Girls' Angle Bulletin

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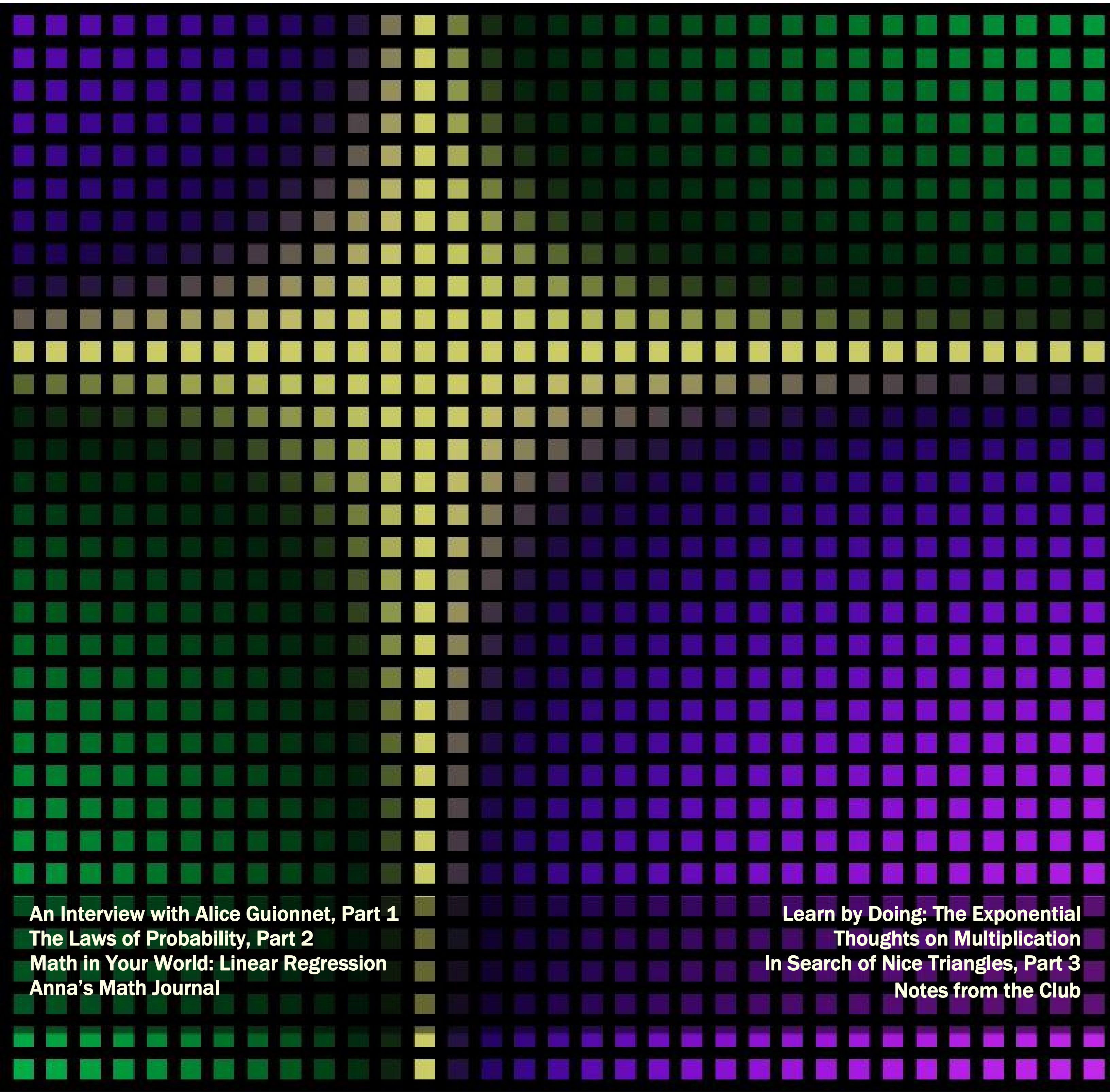
To Foster and Nurture Girls' Interest in Mathematics



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From the Founder

As you read this Bulletin, please question everything. Verify the math for yourself. See if you can predict where things are going and get there first. If you have unanswered questions, send them to us. For example, if you have a hunch about where Anna's Math Journal is heading, work it out and tell us about it. We'd love to hear from you!

- Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Multiplication, In Color!* by C. Kenneth Fan and Toshia McCabe. Created in MATLAB by MathWorks.

An Interview with Alice Guionnet, Part 1

Alice Guionnet is Professor of Mathematics at the Massachusetts Institute of Technology. She received her doctoral degree under the supervision of Gerard Ben Arous.

Ken: When did you decide that you wanted to be a mathematician? What were the influences that led to you becoming a mathematician?

Alice: I was always good in maths but thought I would become an engineer. But in France, we have a very particular college system to become an engineer. After high school we study very intensively for two years and then take very selective exams to enter "schools". Among the best ones are École Polytechnique and École Normale Supérieure, and I decided to go to École Normale (Polytechnique comes with a mandatory military service). A priori you are still free to do whatever you like afterwards but in fact there is an emphasis there to become a faculty. I discovered in École Normale research and I decided to do a PhD in maths, and I loved it. So I stayed.

In some sense, randomness, enables you to "avoid having holes in your data."

Ken: What did you love about math research?

Alice: To create and the beauty of it.

Ken: Can you recall the first nontrivial mathematical thought that you had?

Alice: No, but I do remember very well when I had my first nice idea during my PhD which allowed me to answer the question I was asked to solve. It was during vacation, in Île de Ré, during winter.

Ken: Vacations can be so useful! What mathematics did you find most interesting when you were at the École Normale?

Alice: Probability, PDEs[1], complex analysis, operator algebras... I discovered completely probability, this was amazing. In probability, certainly the law of large numbers and the central limit theorems are one of the most exciting things: the main behavior of large systems is deterministic whereas the fluctuations of a large number of independent variables are always described by the bell curve. In some sense, most of my research is still centered around such questions: how to analyze the macroscopic behavior (like a sum) from the model at the infinitesimal scale (independent variables).

Ken: You study random matrices, but first, let's talk about matrices. I think in high school, matrices are regarded as an array of coefficients of a system of linear equations, and, possibly, as a representation of a linear transformation, usually between Euclidean spaces. What is a matrix to you?

Alice: A matrix is many things. It can be an array of data (and the first appearance of random matrices is in statistics). It is an example of a linear operator, which appears in many fields (physics, etc.).

[1] "PDE" is short for "Partial Differential Equation"

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Alice Guionnet and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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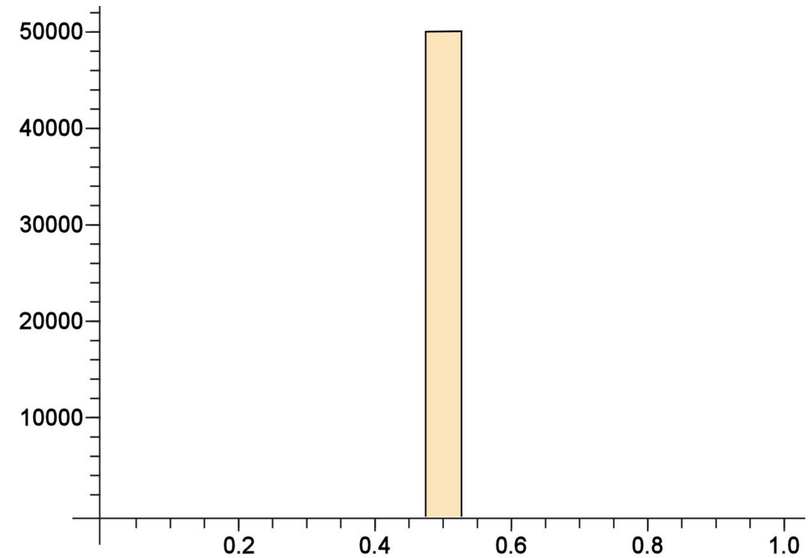
Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

The Laws of Probability[2]

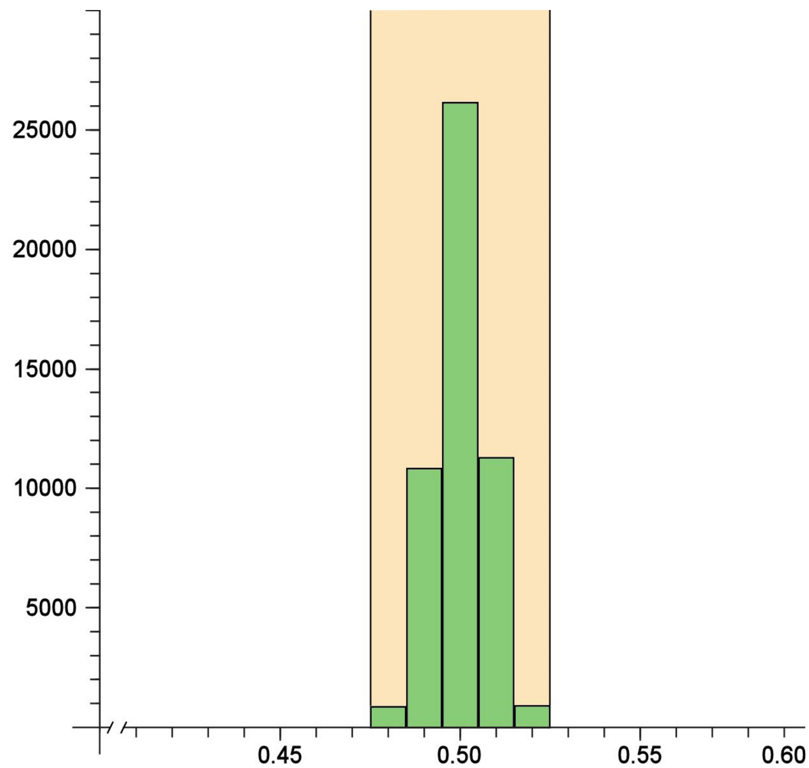
Part 2: Zooming In.
by Elizabeth S. Meckes

Last time we talked about how mathematicians define a fair coin, and the theorem that the limiting proportion of heads when a fair coin is tossed a lot of times is 1/2. Here's a computer simulation of that effect: in the histogram below, the computer did the experiment of tossing a coin 5000 times and counting the proportion of heads. It did this experiment 50,000 times (so the computer tossed 250,000,000 coins), and the histogram below shows the proportions of heads in the 50,000 experiments.



At this resolution, you see exactly what anyone would expect: a big bar at 1/2, meaning that the proportion of heads was between .475 and .525 every time.

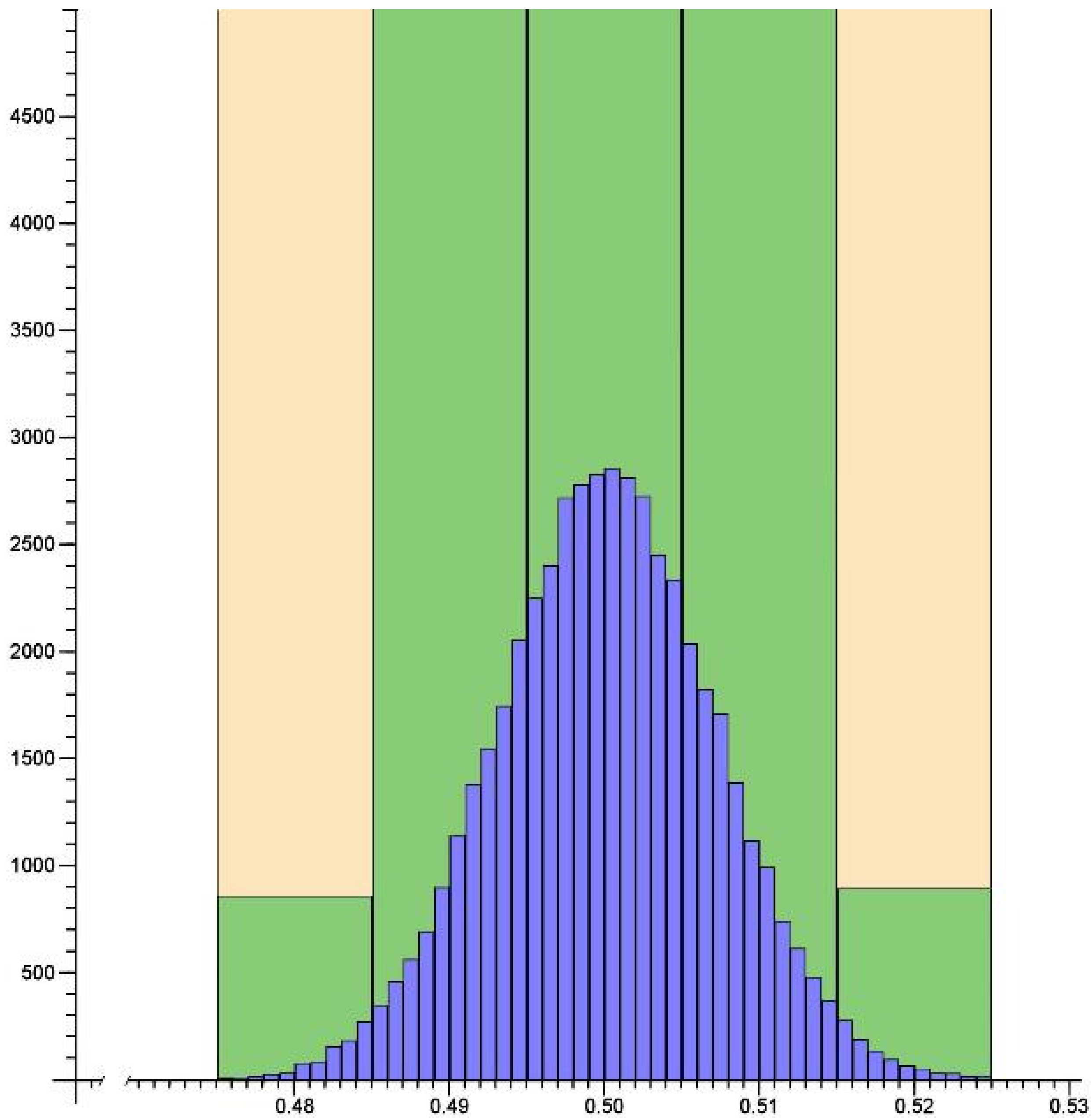
Let's zoom in; we'll make a histogram (bottom right) of what's going on inside that one big bar. This represents exactly the same set of coin tosses, but now the bars correspond to counting points within intervals of length .01, whereas before the bars covered intervals of length .05.



As you can see, it's still reassuringly concentrated around 1/2: there still don't seem to be any outcomes with fewer than 47.5% or more than 52.5% heads. But, because we've zoomed in enough, we're starting to be able to observe the fact that we're not going to get *exactly* half heads and half tails. We can see that the picture looks reasonably symmetric, which seems natural enough: it's just as likely to get a few more heads than tails as vice versa. We can also see that the bars drop off from the one that covers 1/2; the uneven outcomes got more unlikely as they got more uneven.

[2] This content was supported in part by a grant from MathWorks.

Okay, now let's really zoom in:



Whoa! Amazing! That beautiful curve was hidden in that first picture, but because we were looking at a coarse scale, we didn't see it.

The curve you see there is what people often call the "bell-shaped curve" (because it looks like the outline of a bell), or sometimes, the Gaussian curve, for the mathematician Carl Friedrich Gauss. He wasn't the first to see it, though – this curve has shown up as scientists observed and recorded findings about a huge number of features of the natural world. And lurking behind that ubiquitous curve is the **central limit theorem**.

Remember the law of large numbers from last time: if H_n is the number of heads in n tosses of a fair coin, then

$$\mathbf{P}\left[\lim_{n\to\infty}\frac{H_n}{n}=\frac{1}{2}\right]=1.$$

That theorem basically corresponds to the first picture. To zoom in, the first thing we have to do is to center our attention at 1/2. Just like when you zoom in on an online map, you need to move the spot you're interested in to the center; here we move the big bar in the first picture to the center of the number line (namely, 0), and consider

$$\frac{H_n}{n} - \frac{1}{2}$$

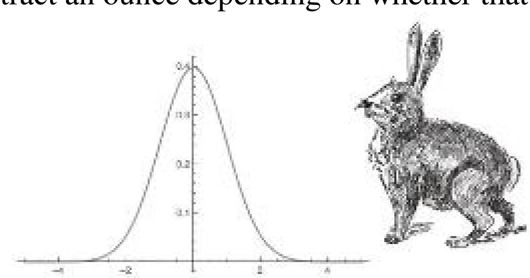
Now that we've centered things, we have to figure out how to turn the idea of zooming in into a mathematical operation we can do. Each time we zoomed in, we looked at a shorter interval on the *x*-axis, but we stretched it out so we could see it, making it have the same physical length as the original histogram. That means that we *multiplied* by larger and larger numbers, but only looked at the interval around our point of interest that fit the physical width of our original histogram. It turns out that for H_n, the right zoom factor is $\sqrt{n}$, and so finally the central limit theorem is a theorem about the random variables

$$X_n = \sqrt{n}\left(\frac{H_n}{n} - \frac{1}{2}\right)$$

Specifically, the central limit theorem says that X_n *converges weakly to the standard Gaussian distribution*, as n tends to infinity. There's a specific technical meaning to that, of course, but what it really means is that as n and the number of experiments we do grow, when we plot histograms at the kind of scale that we did in the third picture, they smooth out into the bell-shaped curve.

So why does all this mean that the bell-shaped curve occurs so often in nature? Well, one of the things about the central limit theorem (and many other theorems in probability) is that it is quite robust, in the sense that if you change the hypotheses a little, the same basic result is still true. In fact, you can push pretty much every aspect of the situation I've described above. The coin tosses don't have to be fair, and they don't even have to be unfair in the same way. Each coin can land on heads with a different probability, as long as those probabilities aren't too different (and they shouldn't be 1 or 0 – no two-headed or two-tailed coins!). The coin tosses don't need to be genuinely independent, either; as long as they don't depend too much on each other, we're okay. And actually, the coin tosses don't even have to be coin tosses! The random variable H_n can be a sum of weakly dependent random variables with pretty much any description you like – they can even be random vectors living in infinite-dimensional spaces! But back down to earth – why do we see the bell-shaped curve in observational data?

Think of it this way: any time you have a feature of something, say the weight of an adult female rabbit, it depends on a lot of little things. For (ridiculously over-simplified) instance, set the hypothetical rabbit's weight at the average weight of an adult female rabbit, but then add on an ounce if its mother was particularly large and take one away if she was particularly small. Now do the same for the father. Now either add or subtract an ounce depending on whether that rabbit's neighborhood is particularly full of lettuce or rather lacking in it. And so on and so forth; the weight of the rabbit is now seen as a random variable which is a sum of a lot of (basically independent??) things, and so the distribution of rabbit weights should look like a bell-shaped curve around the average.



Linear Regression

by Ken Fan
edited by Jennifer Silva

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

In her visit to Girls' Angle this fall, financial modeler Jinger Zhao used **linear regression** to model the connection between girls' wingspan and height. Using the model, she accurately predicted the wingspans of two girls from their heights. Let's explore linear regression by studying this example in greater detail.

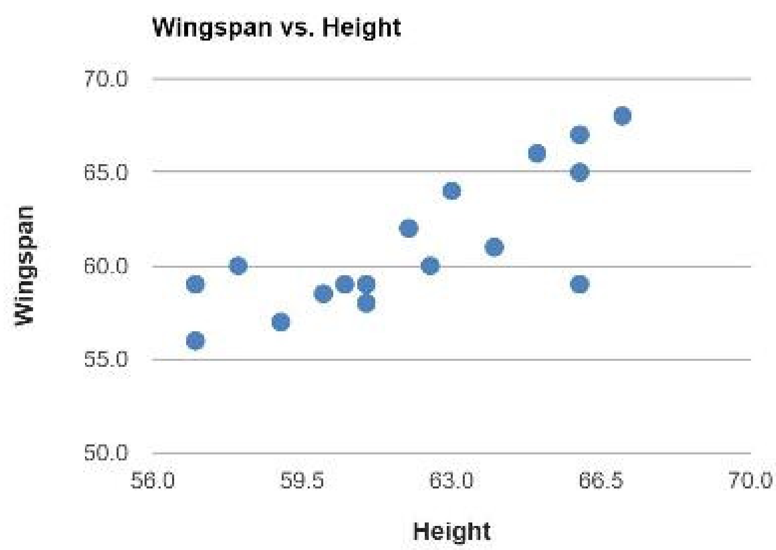
Data Collection

First, 17 girls measured their height and wingspan (or "arm span," which is the distance from the tips of the fingers on one hand to the tips of the fingers on the other, with arms outstretched as wide as possible). We rounded measurements to the nearest half-inch. This data is shown in the table below.

Height	Wingspan
62.5	60.0
61.0	59.0
59.0	57.0
60.5	59.0
61.0	58.0
66.0	59.0
62.0	62.0
66.0	67.0
64.0	61.0
63.0	64.0
65.0	66.0
57.0	56.0
58.0	60.0
67.0	68.0
57.0	59.0
66.0	65.0
60.0	58.5

Height and wingspan (in inches) of 17 girls.

We then created a scatter plot of the data. Our scatter plot treated each girl as a point on a graph whose horizontal axis corresponded to height and whose vertical axis corresponded to wingspan. We made the graph by hand at the club, but the picture below was created automatically by Google Spreadsheets. Please take note that the vertical and horizontal scales are not the same, even though both represent inches.



Does wingspan seem to roughly increase with height?

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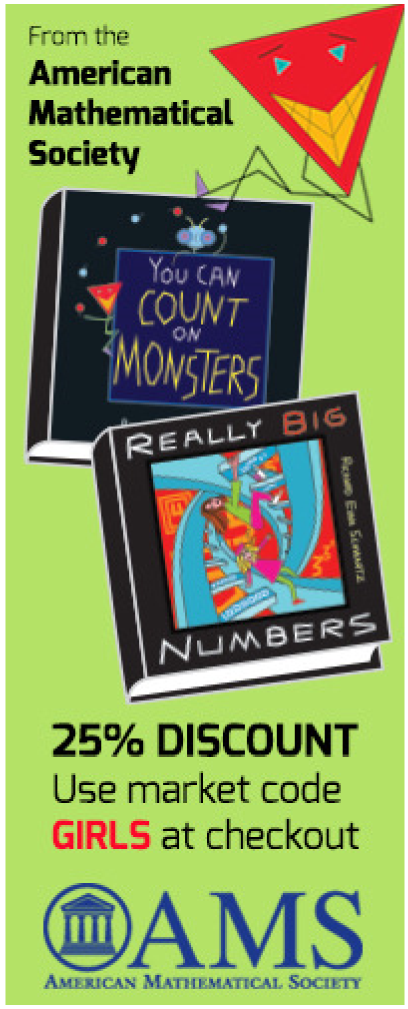

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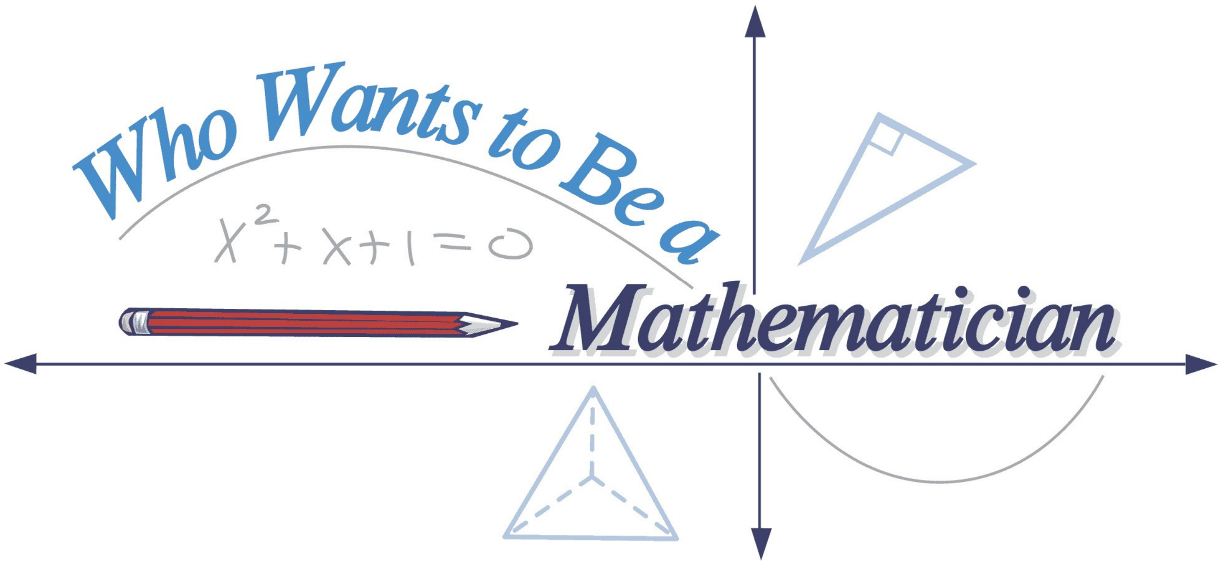
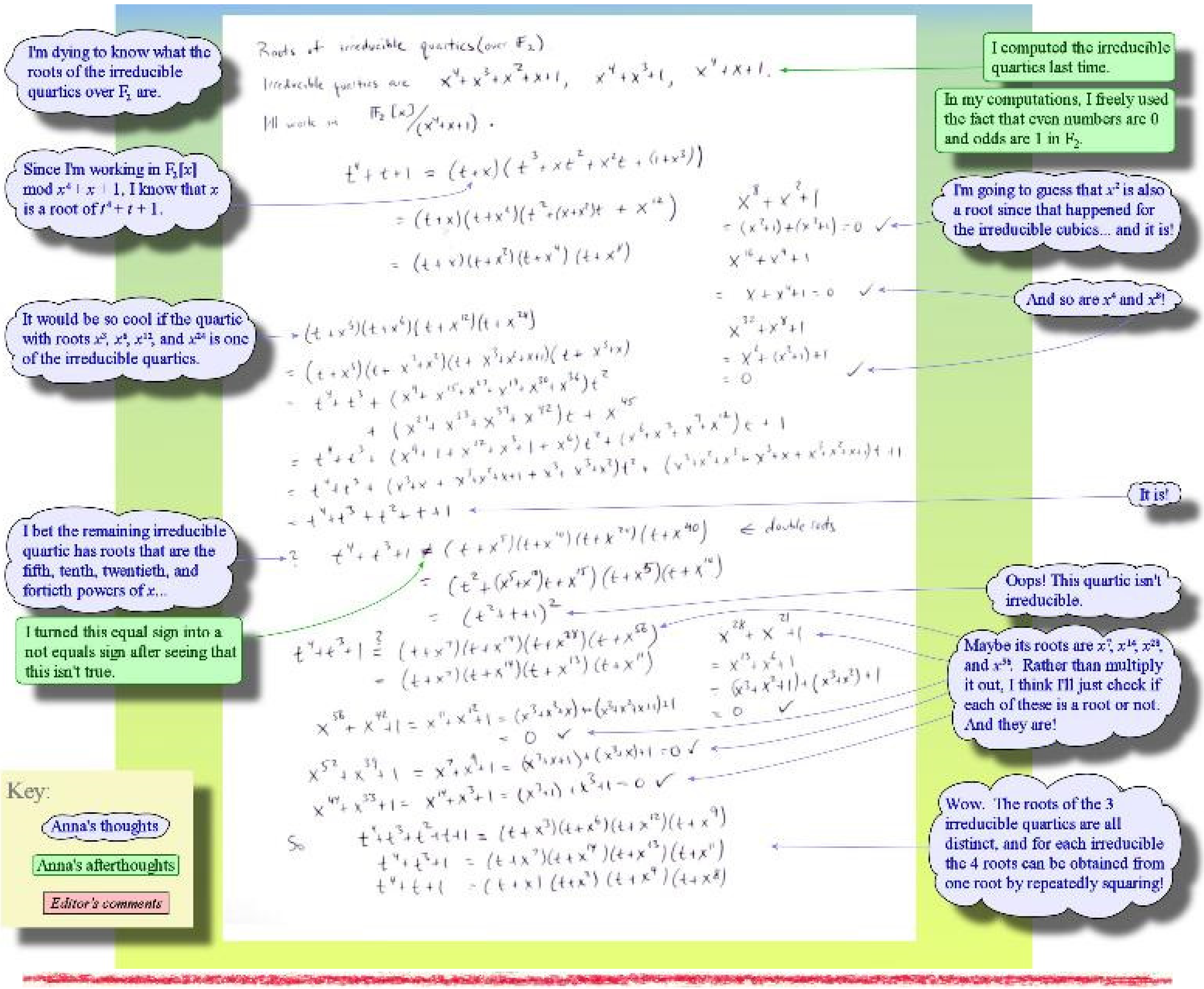


Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues thinking about irreducible polynomials over the finite field with 2 elements.



Roots of irred. quartics include all elements of $\mathbb{F}_2[x]/(x^4+x+1)$

except: $1, x^5, x^{10}$, and 0.

x^5, x^{10} are roots of irred. quadratic t^2+t+1.

The roots of the quartics cover all elements of the finite field with 16 elements except for 0, 1, x^5 and x^{10}. Hm. Curiously, x^5 and x^{10} are roots of the unique irreducible quadratic.

On to quintics!

Actually, before doing quintics, I wonder if it is generally true that if r is a root of an irreducible polynomial, then r^2 is also a root.

"Square" Lemma. $p(r) = 0 \Rightarrow p(r^2) = 0$?

$p(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n$, $a_k \in \mathbb{F}_2$.

$p(r) = a_0 + a_1 r + a_2 r^2 + \cdots + a_n r^n = 0$

$p(r^2) = a_0 + a_1 r^2 + a_2 r^4 + \cdots + a_n r^{2n} \overset{?}{=} 0$

I don't know if $p(r^2) = 0$, but if $p(r) = 0$, then certainly $p(r)^2 = 0$, and $p(r)^2$ would contain many of the terms in $p(r^2)$. Perhaps I should compare them.

$0 = p(r)^2 = (a_0 + a_1 r + a_2 r^2 + \cdots + a_n r^n)^2$

$= (a_0 + a_1 r + a_2 r^2 + \cdots + a_{n-1} r^{n-1})^2 + a_n^2 r^{2n}$ $\quad (x+y)^2 = x^2 + y^2$ in $\mathbb{F}_2$

$= (a_0 + a_1 r + a_2 r^2 + \cdots + a_{n-1} r^{n-1})^2 + a_n r^{2n}$

$= a_0 + a_1 r^2 + a_2 r^4 + \cdots + a_{n-1} r^{2n-2} + a_n r^{2n}$!

Wait a sec! In F_2, the square of a sum *is* equal to the sum of the squares! All the cross terms are zero. So the "square" lemma is true!

OK, *now* onto quintics!

irred. quintics. work in $\mathbb{F}_2[x]/(x^5+x^2+1)$

Want $x, x^2, x^3, x^4, \ldots, x^{31}$ to be all nonzero elements of the field.

True since 31 is prime. In fact $x^{31} = 1$.

The "square" lemma should help a lot in finding the roots of the irreducible quintics. I should be able to organize the roots by powers of x where each set of 5 roots of an irreducible quintic is closed under squaring.

1. x, x^2, x^4, x^8, x^{16}
2. $x^3, x^6, x^{12}, x^{24}, x^{17}$
3. $x^5, x^{10}, x^{20}, x^9, x^{18}$
4. $x^7, x^{14}, x^{28}, x^{25}, x^{19}$
5. $x^{11}, x^{22}, x^{13}, x^{26}, x^{21}$
6. $x^{15}, x^{30}, x^{29}, x^{27}, x^{23}$

Claim. These are the 30 roots of the 6 irred. quintics over $\mathbb{F}_2$.

Only elements missing are 0 and 1.

I just need to make sure I get 5 different powers in each set. These look good. I bet these 6 sets are the roots of the 6 irreducible quintics.

I know x is a root of the polynomial I am modding out by, and so by the "square" lemma the first row of roots must be the roots of $t^5 + t^2 + 1$.

x is a root of t^5+t^2+1 so $t^5+t^2+1 = (t+x)(t+x^2)(t+x^4)(t+x^8)(t+x^{16})$

I computed the 6 irreducible quintics over F_2 last time.

Let's see if I can find which row of roots are those of $t^5 + t^3 + 1$. I guess I'll just plug the first element in each row into the polynomial and see if I get 0.

t^5+t^3+1: $\quad x^{15}+x^9+1 = (x^2+1)^3 + (x^2+1)x^4+1$

$= x^6+x^4+x^2+1+x^6+x^4+1$

$= x^2$

x^3 ... Nope!

$x^{25}+x^{15}+1 = (x^2+1)^5+(x^2+1)^3+1$

$= x^{10}+x^8+x^2+1+x^6+x^4+x^2+1+1$

$= (x^2+1)^2+x^4(x^2+1)+(x^2+1)x+x^4+1$

$= x^4+1+x^6+x^3+x^3+x+x^4+1$

$= x^5+x = x^2+x+1$

x^5 ... Nope!

$x^{35}+x^{21}+1 = x^4+x^{21}+1$

$= x^4+x(x^2+1)^4+1$

$= x^4+x(x^8+1)+1 = x^4+x^9+x+1$

$= x^4+(x^2+1)x^4+x+1$

$= x^6+x+1$

$= x(x^2+1)+x+1 = x^3+1$

x^7 ... Nope!

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

$$x^{55}+x^{33}+1 = x^{24}+x^{2}+1$$
$$= (x^2+1)^4x^4+x^2+1$$
$$= (x^8+1)x^4+x^2+1$$
$$= x^{12}+x^4+x^2+1$$
$$= (x^2+1)^2x^2+x^4+x^2+1$$
$$= (x^4+1)x^2+x^4+x^2+1$$
$$= x^6+x^2+x^4+x^2+1 = x(x^2+1)+x^4+1 = x^4+x^3+x+1$$

x^{11}... Nope!

$$x^{75}+x^{45}+1 = x^{13}+x^{14}+1 = (x^2+1)^2x^3+(x^2+1)^2x^4+1$$
$$= (x^4+1)(x^3+x^4)+1$$
$$= x^8+x^7+x^4+x^3+1$$
$$= (x^2+1)(x^3+x^2)+x^4+x^3+1$$
$$= x^5+x^4+x^3+x^2+x^4+x^3+1$$
$$= x^5+x^2+1 = 0$$

x^{15}... There it is!

⇒ So 6 corresponds to t^5+t^3+1.

That was kind of long. Hm. I think I'll compute the first 5 powers of the first elements in each row and then determine which irreducible it satisfies instead. That might go faster.

$$x^3$$
$$x^6 = x(x^2+1) = x^3+x$$
$$x^9 = x^6+x^4 = x^4+x^3+x$$
$$x^{12} = x^7+x^6+x^4 = x^2(x^2+1)+x^3+x+x^4 = x^3+x^2+x$$
$$x^{15} = x^6+x^5+x^4 = x^3+x+x^2+1+x^4 = x^4+x^3+x^2+x+1$$

⇒ x^3 is root of $t^5+t^4+t^3+t^2+1$

There are many ways the details of this computation can go, so don't pay too much attention to the specific way it is presented here. I'm sure if I did this again, it would look different. In fact, you might want to ignore my work here completely and figure out the roots of the 6 irreducible quintics yourself, then compare the final answer.

$$x^5 = x^2+1$$
$$x^{10} = x^5+x^4+x^2 = x^4+1$$
$$x^{15} = x^4+x^3+x^2+x+1$$
$$x^{20} = x^9+x^8+x^7+x^6+x^5 = x^4+x^3+x+x^3+1+x^3+x^4+x^2+x^3+x+x^2+1$$
$$= x^3+x^2$$
$$x^{25} = x^8+x^7 = x^2+1+x^3+x^4+x^2 = x^4+x^3+1$$

⇒ x^5 is root of $t^5+t^4+t^2+t+1$

$$x^7 = x^4+x^2$$
$$x^{14} = x^8+x^4 = x^4+x^3+x^2+1$$
$$x^{21} = x^4+x^3$$
$$x^{28} = x^8+x^7+x^6+x^5 = x^9+x^3+x^2 = x^4+x^2+x$$
$$x^{35} = x^4$$

⇒ x^7 is root of $t^5+t^3+t^2+t+1$

$$x^{11} = x^2+x+1$$
$$x^{22} = x^5+x^4 = x^4+x^2+1$$
$$x^{33} = x^2$$
$$x^{44} = x^{13} = x^4+x^3+x^2$$
$$x^{55} = x^{24} = x^7+x^6 = x^4+x^2+x^3+x$$

⇒ x^{11} is a root of $t^5+t^4+t^3+t+1$

1.	t^5+t^2+1	x, x^2, x^4, x^8, x^{16}
2.	$t^5+t^4+t^3+t^2+1$	$x^3, x^6, x^{12}, x^{24}, x^{17}$
3.	$t^5+t^4+t^2+t+1$	$x^5, x^{10}, x^{20}, x^9, x^{18}$
4.	$t^5+t^3+t^2+t+1$	$x^7, x^{14}, x^{28}, x^{25}, x^{19}$
5.	$t^5+t^4+t^3+t+1$	$x^{11}, x^{22}, x^{13}, x^{26}, x^{21}$
6.	t^5+t^3+1	$x^{15}, x^{30}, x^{29}, x^{27}, x^{23}$

That wasn't so bad. So here are the irreducible quintics along with their roots. These 30 roots cover all elements of the field with 32 elements except for 0 and 1! There's got to be a big pattern lurking here. What is it???

ABB 12.26.15

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Learn by Doing

The Exponential
by Girls' Angle Staff

When you get to a problem, try to solve it before reading further. A few of the problems are followed by a solution. Subscribers and members are welcome to email us with any thoughts and questions.

Master Lock makes a combination lock that uses directions in its combination. It is called the Speed Dial. The front of the lock has a button that can be moved up, down, left, and right. The lock opens by moving the button according to a specific sequence of directions. For example, a combination might be up-left-down. According to Master Lock, the combination to the lock can be set to any sequence of up/down/left/right directions.

Let's ask: How many combinations are possible with a sequence of a given length?

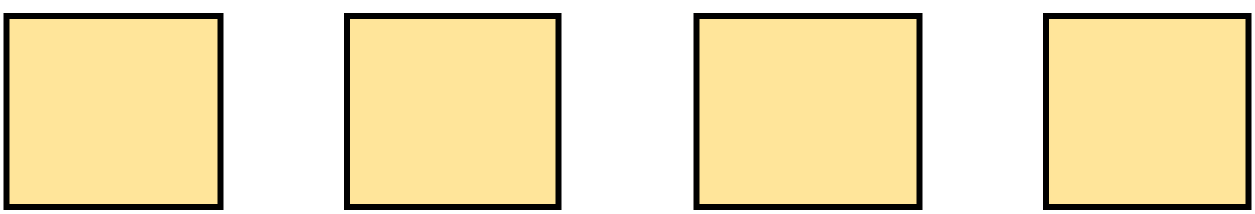
Problem 1. If we use a combination that consists of just a single direction, how many combinations are possible?

Since a direction can take one of four possibilities (up, down, left, or right), there are exactly 4 possible 1-direction combinations.

Problem 2. If we use a combination that consists of exactly 2 directions, how many combinations are possible?

We'll answer Problem 2 by listing all of the possibilities and then counting the items on the list. When you make such a list, you have to be careful about two things. One is that you don't want to miss any combinations. The other is that you don't want to list a combination more than once. To avoid these pitfalls, it helps to devise a strategy.

The strategy we'll use is to organize the set of 2-direction combinations into groups that are smaller and more manageable. Every 2-direction combination has a second direction. This second direction must be up, down, left, or right. So let's organize the 2-direction combinations into 4 bins, one for each of the 4 possible second directions:



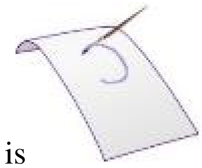
Bin 1. 2^{nd} direction: Up.

Bin 2. 2^{nd} direction: Down.

Bin 3. 2^{nd} direction: Left.

Bin 4. 2^{nd} direction: Right.

For example, we'll put all of the 2-direction combinations that end with up into Bin 1. Since every combination must have a second direction that is either up, down, left, or right, we know that every 2-direction combination will show up in exactly one of these 4 bins. To count the total number of 2-direction combinations, we could then count the number of combinations in each bin and add up the tallies.



To fill the bins, we must list every 2-direction combination whose second direction corresponds to the bin we're filling. Once the second direction of a 2-direction combination is specified, we only need to specify the first direction in order to get the entire combination. Since there are 4 possibilities for the first direction, each bin will contain a total of 4 combinations.

Here are the bins filled with combinations:

Bin 1. 2nd direction: Up.	Bin 2. 2nd direction: Down.	Bin 3. 2nd direction: Left.	Bin 4. 2nd direction: Right.
UU DU	UD DD	UL DL	UR DR
LU RU	LD RD	LL RL	LR RR

Now we can count all of the 2-direction combinations. Notice that there are 4 bins each containing 4 combinations. If we count the number of combinations bin by bin (instead of combination by combination), we see that the total number of 2-direction combinations is equal to $4 + 4 + 4 + 4$, or 4×4.

Problem 3. Find another way to organize a list of the 2-direction combinations.

On to 3-direction combinations! There are so many 3-direction combinations that it starts getting difficult to create a list of them without a strategy. In fact, if we devise a strategy, we might discover that we can count the number of 3-direction combinations without actually having to list them.

Problem 4. Come up with a strategy to list 3-direction combinations. Explain how your strategy will succeed in listing all of them exactly once. If you are confident in your answer, skip to Problem 5.

Observe that *any 2-direction combination can be extended to a 3-direction combination by adding a third direction (which can be up, down, left, or right), and every 3-direction combination can be built from a unique 2-direction combination in this way.*

This suggests organizing 3-direction combinations into bins where each bin contains the 3-direction combinations with a specific 3rd direction. We would then have 4 bins, one for each of the possible 3rd directions. Look inside the bin corresponding to combinations that end with the direction up. Notice that if we ignore the 3rd direction, we get an exact list of all 2-direction combinations. In fact, in any bin, ignoring the 3rd direction produces a list of all of the 2-direction combinations. Therefore, there are 4 bins each with 4×4 combinations.

Problem 5. Convince yourself that the number of 3-direction combinations is $4 \times 4 \times 4$.

In actual use, most locks will be set to combinations with more than 3 directions. A typical lock might have a combination that consists of 10 directions.

Problem 6. Show that there are $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ 10-direction combinations.

Problem 7. Let n be a positive whole number. How many n-direction combinations are there?

The answer to Problem 7 is that we must multiply n 4s together. Exponential notation is a convenient way to express this very concept of multiplying several copies of the same number. In exponential notation, the product of n 4s is written 4^n. The number being repeatedly multiplied is written with a superscript that tells how many times to multiply that number. In general, if b is any number and n is any positive integer, then b^n stands for the product of n factors of b. In b^n, the number b is called the **base** and the number n is called the **exponent**.

See also Notation Station on pages 13-14 of Volume 3, Number 2 of this *Bulletin*.

Using exponential notation, the answer to Problem 5 can be written as 4^{10}. Do you prefer writing 4^{10} or $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$?

Problem 8. What is 2^3? What is 3^2? What is 1^{1000}? What is $(-1)^{999}$? What is $(1/2)^3$? What is 10^5? (Here, we are asking you to express these exponentials as numbers that do not use exponentials. For example, the answer to "What is 5^2?" would be "25.")

Problem 9. In a traditional Master Lock, a combination is a sequence of 3 integers between 0 and 39, inclusive. What is the smallest value of n for which there are more n-direction Speed Dial combinations than there are combinations for a traditional Master Lock?

Problem 10. Suppose a car license plate number consists of any combination of 6 symbols, where a symbol can be a capital letter of the alphabet or a digit. How many distinct license plate numbers are possible?

Problem 11. Chinese pulled-noodles (also known as lamian) are made by taking a wad of dough, stretching it into a long stick, folding this stick over, stretching again, folding the result, stretching again, etc. The result is one long noodle, doubled over numerous times. When the ends are cut off, you get a lot of noodles. If the noodle is folded over 10 times, how many noodles do you get? A master chef can make just over 1000 noodles in a minute. How long would it take a master chef to make just over one million noodles?

If you have difficulty with Problems 12-14, think about each problem using specific, small values for n and m:

Problem 12. Let n and m be positive whole numbers, and let b be any number. Explain why

$$b^n \times b^m = b^{n+m}.$$

Problem 13. Let n be a positive whole number, and let a and b be any numbers. Explain why

$$a^n \times b^n = (ab)^n.$$

Problem 14. Let n and m be positive whole numbers, and let b be any number. Explain why

$$(b^n)^m = b^{nm}.$$

Problem 15. So far, our exponents have been restricted to positive integers. Extend the definition of b^n so that n can be any integer by insisting that the identity in Problem 12 remains valid. Specifically, define b^n for nonzero b and *any* integer n in such a way that $b^n \times b^m = b^{n+m}$.

Thoughts On Multiplication

by Addie Summer | edited by Jennifer Silva

All friends begin as strangers. Even after a couple of encounters with a future friend, we may still not remember our acquaintance's name. Weeks could pass before we discover a mutual love of Italy. Eventually, we're able to predict whether our friend would prefer visiting Walden Pond or the Boston Common. Yet years into a friendship, we can still be surprised to learn that our friend was in the Borghese Gallery on the very same day that we were there ten years before we first met!

If you haven't met multiplication, I'd like to introduce you, because multiplication is a great friend to have.

First Encounters

My first encounter with multiplication was as repeated addition of the same number. For example, 3 multiplied by 5 was defined as 3 + 3 + 3 + 3 + 3; we add 5 threes together. I also learned that "times" is a nickname for "multiplied by," as I would often read or hear people say "5 times 3" instead of "3 multiplied by 5." Maybe that nickname developed because people would ask, "Add 3 how many times?" and the answer would come back, "5 times."

I could see that if I was ever going to know multiplication well, I'd have to be friends with addition. If you're not on good terms with addition yet, take a look at Bjorn Poonen's 3-part series on the meaning of addition starting in Volume 3, Number 3 of this *Bulletin*.

But *why* is multiplication a good friend to have? Is repeated addition by the same number something so special that we should give it its own identity?

When you first become aware of someone destined to become your friend, it can be uncanny how you start to notice that person everywhere. And so it was with multiplication.

I'm hosting a party for 50 guests and I want to make sure I have at least 3 napkins per guest. How many napkins do I need? I need 3 napkins 50 times, or 50 times 3.

I've got 3 gallons of gasoline left in my car and I get 45 miles per gallon. How many more miles can I drive? I can drive 45 miles 3 more times, so I can go 3 times 45 more miles.

I have to print 17 copies of my 8-page report for all of my classmates. How many sheets of paper do I need to put in the printer? I have to print 8 sheets 17 times, so I need 17 times 8 sheets of paper.

I have to purchase 72 California rolls for a friend's graduation concert and each roll costs $5.25. How much money do I need? I will have to pay $5.25 a total of 72 times, so I need 72 times $5.25.

My suitcase is 22 inches long and there are 2.54 centimeters in an inch. I need to convert my suitcase's length to centimeters. How long is it in centimeters? Since every inch is 2.54 centimeters, I must add 2.54 a total of 22 times, so its length in centimeters is 22 times 2.54.

Multiplication would show up in my life on numerous occasions each day. In fact, I would estimate that I run into multiplication an average of 30 times per day. I wonder how many times I come across multiplication each year. Hey, that's another multiplication problem!

At first, I would compute the result of multiplying by adding over and over. For example, to compute 5 times 3, I would add

3 + 3 = 6 (that's 2 times 3).

3 + 3 + 3 = 6 + 3 = 9 (that's 3 times 3).

3 + 3 + 3 + 3 = 9 + 3 = 12 (that's 4 times 3).

3 + 3 + 3 + 3 + 3 = 12 + 3 = 15 (that's 5 times 3)!

Sometimes, it would take me a while to compute the result of multiplying two numbers (which is also called a "product").

Over time, I found myself computing products so often that I decided to make a handy multiplication table and carry it around so that I didn't have to redo computations over and over. The first multiplication table I built was this 12 by 12 table:

12 By 12 Multiplication Table

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8												
9												
10												
11												
12												

Actually, that's only two-thirds of my first multiplication table. Will you complete it?

I computed so many products that I started to memorize them just through everyday familiarity. I suppose it is similar to the way we begin to remember things like who our friend's friends are, without having to really try. This enabled me to compute products a bit faster. For example, now I can just tell you what 4 times 3 is; I don't have to add up 4 threes anymore.

Then, one day, I decided to make an effort to memorize the products in the multiplication table above. At first, I did this just as a kind of memorization game, like the way some people memorize the capitals of all fifty states.

Why try to memorize when we have calculators?

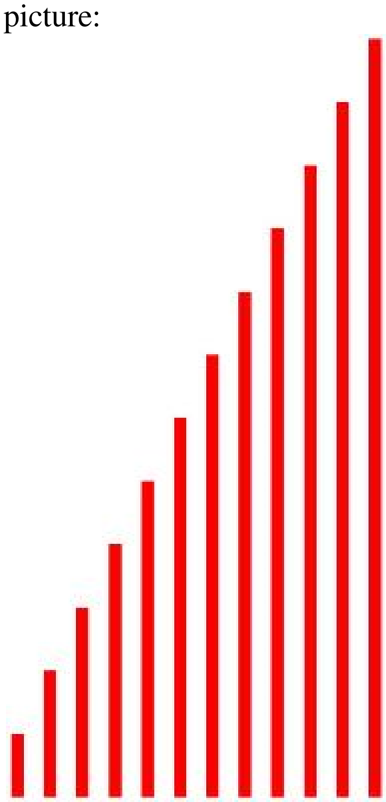
Memorization brought some benefits that I didn't anticipate. Perhaps the most important benefit is that it made it easier for me to notice patterns.

I have a friend who has 4 children: Abby, Emily, Jenny, and Polly. They made a hilarious picture for their holiday greeting card. It was a vertical human pyramid. From bottom to top, it showed Jenny, Emily, Polly, and Abby. Funny, no? Well, of course you won't get the joke. You don't realize that the girls were arranged from smallest to biggest, with Jenny, the smallest, on the bottom looking strained, and Abby, the biggest, precariously balanced on top!

What does this have to do with memorization, numbers, and multiplication? Well, it's the exact same situation as when I write down the numbers 2, 4, 6, and 8. As plain characters, there's no natural order to them. But when we know that "2" stands for the number of hearts here: ♥♥, "4" stands for the number of hearts here: ♥♥♥♥, "6" stands for the number of hearts here: ♥♥♥♥♥♥, and "8" stands for the number of hearts here: ♥♥♥♥♥♥♥♥, then we can recognize the pattern: 2, 4, 6, 8 are in increasing numerical order. There's no way to *deduce* that, say, "4" stands for the number of hearts here: ♥♥♥♥. That is a convention we have to learn. The quicker you can associate the numbers with the symbols, the more likely it is that you'll be able to see the pattern represented by 2, 4, 6, 8. If you don't have these things memorized, 2, 4, 6, 8 will slip by you like any sequence of four numbers, pattern unnoticed.

Imagine how difficult it would be to enjoy a story if, each time you heard a name, you had to stop and ask, "who's that?" If every time we saw "6," we had to reacquaint ourselves with the fact that 6 represents this number of hearts: ♥♥♥♥♥♥, it would be difficult for us to follow the larger story. In mathematics, the larger story is a tapestry of patterns that weave in and out among each other.

For example, look at this picture:



In a single glance, you can see a pattern to the way these red sticks are arranged. They are evenly spaced and go from shortest to longest with great regularity.

But do you recognize that this is exactly the pattern exhibited by the numbers in the second row (headed by "**2**") of the multiplication table: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24? In fact, the pattern of red sticks is essentially the pattern that can be found in any row of the multiplication table.

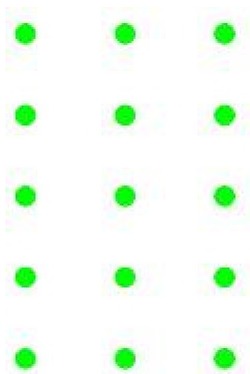
The pattern in the multiplication table is harder to see than the stick pattern because it is obscured by a sea of numerical symbols, whereas the pattern in the red sticks appeals directly to our built-in visual system. The quicker we can translate the numerical symbols to the quantities that they stand for, the easier it will be for us to discover patterns.

Of course, it is also possible to detect patterns through understanding, and the interplay between observation, understanding, and memorization is fascinating. To engage in these three important modes with multiplication, you have to spend quality time with it; that means taking some time to ponder multiplication and compute products *without* the aid of a calculator.

Quality time with multiplication

Multiplication has turned out to be a friend with endless surprises and beautiful qualities. Let's look at one striking quality of multiplication that can be found by playing around with rectangular arrangements of dots.

Since 5 times 3 is 3 + 3 + 3 + 3 + 3, we can illustrate it as the number of dots in the rectangle of green dots shown below.



Each row has 3 dots, and there are 5 rows. If we count the total number of dots row by row, we will compute 3 + 3 + 3 + 3 + 3, which is 5 times 3.

On the other hand, we could also count the same rectangle of dots column by column. There are 3 columns, each containing 5 dots. Therefore, the total number of dots is also equal to 5 + 5 + 5, or 3 times 5.

Since we're counting the same thing, it must be that 5 times 3 is equal to 3 times 5! In general, you can switch the order of multiplication without affecting the product: i.e., a times b equals b times a, for any numbers a and b. This amazing property of multiplication is called **commutativity**.

Patterns upon patterns

Here's another stunning multiplication pattern. To see it, first focus on the diagonal of the multiplication table that runs from the upper left corner down to the lower right. Our table only contains space for the first 12 numbers on this diagonal, but the diagonal continues on forever. The numbers on this diagonal are the numbers that can be expressed as a number times itself. The first few such numbers are 1, 4, 9, 25, 36, etc. These numbers are known as the **perfect squares**.

Now pick any one of these perfect squares and look at the diagonal that runs through that number in the multiplication table perpendicular to the diagonal of perfect squares. Here's the pattern: as you move along this diagonal away from the perfect square, the numbers fall off exactly by amounts equal to the perfect squares! That is, each such diagonal contains upside-down versions of the diagonal of perfect squares. Can you verify and explain that?

Spend some quality time with multiplication. What patterns can you find?

In Search of Nice Triangles, Part 3

by Ken Fan | edited by Jennifer Silva

Emily and Jasmine continue their quest for nice triangles, this time seeking those with integer side lengths and a 120° angle.

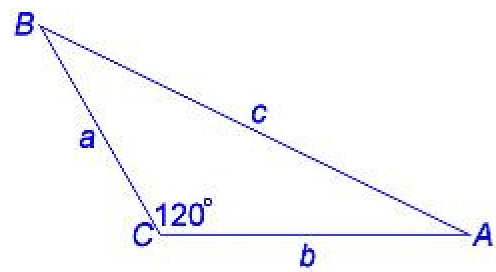
Emily and Jasmine are sitting at a booth in Cake Country. Jasmine watches Emily down her last gulp of hot chocolate.

Jasmine: Emily? Are you ready to work through the 120° case?

Emily: Mmm! Mr. ChemCake makes the best hot chocolate. I'm ready!

Jasmine: It *is* delish! I can only imagine that the 120° case will work out using the same method we used to handle the Pythagorean and 60°-angle cases. I'm not expecting any surprises.

Emily: I agree, but let's do it anyway to be thorough.



Jasmine: Okay.

Emily: If we label the triangle's sides a, b, and c, with c opposite the 120° angle, the law of cosines tells us that $c^2 = a^2 + b^2 + ab$, since the cosine of 120° is -1/2.

Jasmine: And, as before, we seek rational solutions (a/c, b/c) to the equation $1 = x^2 + y^2 + xy$, which is the equation of an ellipse. From one rational solution, we should be able to get the others by looking at where the lines through that solution with rational slope intersect our ellipse.

Emily: We might as well check if there's a solution where $x = 0$ or $y = 0$ to make computations simpler. [Emily computes.] If $x = 0$, then y can be 1 or -1. If $y = 0$, then x can be 1 or -1.

Jasmine: So our ellipse passes through the four points (1, 0), (0, 1), (-1, 0), and (0, -1). Do you have a preference for a particular one of these?

Emily: Actually, I do. Remember how we had to handle the vertical line through our solution separately from the other lines because vertical lines do not have a well-defined slope?

Jasmine: Yes …

Emily: We're really interested in solutions where both coordinates are positive because those represent actual lengths; if we use (-1, 0), we immediately know that the vertical line through it is not of interest, since all points on that vertical have a negative x-coordinate.

Jasmine: Good idea! Then let's consider lines of slope m through (-1, 0). These are the lines given by the equation $y = m(x + 1)$.

Emily: We have to find the simultaneous solutions to $1 = x^2 + y^2 + xy$ and $y = m(x + 1)$.

Emily and Jasmine separately work out the details.

Try to work out the solution yourself before reading further!

Jasmine: Did you get $x = \frac{1-m^2}{m^2+m+1}$ and $y = \frac{m^2+2m}{m^2+m+1}$?

Emily: Yes, I did! And we need both x and y to be positive.

Jasmine: This really is unfolding like the 60° case! Let's see, for x to be positive …

Emily: Jasmine?

Jasmine: What's up?

Emily: I just realized that we can figure out the range of slopes m that yield points with positive coordinates by thinking geometrically. We want our lines that pass through (-1, 0) to have another intersection with our ellipse $1 = x^2 + y^2 + xy$ in the first quadrant where points have positive coordinates.

Jasmine: Right …

Emily: But we just saw that the ellipse intersects the axes at (1, 0), (0, 1), (-1, 0), and (0, -1). If we imagine the slope m going from negative infinity to infinity, when m reaches 0, that's when it intersects the ellipse at (1, 0); then as m increases from 0, the line sweeps through the points of the ellipse in the first quadrant until it passes through the point (0, 1). Since the slope of the line through (-1, 0) and (0, 1) is 1, the values of the slope where we get points in the first quadrant will be values of m between 0 and 1!

Jasmine: Very nice! That saves us some algebra.

Emily: So we're interested in the points $(\frac{1-m^2}{m^2+m+1}, \frac{m^2+2m}{m^2+m+1})$ for $0 < m < 1$.

Jasmine: Wait a sec.

Emily: What's up?

Jasmine: Aren't those the same denominators we got for the 60° case?

Emily: Let me check – I've got my notes for that in my backpack.

Emily pulls out her math journal.

Emily: For the 60° case, we found the solutions $x = \frac{m^2+2m}{m^2+m+1}$ and $y = \frac{1+2m}{m^2+m+1}$. So, yes, the denominators are the same, but before, we wanted $m > 0$.

Jasmine: Gosh, the y-coordinate we found is *exactly* the same expression as the x-coordinate we found for the 60° case. That's got to be telling us something. What could it mean?

Emily: Hmm. You're suggesting a connection between solutions of the 60° case and the 120° case. In the 60° case, we were finding solutions to $1 = x^2 + y^2 - xy$. In the 120° case, we're looking for solutions to $1 = x^2 + y^2 + xy$.

Jasmine: The two equations differ in only one sign. Hey! If (s, t) is a solution to $1 = x^2 + y^2 - xy$, then (s, -t) is a solution to $1 = x^2 + y^2 + xy$.

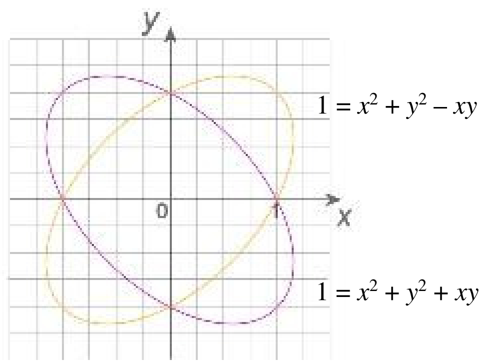
Emily: That's it – I can't believe it! We did all this work when, in effect, we already solved the 120° case when we solved the 60° case!

Jasmine: The transformation that sends the point (s, t) to (s, -t) is a reflection in the x-axis, so the ellipse $1 = x^2 + y^2 + xy$ must be a reflection of the ellipse $1 = x^2 + y^2 - xy$ in the x-axis.

Emily: Wait. There's still a problem, because the reflection takes points in the first quadrant and sends them to the fourth, but we're interested in solutions in the first quadrant. To get solutions to the 120° case from the 60° case, we would have to take the solutions in the fourth quadrant of the 60° case, and then reflect those over the x-axis. We ignored those solutions.

Jasmine: That's true. Gee, there is so much symmetry in this problem. I feel like drawing a good graph of both ellipses.

Jasmine produces this figure on a piece of graph paper:



Jasmine: I used purple for the 120° case and orange for the 60° case.

Emily: Good drawing! You can see in your drawing that flipping over the x-axis swaps the ellipses.

Jasmine: It looks like we should also be able to swap them by rotating 90° about the origin. A rotation by 90° in the counterclockwise direction about the origin sends the point (s, t) to (-t, s).

Emily: Yes, that does work because if (s, t) is a solution to one of our equations, then (-t, s) is a solution to the other. In fact, this transformation can be thought of as a reflection in the x-axis followed by a reflection in the line $y = x$, and reflecting in the line $y = x$ is the same as swapping the horizontal and vertical coordinates. Each of our equations is symmetric in x and y, so swapping horizontal and vertical coordinates preserves both ellipses.

Jasmine: Unfortunately, this rotation sends the first quadrant to the second quadrant. I wish there were a transformation that sends one ellipse to the other *and* maps the first quadrant to itself.

Emily and Jasmine stare at the figure for a good while.

Jasmine: I think I see something!

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 17 - Meet 8
November 5, 2015

Mentors: Karia Dibert, Anna Ellison, Neslly Estrada, Debbie Seidell, Isabel Vogt, Jane Wang (Head)

Vigenère ciphers, envelopes of families of straight lines, functions, and fundamental groups were some of this meet's topics.

Session 17 - Meet 9
November 12, 2015

Mentors: Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Isabel Vogt, Jane Wang (Head)

Some members sought the simplest description for a mystery function consistent with given values of the function on a subset of its domain. Try your hand at this for the three mystery functions below. The domain of all 3 functions is the set of positive integers.

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$f(x)$	2/3	1/2	2/5	1/3	2/7	1/4	2/9	1/5	2/11	1/6	2/13	1/7	2/15	1/8	2/17	1/9	2/19	1/10

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$g(x)$	1	1	2	1	2	2	3	1	2	2	3	2	3	3	4	1	2	2	3	2

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$h(x)$	1	3	2	9	5	6	7	27	4	15	11	18	13	21	10	81	17	12	19	45

Session 17 - Meet 10
November 19, 2015

Mentors: Bridget Bassi, Anna Ellison, Neslly Estrada, Jennifer Matthews, Anuhya Vajapeyajula, Jane Wang (Head)

Modular origami and the arithmetic-geometric mean inequality took center stage at this meet. Can you devise a way to fold the arithmetic mean of two lengths using origami? How about the geometric mean?

Session 17 - Meet 11
December 3, 2015

Mentors: Bridget Bassi, Karia Dibert, Neslly Estrada, Isabel Vogt, Jane Wang (Head)

Visitors: Catherine Kennedy and Jinger Zhao of TwoSigma

Jinger Zhao uses data to predict the future. She is a modeler at TwoSigma, a hedge fund in NYC. She holds degrees in neuroscience and computer science.

Mathematics has been her companion from a very early age. Already at age 6, her parents would keep her busy by giving her math challenges. For example, they'd write numbers on a piece of paper and ask her to multiply them.

One day, they asked her to write down the perfect squares from 1 to 100. She did and noticed something. She focused on the difference between consecutive perfect squares, such as $8^2 = 64$ and $9^2 = 81$. Their difference is 81 – 64, or 17. She noticed that 17 is 8 + 9. In other words, she saw that the difference between the nth and the $(n + 1)th$ perfect square was always equal to the sum of n and $n + 1$. She asked her parents if this was just a coincidence or if it was always true. They shrugged.

Later, when Jinger knew some algebra, she was able to prove the algebraic identity

$$(n + 1)^2 - n^2 = (n + 1) + n,$$

and see that this fact was no coincidence. It is a universal truth, and it turned her on to math.

To illustrate how she uses math to make predictions, she collected the wingspan (a.k.a. arm span) and height of the girls, save for 2 of them, and plotted this data against each other. She used linear regression to make a linear model connecting wingspan to height and used her model to predict the wingspan of the 2 girls left out of the original data set. For details, see Math In Your World on page 9.

Jinger introduced and explained a number of statistical concepts, including the arithmetic mean, variance, standard deviation, and covariance, applying each concept to the wingspan vs. height data.

Jinger concluded her presentation by describing a number of additional applications of these statistical methods, including her analysis of marathon runners. She and her husband both run marathons, and when they ran their first one together, she beat him. Afterward, she suggested that she won because he started off too fast and pooped out at the end. She hypothesized that men, in general, would start off too fast only to poop out at the end. She analyzed runner data and discovered that the data confirms this hypothesis.[1]

Other applications she mentioned: predicting election results, examining how test scores correlate with gender or demographics, predicting stock market moves based on the previous day's emotions as computed by tweets on Twitter, predicting sales at Best Buy based on parking lot coverage, and correlating Fandango ratings with other movie ratings.

Session 17 - Meet 12 Mentors: Bridget Bassi, Karia Dibert, Anna Ellison,
December 10, 2015 Neslly Estrada, Jane Wang (Head)

We held our traditional end-of-session Math Collaboration!

Here's one of the problems from the Math Collaboration:

Fill in the blanks in the template below with the digits 0 through 9 so that the expression is maximized:

$$\_\,\_\,\_ \times \_\,\_ + \_\,\_ \times \_ + \_\,\_$$

(Consecutive blanks correspond to digits in the same number. For instance, the last two blanks represent at 2 digit number.)

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[1] See http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2516575.

Calendar

Session 17: (all dates in 2015)

September	17	Start of the seventeenth session!
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	Jinger Zhao, Two Sigma
	10	

Session 18: (all dates in 2016)

January	28	Start of the eighteenth session!
February	4	
	11	Anna Frebel, Department of Astronomy, MIT
	18	No meet
	25	
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________