Girls' Angle Bulletin

October/November 2015 • Volume 9 • Number 1

To Foster and Nurture Girls' Interest in Mathematics

From the Founder

Special thanks to MIT graduate student Jane Wang for assuming the responsibilities of Head Mentor this semester at Girls' Angle. The Head Mentor is responsible for developing the mathematical content of our meets and keeping track of where each member is in relation to mathematics.

- Ken Fan, President and Founder

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

Individuals

Bill Bogstad
Doreen Kelly-Carney
Robert Carney
Lauren Cipicchio
Lenore Cowen
Merit Cudkowicz
David Dalrymple
Patricia Davidson
Ingrid Daubechies
Anda Degeratu
Kim Deltano
Eleanor Duckworth
John Engstrom
Courtney Gibbons
Vanessa Gould
Rishi Gupta
Andrea Hawksley
Delia Cheung Hom and Eugene Shih
Mark and Lisel Macenka
Brian and Darlene Matthews
Toshia McCabe
Mary O'Keefe
Stephen Knight and Elizabeth Quattrocki Knight
Junyi Li
Beth O'Sullivan
Elissa Ozanne
Robert Penny and Elizabeth Tyler
Malcolm Quinn
Jeffrey and Eve Rittenberg
Christian Rudder
Craig and Sally Savelle
Eugene Sorets
Sasha Targ
Diana Taylor
Marion Walter
Patsy Wang-Iverson
Andrew Watson and Ritu Thamman
Brandy Wiegers
Brian Wilson and Annette Sassi
Lissa Winstanley
The Zimmerman family
Anonymous

Nonprofit Organizations

The desJardins/Blachman Fund, an advised fund of Silicon Valley Community Foundation
Draper Laboratories
The Mathematical Sciences Research Institute

Corporate Donors

Adobe
Akamai Technologies
Big George Ventures
John Hancock
Maplesoft
Massachusetts Innovation & Technology Exchange (MITX)
MathWorks, Inc.
Microsoft
Microsoft Research
Nature America, Inc.
Oracle

For Bulletin Sponsors, please visit girlsangle.org.

Girls' Angle Bulletin

The official magazine of
Girls' Angle: A Math Club for girls
Electronic Version (ISSN 2151-5743)

Website: www.girlsangle.org
Email: girlsangle@gmail.com

This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Inspired by Takens embeddings by C. Kenneth Fan.

An Interview with Elizabeth Munch

Elizabeth Munch is an Assistant Professor in the Department of Mathematics and Statistics at the University at Albany, State University of New York. She received her doctoral degree in mathematics from Duke University under the supervision of John Harer. In addition to her degree in mathematics, Prof. Munch also has a degree in harp performance from the Eastman School of Music.

Ken: Could you please give us a brief autobiographical sketch, perhaps emphasizing aspects that led to your becoming a mathematician? Also, I know you also have a degree in harp performance from the Eastman School of Music and I'm curious if there was a time when you were trying to decide between a career in music and one in math?

Elizabeth: I took a very nonstandard road to get where I am now. When I was in high school, I loved both math and music and was trying to decide what to do in college. I was involved in many activities with both. I played with the Rochester Philharmonic Youth Orchestra and the Hochstein Youth Orchestra. I already had a decent amount of work playing harp at weddings and parties. I was also on my high school's math league team, and was on the Upstate New York ARML team for a few years. I was several years advanced in math in school, so I was taking multivariable calculus as a senior. I ended up deciding on going to college in harp performance for a few reasons. First, I was already studying harp with one of the best professors in the country, Kathleen Bride, at Eastman and wanted to see where it would take me. Maybe it was also because I always felt like I fit in better with the music crowd than the math crowd. Maybe I was just stubborn and wanted to prove to everyone that I could succeed as a musician.

In any case, I ended up at Eastman as only a music major. For me, music changed when it went from being a hobby to being a job. I just wasn't as happy with spending day after day in a practice room. I was too much of a perfectionist, and that combined with stage fright made high pressure performances incredibly rough. However, luckily for me, Eastman is part of the University of Rochester. So, I was able to start taking other classes to try to figure out what I wanted to do if I didn't want to do music. I started with language classes and astronomy since I think I was still too stubborn to admit that I had made the wrong choice. Well, maybe I should clarify that. I still believe going to music school was the right choice for me at the time and I would not have landed where I am now without that experience. However, after finally starting to take math classes, I decided to add a math degree to my music degree (I did finish both!).

Ken: Wow, doing both sounds quite challenging! Was it difficult?

Elizabeth: The hardest part with trying to do a dual degree was the fact that the two programs were on different campuses, so I spent a lot of time on a bus going the 10 miles back and forth between the downtown Eastman Campus and the main River Campus. There were also issues with scheduling since, for example, I had to be in orchestra every semester which met MWF afternoons from (I may be misremembering) 1-3 pm. This meant that a lot of classes that I wanted or needed for my math degree were not available to me since I couldn't be on

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Elizabeth Munch and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

Girls!

Learn Mathematics!

Make new Friends!

Meet
Professional Women
who use math in their work!

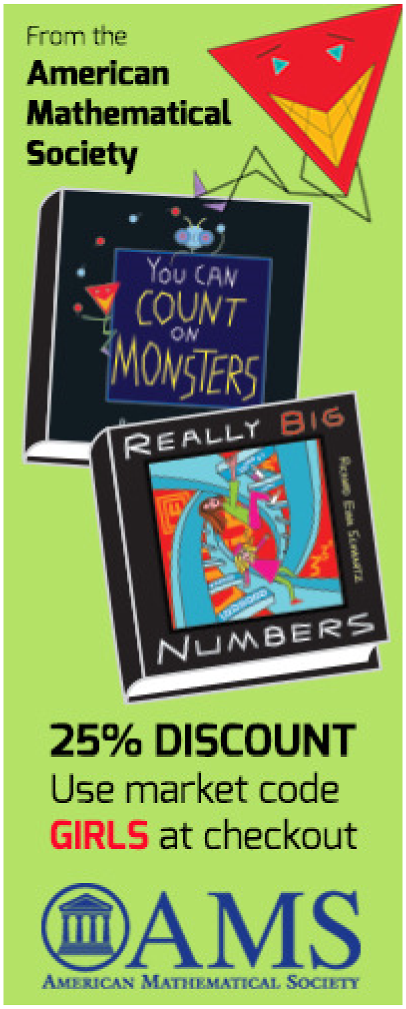
Improve how you Think and Dream!

Girls' Angle

A math club for ALL girls, grades 5-12.

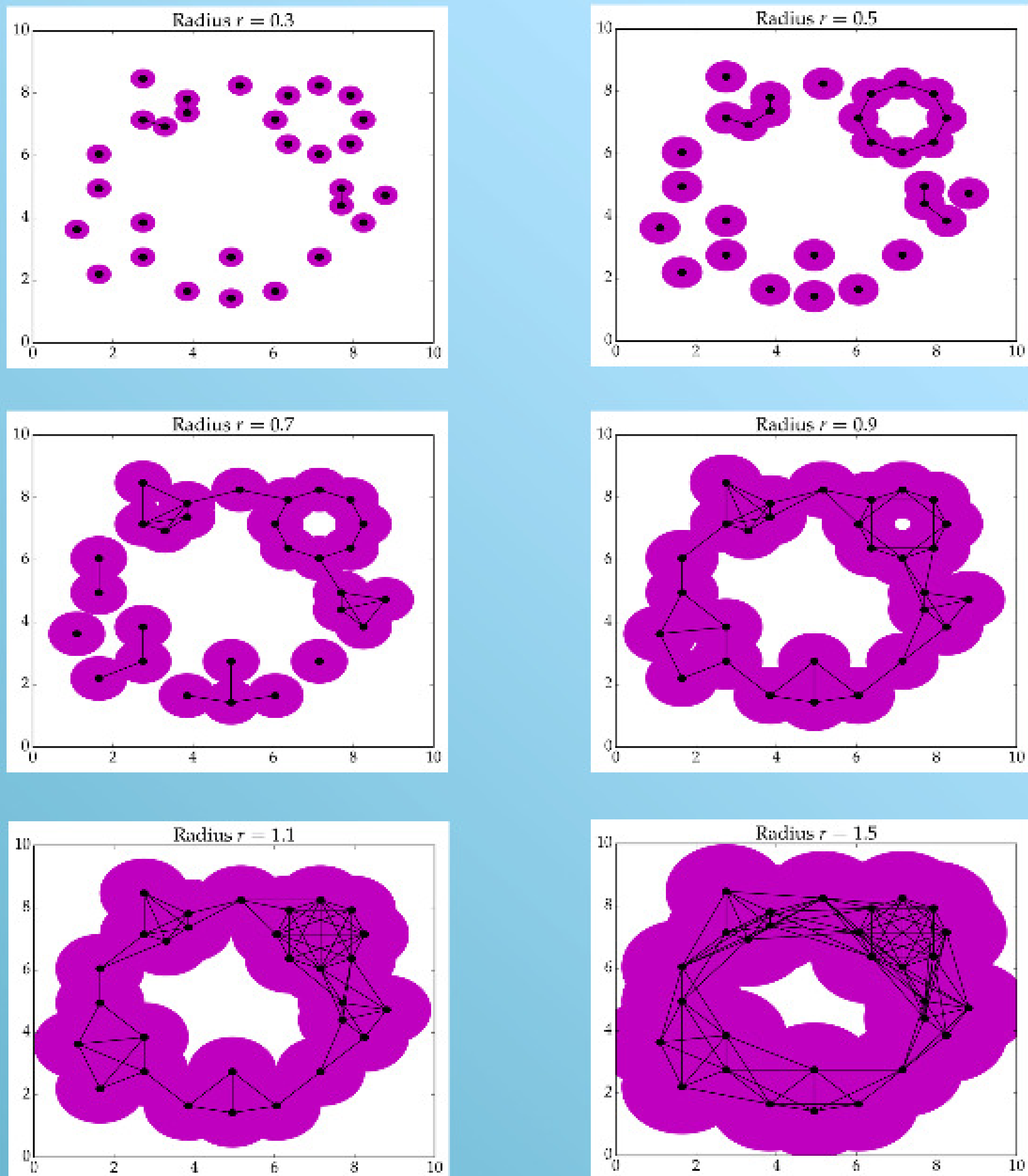
girlsangle@gmail.com
girlsangle.org

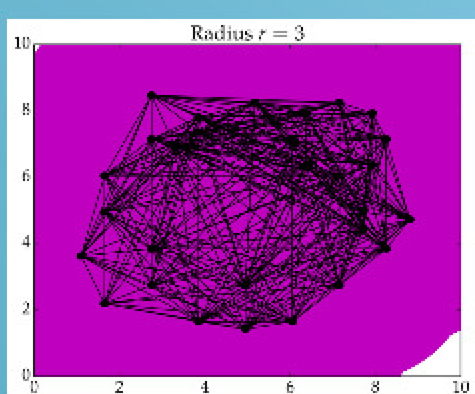
Girls' Angle



The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

Content Removed from Electronic Version





A series of Rips complexes.

Each image consists of the same point cloud. In each image, purple disks of radius r are centered on each data point. An edge is drawn between two data points if their disks overlap. Successive images show disks of increasing radius. As the radius increases, the topology of the purple region changes. Images courtesy of Prof. Munch.

The Laws of Probability[1]

Part 1: What Makes a Coin Fair?
by Elizabeth S. Meckes

Everyone knows what it means to toss a fair coin, right? It means that it's equally likely to land on heads or tails. But what does that really mean? You toss it once, it lands on heads, so what? Is it fair? Is it unfair? How do you know?

At this point, you're probably going to tell me that I should toss it a bunch of times. If it lands on heads every time, we're pretty sure it's not a fair coin. We know what should happen, and it pushes us a little closer to knowing what fair means: if we toss the coin a lot of times, we should get about equal numbers of heads and tails. And that's perfectly fine for a Saturday afternoon, but not very satisfying to a mathematician.

There's a big difference between what we mean when we talk about "laws" in physics and when we talk about "laws" in mathematics. In physics, we're trying to describe the reality that we see, and to do it accurately enough to be able to make valid predictions. But in math, even though we often start with real, physical observations like coin tosses, our mindset is different. We want to come up with some axioms (statements we will assume) which seem reasonable based on our observations and are as simple as possible; then we want to see how much we can prove. Our goal is to start from these very simple assumptions, things we feel comfortable assuming, and prove that the more complicated things we think we've observed follow just from those axioms.

Understanding what a fair coin is is a great way to see the difference between mathematical and physical laws at work. The idea that I can't predict whether the coin lands on heads or tails is very hard to turn into a mathematical axiom; it's not even clear how to test it by experiment. The suggestion I imagined you making before, that I should check fairness by tossing the coin a lot, led us to the general idea that a coin is fair if when you toss it a lot of times, it lands on heads about half the time. But that's still awfully fuzzy. We could make it sound a bit math-ier by saying that if H_n is the number of times out of n tosses that the coin lands on heads, then we should have $\lim_{n\to\infty} H_n / n = 1/2$. But really I'm just conning you with fancy language and notation. If I toss the coin n times, I get a certain number for H_n. And then if I do it again, I get a different number: H_n is random! Even if I could toss a coin an infinite number of times in order to take the limit, how do I know I'd get the same thing if I did the whole process again?

The answer that probabilists have settled on is that going through limits is a bad way to define fair. Instead, we assume that we can assign numbers called "probabilities" to events in a way that satisfy a small set of axioms which are so simple and so intuitively reasonable that we don't mind taking them as a starting point. Then, we prove the limiting statement above: that if you toss the coin a lot of times, the limiting proportion of times it lands on heads tends to 1/2.

So, what are these axioms? The first one is that I can assign a numerical probability, which I'll call $\mathbf{P}(E)$ to any **event** E. Sticking just with coin tossing, an event is anything I can describe in terms of the outcomes of a series of coin tosses. So E could be the event that the first three tosses are heads, heads, tails. Or it could be that the seventh toss is tails. Or it could be that every other toss is a heads (forever – this is math, so I can have an infinite sequence of tosses). I moreover assume that for any event E, $\mathbf{P}(E)$ is between 0 and 1 (including possibly 0 or 1). For example, if E is the event that the first toss is heads, and I'm trying to talk about a fair coin, then $\mathbf{P}(E)$ should be 1/2.

[1] This content was supported in part by a grant from MathWorks.

My second axiom is very simple: if E is just the event that something, anything, happens, then $\mathbf{P}(E) = 1$. And here's the third and final one, which is as complicated as it gets: if I have a bunch of different events $E_1, E_2, \ldots$ with no overlap, then I can figure out the probability that one of them happens by adding up the individual probabilities. This has to work even if there are infinitely many E_k.

And that's it. Those are the properties that something I call probability has to have. Now, back to our fair coin. Like we said above, if E is the event that the first toss is heads, then $\mathbf{P}(E)$ should be 1/2. And if E_2 is the event that the second toss is heads, then $\mathbf{P}(E_2)$ should be 1/2. And so on; each individual toss should be equally likely to be heads or tails. But there's one other important feature of a fair coin: **independence**. How the toss came out on the first try shouldn't tell you anything about what's going to happen next, and vice versa. For our coin tossing, this means that all of the possible strings of outcomes of a given length should be equally likely: e.g., the first three trials have eight total possible outcomes, as shown at right, and each has probability 1/8.

Phew. Okay, now we really know what a fair coin is. So what about tossing it a lot of times? We can start from just the three axioms above and prove what's called the strong law of large numbers. In symbols, if H_n is the number of heads in the first n tosses of a fair coin, then the strong law of large numbers says that

$$\mathbf{P}\left[\lim_{n\to\infty}\frac{H_n}{n}=\frac{1}{2}\right]=1.$$

What this means is that it's essentially certain that in an infinite sequence of independent tosses of a fair coin, the limiting proportion of heads would be 1/2. I really have to have that cheater word "essentially" there: it's of course possible that the limit might be something else (or even not exist). In principle, I could toss a fair coin forever and get heads every single time. But what the strong law of large numbers says is that the probability that that will happen is zero. It's not that it can't happen, but it won't.

So caveats and technicalities aside, modern mathematics has triumphed: we can start with very simple, very reasonable assumptions about how anything called probability should work, and our intuition about what fairness should mean becomes a theorem we can prove.

The Benefit of Being Off

by Lightning Factorial
edited by Jennifer Silva

"Stop going for the bull's-eye!"

I checked my dart-throwing motion and turned to face the audience.

"Who said that?" I asked.

A young woman waved back.

"Oh, hi Addie!" It was Addie Summer. "Did you just tell me *not* to aim at the bull's-eye?"

"Yes!" she replied.

"Do you want me to lose?" I asked incredulously. "The bull's-eye is worth 50 points."

"If you can hit it," commented Addie.

"But even if I'm off a little bit, I still get 25 points," I responded.

"I've been keeping track of where your darts land. According to my computations, you have a higher expected score if you aim a little bit below and to the left of the bull's-eye," said Addie rather matter-of-factly.

"Really? Is that so?" I said in disbelief.

"It is. Do the math."

Thus playing darts led me back to mathematics. I've since thought about Addie's advice and reluctantly concluded that she's right. Here's why:

The idea can be illustrated clearly with a simplified version of darts. Imagine a "dartboard" that consists of 5 square targets in a row with the following point values:

6	0	10	0	6

If I could hit the 10-point middle target every time, then I should definitely aim at it with every throw. But the point Addie was making is that I, like most casual dart players, am unable to hit what I'm aiming at every single time (if ever!).

To better model my dart throwing, let's suppose that in this simplified dart game, I hit my target with frequency X, where X is a number between 0 and 1, inclusive. For example, if $X = 1$, that means that I can always hit my target, but if $X = 1/2$, it means that I hit my target only half the time. And suppose that my dart lands just to the left of my target with frequency Y. I'll assume that I miss just to the right exactly as often as I miss just to the left, so the frequency that I miss just to the right is also Y. Finally, I'll assume that I'm skilled enough that I never miss further off than just to the right or left of my target, that is, $Y + X + Y = 1$.

Using this information, I can figure out what my average score will be if I aim for different parts of this simplified dartboard.

Suppose, for instance, that I aim directly at the 10-point target. With N dart throws, I can expect XN of them to hit the target; YN of them will hit the 0-point target on the left, and YN of them will hit the 0-point target on the right. Therefore, with N dart throws, I would expect to score $10XN$ points, or $10X$ points per dart on average.

On the other hand, suppose I deliberately aim for the leftmost 6-point target. In this case, a fraction X of my throws will hit the 6-point target, a fraction Y will land off the left side of the dartboard, and a fraction Y will land on the left 0-point target. Each dart I throw would now be worth $6X$ points on average.

The figure below gives the expected points per dart depending on where I aim.

$6X$	$16Y$	$10X$	$16Y$	$6X$

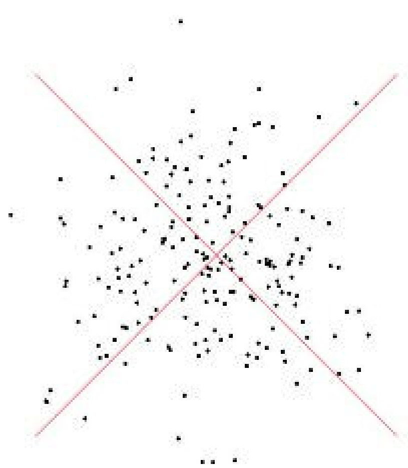
Notice that aiming for one of the 0-point targets makes each of my darts worth, on average, $16Y$ points. What Addie was suggesting is that it is possible for $16Y > 10X$, meaning that I'd be better off aiming at the 0-point target instead of the 10-point target!

Let's figure out when $16Y > 10X$.

Recall that we are also assuming that $X + 2Y = 1$. Therefore $X = 1 - 2Y$. If we substitute $1 - 2Y$ for X in our inequality, the inequality becomes $16Y > 10(1 - 2Y) = 10 - 20Y$. Rearranging terms and simplifying, this can be rewritten $Y > 5/18$.

Since $X = 1 - 2Y$, the inequality $Y > 5/18$ is equivalent to $X < 4/9$.

In other words, if I hit my target less than 4/9 of the time, it would indeed be better for me to aim off-target!

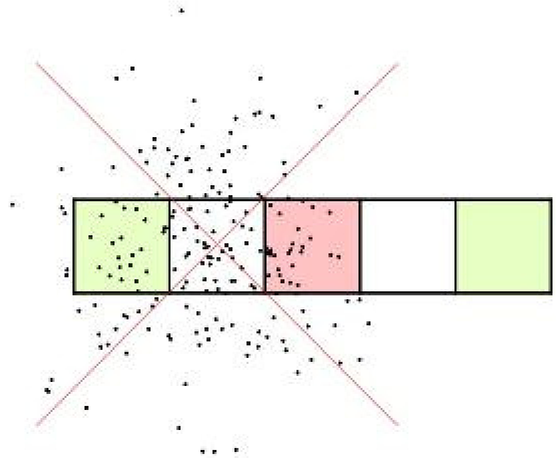


In actuality, if I throw hundreds of darts at a target, the dart hole pattern created will look something like the picture on the left. To obtain a more realistic sense of what happens when you throw darts, take a large sheet of paper and mark it with a bull's-eye. I used a big red X for my bull's-eye. Affix the paper to a wall (that nobody minds you throwing darts at!). Then throw several darts, always aiming for the bull's-eye. Make sure to stand the same distance from the bull's-eye that you plan to stand from a dartboard when you play darts. The resulting pattern of holes provides a good sampling of what happens when you aim at a specific target. The picture at left shows the results of my throwing 200 darts, aiming at the center of the red X.

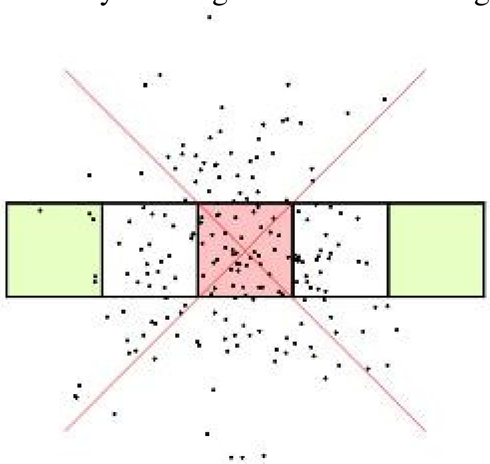
Using this dart hole pattern, I can make a more realistic computation for the average value of a dart when I aim at a particular location. I superimpose the dart hole pattern over the dartboard, placing the center of the red X over the target point that I intend to aim at. I then tally up the score I would get with the 200 throws that make up the dart hole pattern and divide the result by 200 to get the average points per dart.

Let's go through this procedure on our simplified dartboard.

The setup for this process is shown at right for the case where I aim at the center of the left 0-point target. As you can see, many throws would not even land on the dartboard! Darts that land off of the dartboard as well as darts that land in the white target contribute no points to my score. Darts that land in the green targets contribute 6 points each, and darts that land in the red target contribute 10 points each.



I count 22 dart holes in the left green target, 27 dart holes in the red target, and no dart holes in the right green target, giving me a total score of 22 × 6 + 27 × 10 = 402 points. I then divide by 200 to get 2.01 as the average points per dart.

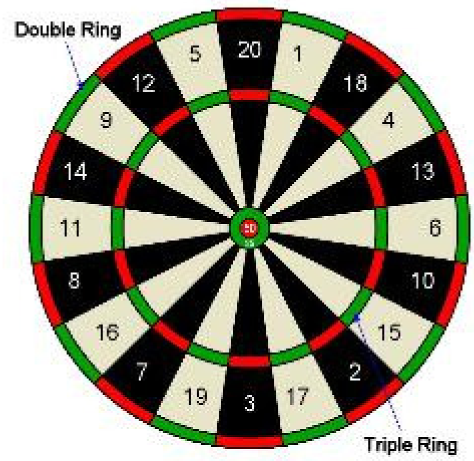


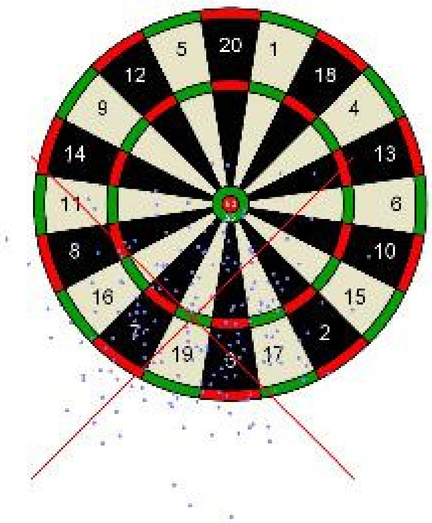
On the other hand, if I aim for the center of the red target, as shown at left, I count 5 dart holes in the left green target, 43 in the red target, and none in the right green target. All other dart holes contribute nothing to my score. So with these 200 dart throws, my score would be 5 × 6 + 43 × 10 = 460 points, or 2.3 points per dart on average.

It turns out that with this simplified dartboard and a more realistic count, it would be better for me to aim at the middle of the red target instead of the middle of the left 0-point target, but not by much.

To determine the ideal spot for me to aim at to maximize my score, I should perform this computation several times, moving the center of the red X all over the dartboard. I then can aim at the point that yields the largest average points per dart. Mathematically, we can define a function s that takes a point P and returns the average points per dart when I aim at P. With P as the center of the red target, we just computed that $s(P) = 2.3$; when P is the center of the left 0-point target, we found that $s(P) = 2.01$. The operation we perform to compute s is known as **convolution**. We can say that s is the "convolution of the dart hole distribution with the dartboard score values." One can think of the dartboard score values as giving the points per dart as a function of where you should aim *if you can always hit your target*, and s as giving the points per dart as a function of where you should aim when accounting for the distribution of how you miss the target.

The only difference between our simplified example and a regulation dartboard (illustrated at right) is that a regulation dartboard presents a more complex scoring map. The central red circle is worth 50 points and the small green ring around it is worth 25 points. The twenty pizza slices are worth points as shown. However, there are two rings, the double ring and the triple ring. Putting a dart in the double ring is worth twice the value of the corresponding pizza slice, whereas putting a dart in the triple ring is worth three times the value of the corresponding pizza slice. The most valuable region on the dartboard is the red triple ring section in the 20-point sector, which is worth 60 points.





Although there's a more complex scoring system, the same principles apply. We take our dart hole distribution and place the center of the X over the point at which we intend to aim. We compute the score we'd receive by examining where each dart hole is located. Finally, we divide the total score by the number of dart holes to get the average points per dart. We do this by placing the center of the X over many different points and finding the point that yields the largest average points per dart. That point is where we should aim when we play. The illustration at left shows the set-up I'd use to compute the average points per dart if I were to aim at the triple ring region inside the 19-point sector.

If you look closely at the pattern of dart holes (the blue dots in the illustration above left), you'll see that although there's a hole in the 5-point and 20-point sectors, there is no hole in the 9-point sector. You might feel that this does not accurately reflect the truth, especially since the 9-point sector seems closer to the center of the red X than the 5-point sector. Having no holes in the 9-point sector may be just a result of dumb luck. We might suspect that if we created a dart hole pattern with many more dart throws, we'd eventually see some holes in the 9-point sector. Indeed, the more darts you throw, the more accurately you'll be able to determine the average points per dart. Unfortunately, the more darts you throw, the more tedious the computations become.

So, rather than use an actual dart hole pattern obtained by having you throw millions of darts at a wall, we can instead make some assumptions about how the dart holes will be distributed and encode this information in a **density function**. For each point *P*, the density function returns the limit of the fraction of dart holes in a circle centered at *P* divided by the area of the circle, as the radius of the circle tends to 0 and the number of dart throws increases without bound. By modeling your dart throwing with an appropriate mathematical function, we can eliminate quirks that result from peculiarities of chance and we can make computations without having to trouble you with the task of throwing millions of darts. However, by replacing an actual sampling of dart holes with a mathematical model, we must bear in mind that we might introduce simplifying assumptions that are at odds with reality.

Using a density function is exactly what researchers Ryan Tibshirani, Andrew Price, and Jonathan Taylor did to determine optimal targeting in darts.[1] They assumed a **Gaussian** distribution of dart holes. The Gaussian distribution is informally called the "bell curve," because a graph of its density function resembles a church bell. The skill level of a player is reflected in the concentration of dart holes near the target. The more concentrated, the higher the skill level. The authors found that for an unskilled player such as me, the optimal place to aim is in the 8-point sector, about a sixth of the way from the center toward the rim of the dartboard.

Take it to Your World

Make a dart hole distribution and compute your ideal dartboard aiming spot.

Read the paper by Tibshirani, Price, and Taylor from the footnote below. It contains neat "heat maps" that represent the function we denoted by s – the average points per dart as a function of aiming location – for various skill levels. As you'd expect, the more skilled the player, the more s looks like the dartboard scoring map.

---

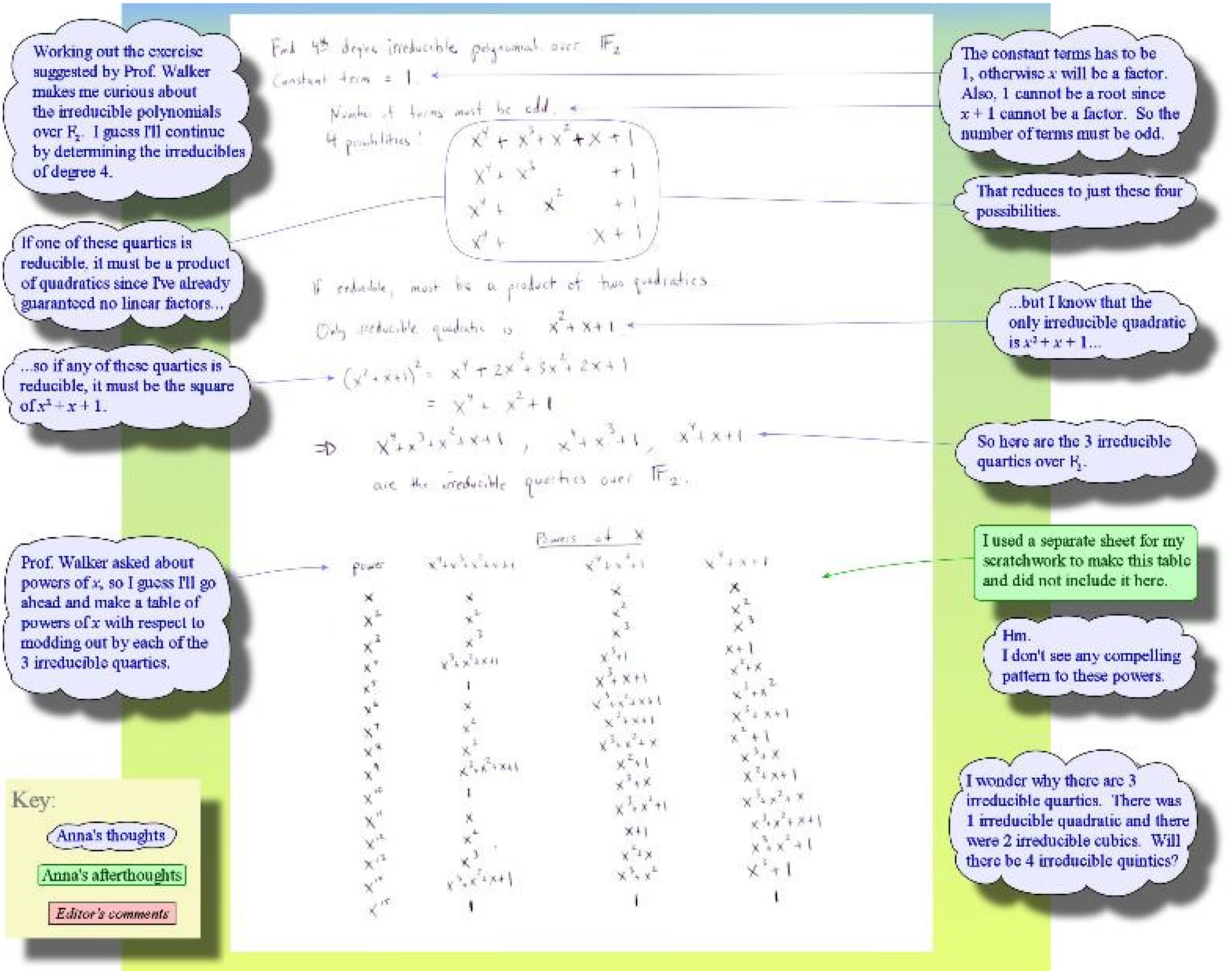
[1] Tibshirani, R. J., Price, A. and Taylor, J. (2011). A statistician plays darts. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 174, 213-226.

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues thinking about irreducible polynomials over the finite field with 2 elements.



power	$x^4+x^3+x^2+x+1$	x^4+x^3+1	x^4+x+1
x	x	x	x
x^2	x^2	x^2	x^2
x^3	x^3	x^3	x^3
x^4	x^3+x^2+x+1	x^3+1	$x+1$
x^5	1	x^3+x+1	x^2+x
x^6	x	x^3+x^2+x+1	x^3+x^2
x^7	x^2	x^2+x+1	x^3+x+1
x^8	x^3	x^3+x^2+x	x^2+1
x^9	x^3+x^2+x+1	x^2+1	x^3+x
x^{10}	1	x^3+x	x^2+x+1
x^{11}	x	x^3+x^2+1	x^3+x^2+x
x^{12}	x^2	$x+1$	x^3+x^2+x+1
x^{13}	x^3	x^2+x	x^3+x^2+1
x^{14}	x^3+x^2+x+1	x^3+x^2	x^3+1
x^{15}	1	1	1

5^{th} degree irreducibles over $\mathbb{F}_2$

For a 5th degree irreducibles, the constant term must still be 1 and the number of terms must still be odd. That's true for all irreducible polynomials over $\mathbb{F}_2$ of degree greater than 1. That reduces to 8 possibilities.

→ Constant term 1, Number of terms odd ⇒ 8 possibilities

Of these 8, reducible ones factor as a quadratic times a cubic (both irreducible)

x^2+x+1 $\quad$ x^3+x^2+1, x^3+x+1 ⇒ 2 polynomials

$(x^2+x+1)(x^3+x^2+1) = x^5+x+1$

$(x^2+x+1)(x^3+x+1) = x^5+x^4+1$

There's only 1 irreducible quadratic and 2 irreducible cubics, so of the 8 quintics with no linear factor, only 2 are irreducible.

6 irreducibles:

$x^5+x^4+x^3+x^2+1$

$x^5+x^4+x^3+x+1$

$x^5+x^4+x^2+x+1$

$x^5+x^3+x^2+x+1$

x^5+x^3+1

x^5+x^2+1

That leaves 6 irreducible quintics, not 4. Hm. Why 6?

The big questions on my mind are:

What are the irreducible polynomials over $\mathbb{F}_2$?

How many irreducible polynomials of degree n are there over $\mathbb{F}_2$?

→ What are the irreducible polynomials over $\mathbb{F}_2$?

→ How many irreducible polynomials of degree n are there over $\mathbb{F}_2$?

Hm. Why 6 irreducible quintics? Each quintic has 5 roots, so these 6 irreducibles together have 30 roots...assuming there are no common roots. Wait...can two irreducible quintics share a root? Actually, what are the roots of these irreducible polynomials I've been computing?

I might as well start with the lone irreducible quadratic.

$\dfrac{\mathbb{F}_2[x]}{(x^2+x+1)} = \{0, 1, x, x+1\}$ $\quad$ $t^2+t+1 \stackrel{?}{=} (t+x)(t+x+1)$

$= t^2 + (x+x+1)t + x(x+1)$

$= t^2+t+1$ ✓

The roots are x and $x+1$. (Gee, I'm getting used to the luxury of being able to ignore signs over $\mathbb{F}_2$.) This makes sense because x has to be a root, and since 0 and 1 are not roots, the other root must be $x+1$.

Now for the cubics...

$\dfrac{\mathbb{F}_2[x]}{(x^3+x+1)} = \{0, 1, x, x+1, x^2, x^2+1, x^2+x, x^2+x+1\}$

a	$x \cdot a$	$x(x \cdot a)$
0	x	0
1	$x+1$	$x+1$
x^2	x^2+x	$x^3+x^2 = x^2+1$
x^2+1	x^3+x+1	$x^4+x^3+x^2+x+1 = x^2+x$

Let's see. t^3+t+1 has x as a root, so $t+x$ is a factor, and so t^3+t+1 must be $t-x$ times $t^2+\ldots xt+\ldots x^2$.

~~$t^3+t+1 = (t+x)(t^2+xt+x^2)$~~

$= (t+x)($

And t^2+xt+x^2 has roots... wait a sec...I must have done something wrong.

$t^3+t+1 = (t+x)(t^2+xt+x^2+1)$

Oh, t^3+t+1 is $t+x$ times t^2+xt-x^2+1.

$= (t+x)(t+x^2)(t+x^2+x)$

$= (t+x)(t+x^2)(t+x^4)$

Since I computed the powers of x, i decided to write the roots as powers of x.

$(t+x+1)(t+x^2+1)(t+x^2+x+1) = (t+1)^3+(t+1)+1$

$= t^3+3t^2+3t+1+t+1+1$

$= t^3+t^2+1$

$= (t+x^3)(t+x^6)(t+x^{12})$

If the two irreducible cubics have distinct roots, then the other cubic must have roots $x+1$, x^2+1, and x^2+x+1... And it does!

Hmm! The exponents of the roots are geometric sequences. I'll have to think more on this next time!

ABB 10.26.15

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

In Search of Nice Triangles, Part 2

by Ken Fan | edited by Jennifer Silva

Emily: Jasmine? Are you in the mood to think about triangles?

Jasmine: You read my mind!

Emily: Last time, we saw that the only triangles with integer side lengths and angles that measure a rational multiple of 360° are the equilateral ones with integer side lengths.

Jasmine: Right.

Emily: I'm eager to explore what triangles exist if we weaken our conditions on either the side lengths or the angle measures.

Jasmine: Same here. What do you say we look for triangles with integer side lengths, but only two angles that are a rational multiple of 360°?

Emily: Okay!

Jasmine: From last time, we know that if all the side lengths of a triangle are integers, then the cosines of all of the angles must be rational numbers.

Emily: Yes, that follows from the law of cosines.

Jasmine: And we know exactly which "nice" angles have rational cosines, thanks to the Chebyshev polynomials and the rational root theorem. Up to multiples of 360°, they are the angles which have degree measures 0, 60, 90, 120, 180, 240, 270, and 300.

Emily: Since the angles in a triangle add up to 180°, we can ignore angles that are 180° or larger; the 0° angle can't be part of a triangle, so that leaves only the 60°, 90°, and 120° angles to play with.

Jasmine: If two of the angles measure 60°, then the third angle also measures …

Emily: Hold on! I feel so silly!

Jasmine: What?

Emily: The angles of a triangle add up to 180°!

Jasmine: Yeah?

Emily: That means that if two of the angles are rational multiples of 360, so will be the third!

Jasmine: Oh yeah! That puts us right back in the situation we studied last time. There's no such thing as a triangle that has exactly two angles that measure a rational multiple of 360°.

Emily: Maybe there are interesting nice triangles with integer side lengths but only one angle that measures a rational multiple of 360°.

Jasmine: Let's find out!

Emily: The one nice angle must measure 60°, 90°, or 120°.

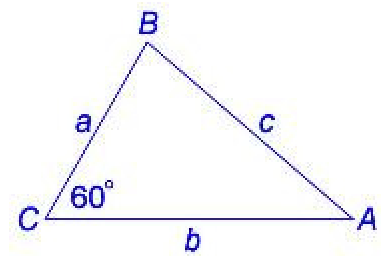
Jasmine: If the special angle is 90°, then we're looking for Pythagorean triples.

For more on Pythagorean triples, see pages 22-24 of Volume 8, Number 3 of this *Bulletin*.

Emily: Ah, that's right. That's been well studied.

Jasmine: Let's concentrate on the other cases then, starting with triangles with a 60° angle.

Emily: Okay. Here's a figure.



Emily draws the figure at right.

Jasmine: We want a, b, and c to be positive integers. From the law of cosines, we know that $c^2 = a^2 + b^2 - 2ab \cos 60°$. Since $\cos 60° = 1/2$, this simplifies to $c^2 = a^2 + b^2 - ab$.

Emily: This is almost like looking for Pythagorean triples. For Pythagorean triples, we'd be looking for positive integer solutions to the equation $c^2 = a^2 + b^2$.

Jasmine: It's so similar that I bet we can solve our problem by tweaking one of the methods for finding the Pythagorean triples.

Emily: Do you remember how to find Pythagorean triples?

Jasmine: I remember one way. First, one divides the equation $c^2 = a^2 + b^2$ by c^2 to obtain the equation $1 = (a/c)^2 + (b/c)^2$. This shows that every Pythagorean triple is similar to a right triangle with rational leg lengths and hypotenuse 1. So we look for points with rational coordinates on the unit circle $x^2 + y^2 = 1$.

Emily: How are those found?

Jasmine: One way is to consider lines through the point (-1, 0). Non-vertical lines through (-1, 0) intersect the unit circle $x^2 + y^2 = 1$ twice, once at (-1, 0) and once at some point (p, q). If p and q are rational numbers, then the slope of the line is a rational number. And, as it turns out, if the slope of the line is a rational number, p and q will be rational numbers as well. So in this way, rational points on $x^2 + y^2 = 1$ are parametrized by rational numbers m: for each such m, we look at where the line through (-1, 0) with slope m intersects $x^2 + y^2 = 1$.

Emily: I see. Let's try to modify the argument to solve our problem.

Jasmine: Okay. First, we divide the equation $c^2 = a^2 + b^2 - ab$ by c^2 and obtain the equation $1 = (a/c)^2 + (b/c)^2 - (a/c)(b/c)$.

Content Removed from Electronic Version

Content Removed from Electronic Version

Content Removed from Electronic Version

Learn by Doing

Finite Fields
by Girls' Angle Staff

In last issue's interview, Judy Walker suggested some exercises that use finite fields. Anna, of Anna's Math Journal, took up Judy's suggestion and continues her exploration in this issue. This installment of Learn by Doing is intended for readers who are unfamiliar with finite fields but would like to follow Anna's investigation and learn enough about finite fields to start doing some explorations on their own.

Typically, finite fields are introduced only after introducing Galois theory because Galois theory provides powerful tools that can be used to understand their structure. Here, we'll attempt a different approach that requires little by way of prerequisites. For the last problems, it will help if you know about polynomials, the Euclidean algorithm, and Bézout's theorem.

First, what is a field? The rational numbers, the real numbers, and the complex numbers are all examples of a field. All three sets are equipped with operations of addition and multiplication, and both operations enjoy many useful properties. If we isolate some of the common properties of these three examples, we obtain the definition of a field.

So a field is a set that has two binary operations, called addition and multiplication, which are denoted by "+" and juxtaposition, respectively. For any elements a, b, and c in the field, we must have:

0. Closure of addition and multiplication: $a + b$ and ab are in the field.
1. The commutative laws of addition and multiplication: $a + b = b + a$ and $ab = ba$.
2. The associative laws of addition and multiplication: $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.
3. The distributive law: $a(b + c) = ab + ac$.
4. Existence of an additive identity: there exists an element, denoted 0 and called "zero," such that $a + 0 = 0 + a = a$.
5. Existence of additive inverses: for any a in the field, there exists an element, denoted $-a$, such that $a + (-a) = (-a) + a = 0$.
6. Existence of a multiplicative identity: there exists an element different from 0, denoted 1 and called "one," such that $a1 = 1a = a$.
7. Existence of multiplicative inverses for nonzero elements: for any $a \neq 0$ in the field, there exists an element, denoted a^{-1} or $1/a$, such that $aa^{-1} = a^{-1}a = 1$.

These properties are collectively referred to as the "field axioms."

Problem 1. Let **Q** the set of rational numbers together with the usual operations of addition and multiplication. Check that all field axioms are satisfied. Convince yourself that the set of real numbers and the set of complex numbers with the usual addition and multiplication are fields.

Problem 2. Let **Z** the set of integers together with the usual operations of addition and multiplication. **Z** is not a field. Which field axioms fail?

Problem 3. Notice that $\mathbf{Z} \subset \mathbf{Q}$. Show that **Q** is the smallest field that contains **Z** in the sense that if $\mathbf{Z} \subset F \subset \mathbf{Q}$ and F is a field, then $F = \mathbf{Q}$.

Problem 4. Let **Q**[$\sqrt{2}$] be the set of real numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers (with the usual operations of addition and multiplication). Show that **Q**[$\sqrt{2}$] is a field.

Problem 5. Use the field axioms to show that in any field, multiplication by 0 always produces 0. That is, for all x in the field, we must have $x0 = 0$. (Hint: Use the distributive law and the fact that $0 + 0 = 0$.)

Problem 6. Show that for any field element a, we have $(-1)a = -a$. (Note: If your reaction to this and the previous question is that there's nothing to prove, it probably means that you are recognizing these facts from your experience with multiplication of real numbers. The point of problems 5 and 6 is that these facts are true in any field because they follow from the field axioms. There is no field axiom that explicitly states that you can get the additive inverse of a by multiplying it with the additive inverse of 1, so it is something that requires proof. That is, show that $(-1)a = -a$ is a logical implication of the field axioms.)

Problem 7. In fact, show that for any a and b in a field, $(-a)b = -(ab)$.

In a field, we generally write $a - b$ for $a + (-b)$. We also use standard symbols for integers as shorthand for elements in a field obtained by repeated addition of 1 or -1. (E.g. 2 is 1 + 1 and -3 is -1 – 1 – 1.)

A **finite field** is a field that has a finite number of elements.

Problem 8. Perhaps the most sensible first question to ask about finite fields is, "Do they exist?" To answer, one might begin by trying to construct the smallest possible one. Because 0 and 1 must be distinct, there is no field with 1 element. But perhaps there is a field with just the 2 elements 0 and 1. Let $\mathbf{F}_2 = \{0, 1\}$. Define an addition and multiplication on $\mathbf{F}_2$ that makes it a finite field. To assist, here are addition and multiplication tables already filled in with entries that are dictated explicitly by the field axioms. (Here, "×" stands for multiplication.)

+	**0**	**1**
0	0	1
1	1	

×	**0**	**1**
0		0
1	0	1

Be sure to check that all the field axioms hold.

Note that there is only one way to complete the addition and multiplication tables to create a field with 2 elements.

Problem 9. Make addition and multiplication tables for a field with the elements 0, 1, and a.

Problem 10. Make addition and multiplication tables for a field with the elements 0, 1, p, and q.

The last two problems probably required a good bit of work to do completely. It would be rather tedious if all finite fields had to be constructed by explicitly showing their addition and multiplication tables and then checking that all the field axioms are satisfied. So let's explore other, more efficient ways to build finite fields.

Problem 11. (Modular arithmetic.) The set of integers **Z** with standard addition and multiplication satisfies most of the field axioms. We will exploit this by modifying **Z** to create a number of finite fields. If you are familiar with modular arithmetic, you'll recognize it here.

A. Fix a positive integer N. Define an equivalence relation ~ on **Z** by declaring that $a \sim b$ if and only if N divides $b - a$. (Show that this is an equivalence relation.) Let **Z**/N**Z** denote the set of equivalence classes in **Z** with respect to ~. If a is in **Z**, denote by $\overline{a}$ the equivalence class of a. (For a brief intro to equivalence relations, see page 10 of Volume 1, Number 3 of this *Bulletin*.)

B. Let a, b, c, and d be in **Z**. Suppose that $a \sim b$ and $c \sim d$. Show that $a + c \sim b + d$ and $ac \sim bd$.

Part B shows that it is sensible to define binary operations of addition and multiplication by the formulas $\overline{a} + \overline{b} = \overline{a+b}$ and $\overline{a}\overline{b} = \overline{ab}$.

C. Show that so defined, addition and multiplication satisfy the field axioms if and only if N is a prime number. (As you do this, note how commutativity and associativity follow from commutativity and associativity of integer addition and multiplication, which you can assume.) Which field axioms fail when N is composite?

Thus, for any prime number p, there exists a finite field with p elements.

Problem 12. Compare **Z**/4**Z** to the field you constructed in Problem 10.

Let E and F be fields. We say that they are **isomorphic** if there exists a bijective map $f : E \to F$ such that $f(0) = 0$, $f(1) = 1$ (note that in these equations, the 0 and 1 on the left side of the equal sign are the additive identity and multiplicative identity in E, whereas on the right side of the equal sign, they are the additive identity and multiplicative identity in F), and, for all x and y in E, we have $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$.

Problem 13. Let F be a finite field. Show that F contains a field isomorphic to **Z**/p**Z** for some prime number p. (Hint: Consider the sequence 1, 1 + 1, 1 + 1 + 1, ... Because F is finite, this sequence cannot produce new elements of F forever.)

Let F be any field. Denote by $F[x]$ the polynomials in x with coefficients in F. That is,

$$F[x] = \{\ c_0 + c_1x + c_2x^2 + \ldots + c_dx^d \mid d \text{ is a nonnegative integer and } c_k \text{ in } F \text{ for all } 0 \le k \le d\ \}.$$

Define addition and multiplication in $F[x]$ in the usual way. That is, if $a_0 + a_1x + \ldots + a_dx^d$ and $b_0 + b_1x + \ldots + b_ex^e$ are in $F[x]$, then

$$(a_0 + a_1x + \ldots + a_dx^d) + (b_0 + b_1x + \ldots + b_ex^e) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \ldots$$

and

$$(a_0 + a_1x + \ldots + a_dx^d)(b_0 + b_1x + \ldots + b_ex^e) = a_0b_0 + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \ldots$$

We are going to mimic the construction of **Z**/N**Z** replacing **Z** with $F[x]$ and the modulus N with a polynomial $p(x)$. (Before reading further, can you guess what condition will be needed on $p(x)$ to obtain a field?)

Problem 14. A. Fix a polynomial $p(x)$ in $F[x]$. Define an equivalence relation on $F[x]$ by declaring that $a(x)$ is equivalent to $b(x)$ if and only if $b(x) - a(x)$ is divisible by $p(x)$. (Show that this is an equivalence relation.) Let $F[x]/(p(x))$ denote the set of equivalence classes in $F[x]$ with respect to this equivalence relation. If $a(x)$ is in $F[x]$, denote by $\overline{a(x)}$ the equivalence class of $a(x)$.

Let $f(x)$ and $g(x)$ be polynomials in $F[x]$. We say that $f(x)$ is divisible by $g(x)$ (or, $g(x)$ divides into $f(x)$) if there exists a polynomial $q(x)$ in $F[x]$ such that $f(x) = q(x)g(x)$.

B. Let $a(x)$, $b(x)$, $c(x)$, and $d(x)$ be in $F[x]$. Suppose that $a(x)$ is equivalent to $b(x)$ and $c(x)$ is equivalent to $d(x)$. Show that $a(x) + c(x)$ is equivalent to $b(x) + d(x)$ and $a(x)c(x)$ is equivalent to $b(x)d(x)$.

Part B shows that we can define an addition and multiplication in $F[x]/(p(x))$ by using the formulas $\overline{a(x)} + \overline{b(x)} = \overline{a(x) + b(x)}$ and $\overline{a(x)}\ \overline{b(x)} = \overline{a(x)b(x)}$.

C. Show that so defined, addition and multiplication satisfy all the field axioms except that nonzero polynomials do not always have a multiplicative inverse.

D. Show that every element of $F[x]/(p(x))$ can be expressed as $\overline{a(x)}$ where $a(x)$ is a polynomial of degree less than the degree of $p(x)$. (Hint: Use polynomial division and look at the remainder.)

A polynomial $p(x)$ is said to be **irreducible** if and only if $p(x)$ cannot be written as a product of two polynomials each of degree 1 or greater.

E. Show that all field axioms are satisfied in $F[x]/(p(x))$ if $p(x)$ is irreducible. (Suggestion: Adapt the Euclidean algorithm for finding the greatest common factor of two integers to polynomials and use it to prove a polynomial version of Bézout's theorem. For more on Bézout's theorem, see p. 16 of Volume 6, Number 5 of this *Bulletin*.)

Problem 15. Let $p(x) = x^2 + x + 1$ in $\boldsymbol{F}_2[x]$. Show that $p(x)$ is irreducible. Show that $\boldsymbol{F}_2[x]/(p(x))$ is a finite field with 4 elements. (Also, take a look at Anna's Math Journal in the previous issue of this *Bulletin*.)

Problem 16. Can you construct a field with 32 elements? (Hint: Find an irreducible polynomial of degree 5 in $\boldsymbol{F}_2[x]$. Also, take a look at Anna's Math Journal in this issue of the *Bulletin*.)

Problem 17. Construct a field with 9 elements.

Problem 18. Prove that all finite fields have p^n elements for some prime number p and positive integer n.

Problem 19. Prove that for any prime number p and positive integer n, there exists a field with p^n elements.

Problem 20. Prove that any two finite fields of the same size are isomorphic.

Remember, subscribers are always welcome to email us with any thoughts and questions!

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 17 - Meet 1 September 17, 2015	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison, Alexandra Fehnel, Jennifer Matthews, Wangui Mbuguiro, Debbie Seidell, Sara Sussman, Isabel Vogt, Jane Wang (Head)

Permutohedrons can be good stepping stones into higher dimensional thinking. If you have students who are comfortable with equations for straight lines and planes, first challenge them to describe in detail the 2-dimensional permutohedron, which is the convex hull of the 6 points obtained by permuting the coordinates of (1, 2, 3). Next, ask them to describe in detail the 3-dimensional permutohedron, which is the convex hull of the 24 points obtained by permuting the coordinates of (1, 2, 3, 4). And when they've provided a good description of that, move on to the 4-dimensional permutohedron, which is the convex hull of the 120 points obtained by permuting the coordinates of (1, 2, 3, 4, 5).

In general, the n-dimensional permutohedron is the convex hull of the $(n + 1)!$ points obtained by permuting the coordinates of $(1, 2, 3, ..., n, n + 1)$ in Euclidean $(n + 1)$-dimensional space. Challenge students to find, for each k-dimensional face of the permutohedron, linear equations whose solution set intersects the permutohedron in that face.

Session 17 - Meet 2 September 24, 2015	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Debbie Seidell, Sara Sussman, Isabel Vogt, Jane Wang (Head)

Spirographs became the launch point of at least three different mathematical journeys. Some members began thinking about stars that can be formed by connecting each dot in a circular arrangement of n dots to the dot k over in the clockwise direction, for some fixed k. Other members investigated the geometric effect of different gear ratios. And another group of members began to work out an algorithm for finding the center of a circle. That is, how can you find the center of a circle if you're given a circle without its center marked?

The first journey (concerning n-pointed stars) is equivalent to understanding the solubility of linear equations in one variable modulo n, an extremely important topic in algebra and number theory. For more on such stars, check out the series Star Tips in this *Bulletin*, Volume 8, Numbers 1-4.

The third journey led members to make the following conjecture: If equally spaced parallel line segments are drawn all the way across the face of a circle, including as many lines as will fit, then the center of the circle will lie on or between the two that are longest. Can you prove or disprove this conjecture?

Session 17 - Meet 3 October 1, 2015	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Jennifer Matthews, Debbie Seidell, Isabel Vogt, Jane Wang (Head)

Function madness erupted in a game that pitted two teams against each other. Each team invented a function (from the set of real numbers to itself) for the other team to try to figure out. To figure out the functions, a team could ask for the output of the function on any input.

Session 17 - Meet 4 October 8, 2015	Mentors:	Bridget Bassi, Karia Dibert, Neslly Estrada, Jennifer Matthews, Debbie Seidell, Sara Sussman, Anuhya Vajapeyajula, Isabel Vogt, Jane Wang (Head), Sibo Wang

Many problem solving efforts at this meet shared an important common theme: finding the simplest case that is not yet understood. For example, consider the problem of determining the number of 5-dimensional faces of a 6-dimensional permutohedron. We recognize that this question is an instance of the general question, "how many (n – 1)-dimensional faces does the n-dimensional permutohedron have?" If we are struggling to answer the original question, then there is a great deal of wisdom in putting aside the original question and replacing it with the case where $n = 1$. When the $n = 1$ case is understood, then proceed to the case of $n = 2$, and so on. It often happens that working systematically from the simplest case leads to more rapid understanding of the general case than stubbornly sticking to an attempt to resolve a more advanced case before the simpler cases have been examined. Another advantage is that patterns are easier to detect in data collected systematically rather than haphazardly.

Session 17 - Meet 5 October 15, 2015	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Jennifer Matthews, Debbie Seidell, Jane Wang (Head)

Some members built models of polyhedra using straws and string. Others explored criteria for when a shape can be tiled with dominos.

Session 17 - Meet 6 October 22, 2015	Mentors:	Bridget Bassi, Anna Ellison, Jennifer Matthews, Debbie Seidell, Isabel Vogt, Jane Wang (Head)

Some members worked on creating an algorithm for how to make a jam sandwich.

Another question that arose at this meet: can a mesh consisting of regular hexagonal cells made of rigid edges but flexible joins flex into a 3-dimensional shape, or will it be rigid?

Session 17 - Meet 7 October 29, 2015	Mentors:	Bridget Bassi, Karia Dibert, Neslly Estrada, Isabel Vogt, Jane Wang (Head), Anuhya Vajapeyajula

Jane opened the meet by presenting some interesting and sometimes spooky estimation and approximation problems. She discussed the value of estimation and some techniques for making good estimations. Good estimation involves identifying the most important influences and thinking in terms of orders of magnitude.

For some problems, approximation is extremely useful. For example, the motion of a simple pendulum is governed by a differential equation of the form $\frac{d^2x}{dt^2} + k \sin x = 0$, where k is a constant and x is a function of t. The solution to this differential equation cannot be expressed in terms of elementary functions. However, if the pendulum is only slightly perturbed, x will be small, and for small values of x, $\sin x$ is closely approximated by x. By approximating $\sin x$ with x for small values, useful approximate solutions can be obtained to the differential equation.

Calendar

Session 17: (all dates in 2015)

September	17	Start of the seventeenth session!
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	Jinger Zhao, Two Sigma
	10	

Session 18: (all dates in 2016)

January	28	Start of the eighteenth session!
February	4	
	11	
	18	No meet
	25	
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________