there's a simpler way to express these things. By the way, from our previous results, we've actually shown that the only tip that is approached from the south is tip k.

Emily: You're right! But we still have to check that if $k < x < n$, we won't rotate through due north as we turn to face in the direction of tip $x + k$. Knowing that we didn't approach tip x from the south means that due north must be within a 180° right turn from the direction that points from tip $x - k$ to tip x. If we do turn through due north at tip x, we would have to end up facing a direction that points at least slightly to the east. And since tip x is already east of the vertical through tip 0, tip $x + k$ would also be east of the vertical through tip 0. That means that if we were to travel around the perimeter of the convex polygon in the clockwise direction from tip x to tip $x + k$, we would traverse the entire section from tip 0 to tip k. But that's impossible, because tip $x + k$ cannot be more than k tips farther around the perimeter from tip x. So we can't rotate through due north at tip x!

Jasmine: Whew! I think that does it! So the bottom line is that after turning away from due north at tip k, the only tips we will encounter on our walk that contribute to our full-turn count are the tips 0 through $k - 1$. There are k of those, so we end up doing k full turns as we walk a full lap around the star track!

Emily: Therefore, $180n - S = 360k$, or $S = 180n - 360k = 180(n - 2k)$!



Turning at tip x when $k < x < n$.

Jasmine: That's it! Our conjecture is proven!

Emily: Sweet! Let's celebrate with a cup of hot cocoa.

Jasmine: Wait a sec – we've only done the case of irreducible (n, k)-stars. What about reducible (n, k)-stars?

Emily: Oh yeah.

Jasmine: If the (n, k)-star is reducible, it means that the greatest common factor of n and k is greater than 1. Let d be the greatest common factor of n and k.

Emily: Earlier, we saw that the (n, k)-star is a union of d irreducible $(n/d, k/d)$-stars.

Jasmine: The formula we just derived shows that the sum of the tip angles in each $(n/d, k/d)$-star is $180(n/d - 2k/d) = 180(n - 2k)/d$. Therefore, the total of all of the tip angles in an (n, k)-star is d times $180(n - 2k)/d$, which turns out to be $180(n - 2k)$ again!

Emily: Yay! It's kind of neat how the spikiest n-vertex star, when n is odd, has tip angles that add up to 180°. When n is odd, that's the $(n, (n - 1)/2)$-star, which includes the triangle and the 5-pointed stars that launched us on this journey!

Jasmine: Yes, and I think it's neat that k, our "spikey factor," turns out to be the number of full circles we turn through as we complete one lap of the star track. It's like there are $180n$ potential degrees that our star tips can grab; the bigger k is, the more of those degrees are put into turning, so the tips get sharper and sharper.

Emily: Hey, how about that hot cocoa?

Jasmine: Let's go!

Meditate$^{\text{Math}}$

The Chocolate Bar Diagonal

A rectangular chocolate bar is patterned by an *N* by *M* grid, dividing the bar into *NM* identical bite-sized pieces. If you slice a knife along a diagonal, corner to corner, how many bite-sized pieces will be split in two?

The table on the next page gives the answer for various values of *N* and *M*.

Find a quiet place to meditate upon this diagram. Ignore time. Just observe. What do you see? What can you explain? Record your observations. Jot down any explanations.

If you'd like a little help to get going, here are a few observations and questions to ponder. Pick one at random and try to explain it. The facts are listed in no particular order.

1. The entries in the first row correspond to the column number.

2. The entries in the second row list each even number twice and in numerical order.

3. Every 3 consecutive entries in the third row follow the pattern

$$3n \qquad 3n + 1 \qquad 3n$$

4. What patterns do you see within a row?

5. The table is symmetric about the northwest-southeast diagonal.

6. What sequence is formed by the numbers along the northwest-southeast diagonal?

7. What sequence is formed by the numbers just above the northwest-southeast diagonal?

8. What sequences are formed by the numbers going down the various northwest-southeast diagonals of the table?

9. Can you find a formula for the entry in the *N*th row and *M*th column?

10. On some northeast-southwest diagonals, all entries are equal. When does this happen?

11. When are two consecutive entries in the same row or column equal?

12. If the table is extended infinitely to the left and down, is there a bound on the difference between consecutive entries in the same row?

13. What is the smallest entry in any row?

14. How many times will a specific number appear in this table?

M \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16	18	18	20	20	22	22	24	24	26
3	3	4	3	6	7	6	9	10	9	12	13	12	15	16	15	18	19	18	21	22	21	24	25	24	27
4	4	4	6	4	8	8	10	8	12	12	14	12	16	16	18	16	20	20	22	20	24	24	26	24	28
5	5	6	7	8	5	10	11	12	13	10	15	16	17	18	15	20	21	22	23	20	25	26	27	28	25
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25	25	26	27	28	25	30	31	32	33	30	35	36	37	38	35	40	41	42	43	40	45	46	47	48	25

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues her investigation into cross sections of the surface $z = xy$.

Last time, I looked at cross sections of $z = xy$ with vertical planes. Now I'll look at non-vertical planes, starting with horizontal ones.

$z = xy$

$z = c$, c constant

$xy = c$ hyperbola if $c \neq 0$. perpendicular lines if $c = 0$.

Horizontal cross sections are generically hyperbolas with asymptotes the x and y-axes. If $c = 0$, the cross section *is* the x and y axis.

I should have said that the *projection* of the cross section to the xy-coordinate plane has asymptotes the x and y-axes.

Next, non-vertical, non-horizontal planes.

Let P be a plane not parallel to xy-coord. plane nor perpendicular to xy-coord. plane.

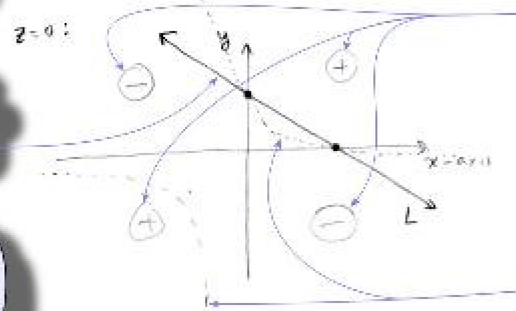
P intersects the xy-coordinate plane in a line. I'll start with the generic case where this line cuts both axes, but not at the origin.

Let $L = P \cap xy$-coord. plane.

Generic case where L intersects both the x-axis and y-axis but not at the origin

I'll look at the projection of the cross section to the xy-coordinate plane. I'll make a picture. Here's L.

$z = 0$:



Suppose the plane P is above the xy-coord. plane on the side of L with the origin

If P sits above the xy-coordinate plane on the side of L that the origin sits on, I can rule out whole regions where the sign of xy isn't consistent with the points on the plane P.

Hyperbola?

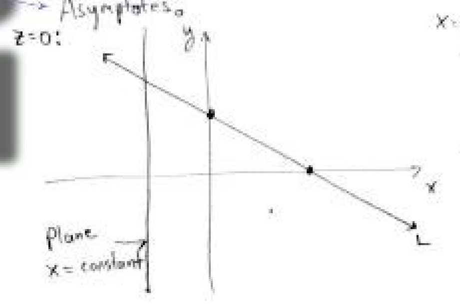
I bet the projection is a hyperbola.

I'm beginning to think that all cross sections are conic sections of some sort. If so, these have to be hyperbolas because they're split into two pieces. So, what are the asymptotes?

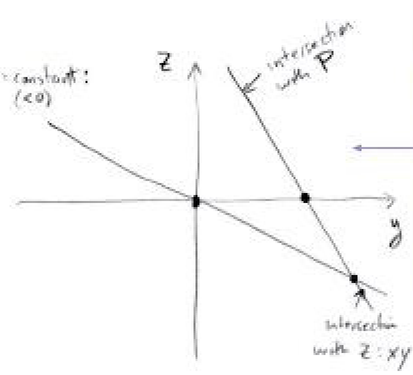
Asymptotes?

I should later try to show that cross sections are always conics ...and that projections of conics are conics.

$z = 0$:



x = constant: (<0)



Oh! I think I see! I can use the doubly ruled nature of $z = xy$ to find the asymptotes! If I look at the intersections with planes of the form x = constant, the intersection with $z = xy$ will be a line of slope that constant. The intersection with P will also be a line of some fixed slope m independent of the constant. These lines will always meet unless the constant happens to be m! So $x = m$ must be the asymptote!

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

To recap...

Vertical asymptote of hyperbolic projection is $x = m_x$ where m_x is slope of the line that is the intersection of P with any plane of the form $x =$ constant.

Actually, I'll be more specific by using a specific equation for the plane P.

If P is represented by $z = ax + by + c$.

If $x = 0$, $z = by + c$ $\Rightarrow$ slope b
$\Rightarrow$ asymptote $x = b$

If $y = 0$, $z = ax + c$ $\Rightarrow$ slope a
$\Rightarrow$ asymptote $y = a$.

The horizontal asymptote of the projection can be found in an entirely analogous way, so must be $y = a$.

If I rotate P about L, I should get a whole family of hyperbolas. If P is given by $z = ax + by + c$, then other planes that contain L, which is the line in the xy-coordinate plane with equation $ax + by + c = 0$, will have equation $z = t(ax + by + c)$, where t is a constant.

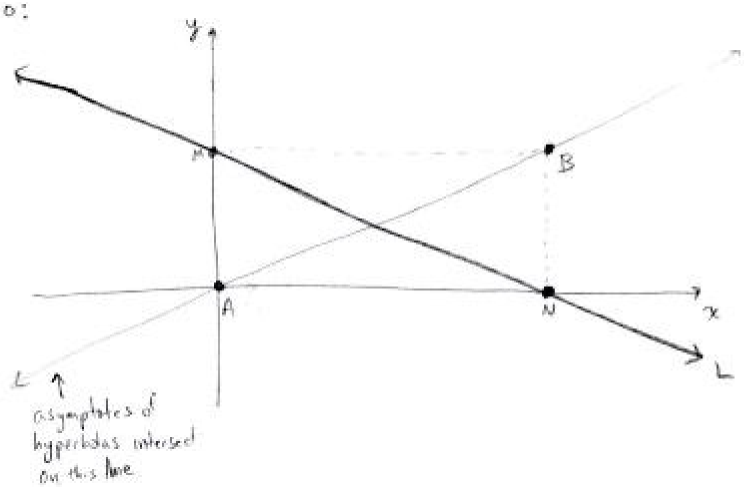
As P rotates about L, get family of hyperbolas

$z = t(ax + by + c)$ Asymptotes $x = tb$ $y = ta$

Let I = intersection of asymptotes $= (tb, ta, 0)$

I is also the "center" of the hyperbolic projection of the cross section to the xy-coordinate plane.

$z = 0$:



When I = A or I = B cross-section is two intersecting lines (degenerate hyperbola)

As I crosses A or B the projection of the cross-section will morph from "quadrant I, III" hyperbolas to "quadrant II, IV" hyperbolas.

ABB 2.26.15

I've almost got a complete picture of all cross sections of the surface $z = xy$. All that remains to check are the cases where L is not generic. Those are: when L is parallel to an axis or when L passes through the origin.

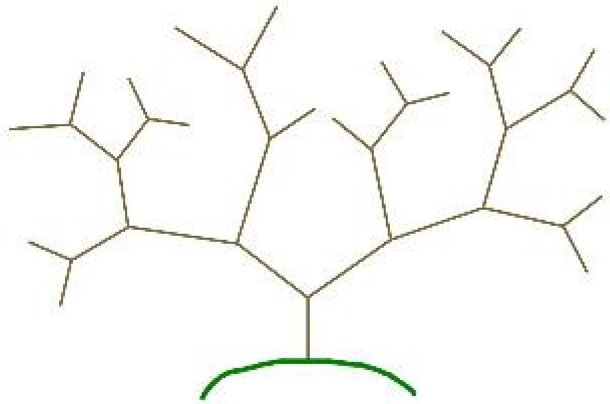
I also need to verify my assumptions about cross sections always being conics and that projections of conics are conics.

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

The Mad Hatter’s Number Tree

by Konstanze Rietsch[1]

“What kind of game is it you are playing here?” said Alice to the Mad Hatter who had just reached for the quill resting on the table next to his steaming cup of tea. “We are playing the grow-your-own-numbers game!” the Mad Hatter replied, and proceeded to dip his quill in ink and draw a binary tree on the tablecloth.



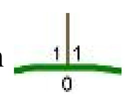
Numbers don’t grow on trees! Alice thought.

“All numbers grow on trees!” the Mad Hatter declared, as if he had read Alice’s mind, and began adding numbers to his tree. Alice watched for a while and then she exclaimed, “But you are only getting very special numbers!”

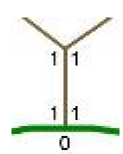
The Rules of the Game or How Numbers Grow On Trees

At every branch point, there are exactly 3 branches!

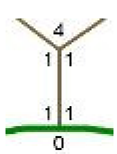
How to draw the tree: At the end of every branch precisely two new branches start, and new branches never intersect or meet up again with old branches.

Around every branch point 3 numbers grow starting with 

and following these two rules:

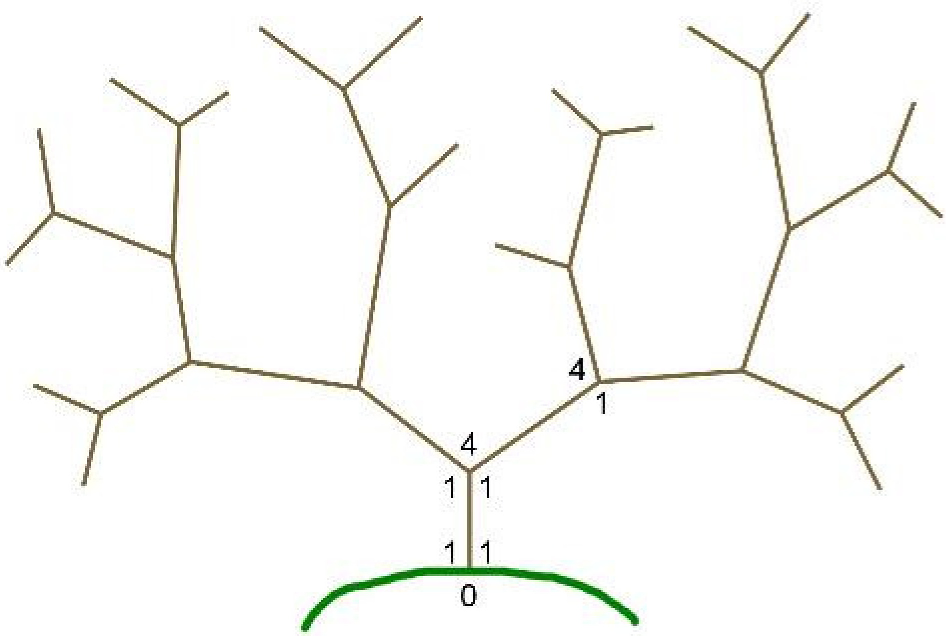
1. The numbers stay the same as you go up along a branch to the next new branching point. For example: 

2. The new number in between the two new branches is the sum of the four numbers along the branch below minus the number just below the start of the previous branch.

So 

because 1 + 1 + 1 + 1 – 0 = 4.

[1]This content supported in part by a grant from MathWorks.



Grow out the number tree above. Continue as far as you like.

Is Alice right? What special numbers is the Mad Hatter getting?
More difficult: How are the three numbers around any branching point related?

The Mad Hatter and Alice allude to characters in *Alice's Adventures in Wonderland* and *Through the Looking Glass* by Charles Dodgson (a.k.a. Lewis Carroll), an English author and mathematician who lived during the 19th century.

The Derivative, Part 2

by Ken Fan | edited by Jennifer Silva

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Gibbons, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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Learn by Doing

Pythagorean Triples
by Girls' Angle Staff

A right triangle is a triangle with a right angle. Because the measures of the three angles in a triangle add up to 180°, a triangle can have at most one right angle. In a right triangle, the side opposite the right angle is the **hypotenuse** and the sides that meet at the right angle are the **legs**.

Let c be the length of the hypotenuse of a right triangle and let a and b be the lengths of its legs.

The famous Pythagorean theorem says that $c^2 = a^2 + b^2$.

Conversely, it is also true that if a, b, and c are positive and satisfy the equation $c^2 = a^2 + b^2$, then there is a right triangle whose hypotenuse is of length c and whose legs are of lengths a and b.

Here, we won't prove the Pythagorean theorem or its converse. Instead we will assume both and find right triangles with integer side lengths.

Problem 1. Show that a triangle with side lengths 3, 4, and 5 is a right triangle. In other words, show that $5^2 = 3^2 + 4^2$.

Problem 2. Try to find several more triples of positive integers a, b, and c that satisfy the Pythagorean relation $c^2 = a^2 + b^2$.

We will call a triple of integers (a, b, c) a **Pythagorean triple** if and only if $c^2 = a^2 + b^2$. Note that we're allowing negative integers as well as zero to be members of Pythagorean triples.

Problem 3. Suppose (a, b, c) is a Pythagorean triple. Let t be a number such that ta, tb, and tc are all integers. Show that (ta, tb, tc) is also a Pythagorean triple.

Because of Problem 3, if (a, b, c) is a Pythagorean triple and d is a common factor to a, b, and c, then $(a/d, b/d, c/d)$ is also a Pythagorean triple. We will call a Pythagorean triple (a, b, c) **primitive** if and only if a, b, and c share no common factor bigger than 1.

We will develop two methods to find *every* Pythagorean triple. Because every Pythagorean triple is a multiple of a primitive Pythagorean triple, we will focus on finding the primitive ones.

The first method we will develop is a "parametric" one. You'll need to know how to write equations for straight lines in the xy-coordinate plane.

Problem 4. Suppose (a, b, c) is a primitive Pythagorean triple. Show that $(a/c, b/c)$ is a point on the unit circle centered at the origin, i.e., a point on the graph of the equation $x^2 + y^2 = 1$ in the xy-coordinate plane. Notice that $(a/c, b/c)$ is a point whose coordinates are rational numbers. (A rational number is a number that can be expressed as the ratio of two integers.)

Problem 5. Conversely, suppose that (p, q) is a point with rational coordinates on the unit circle. Show that there is a unique positive integer c for which (cp, cq, c) is a *primitive* Pythagorean triple.

Problems 4 and 5 inform us that finding primitive Pythagorean triples is equivalent to finding rational points (i.e. points whose coordinates are rational numbers) on the circle $x^2 + y^2 = 1$.

So let's find rational points on the circle $x^2 + y^2 = 1$.

Problem 6. Note that the 4 points where the circle intersects the coordinate axes are rational points. Let P be the point (-1, 0).

Problem 7. Write down the equation of the line with slope m that passes through P.

Problem 8. The only line through P that cannot be expressed in Problem 7 is the vertical one because vertical lines have undefined slope. But this means that a line from Problem 7 must intersect the circle $x^2 + y^2 = 1$ in two places, P and another point which we will call P_m. Find the coordinates of P_m in terms of m.

Problem 9. Show that P_m is a rational point if and only if m is a rational number.

Problems 8 and 9 show that the map that sends m to P_m is a map from the real number line to the circle $x^2 + y^2 = 1$, minus the point P. Furthermore, rational points on the real number line are mapped onto rational points on the circle. (This map is an example of a **birational** map.)

Let Q_m denote the primitive Pythagorean triple associated to P_m as in Problem 5.

Problem 10. Complete the following table.

m	P_m	Q_m
1/2	(3/5, 4/5)	(3, 4, 5)
1/3	(4/5, 3/5)	(4, 3, 5)
2/3	(5/13, 12/13)	(5, 12, 13)
1/4		(15, 8, 17)
3/4		
1/5		
2/5		
3/5		
4/5		

Problem 11. Let $m = p/q$, where p and q are integers with no common factor. By substituting p/q for m in your formula for P_m and clearing denominators, deduce that $(q^2 - p^2, 2qp, p^2 + q^2)$ is a Pythagorean triple.

Problem 12. For relatively prime integers p and q, the Pythagorean triple $(q^2 - p^2, 2qp, p^2 + q^2)$ is not necessarily primitive. Find conditions on p and q that guarantee primitivity.

Now we'll explore a second method for generating primitive Pythagorean triples that seems to have first been published by Berggren in 1934.

We will define several functions that map ordered triples (a, b, c) to ordered triples.

Problem 13. Note that if (a, b, c) is a Pythagorean triple, then so is (b, a, c).

Problem 13 motivates the definition of our first map S. Let $S((a, b, c)) = (b, a, c)$.

Problem 14. Note that if (a, b, c) is a Pythagorean triple, then so are $(\pm a, \pm b, \pm c)$ for any combination of sign choices.

Problem 14 motivates the definition of the following 3 maps N_1, N_2, and N_3:

$$N_1((a, b, c)) = (-a, b, c)$$
$$N_2((a, b, c)) = (a, -b, c)$$
$$N_3((a, b, c)) = (a, b, -c)$$

Finally, define $F((a, b, c)) = (-a - 2b + 2c, -2a - b + 2c, -2a - 2b + 3c)$.

Problem 15. Suppose (a, b, c) is a Pythagorean triple. Show that $F((a, b, c))$ is also a Pythagorean triple.

Problem 16. Show that $F(F((a, b, c))) = (a, b, c)$. In other words, F is its own inverse.

Problem 17. Starting with the Pythagorean triple (1, 0, 1), apply the various maps S, N_1, N_2, N_3, and F over and over, applying them even to the results of applying them, and the results of those, and so on. What do you notice? Make a conjecture and try to prove it.

The remaining problems relate the two methods for finding Pythagorean triples.

Problem 18. Define functions n_k (from nonzero rational numbers to rational numbers) for $k = 1$, 2, and 3, by the relationship

$$Q_{n_k(m)} = N_k(Q_m).$$

Determine n_1, n_2, and n_3.

Problem 19. Define the function s (which will map from the set consisting of all rational numbers other than -1 to itself) by the relationship $Q_{s(m)} = S(Q_m)$. Find s.

Problem 20. Define the function f (which will map from the set consisting of all rational numbers other than 1/2 to itself) by the relationship $Q_{f(m)} = F(Q_m)$. Find f.

Problem 21. Compute $n_k(n_k(m))$, $s(s(m))$, and $f(f(m))$. Does your result surprise you?

Exploring Exponents

by Addie Summer

Study exponents carefully and you soon run into a lot of fascinating math. Here, we'll take the first steps on this journey and hint at the treasures that await.

Before you learn about exponents, you have to be comfortable with multiplication.

Step 1. Origins

A square has side length s. What is its area?
A cube has side length s. What is its volume?

The answers are $s \times s$ and $s \times s \times s$, respectively.

Here's another series of questions:

A license plate consists of 2 capital letters. How many license plates are possible?
A license plate consists of 3 capital letters. How many license plates are possible?
A license plate consists of 4 capital letters. How many license plates are possible?

The answers are 26×26, $26 \times 26 \times 26$, and $26 \times 26 \times 26 \times 26$, respectively.

In all the answers above, we are multiplying a number by itself a certain number of times. Exponents are a shorthand notation for this kind of operation. Instead of $s \times s \times s$, we can write s^3. Instead of $26 \times 26 \times 26 \times 26$, we can write 26^4. The number being multiplied repeatedly is called the **base** and the superscript on the base is called the **exponent**. The exponent tells how many factors of the base there are in the product being represented. (Also, see *Notation Station* on page 13 of Volume 3, Number 2 of this *Bulletin*.)

In general, if n is a positive integer, a^n stands for $a \times a \times a \times \cdots \times a$, where the product has n factors of a.

Step 2. Basic properties

The two identities below are equalities between expressions that consist of products of some number of factors of a. To verify each identity, count the number of factors of a on both sides and check that they are the same.

$$a^{n+m} = a^n a^m \qquad\qquad (a^n)^m = a^{nm}$$

Step 3. Extending to positive rational exponents, but only for positive, real number bases

We know what it means to write a^n when n is a positive integer. We'd like to extend the notation so it makes sense with positive rational exponents.

What is a meaningful way to extend exponents to positive rational numbers?

A meaningful way to extend exponents to positive rational numbers is to try to extend in such a way that the two basic properties $a^{n+m} = a^n a^m$ and $(a^n)^m = a^{nm}$ continue to hold.

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 16 - Meet 1 January 29, 2015	Mentors:	Rediet Abebe, Bridget Bassi, Karia Dibert, Lydia Goldberg, Hannah Larson, Jennifer Matthews, Sophia Tabchouri, Isabel Vogt, Jane Wang

Some girls worked out a parametrization of Pythagorean triples. To work out this method and explore another, please see this issue's *Learn by Doing* on page 22.

Here's a question from this meet:

> What is the largest number you can express using 5 ones, addition, subtraction, multiplication, division, exponents, parentheses, and at most one factorial?

If you use an exponent, both the base and the exponent must be expressions involving some of the 5 ones that you are allowed. (For example, you cannot cube a number unless you can express the exponent of 3 by using some of the ones for that purpose.) For example, the following is an allowed expression:

$$(1 + 1)^{(1 + 1 + 1)!}.$$

This evaluates to 64, but it is not the largest number that can be so expressed. Can you find the largest?

A related question is, what are all the possible numbers that can be expressed using 5 ones, addition, subtraction, multiplication, division, exponents, parentheses, and at most one factorial? Mathematics provides good training for systematically listing all possibilities.

More generally, let $M(n)$ be the largest number that you can express using n ones, addition, subtraction, multiplication, division, exponents, parentheses, and at most one factorial. What can you say about $M(n)$?

Session 16 - Meet 2 February 5, 2015	Mentors:	Rediet Abebe, Bridget Bassi, Karia Dibert, Alexandra Fehnel, Hannah Larson, Wangui Mbuguiro, Sophia Tabchouri, Isabel Vogt, Jane Wang, Sibo Wang

One of our members explored the differences between absolutely convergent and conditionally convergent series. If you're unfamiliar with these notions and are in the mood for a challenge, here is a very terse introduction. For each positive integer n, let a_n be a number. We can think of a_n as the sequence of numbers $a_1, a_2, a_3, \ldots$. Let s_n be the sum of the first n terms of this sequence, that is, let $s_n = a_1 + a_2 + a_3 + \ldots + a_n$. By definition, the infinite series

$$a_1 + a_2 + a_3 + a_4 + a_5 + \ \ldots$$

converges to s if and only if the limit, as n tends to infinity, of s_n exists and is equal to s. (A sequence c_n converges to c as n tends to infinity if and only if for all $\varepsilon > 0$, there exists N such that $| c_n - c | < \varepsilon$ for all $n > N$.)

Here is the salient example used to illustrate conditional convergence: $a_n = -(-1)^n/n$. Here's the big challenge: convince yourself that by rearranging the terms of a_n, you can create different series that converge to different numbers. In fact, show that for any real number r, it is possible to rearrange the terms of a_n so that the resulting series converges to r. (A sequence b_n is a rearrangement of the sequence a_n if and only if there exists a one-to-one and onto map p from the positive integers to itself such that $b_n = a_{p(n)}$.)

Hint: start by showing that the sum of the reciprocals of the odd positive integers diverges. That is, if you let $t_n = 1 + 1/3 + 1/5 + 1/7 + \ldots + 1/(2n - 1)$, then the limit, as n tends to infinity, of t_n is infinity.

Another hint: why does the convergence of a series not depend on its first finite number of values? That is, elimination or alteration of a finite number of terms in a series does not affect whether it converges or not.

For a more detailed introduction to this topic, consult a text on analysis, such as *Introduction to Calculus and Analysis, Volume 1*, by Courant and John or *Principles of Mathematical Analysis*, by Rudin. And remember, all members and subscribers are invited to email us with any questions or bring questions up at the club!

Session 16 – Meet 3 February 12, 2015	Mentors:	Bridget Bassi, Karia Dibert, Sophia Tabchouri, Anuhya Vajapeyajula, Jane Wang

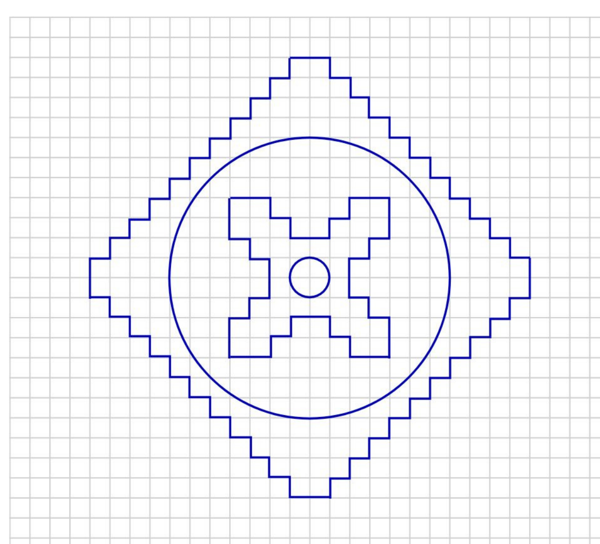
Some members played the Define this Game (see page 8 of Volume 4, Number 4 of this *Bulletin*).

Also, one member asked the following excellent question: Is the ratio of circumference to diameter the same for all circles? If so, why?

Session 16 - Meet 4 February 26, 2015	Mentors:	Rediet Abebe, Karia Dibert, Lydia Goldberg, Anuhya Vajapeyajula, Jane Wang, Sibo Wang

For half of this meet, we engaged in a mathematical art gallery activity. Each member was given a 2D design on graph paper. She had some amount of time to produce a written description of the design. Using the written descriptions, other members would try to replicate the design up to congruence. Next meet, we plan to have an "art gallery showing" of all the drawings together with an "analysis and critique."

The main purpose of this activity is to develop and improve proficiency in precision writing. We also hope that participants will learn new vocabulary and new ways to express concepts in planar geometry.



An example of a design for the art gallery activity.

Calendar

Session 15: (all dates in 2014)

September	11	Start of the fifteenth session!
	18	
	25	No meet
October	2	Emily Pittore, iRobot
	9	
	16	
	23	
	30	
November	6	
	13	Cornelia A. Van Cott, University of San Francisco
	20	
	27	Thanksgiving - No meet
December	4	
	11	

Session 16: (all dates in 2015)

January	29	Start of the sixteenth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	Bathsheba Grossman, Artist
	19	
	26	No meet
April	2	
	9	
	16	
	30	
May	7	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____