Girls' Angle Bulletin

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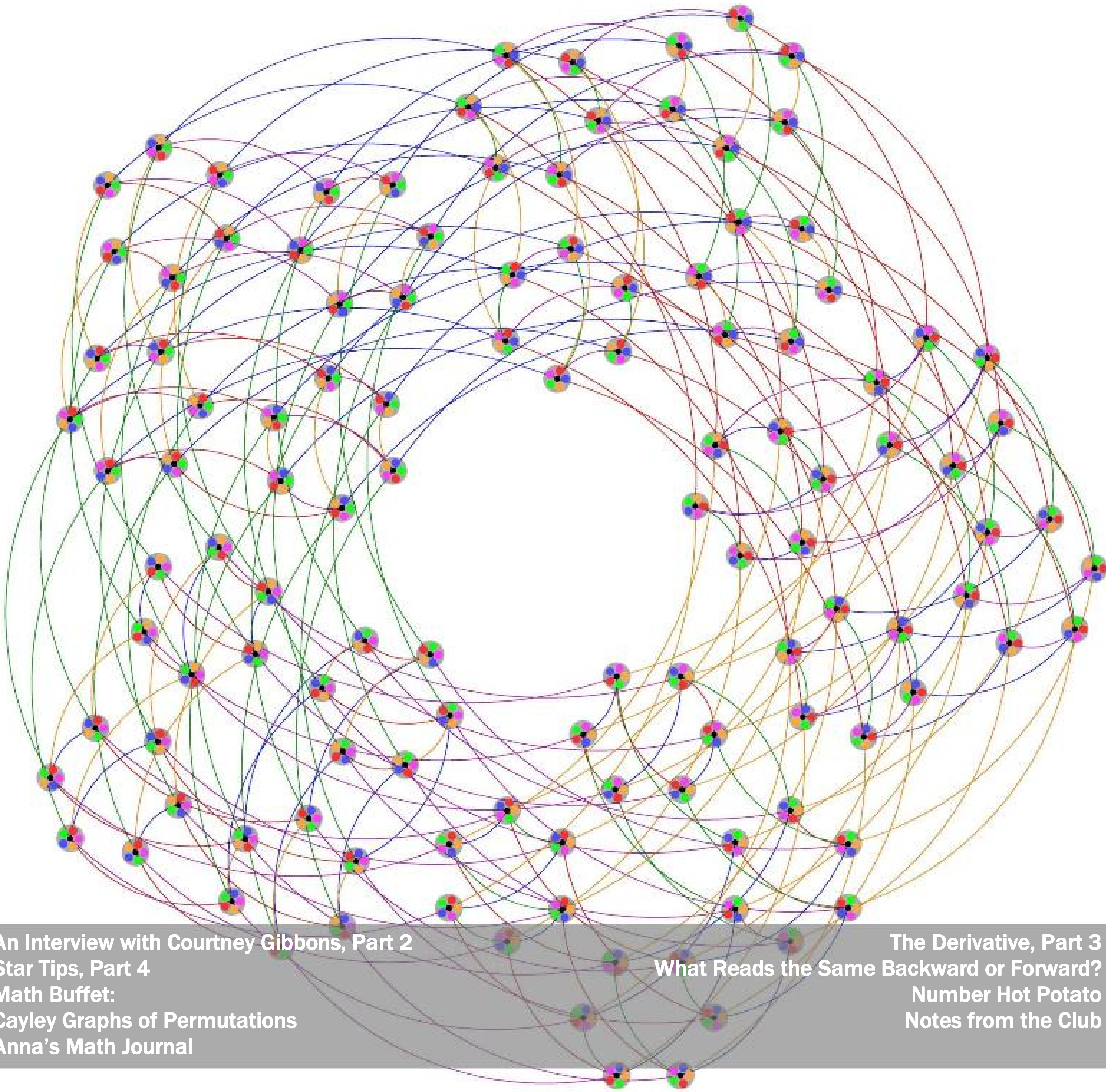
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

This issue's Math Buffet features some Cayley graphs created by our members and staff. We encourage you to construct one of your own. If you do, we welcome you to share it with us.

- Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: A Cayley graph of S_5, by C. Kenneth Fan. Courtney Gibbons' discussion of Cayley graphs in her interview (see page 5) inspired those featured in this issue's Math Buffet on page 12.

An Interview with Courtney Gibbons, Part 2

Ken: You list homological algebra as one of your areas of interest. What is homological algebra? Will you take us on a guided tour that leads from a typical high school algebra class to homological algebra?

Courtney: Oh boy! This will be a fun trip. First, let's look at the word **homology**. If we break it into parts, it means "the study of similarity" (roughly – remember, I'm a mathematician, not a linguist). So we're going to end up talking about how to measure how similar two different algebraic objects are. But let's start back at high school algebra.

In high school algebra, you're primarily concerned with polynomials in one (or maybe two) variables, like

$$p(x) = x^2 - x - 1$$

or

$$q(x, y) = xy^2 - x^2 + y^2 + 1.$$

One of the things you do in high school with polynomials like these is try to factor them. For example,

$$p(x) = \left(x - \frac{1+\sqrt{5}}{2}\right)\left(x - \frac{1-\sqrt{5}}{2}\right),$$

which you can check by expanding. The polynomial $q(x, y)$ also factors:

$$q(x, y) = (x + 1)(y^2 - x + 1).$$

You may have also done some polynomial division in your high school algebra class. For example, if you wanted to see if $x - 1$ divides into $x^2 - x - 1$, you could set the problem up like a long division problem. You'd end up with

$$(x^2 - x - 1) \div (x - 1) = x, \text{ remainder -1.}$$

Since you get a nonzero remainder, $x - 1$ doesn't divide evenly into $x^2 - x - 1$. That is, $x - 1$ isn't a factor of $p(x)$.

Sometimes, you have a polynomial (or a set of polynomials) that is really important in some application, and you want to work modulo that polynomial. All that means is that I'm going to start lumping things together by their remainder when I divide by, for example, $x - 1$. It's just like how 2 am and 2 pm appear the same clocks that repeat every 12 hours. On such clocks, two times appear the same if they differ by a multiple of 12 hours. Similarly, when I work "modulo $x - 1$," I think of $p(x)$ and -1 as representing the same thing since they differ by a multiple of $x - 1$.

Now, we could use the remainder to start measuring how similar two polynomials are. For example, if the remainder is 0, this tells us the polynomials are really closely related (one is a factor of the other!). If the remainder is nonzero but has degree 0, this means the polynomials are pretty closely related – one is almost a factor of the other, but it's off by adding a constant. The higher the degree of the remainder term, the less similar the polynomials are.

In the work that I do, I work with abstract objects called **modules**, but they're actually built from polynomials. First, I start with one module, and I try to approximate it with a special type of module called a **free module**. The way that I measure how similar my module is to its approximation is by doing what is analogous to polynomial division – except I don't just do it once! I start by dividing two modules, and this gives me a remainder term. But the two modules I started with give me a recipe for dividing two more modules... and so on, and so on... possibly infinitely! Each of the remainder terms tells me about how close my starting module is to being a free module. By following my recipe, I'm making sure I'm getting the best possible remainders at each step. What I'm describing in this paragraph is the process of

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We're committed to producing quality math educational content and make every effort to provide this content to you for free.

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For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Gibbons, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

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Thank you and best wishes,
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Star Tips, Part 4

by Ken Fan | edited by Jennifer Silva

Emily: Jasmine?

Jasmine: Yeah?

Emily: Did you see the cover of the last issue of the *Girls' Angle Bulletin*?

Jasmine: Yes, I did. It had a 17-pointed star on it. I think it was a regular (17, 7)-star, to be exact.

Emily: Right. But inside that (17, 7)-star there is a (17, 6)-star, a (17, 5)-star, a (17, 4)-star, all the way down to a blue (17, 1)-star in the center.

Jasmine: Really? Let me check. I've got a copy in my backpack.

Jasmine pulls out her copy and examines the cover.

Jasmine: You're right!

Emily: It made me wonder, if one draws a regular (17, 8)-star, would it contain the (17, k)-stars for all values of k from 1 to 7? Or more generally, if one draws a regular (n, m)-star, will it contain all of the regular (n, k)-stars for values of k from 1 to m – 1?

Jasmine: That sounds plausible. Let's try to prove it!

Emily: Okay!

Jasmine: Hmm. But how?

Emily: Maybe we can start by trying to make the observation more precise. Let's call the number of star tips n. Let m be a positive integer less than or equal to $\lfloor (n-1)/2 \rfloor$, because that's the largest spikey factor possible. Suppose k is a positive integer less than m. We want to show that the regular (n, m)-star contains a regular (n, k)-star.

Jasmine: Can you also be precise about exactly how you think the regular (n, k)-star is contained in the regular (n, m)-star? If you can, then I think we'd be able to prove this by showing that what you think is a regular (n, k)-star actually *is* a regular (n, k)-star.

Emily: Yes, that makes sense. So let me see if I can say exactly what the vertices of the (n, k)-star are. The vertices come from the intersections of edges in the (n, m)-star. But which intersections?

Jasmine: Let's label the vertices of the (n, m)-star clockwise from 0 to $n - 1$, as we've been doing.

Emily: Okay. In that case, the edges of the (n, m)-star are the line segments that connect each vertex x to the vertex $x + m$, modulo n. Looking at the (17, 7)-star on the *Bulletin* cover, it appears that the vertices of the (17, 1)-star it contains are the intersections of the edges that connect vertex x to vertex $x + m$ and vertex $x + 1$ to vertex $x + 1 + m$. And the vertices of the (17, 2)-star within the (17, 7)-star are the intersections of the edges that connect vertex x to vertex $x + m$ and vertex $x + 2$ to vertex $x + 2 + m$.

Jasmine: Yes, that looks right. Let's call the edge that connects vertex x to vertex $x + m$, E_x. It seems that the vertices of what we hope will be a regular (n, k)-star will be the intersections of the edges E_x and E_{x+k} for each x from 0 to $n - 1$. We know that these edges intersect because the endpoints of E_{x+k} are on opposite sides of the line containing E_x, since $m \le \lfloor (n-1)/2 \rfloor$.

Emily: If those intersections are the vertices of our hoped-for regular (n, k)-star, then at the very least, they must also form the vertices of a regular n-gon.

Jasmine: That follows from symmetry. The regular (n, m)-star has a $2\pi/n$-radian rotational symmetry about its center. We know that E_x and E_{x+k} do not contain the center of symmetry since the edges of a regular (n, k)-star don't pass through the center, so the intersection of E_x and E_{x+k} cannot be at the center. From one of these intersections, we get all the others by rotating n times by $2\pi/n$ radians about the center. Therefore, these n vertices are the vertices of a regular n-gon.

Emily: Nice. I love symmetry!

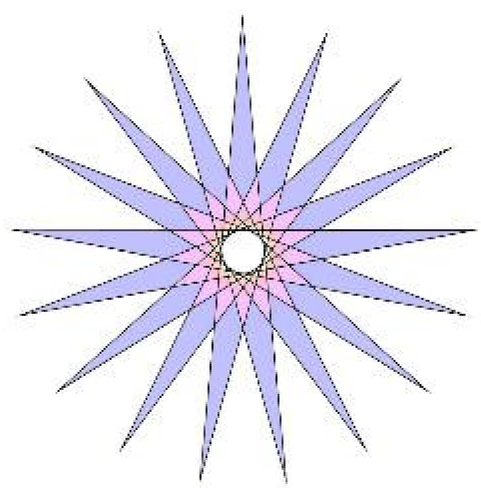
Jasmine: But we still have to show that the edges of the (n, m)-star connect these vertices as they should in an (n, k)-star.

Emily: True. Let's see. Let's denote by V_x the intersection of E_x and E_{x+k}. Since V_{x+k} is the intersection of E_{x+k} and E_{x+2k}, both V_x and V_{x+k} are on E_{x+k}. And these are the only two vertices on E_{x+k}, because there can only be two points on E_{x+k} that are the same distance from the center.

Jasmine: Excellent, that does it! Vertex V_x is connected by edge E_{x+k} to vertex V_{x+k}, and vertex V_{x+k} is exactly the vertex that is k over from vertex V_x. That is just what we need to declare that these vertices are strung together by the edges of the (n, m)-star to form an (n, k)-star!

Emily: Cool! So if I draw a regular (17, 8)-star, I should get 17-pointed stars with all the other spikey factors for free! It makes me want to make one.

Emily produces the image at right.



Jasmine: Wow, the (17, 7)-star inside of it is much smaller. It appears to be about one-third the size of the (17, 8)-star. You can barely make out the (17, 2)-star. I wonder how the sizes of the stars shrink.

Emily: We should be able to figure that out, since we've described the vertices of all of the stars.

Jasmine: The vertices of each of our regular stars sit on concentric circles, so let's compute the ratios of the radii of those circles.

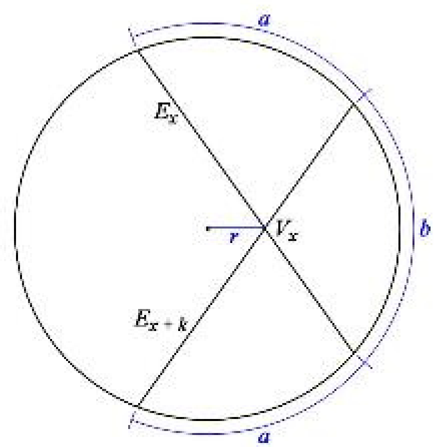
Emily: That sounds like a good plan. Actually, we can get the ratio of the radius of a regular (n, m)-star to the radius of the regular (n, k)-star inside it by comparing both to a containing regular (n, $\lfloor (n-1)/2 \rfloor$)-star.

Jasmine: I'm getting tired of saying $\lfloor (n-1)/2 \rfloor$ over and over, so let's define S to be $\lfloor (n-1)/2 \rfloor$. That's a capital S to stand for the largest possible spikey factor. What you're suggesting is that we focus on computing the ratio of the radius of the regular (n, S)-star to the radius of the regular (n, k)-star that it contains.

Emily: Right.

Jasmine: Okay, I'm good with that. And since we're dealing with ratios, we might as well normalize so that the regular (n, S)-star has radius 1.

Emily: Sure. In that case, what we have to do is figure out the distance of V_x from the center.



Jasmine: Why don't we just use coordinate geometry?

Emily: Eek! Are you sure? That might get messy.

Jasmine: Well, here's the situation.

Jasmine draws the figure at right.

Jasmine: We can introduce our coordinate axes so that the center is at the origin and V_x is on the positive horizontal axis. Then all we have to do, by symmetry, is find the equation of the line that contains either E_x or E_{x+k} and compute its horizontal intercept!

Emily: That doesn't seem so bad. In your figure, arc angle a measures $2\pi k/n$ radians, and arc angle $a + b$ measures $2\pi S/n$ radians. So arc angle b is $2\pi(S - k)/n$ radians.

Jasmine: In polar coordinates, the endpoints of E_x are at $a + b/2$ radians and $-b/2$ radians; since we're normalizing the radius of the (n, S)-star, both are at distance 1 from the center. So the coordinates of the endpoints of E_x are ($\cos(a + b/2)$, $\sin(a + b/2)$) and ($\cos(-b/2)$, $\sin(-b/2)$). For the horizontal and vertical coordinates, I'll use p and q respectively, because we're already using x to denote something else. Using the point-slope form of a line, I find that the equation of the line that contains E_x is

$$\frac{q-\sin(-b/2)}{p-\cos(-b/2)}=\frac{\sin(a+b/2)-\sin(-b/2)}{\cos(a+b/2)-\cos(-b/2)}.$$

Emily: To find the x-intercept, we just have to plug in $q = 0$ and solve for p.

Emily and Jasmine separately carry out the computation and simplify using trigonometric identities.

Jasmine: I get $r = \cos(\pi S/n) / \cos(\pi k/n)$. Is that what you got?

Emily: Yes, I got that too! It works out to be a relatively simple expression.

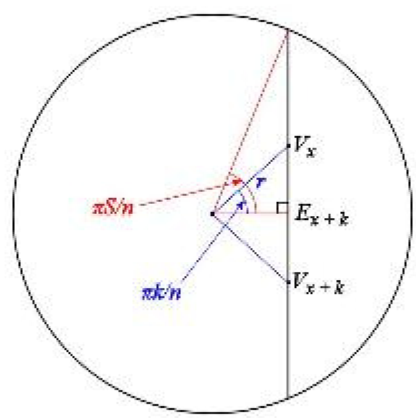
Jasmine: And when you let $k = S$, you get $r = 1$, which is what we should get since the ratio of the radius of the regular (n, S)-star to itself should be 1.

Emily: And as k increases from 1 to S, the denominator, $\cos(\pi k/n)$, decreases, so r increases. This means that the stars are ordered outward from the center by their spikey factor.

Jasmine: This formula is so straightforward. There's got to be a way to see it directly.

Emily: Yeah, totally!

Emily and Jasmine try to find a more direct way of seeing the formula.



Emily: I've got it!

Emily shows Jasmine the figure at right.

Emily: We already showed that on E_{x+k} there are two connected vertices of the (n, k)-star, namely V_x and V_{x+k}. These two vertices are separated by an angle of $2\pi k/n$ radians, so the indicated blue angle in the figure measures $\pi k/n$ radians. Similarly, the endpoints of E_{x+k}, which are connected vertices in our (n, S)-star, are separated by an angle of $2\pi S/n$ radians, so the indicated red angle in the figure measures $\pi S/n$ radians. But the distance from the center of the circle to E_{x+k} is equal to both $\cos(\pi S/n)$ and $r \cos(\pi k/n)$, therefore $\cos(\pi S/n) = r \cos(\pi k/n)$. Dividing both sides by $\cos(\pi k/n)$ yields the formula!

Jasmine: Nice! Why didn't we see that earlier? It would have saved us a lot of computation.

Emily: It sure would have! In any case, now we know that if we draw the spikiest regular n-pointed star, the radius of the next spikiest n-pointed star inside of it will have a radius that is smaller by a factor of $\cos(\pi S/n) / \cos(\pi(S-1)/n)$.

Jasmine reaches for a calculator. Emily gives her a quizzical look since she doesn't know what Jasmine would want to compute.

Jasmine: I'm just curious to know what that ratio is when $n = 17$. When n is 17, S would equal 8, so I have to compute $\cos(8\pi/17) / \cos(7\pi/17)$. What? Oh, I forgot to set my calculator to radians! There we go – I get approximately 0.337.

Emily: That's really close to 1/3. Isn't that what you said you thought the ratio looked like?

Jasmine: It is. Hmm, wait a second. Does $\cos(\pi S/n) / \cos(\pi(S - 1)/n)$ converge to some positive constant as n tends to infinity?

Emily: Interesting question! As n tends to infinity, both S/n and $(S - 1)/n$ tend to 1/2 because S is approximately $n/2$. And the cosine of $\pi/2$ is 0. Uh oh ... both the numerator and denominator tend to 0 as n tends to infinity.

Jasmine: Perhaps we can use l'Hôpital's rule.

Emily: But $S = \lfloor (n-1)/2 \rfloor$, so $\cos(\pi S/n) / \cos(\pi(S - 1)/n)$ isn't a continuous function of n, let alone differentiable, not to mention that n is restricted to be a positive integer.

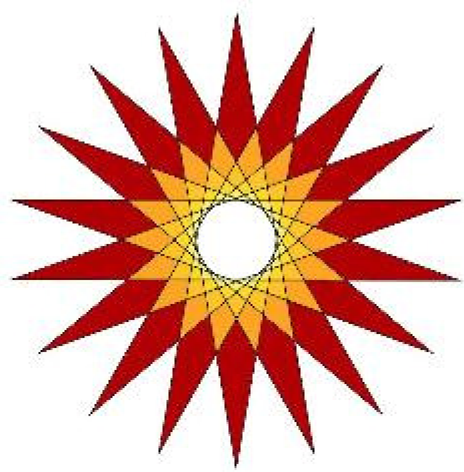
Jasmine: We can split it into the cases where n is even and n is odd. We'll get two different sequences and we can think of n as a continuous variable in each case. If l'Hôpital's rule gives us a limit, then it would have to be the limit for the sequence as well.

Emily: I agree. So when n is even, $n = 2S + 2$ and we're looking for the limit, as n tends to infinity, of $\cos(\pi(n - 2)/(2n)) / \cos(\pi(n - 4)/(2n))$. By l'Hôpital's rule, this is the limit, as n tends to infinity, of $(1/2) \sin(\pi(n - 2)/(2n)) / \sin(\pi(n - 4)/(2n))$, and that is equal to 1/2!

Jasmine: And when n is odd, $n = 2S + 1$, so we need to compute the limit, as n tends to infinity, of $\cos(\pi(n - 1)/(2n)) / \cos(\pi(n - 3)/(2n))$. Using l'Hôpital's rule, I get a limit of 1/3.

Emily: That's funny! So when n is odd, the $(n, S - 1)$-star is about one-third the size of the containing (n, S)-star, but when n is even, it'll be about half the size. Being close to 1/3 was no coincidence when $n = 17$.

Jasmine: I really want to draw a regular (18, 8)-star just to see that the (18, 7)-star inside of it is about half its size.



Jasmine draws the figure at right.

Jasmine: Math never lies!

Emily: While you were making that drawing, I decided to go ahead and compute the limit, as n tends to infinity, of the radius of the $(n, (S - k))$-star contained inside a regular (n, S)-star of radius 1. It's kind of neat. I found that when n is even, the radius of the $(n, S - k)$-star tends to $1/(k + 1)$ and when n is odd, the radius of the $(n, S - k)$-star tends to $1/(2k + 1)$.

Jasmine: How amusing! Who would have imagined that thinking about n-pointed stars would tie in to the reciprocals of the positive integers?

Math Buffet

Cayley Graphs of Permutations
Layout by Toshia McCabe

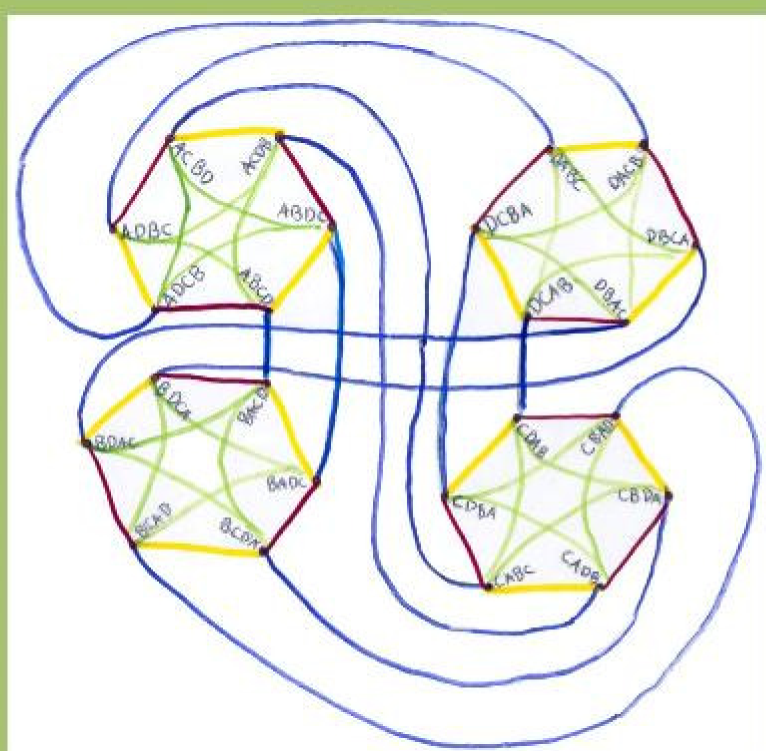
A **graph** is a collection of **nodes** that are connected by **edges**. For example, we can make a graph by declaring the nodes to be countries and placing an edge between two nodes if the corresponding countries border each other. Graphs are typically drawn by indicating nodes with points and edges with line segments or curves.

Fix a set S of n distinct objects. There are $n!$ different orders in which we can line these objects up in a row. Such arrangements are called **permutations** of S. If $S = \{A, B, C\}$, for example, then the $3! = 6$ permutations of S are: ABC, ACB, BAC, BCA, CAB, and CBA.

There's a lot of structure to the set of permutations of S. One way to visualize this structure is to form a **Cayley graph** of the permutations. To make a Cayley graph, draw a node for each permutation. Notice that by rearranging the positions of the objects, you can change any permutation of S to any other. For example, by switching the last two letters, we can change ABC to ACB. For the edges, select a few of the various ways to change the positions of the objects in a permutation of S. If a node Y can be obtained from node X via one of your position permutations, draw an arrow from node X to node Y. Make all arrows corresponding to the same position permutation the same color.

In this Math Buffet, we feature Cayley graphs of the permutations of S for various sets S created by Girls' Angle members and staff. In some cases, arrowheads are omitted.

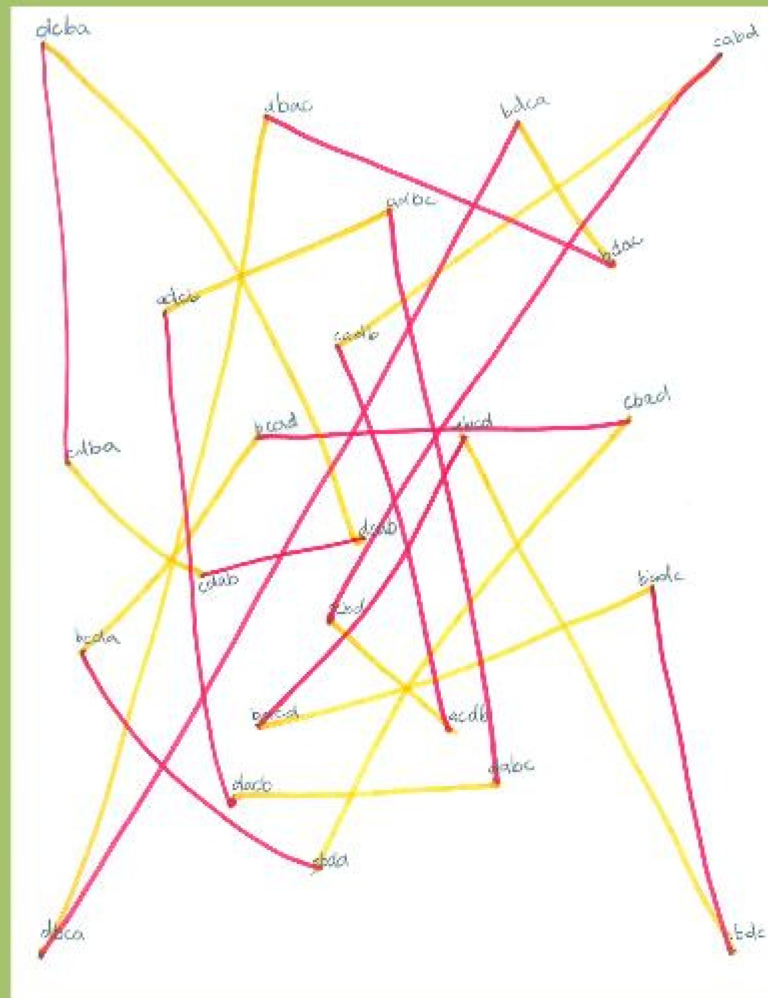
If you are familiar with group theory, note that Cayley's theorem implies that the Cayley graph of any group can be realized as a subgraph of the Cayley graph of permutations.

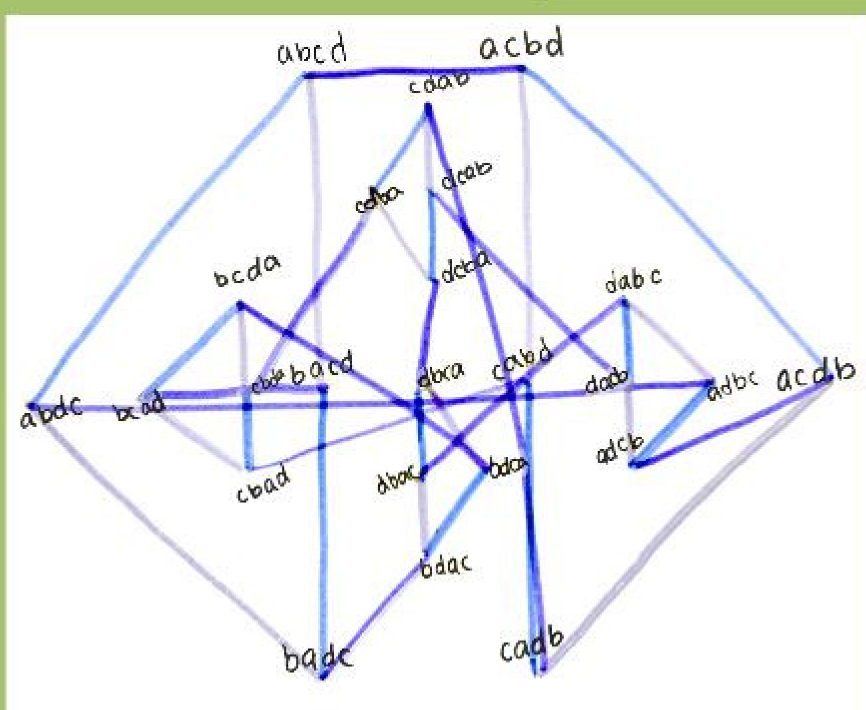


This Cayley graph of the permutations of {A, B, C, D} shows four hexagonal subgraphs that have the structure of a Cayley graph of the permutations of a 3 element set. Each hexagonal structure is connected to the others by two edges. Design by **Sue D. Nym**.

Can you see that the Cayley graph of the permutations of {a, b, c, d} shown at right consists of 6 disjoint 4-edge loops, where each loop consists of an alternating pattern of yellow and red edges?

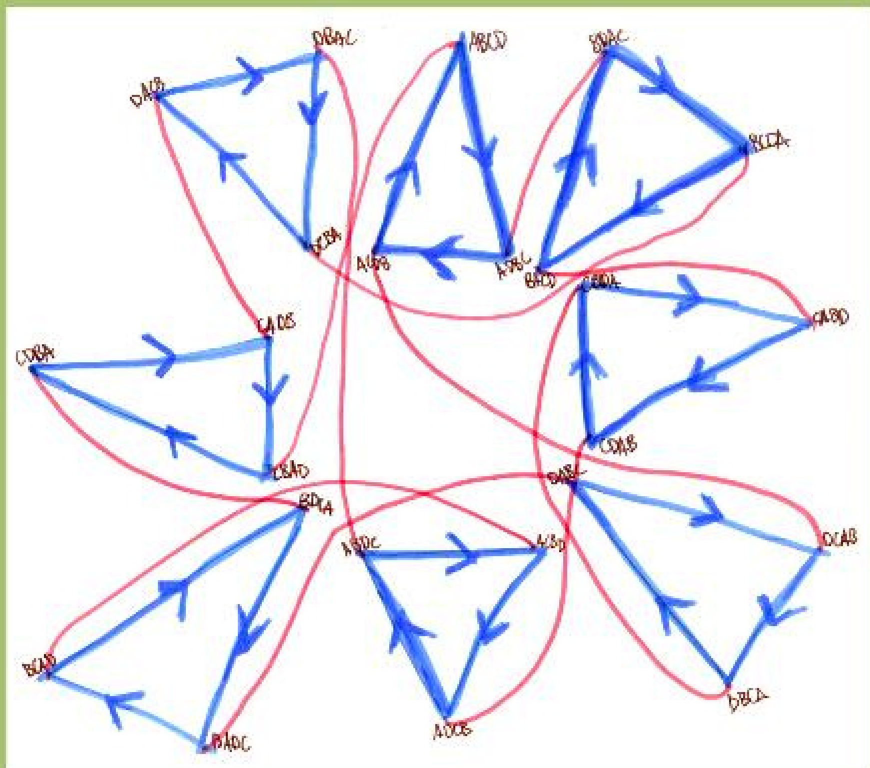
Design by **Pikachu**.





The Cayley graph of the permutations of {a, b, c, d} shown at left has edges that are generated by the so-called “adjacent transpositions.” Adjacent transpositions swap the letters in adjacent positions.

Design by **Fallow**.

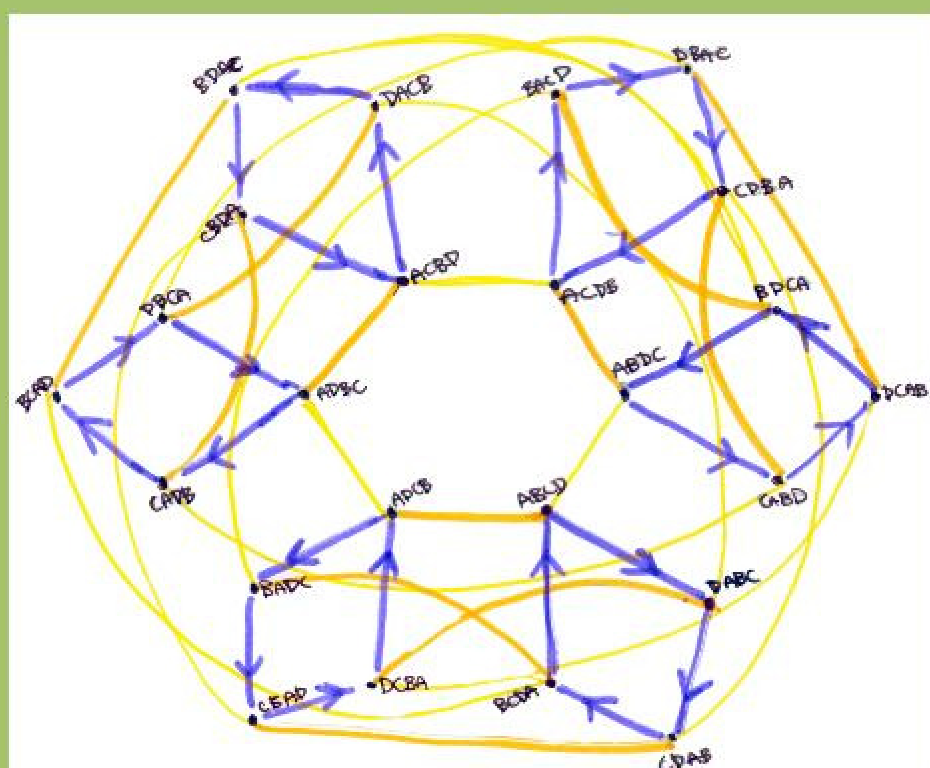


Blue triangles are woven together by red edges in the Cayley graph of the permutations of {A, B, C, D} shown at left. How many patterns and symmetries can you find? There are many!

Design by **Ombrésa**.

For those who know group theory, the Cayley graph at right contains an orange/blue subgraph that corresponds to a Sylow-2 subgroup of S_4, which is isomorphic to the dihedral group D_4. How can you tell that the orange/blue subgraph represents the Cayley graph of a non-commutative group?

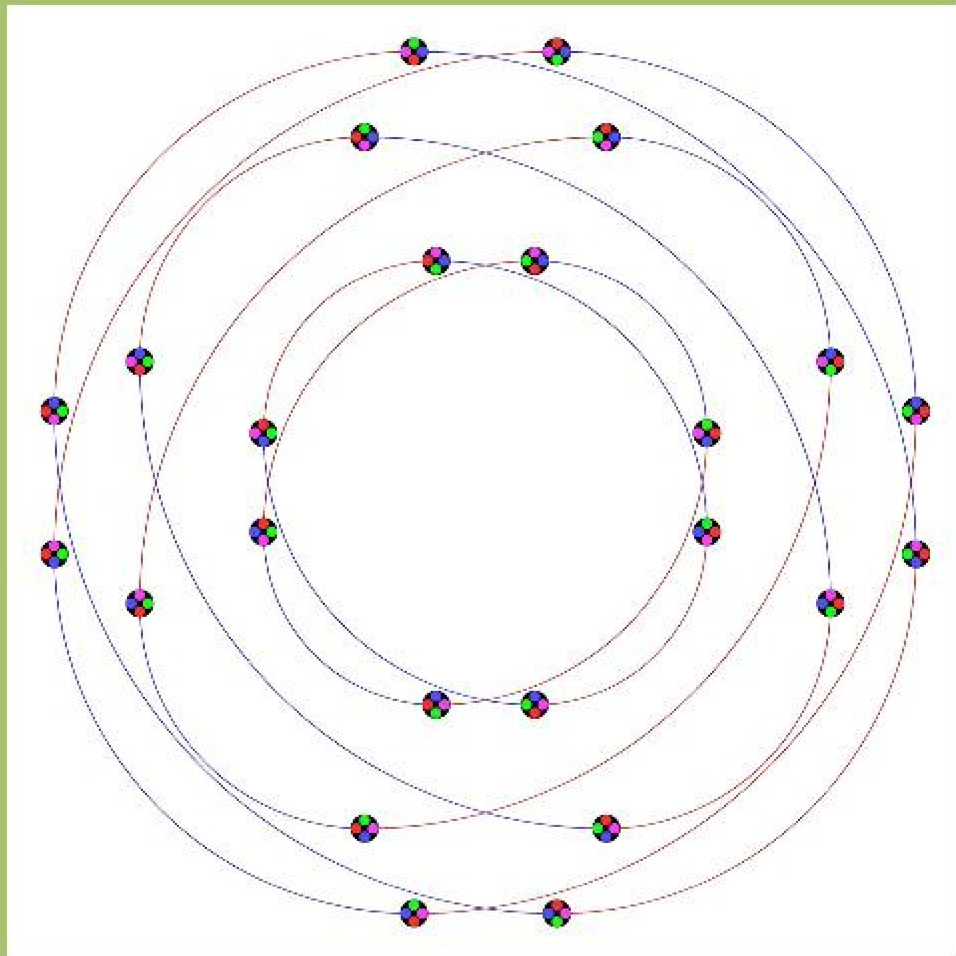
Design by **404 Name Not Found**.

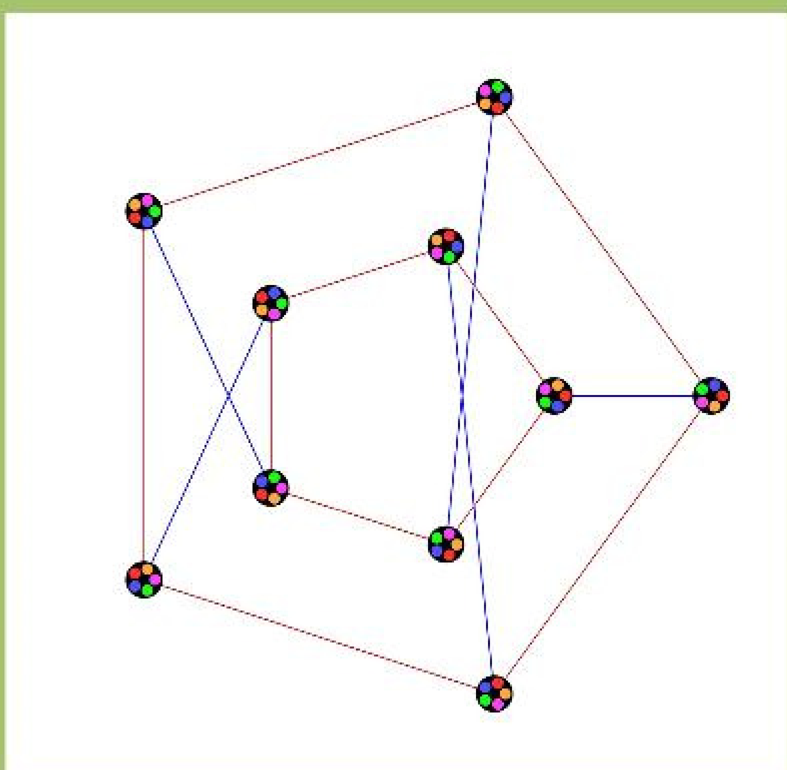


Does this Cayley graph look familiar?

It's essentially the same as one of the others. Can you figure out which one?

Design by Girls' Angle staff.





The Cayley graph at left is a Cayley graph of the dihedral group D_5.

Design by Girls' Angle staff.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues her investigation into cross sections of the surface $z = xy$.

So far, I've looked at cross sections of $z = xy$ with all types of planes except for non-vertical, non-horizontal ones that intersect the xy-coordinate plane in a line parallel to the x or y-axis or a line that contains the origin.

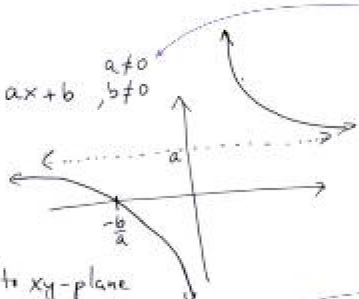
So, I'll look at planes that intersect the xy-coordinate plane in a line parallel to the y-axis.

$z = xy$

Let P be the plane $z = ax + b$, $a \neq 0$, $b \neq 0$

$xy = ax + b$

$x(y - a) = b$



The projections of these cross sections are hyperbolas.

projection of crosssection to xy-plane is hyperbola with asymptotes $x = 0$ and $y = a$.

Actually, the asymptotes of the projection to the xy-coordinate plane are given by the same expressions I found last time, so I didn't really need to put this in a separate case.

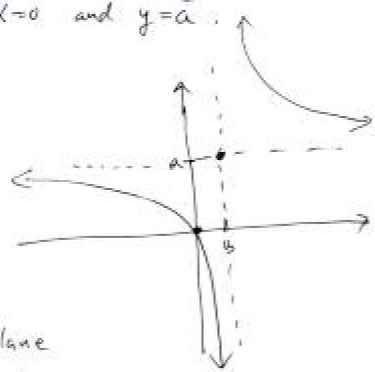
Now for cross sections with planes that contain the origin...

Let P be the plane $z = ax + by$

$xy = ax + by$

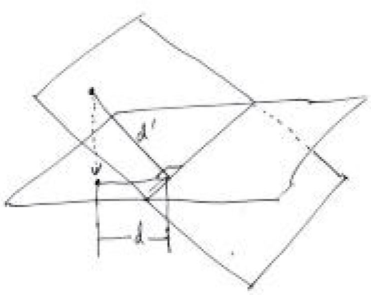
$xy - ax - by = 0$

$(x - b)(y - a) = ab$



These fit the same pattern! The projection of the cross section to the xy-coordinate plane is again a hyperbola with asymptotes given by the same formulas from last time.

projection of cross section to xy-plane is hyperbola with asymptotes $x = b$, $y = a$.



$\frac{d'}{d}$ = const. = stretch factor.

$\Rightarrow$ if projection of cross section to xy-plane is conic, then cross section is conic.

I should make sure that the cross section itself is a conic of the same type as its projection to the xy-plane. If I introduce a coordinate system so that the intersection of the cross sectional plane and the xy-coordinate plane is one of the axes, then the coordinates in the cross sectional plane and the coordinates in the xy-coordinate plane are related by a stretch along the direction normal to the line of intersection. That makes it clear, because if one stretches a conic that way, its character doesn't change.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

To recap...

Summary for $z = xy$. Let P be a plane.

Cross sections are

1. lines if P is vertical and parallel to a coordinate plane.
2. parabolas if P is vertical and not parallel to a coordinate plane.
3. hyperbolas in all other cases.

Furthermore, parabolic cross sections intersect the xy-plane on the x and y-axes.

Hyperbolas all project to hyperbolas in the xy-plane with asymptotes parallel to the x and y-axes.

I also wanted to think about whether it is true or not that cross sections of general quadric surfaces will always be conics, or degenerate conics.

General quadric surface:

$$A_{xx}x^2 + A_{yy}y^2 + A_{zz}z^2 + A_{xy}xy + A_{xz}xz + A_{yz}yz + A_x x + A_y y + A_z z + A_0 = 0$$

Let me write down an equation for the most general quadric surface.

where A_{xx}, A_{yy}, A_{zz}, A_{xy}, A_{yz}, A_{xz}, A_x, A_y, A_z, and A_0 are all constants.

Cross section with plane $Ax + By + Cz + D = 0$

$\Rightarrow$ proj. to some coord. plane is conic

So crosssection is a conic.

Hm. Actually, this works out quit nicely. One of the coefficients A, B, or C must be nonzero. So I can solve for the corresponding variable and substitute into the equation of the general quadric. The degrees of terms won't change, so I will get an equation for a conic. So the projection of the cross section to at least one of the coordinate planes will be a conic, and, hence, so will be the cross section itself.

Linear section of a quadric is a quadric.

In fact, any linear section of a quadric of any dimension will be a quadric!

ABB 4.26.15

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

The Derivative, Part 3

by Ken Fan | edited by Jennifer Silva

Calculus is a subject well covered in textbooks. Instead of giving another textbook treatment of the subject, we aim to provide an illuminating, though non-rigorous, explanation.

We assume that all functions are real-valued functions defined on some set of real numbers, unless otherwise noted.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Gibbons, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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What Reads the Same Backward or Forward?

by Stuart Sidney | edited by Jennifer Silva

This title was the headline of a recent "Word Watch" column in the Sunday *Hartford Courant*. The answer is, a **palindrome** – a word, phrase or sentence that reads the same backward or forward. For instance, *redder* is a word that is a palindrome; a famous sentence that is a palindrome is what, it is jokingly said, the first man said upon meeting the first woman (in the Bible): *Madam, I'm Adam!* (Do you think Adam and Eve, if they existed, *really* communicated in English?) This example illustrates that in determining palindrome status, we ignore punctuation, spaces, and issues of capitalization; all that counts is the order of the letters, here *m-a-d-a-m-i-m-a-d-a-m*.

The same concept makes sense for numbers. A non-negative integer is a **palindromic number** (in the usual base 10) if it reads the same backward and forward. For example, 26,362 is a palindromic number. The only palindromic number that begins or ends in 0 is 0 itself. Every 1-digit number is palindromic, but the only 2-digit palindromic numbers are 11, 22, 33, 44, 55, 66, 77, 88, 99.

How many 3-digit palindromic numbers are there? 4-digit? 5-digit? 6-digit? Can you make and verify a conjecture for n-digit palindromic numbers?

Of course, this notion makes sense in any base. In base 5, for instance, the only available digits are 0, 1, 2, 3, 4. So an example of a base 5 palindromic number is 31413_5; this is the base 10 number $3 \times 5^4 + 1 \times 5^3 + 4 \times 5^2 + 1 \times 5^1 + 3 \times 5^0 = 1875 + 125 + 100 + 5 + 3 = 2108$, where the subscript 5 on 31413 indicates that the number is being expressed in base 5, and the absence of a subscript on 2108 indicates that the number is in base 10, the default base. Notice from this example that a number may be palindromic in one base but not in another. Other examples of this are $1221_4 = 151_8 = 77_{14} = 55_{20} = 33_{34} = 11_{104} = 210_7 = 105_{10}$ and $1991_{10} = 7C7_{16} = 5543_7$ where in base 16 we count 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *A*, *B*, *C*, *D*, *E*, *F*.

How many n-digit palindromic numbers are there in base 5? In base b where $b \geq 2$?

There are lots of properties a palindromic number might or might not have. It might be a perfect square, or cube, or 4th power, and so on. It might be a prime.

Here are the first few palindromic numbers of two or more digits that are perfect squares: $11^2 = 121$, $22^2 = 484$, $26^2 = 676$, $101^2 = 10{,}201$, $111^2 = 12{,}321$, $307^2 = 94{,}249$. Notice that the square root of the palindromic square may or may not be palindromic.

Which of these palindromic squares would still be squares if we used the same sequence of digits but a base other than 10, such as base 12 or 16? Which would be squares for *all* bases greater than 9?

The first few palindromic numbers of two or more digits that are perfect cubes are $7^3 = 343$, $11^3 = 1{,}331$, $101^3 = 1{,}030{,}301$, $111^3 = 1{,}367{,}631$, $1001^3 = 1{,}003{,}003{,}001$. The only known palindromic cube whose cube root is *not* palindromic appears to be $2201^3 = 10{,}662{,}526{,}601$. It is conjectured that the only palindromic 4th powers have fourth roots of the form $1000 \cdots 0001 = 10^k + 1$, which (of course) are palindromic.

Check that, conversely, in *any* base $b > 6$, the 4th power of $10_b^k + 1$ where $k \geq 1$ is in fact palindromic.

Now let's turn our attention to prime numbers that are palindromic. Can 3223 be prime? This is equal to $3 \times 1001 + 2 \times 220$, and both 1001 and 220 are divisible by 11, hence so is 3223. Can 743,347 be prime? It equals $7 \times (10^5 + 10^0) + 4 \times (10^4 + 10^1) + 3 \times (10^3 + 10^2)$, and the terms in parentheses are all divisible by 11, with respective quotients 9091, 10×91, and 10^2. What about 87,433,478? Check whether $8 \times (10^7 + 10^0)$ has an obvious factor other than 8 and $10^7 + 10^0$. Can you generalize what you just did and prove the following theorem?

Theorem. In base 10, the only palindromic prime number with an even number of digits is 11.

What about other bases? Let's review a bit of algebra. Using the distributive rule and then canceling, what do you get if you multiply $(7 + 1) \times (7^2 - 7^1 + 7^0)$?
$(6 + 1) \times (6^4 - 6^3 + 6^2 - 6^1 + 6^0)$? $(12 + 1) \times (12^6 - 12^5 + 12^4 - 12^3 + 12^2 - 12^1 + 12^0)$?
If $b \geq 2$ is any base and $n \geq 4$ is *even*, what do you get if you multiply
$(b + 1) \times (b^{n-2} - b^{n-3} + \cdot\cdot\cdot + b^2 - b^1 + b^0)$? What can you deduce about palindromic prime numbers in base b? Isn't that interesting?

How many palindromic primes are there? Who knows? The ones with three digits (in base 10) are 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, and 929. There is even a known palindromic prime with 39,027 digits!

There's lots more that can be said about palindromic numbers of various special types, but let's instead shift gears and look at the process of generating palindromic numbers from any initial numbers. The procedure is simple. Start with any positive integer. If it is already palindromic, you are done. If it is not, reverse its digits and add this new number to the original number. If the resulting number is palindromic, you are done. If not, continue until (finally) you get a palindromic number.

Let's try some examples. Starting with 75, we next get 75 + 57 = 132 which becomes 132 + 231 = 363: done! 346 becomes 346 + 643 = 989: done! 358 becomes 358 + 853 = 1,211 which becomes 1,211+1,121 = 2,332: done! Sometimes you'll have a number that ends in 0, so its "reverse" number will begin with 0. For instance, 366 becomes 366 + 663 = 1,029 which becomes 1,029 + 9,201 = 10,230, which becomes 10,230 + 03,201 = 13,431: we have arrived, on the way passing a number ending in 0 and its reverse that begins with 0.

Try starting with various 2-digit and 3-digit numbers (less than 196, to be safe!) and see how long it takes to get to a palindromic number. In some cases, it might take quite a while. You might also want to experiment with bases other than 10.

One might well imagine that no matter what number you start with, eventually you'll reach a palindromic number. However, it seems that nobody actually knows whether this is true. If you start with anything up to 195, there's no problem. But 196 seems to be playing a sneaky game. The procedure has been iterated nearly 725,000,000 times starting with 196, resulting in a number of roughly 300,000,000 digits, and it still has no palindromic result. Maybe *you'll* be the one to settle the problem for 196, and perhaps even the one to determine whether the procedure *always* leads to a palindromic number.

Number Hot Potato

by Ken Fan

Number Hot Potato is a variant of Hot Potato. Like Hot Potato, players toss a bean bag around and the last player holding the bean bag when a timer goes off is out. However, unlike Hot Potato, in Number Hot Potato, before tossing the bean bag, the player with the bean bag must state a valid number. Rules for determining whether a number is valid or not are created at the beginning of each round.

At the club, we have played the game with groups of 4-7 girls, but it probably works for larger groups as well. Participants should be about the same math skill level. The game can be made extremely difficult or very easy. It is up to the referees. Number Hot Potato is an intense game that demands a lot of the referees, so it helps to have at least 3.

Materials: a bean bag, a timer, a notepad, and a pencil.

How to Play a Round of Number Hot Potato

Step 1. Tell players how much time they'll have for the round. We recommend anywhere from 20 seconds to a minute.

Step 2. Tell players the criteria for what makes a number valid for this round. We'll say more about this later.

Step 3. Toss the bean bag to a participant and start the timer.

Step 4. If the time runs out at any point during the execution of this step, whoever is holding the bean bag is out and play continues with Step 5. Until time runs out, the person with the bean bag tries to say a number as quickly as possible. If the number is valid, then the player may toss the bean bag to another player. If the number is not valid, the player is out and play continues with Step 5. If no out has occurred and time has not run out, repeat this step.

Step 5. If there is more than one player not yet out, start a new round by going to Step 1.

Step 6. The sole player not yet out is declared the "winner."

Valid Numbers

The criteria for what constitutes a valid number are created by the referees. The referees should chose them to thwart any strategies that emerged in previous rounds or choose them to introduce new challenges. Typically, the game begins with either all numbers valid or the criterion that a number is valid if it has not yet been said during the round. For more examples of such criteria, please see the sample game on the next page.

Tips

Use the notepad and pencil to record the criteria for number validity for each round as well as all numbers spoken by the players. If you have 3 referees, have one handle the timer and have the other two keep track of the numbers and check for their validity.

Sample Game

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 16 - Meet 5 March 5, 2015	Mentors:	Bridget Bassi, Meena Bopanna, Karia Dibert, Alexandra Fehnel, Lydia Goldberg, Jennifer Matthews, Wangui Mbuguiro, Jane Wang

We did a mathematical art gallery activity using an organizational scheme devised by Karia and Jane. (See the description of Session 16, Meet 4 in the previous issue.) Each member managed to see how her written description of a geometric figure was rendered by several others. The purpose of this activity is to bring to the forefront concepts in geometry.

Session 16 - Meet 6 March 12, 2015	Mentors:	Bridget Bassi, Meena Bopanna, Karia Dibert, Wangui Mbuguiro, Sophia Tabchouri, Anu Vajapeyajula, Jane Wang, Sibo Wang

We followed up on the art gallery activity by introducing several ways to communicate geometric objects using coordinates. We also provided a mentor-written description of a figure to illustrate ways to communicate effectively.

One of our members proved the Schröder-Bernstein theorem. The Schröder-Bernstein theorem states that if there exists injective maps from set S to set T and from set T to set S, then there exists a bijection between S and T. Here's a question: if there exist surjective maps from S to T and from T to S, does there exist a bijection between S and T?

> An injective map $f : S \to T$ is a map for which $f(x) = f(y)$ implies $x = y$.
>
> A surjective map $f : S \to T$ is a map such that for every t in T, there exists s in S such that $f(s) = t$.
>
> A bijective map is both injective and surjective.

Session 16 – Meet 7 March 19, 2015	Mentors:	Rediet Abebe, Bridget Bassi, Karia Dibert, Hannah Larson, Jennifer Matthews, Wangui Mbuguiro, Anuhya Vajapeyajula, Jane Wang

One of our members explored ordinals and found examples that showed that ordinal addition is not commutative.

Session 16 - Meet 8 April 2, 2015	Mentors:	Bridget Bassi, Karia Dibert, Lydia Goldberg, Jennifer Matthews, Anuhya Vajapeyajula, Jane Wang, Sibo Wang

In addition to making progress on various ongoing projects, we introduced a new game called Number Hot Potato (see page 24). In a certain sense, Number Hot Potato is a game of wits between participants and the referees. Referees invent more and more elaborate rules while participants devise clever schemes to navigate these rules quickly.

Session 16 - Meet 9 April 9, 2015	Mentors:	Alexandra Fehnel, Lydia Goldberg, Wangui Mbuguiro, Jane Wang
	Visitor:	Bathsheba Grossman, Sculptor

Bathsheba Grossman presented on her math-inspired art. The daughter of two English professors, she was raised in nearby Lexington. She majored in mathematics at Yale. Inspired by the works of Robert Engman, she found her calling in designing abstract 3D forms. When 3D printing appeared, she found an ideal medium for her ideas. Computer assisted design and 3D printing combine many of her talents: computer programming, math, and sculpture.

> *If you want to create something, you have to sit down and play with the tools.*
>
> \- Bathsheba Grossman

The presentation began with a show and tell where she put on display a couple dozen of her designs, including the 3D shadow of a hyper dodecahedron, a Klein bottle (that functions as a bottle opener), sculptures based on the Platonic solids and their duals, the Seifert surface joining Borromean rings, designs based on minimal surfaces, and other items, including a balancing squid and a cuttlefish bottle opener. She also showed us two glass cubes, one with a Calabi-Yao manifold etched within and one containing a Julia set.

Bathsheba explained the actual processes that are used to create these 3D sculptures and she demonstrated Computer-Assisted Design (CAD) software that she uses to develop her sculptural ideas.

Session 16 - Meet 10 April 16, 2015	Mentors:	Bridget Bassi, Karia Dibert, Alexandra Fehnel, Jennifer Matthews, Wangui Mbuguiro, Anuhya Vajapeyajula, Jane Wang

Some of our younger members are coming tantalizing close to settling a version of the "*N* ones" problem that was stated at the first meet of this session.

Some members worked on the following "Angry Cake Eater" problem: Two people love to eat cake. One of them always takes a single slice of cake. The other always takes 2 slices. If the person who always takes 2 slices comes to the cake and finds only 1 slice left, he throws a temper tantrum. The two people are equally likely to be the next person to take slices from the cake. What is the probability that there will be a temper tantrum if the cake starts with N slices?

One of our members explored the universal logic gates.

Session 16 - Meet 11 April 30, 2015	Mentors:	Bridget Bassi, Karia Dibert, Sophia Tabchouri, Jane Wang

Members who made Cayley graphs (see page 12) dove more deeply into the structure of permutations. For example, they figured out how to make the connected component of a Cayley graph of permutations have the connectivity of the vertices and edges of a cube.

We introduced the mathematical technique of induction.

On this page are two samples of the artwork of Bathsheba Grossman, who was our Support Network visitor on April 9.

Above is the "Quin" lamp. At left is the "Gyroid."

Both photos courtesy of Bathsheba Grossman.

Calendar

Session 16: (all dates in 2015)

January	29	Start of the sixteenth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	
	19	
	26	No meet
April	2	
	9	Bathsheba Grossman, Sculptor
	16	
	30	
May	7	

Session 17: (all dates in 2015)

September	17	Start of the seventeenth session!
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	
	10	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls’ Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant’s Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don’t like math, what don’t you like? If you love math, what do you love? What would you like to get out of a Girls’ Angle Membership?

____

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- □ Enclosed is a check for $36 for a 1-year Girls’ Angle Membership.
- □ I am making a tax free donation.

Please make check payable to: **Girls’ Angle**. Mail to: Girls’ Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is \$216 for one session (12 meets)	☐ I will pay on a per meet basis at \$30/meet.
☐ I will pay on a per meet basis at \$20/meet.	☐ I'm including \$36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________