Girls' Angle Bulletin

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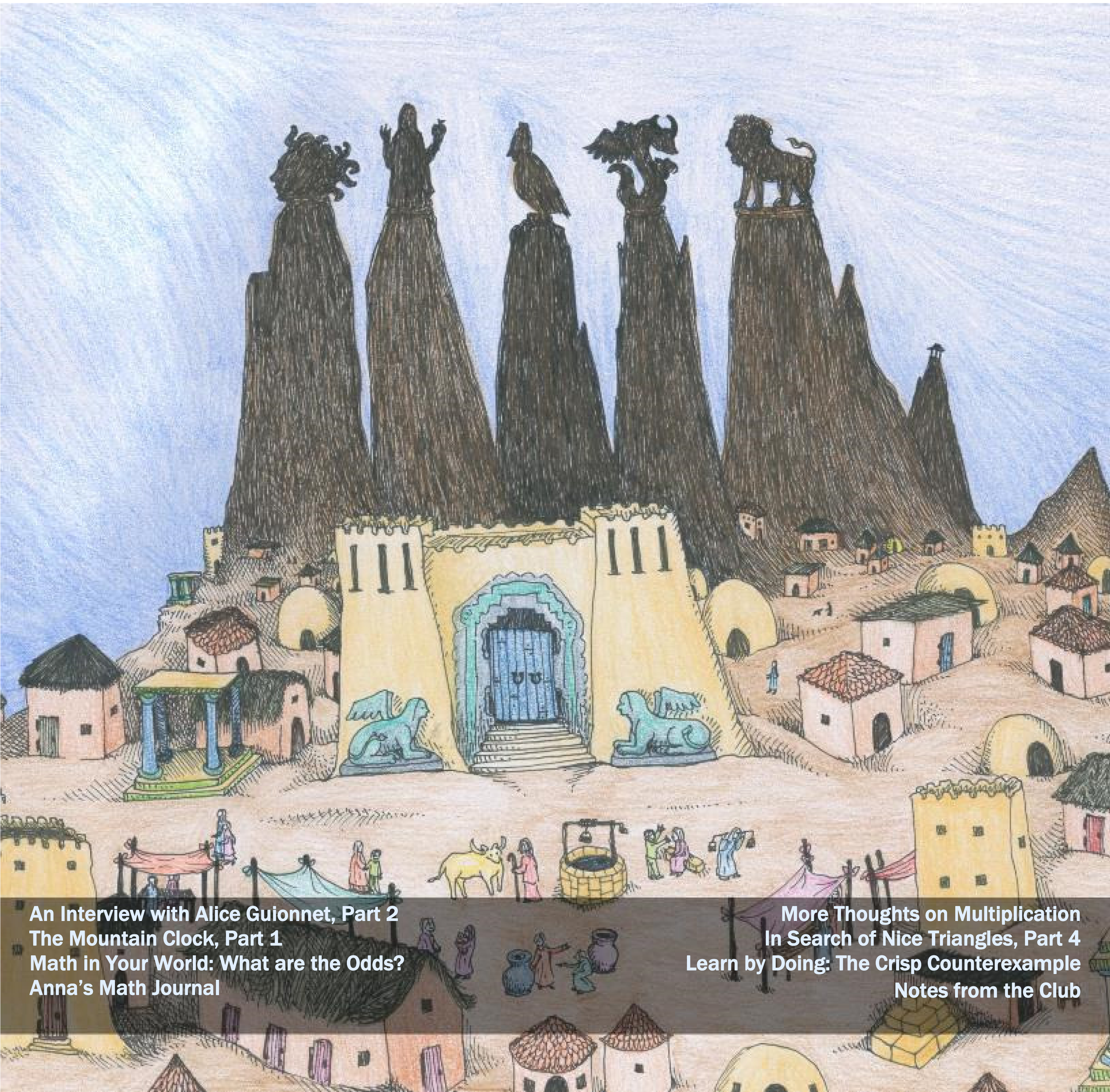
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

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- Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *High Hour* by Julia Zimmerman. To understand this illustration, read *The Mountain Clock* on page 7.

An Interview with Alice Guionnet, Part 2

This is the concluding half of our interview with MIT Professor Alice Guionnet.

Ken: You seem to be saying that in some cases, if you try to extract information using deterministic formulas, the formulas may be so daunting to solve, you get nowhere. But if, instead, you treat the problem randomly, while you may not be getting exact information about the system, you can make progress in the sense that you can deduce information about the statistics of the system.

Alice: Yes, another illustration of this is compressed sensing. For example, think of scanning an image. A scan can take a long time if it scans *everything*. To make it faster, the idea is to take fewer images. But if you take a very specific sample of the image, you could miss the interesting part. However, if you take random samples of the image, with a certain probability, you can recover the image (if you make some assumptions about the nature of the original image).

By using randomness, you can recover the image with very large probability. It's really the law of large numbers. When you take a random sample, you still somehow get all the information. In some sense, randomness, enables you to "avoid having holes in your data."

Ken: Why are you particularly interested the application to operator algebras? What kinds of questions about operator algebras are you interested in addressing through the use of random matrices?

Alice: Random matrices turn out to be approximations of nice operators. In particular, independent matrices approximate free operators, that is, operators which have no relations between them, so that you cannot write a non-trivial word in these operators which would give the identity. Random matrices can thus be used to understand better problems with free operators. The type of questions we dream to solve is about isomorphisms of algebras, like von Neumann algebras. The idea is that classical probability provides tools to compare or measure things, like entropy, dimension etc. Via random matrices, we can get ideas to generalize these concepts to operator algebras, hoping that we will this way define proper and powerful tools. This was successful to some extent, but not easy.

Ken: I suppose that when one speaks of a random matrix, one must specify a distribution for the matrices. How do you decide which distributions are of interest or are worthy of study? Naively, why should certain matrices be weighted as more likely than others?

Alice: Often in probability things are modeled starting from an assumption of independence. Hence, the idea is in general when you want to model a random matrix to take it as independent as possible within the constraints you know that you have. This is at least what the physicist Dyson[1] was saying: you have some constraints imposed by the physics and then take everything else as random as possible. If you have no idea about the distribution of each entry beside their independence, you often take Gaussian variables, as they appear naturally after the central limit theorem.

[1] Freeman Dyson

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Alice Guionnet and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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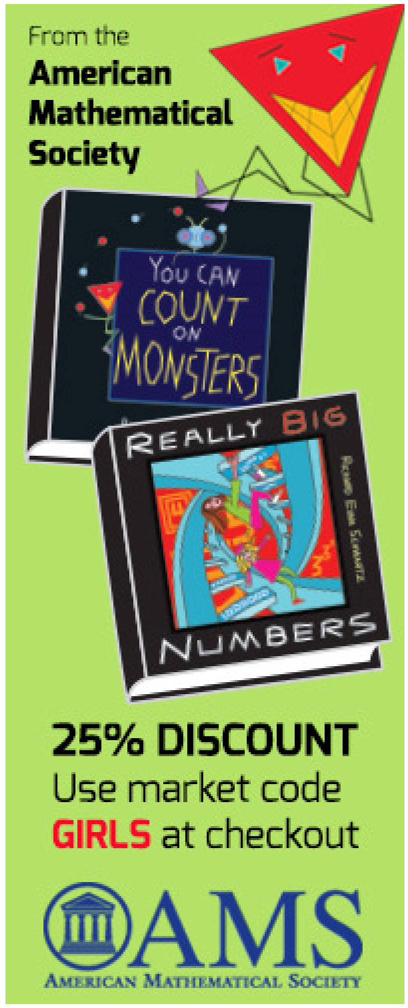

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Girls' Angle

The Mountain Clock, Part 1

by Ken Fan | edited by Jennifer Silva

A long time ago, in a city whose name has long since been forgotten, there lived a remarkable people led by a benevolent King. The King was so fond of ideas that he would say, "Good cities are based on beautiful things, great ones on beautiful ideas."

The King himself transformed his city into the envy of the ancient world by implementing one simple idea. While the peoples of all other kingdoms organized their days into 24 hours, this King decreed a 32-hour day.

"What difference could that possibly make?" you might wonder. "Earth spins at its own steady clip, regardless of how humans choose to tinker with the reportage of time." And that is true, but think about how much of your life is organized around the 24-hour day. How long are the vast majority of lectures, classes, meetings? At what times do people tend to meet? Do you typically hear people say, "Let's meet at 11:23 for lunch tomorrow"? No! They say, "See you at 11!" It is just like length. Mandate a unit of length and, suddenly, all manner of planks, walls, rooms, playing fields, and patios measure whole multiples of that standard.

Besides, reasoned the King, people waste so much time. If people would concentrate harder, everything could be done in three-fourths the time. And so the king decided upon a new hour that lasted 45 of our minutes long. While we take a recommended 8 hours of sleep leaving 16 hourly slots in which to schedule our daily tasks, the fortunate subjects of this ancient King slept a recommended 12 and still had 4 extra slots to play with during their waking hours. In sum, they got more sleep and more done each and every day than everybody else.

Though the city eventually flourished with the implementation of a 32-hour day, the initial adoption was tortuous. All the ancient inventions created to help people keep track of time were based on a 24-hour day, thus were useless under the new mandate. For months after the King's decree people could only guess at the time or ask a neighbor, who would guess or ask another neighbor … People were thrown so out of sync with each other that half the town thought it was 26 o'clock when the other half thought it was 27 o'clock. Businesses were sent into chaos as stores were closed when customers assumed they'd be open. I'm sure you can relate to that feeling you get when you're craving a doughnut; you pull up to the doughnut shop and yank on the door, but the door won't budge because the shop is closed. Romances soured when suitors' dates failed to show up on time, or, more accurately, *disagreed* on the time.

These problems weighed heavily on the King, who became withdrawn, deep in thought, seeking a solution to the crisis he had caused.

The solution came to the King in a flash of insight. He would build a massive clock high up on the mountain. The clock would be visible to all and would sync up the city.

The mountain boasted five great peaks. On each peak, the King would perch a colossal statue. "What is amazing," explained the King to the Royal Council, "is that there are exactly 32 subsets of a set with 5 elements. By associating each hour with a different subset of the 5 statues, we can show the time by putting different subsets of the 5 statues on display."

"Are there really 32 subsets of a 5-element set?" asked a member of the Council.

"Indeed there are," assured the King. "For we can determine a subset by asking each of the five elements, 'Are you in the subset or not, yea or nay?' Since each element answers independently, we have a total of 2 to the 5^{th} subsets; that, counsellors, works out to 32."

"But, King, sir," inquired another, "how will the people know which subsets correspond to each hour?"

"Why don't we enlist the help of the Royal Academy?" suggested another.

"No need, my friend!" responded the King. "I've worked that out, too! We will employ binary numbers."

"Ah, of course!" said the Royal Council members. "You're the King!"

"Each number from 0 to 31 corresponds to a 5-digit binary number," explained the King. "The digits of a binary number are either 0 or 1. Each statue will correspond to one of the binary place values. When the digit in the corresponding place value is 1, the statue will be put on display, and when it is 0, the statue will be pulled back out of sight from the city.

"I will personally train a group of Royal Time Keepers who will be entrusted with moving the statues from one hour to the next."

"Brilliant!" exclaimed the Royal Counsellors.

The King recruited 400 Royal Time Keepers. Applicants had to be equal parts strong, responsible, and able to count in binary. These 400 Royal Time Keepers were split 80 to a statue, and these 80 were, in turn, split into 4 teams of 20. The 4 teams each took 8-hour shifts.

Tryouts were not easy. The time statues were enormous. Every sinew of all 20 members of a time keeper team was needed to quickly push or pull the statue to its appropriate binary state. The best time keeper teams could change the binary state of a statue in less than 30 seconds!

The Royal Time Keepers were idolized. They were physically fit, dependable, and smart. For their important service to the city, they were rewarded with free meals and housing. Children aspired to become Royal Time Keepers. They'd practice pushing heavy things and counting in binary until they could rattle off the binary code for any hour. "What is 17 o'clock?" a parent would quiz. "Frogfish and Lion stare each other down!" "And High Hour?" "That's 31 o'clock, when all statues are on display!" "And the Hour of the Bird Lady?" "That's 4 o'clock!"

Mountain Clock Schedule

Hour	Statue Code				
	Frogfish	**Apple Bearer**	**Bird Lady**	**Dragon**	**Lion**
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	1	0
3	0	0	0	1	1
4	0	0	1	0	0
5	0	0	1	0	1
6	0	0	1	1	0
7	0	0	1	1	1
8	0	1	0	0	0
9	0	1	0	0	1
10	0	1	0	1	0
11	0	1	0	1	1
12	0	1	1	0	0
13	0	1	1	0	1
14	0	1	1	1	0
15	0	1	1	1	1
16	1	0	0	0	0
17	1	0	0	0	1
18	1	0	0	1	0
19	1	0	0	1	1
20	1	0	1	0	0
21	1	0	1	0	1
22	1	0	1	1	0
23	1	0	1	1	1
24	1	1	0	0	0
25	1	1	0	0	1
26	1	1	0	1	0
27	1	1	0	1	1
28	1	1	1	0	0
29	1	1	1	0	1
30	1	1	1	1	0
31	1	1	1	1	1

Order was restored to the city, and the sleep-deprived King managed to get in a good 16 hours of sleep. But this period of calm did not last long.

"Great King, sir! There is discord among the Royal Time Keepers!" came the disturbing news from a Royal Counsellor.

“Discord?” said the King, taken aback. “But they’re all treated the same, and treated extremely well! How can there be discord?”

“The Lion team is complaining about the Frogfish team, sir!”

“Complaining? What is their complaint?” asked the King.

“While the Lion teams toil every hour, the Frogfish teams only toil twice each day. In fact, two of the Frogfish teams just lounge around the mountain or play soccer during their entire shift!”

The King thought quietly to himself. “Oh dear, that would be the case. In fact, the Apple Bearer teams don’t have to work much harder than the Frogfish teams. What can be done?”

Fortunately, this difficulty wasn’t hard for the King to resolve.

“From day to day, let the teams rotate their shifts and statue assignments!” commanded the King.

As you might expect, the different time keeper teams varied in their ability to move the statues. Some were faster than others. And with the new rotation scheme, one never knew which statues would be moved quickly and which more slowly. These variations caused new problems.

The transition from 15 o’clock to 16 o’clock was especially fraught with confusion each day. During one particularly poor transition, the Lion team was fastest, followed by the Apple Bearer, Bird Lady, Dragon, and finally, the Frogfish team. The result was that the mountain clock read 01111 during the fifteenth hour, then changed briefly to 01110, then to 00110, 00010, and 00000, finally settling on 10000. Even though the clock showed these spurious readings for mere seconds, it was always enough to create problems down in the city. A busy courier glanced up at the mountain and saw that it was only the fourteenth hour and not the sixteenth, so she took a quick breakfast. As a result, an important message did not get delivered on time, not to mention the indigestion the courier got when she looked up the mountain and found that time had lurched ahead two hours just over the course of her meal.

The confusion and complaints mounted. Time keeper teams began blaming each other for the city’s problems.

And then came a catastrophe. It happened barely a year after the 32-hour day mandate. A hawk family had taken up nest atop the nose of the Lion statue. The King could not tolerate a hawk family living on the tip of a time statue! He ordered the Royal Time Keepers to take the nest down and move it onto a nearby ledge.

Normally, the third teams would arrive just before the seventeenth hour, but this day, the third Lion team hiked up the mountain early to dislodge the hawk nest. A brave young Royal Time Keeper climbed the statue. With feet planted on the statue’s shoulders and one hand grasping an ear, the Time Keeper made a leap for the nose but slipped and crashed down. He bounced off the rump onto his fellow Time Keepers below and injured several, himself most of all.

Soon it was 17 o’clock, and the uninjured tried to move the statue back but lacked the strength. The Mountain Clock got stuck at 16 o’clock. A fleet-footed Time Keeper ran to the city to inform the King and beseech medical aid. The King immediately dispatched the fourth Lion team to move the Lion statue, as well as the Royal Medics to tend to the injured. But before they could hike up the mountain, the city was once again thrown into time chaos.

“A good clock is essential to a well-oiled society,” thought the King. “There must be a better way.” Unable to resolve the problem, the King stormed into the Royal Council chambers.

“Summon the Dean of the Royal Academy!”

To be continued …

What are the Odds?

by Katherine Cliff
edited by Jennifer Silva

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Last issue, we explored how exponentials pop up in real life, like multiplying 4 together many times to find out how many possible combinations a Speed Dial Master Lock has. But sometimes, we have to be a little choosier about how we multiply our numbers together. Let's look at the lottery as an example.

Remember a few weeks ago when people went crazy over the Powerball lottery? It seemed like everyone over 18 years old bought at least one ticket. Parents were secretly daydreaming about taking three-week cruises through Europe and sending their kids to Ivy League schools without needing to worry about scholarships. Everyone seemed to think, *Someone will win, and it might as well be me!*

How likely is a person to win, though? All we need is a bit of knowledge about the game and some multiplication, and we can give this situation a reality check.

A year ago, the Powerball lottery was actually a bit easier to win. There were 59 numbered white balls and 35 numbered red balls; we'll use these parameters for our first scenario.

The red balls are known as "powerballs." On lottery night, 5 white balls and 1 powerball are drawn; if your ticket matches the white balls (in any order) and the powerball, you win the jackpot. Fortunately for lottery players, this isn't an exponential situation: we don't have to multiply 59 by itself 5 times to find out how many combinations of white balls there are, because a white ball can't be drawn twice.

The idea is this: for the first ball, we have 59 options. Once we have chosen that ball, there are only 58 options left for the second ball, then 57 for the third ball, and so on. That gives us $59 \times 58 \times 57 \times 56 \times 55 = 600{,}766{,}320$ ways to pick 5 white balls from those 59. Note that this method cares about the order that we draw the balls: 1, 2, 3, 4, 5 is considered a different case from 5, 4, 3, 2, 1, because the numbers are in a different order. The lottery isn't concerned about the order of the white balls, so we need to get rid of the repeat cases of the same numbers arranged in different orders. To do this, we look at how many ways there are to arrange a set of 5 balls, using the same kind of thinking as before: first we have 5 balls to choose from, then 4, then 3, etc. We'll have $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$ ways of arranging any given set of 5 balls. So, for any distinct set of 5 balls, there are 120 ways to arrange them; we're counting each outcome for the white balls 120 times. We siphon out all of those repeats by dividing, so we get $600{,}766{,}320 \div 120 = 5{,}006{,}386$ ways of picking those first 5 balls.

This hints at a handy formula for calculating how to choose k objects from a total of n choices when you don't care about the order of the objects. To choose k objects in a specific order, we have $n!/(n-k)!$ ways. (Crosscheck that with our calculation for choosing 5 white balls from 59: $59 \times 58 \times 57 \times 56 \times 55$.) To get rid of

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the repeat outcomes – where we have the same set of objects arranged in different orders – we divide by the number of ways k objects can be ordered, which is $k!$. Our formula for the number of ways k objects can be chosen from n objects is

$$\frac{n!}{(n-k)!k!}.$$

For more on choose notation, see "Notation Station" on page 24 of Volume 8, Number 2 of this *Bulletin*.

Mathematicians love a good shorthand, so we introduce the notation $\binom{n}{k}$ or ${}_nC_k$ for $\frac{n!}{(n-k)!k!}$. It's called a **combination**, and we refer to ${}_nC_k$ as "n choose k."

But wait, there's more! We still have the powerball to consider. That gives us another 35 possibilities for every combination of white balls, so in the end there are a total of 5,006,386 × 35 = 175,223,510 possible ways to choose the 5 white balls and the powerball. Only one of those possible outcomes will be chosen, so the probability of winning is 1 in 175,223,510.

Recall that when outcomes are equally likely, the probability of a favorable outcome occurring is equal to the number of favorable outcomes divided by the total number of possible outcomes.

Those odds are pretty bad! If you and everyone in the 12 most populous states in America wrote your names on slips of paper that were put into large pot and shaken up, then we picked a name *Hunger Games*-style, you would have about the same chances of being picked as of winning the aforementioned lottery. In fact, if each of those slips were an inch long and we lined them up end-to-end, they would stretch from Boston to San Francisco with another 70 miles of slips to spare.

Now that we have a sense of the scale of those chances, let's look at the current version of the lottery. Last year, the Powerball game changed: now there are 69 white balls, and only 26 powerballs. How does that change the probability of winning the jackpot? Our new calculation is

$$\binom{69}{5}\times 26=\frac{69!}{(69-5)!5!}\times 26=292,201,338.$$

That's where that popular "1 in 292 million" statistic comes from. To comprehend those odds, now we'd need everyone from the 31 most populous states in our large name-pot. That's most of the country! In other words, the chances are astronomically small that you'll win the lottery.

Even when people accept that the odds of winning the lottery are awful, they tend to ask hopeful questions. For instance: what if I bought a lot of tickets? Okay, sure. If you buy ten tickets, you increase your chances of winning: now you're at a whopping 10 in 292,201,338 chance of winning – still not great. If you wanted to bring your chances down to a good old-fashioned 50-50 coin flip (i.e., you're equally likely to win as not), you would need to buy 146,100,669 tickets; at \$2 per ticket, that would cost you over \$292 million dollars, and you're still not guaranteed a win. Buying a lot of tickets isn't a lucrative strategy.

The next hopeful question goes something like this: since tickets are only \$2 each and the jackpot is so huge, isn't it "worth it" to play? The answer to this question isn't straightforward. Before we can answer it, we need to understand the idea of expected value.

We'll take a step back from the lottery for a moment. Think about a single fair die, numbered from 1 to 6. Each time we roll the die, we have a 1/6 chance of rolling a 1 (since there is one 1, and six total numbers). In fact, there is a 1/6 probability of rolling a 2, a 1/6 probability of rolling a 3, and so on. Now let's roll that die hundreds of times, then average the values of all of those rolls. What would we expect the average to be? That's the expected value of rolling the die. If we roll the die N times, where N is a large number, we expect each number to come up $N/6$ times, so the average of all of the outcomes would be:

$$\frac{1}{N}\left(1\cdot\frac{N}{6}+2\cdot\frac{N}{6}+3\cdot\frac{N}{6}+4\cdot\frac{N}{6}+5\cdot\frac{N}{6}+6\cdot\frac{N}{6}\right)=\frac{7}{2}.$$

Remember that this is an average over a whole bunch of rolls, so it's okay that we get an expected value that's not even on the die. Notice that the N cancels out in the computation; to compute the expected value we could have simply added up the product of each outcome with its probability of occurring.

Now we're ready to compute the expected winnings for the lottery. The first thing to consider is that the jackpot isn't the only prize you can win. For example, you can win $100 if you match 4 of the white balls or if you match 3 of the white balls and the powerball. The Multi-State Lottery Association provides a table of the probabilities of winning each prize, but their probabilities are approximations. With the information in this article, you can determine the exact probabilities. Can you complete the following table? (For the answers, see the end of this article.)

Prize	How to Win	Probability
Grand Prize	○○○○○ + ●	1 in 292,201,338
$1,000,000	○○○○○	
$50,000	○○○○ + ●	
$100	○○○○	
$100	○○○ + ●	
$7	○○○	
$7	○○ + ●	
$4	○ + ●	3,176,880 in 292,201,338
$4	●	7,624,512 in 292,201,338

As an example, we will show how to compute the last row, which represents the probability of matching just the powerball. To match *just* the powerball, all 5 of your white ball picks must *not* be drawn during the lottery drawing. There are ${}_{64}C_5$ ways to pick 5 white balls from among those that were not drawn. So the probability of missing all the white balls is ${}_{64}C_5/{}_{69}C_5$. The probability of matching the powerball is 1/26. Therefore, the probability of matching *just* the powerball is ${}_{64}C_5/{}_{69}C_5 \times 1/26$, which works out to 7,624,512 in 292,201,338. (Notice that this is less than the probability of matching the powerball, which is 1/26. If you add up the probabilities you compute for each of the prizes that require a powerball match, you should get 1/26. That's a good way to check some of your mathematical computations.)

If we want to calculate the expected value of playing the lottery, we would multiply each prize by the probability of winning the prize, then add up all of those values. Without the grand prize, the expected value is approximately 32 cents. That's terrible! (Check this.) People pay $2 to play for an average winnings (without the grand prize) of only 32 cents!

Can you figure out how big the grand prize needs to be for the expected value of the winnings to be $2?

Spoiler Alert! Try to compute this before reading further.

We find that the grand prize needs to be $490,936,628 in order for the expected value of the winnings to be $2. By that standard, it seems like playing that big lottery in January wasn't all that crazy, since the jackpot was over $1 billion.

Do you know what we forgot? Taxes. The lottery withholds taxes from any winnings larger than $5,000, so we need to reconsider our calculations for the top three prizes. Between federal and state taxes, let's assume that you'll pay 50% of your winnings back to the government. This effectively reduces the value of the top three prizes to half of their stated values. Can you show that the grand prize must then be $1,022,873,256 for a ticket to be worth $2?

But wait! We have to be very careful. These figures represent computations based on the assumption that you are the *sole* winner. If there are multiple winners of the grand prize, then the grand prize is split evenly among each winning ticket. When more people play, the probability that there are multiple winners increases.

If you want to dig a little deeper and refine this number even more, you might consider the difference between taking your winnings as a lump sum (which means you get less money overall) and the 30-year annuity. That's the stuff accountants like to dream about.

Take it to Your World

Design a lottery of your own. Determine the winning outcomes and prizes. Compute the probabilities of winning the various prizes and the expected value of the entire game.

What would be the probability of winning the jackpot in the Powerball lottery if you not only had to pick the 5 white balls and powerball, but you also had to pick the 5 white balls in the order that they are drawn?

Think about how to compute the expected winnings if N tickets are purchased independently of each other. That is, each of the 292,201,338 different ways to pick the 5 white balls and the powerball is equally likely to be chosen for each ticket. How can you account for the possibility of having to share the grand prize?

Here are the probabilities for the Powerball lottery:

Prize	How to Win	Probability
Grand Prize	○○○○○ + ●	1 in 292,201,338
$1,000,000	○○○○○	25 in 292,201,338
$50,000	○○○○ + ●	320 in 292,201,338
$100	○○○○	8,000 in 292,201,338
$100	○○○ + ●	20,160 in 292,201,338
$7	○○○	504,000 in 292,201,338
$7	○○ + ●	416,640 in 292,201,338
$4	○ + ●	3,176,880 in 292,201,338
$4	●	7,624,512 in 292,201,338

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues thinking about irreducible polynomials over the finite field with 2 elements.

I feel like making a table summarizing what I know about roots of irreducible polynomials over $\mathbb{F}_2$.

Size of field	elements are roots of ...
2	x, $x+1$ all irreducible linear t, $t+1$
2^2	t, $t+1$, t^2+t+1 all irred linear and quadratic
2^3	all irred. linear and cubic
2^4	all irred. linear, quadratic, quartic.
2^5	all irred linear, quintic.

Hm... The degrees of the irreducibles divide the exponent in the size of the field... Could that be true in general?

I think I'll state this as a conjecture.

Conjecture 1. The elements of the finite field with 2^n elements are exactly the roots of all irreducible polynomials over $\mathbb{F}_2$ of degrees that divide n.

I've also begun to suspect that irreducible polynomials have completely different roots. Since I'm stating conjectures, I might as well write this too.

Conjecture 2. Different irreducible polynomials over $\mathbb{F}_2$ have completely different roots.

Let N_d be the number of irreducible polynomials over $\mathbb{F}_2$ of degree d.

An irreducible polynomial of degree d has d roots, so the total number of roots all N_d irreducible polynomials of degree d would provide is dN_d, if these conjectures are true.

$$N_1 = 1$$

$$2^n = \sum_{d|n} d N_d .$$

If these conjectures are true, then the number of elements in the finite field with 2^n elements must correspond to the total number of roots of all irreducible polynomials whose degrees divide n. This would be amazing!

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

I feel like using the formula to compute values of N_d.

$$2^2 = 1 \cdot N_1 + 2N_2 = 2 + 2N_2 \Rightarrow N_2 = 1 \quad \checkmark$$

$$8 = 1 \cdot N_1 + 3N_3 \Rightarrow N_3 = 2 \quad \checkmark$$

$$16 = 1 \cdot N_1 + 2N_2 + 4N_4 = 2 + 2 + 4N_4 \Rightarrow N_4 = 3 \quad \checkmark$$

$$32 = 1\,N_1 + 5N_5 \Rightarrow N_5 = 6 \quad \checkmark$$

Since the conjectures are based on the computations of N_d for d = 1, 2, 3, 4, and 5, it's no surprise to recover these values of N_d.

$$64 = 1 \cdot N_1 + 2N_2 + 3N_3 + 6N_6 = 2 + 2 + 6 + 6N_6 \Rightarrow N_6 = 9 \quad ?$$

$$128 = 1 \cdot N_1 + 7N_7 \Rightarrow N_7 = 18 \quad ?$$

But these values are new to me. I suppose I could count the number of irreducible polynomials of degrees 6 and 7 to see if these are correct.

Wait a sec! If the conjectures are true, I should be able to get a simple formula for N_p where p is prime.

$$2^p = 1 \cdot N_1 + pN_p \quad , \; p \text{ prime}$$

If the conjectures are true, this must always be an integer...

$$2^p - 2 \text{ must be divisible by } p \quad \checkmark$$

...and it is, by Fermat's little theorem!

$$N_p = \frac{2^p - 2}{p} \quad ?$$

Fermat's little theorem states that $a^p - a$ is divisible by p for all a, when p is prime.

I should get something nice when the exponent is a product of two primes, since there will only be 4 terms in the sum.

$$2^{pq} = 1\,N_1 + pN_p + qN_q + pq\,N_{pq} \qquad p, q \text{ prime}$$

$$= 2 + (2^p - 2) + (2^q - 2) + pq\,N_{pq}$$

$$pq\,N_{pq} = 2^{pq} - 2^p - 2^q + 2$$

$$\cancel{(2^p - 1)(2^q - 1) + 1}$$

Oops! What am I doing?

$$2^{pq} - 2^p - 2^q + 2 \equiv 2^q - 2 - 2^q + 2 \equiv 0 \pmod{p}$$

This is divisible by pq? Huh, I guess it is, since it is congruent to 0 mod p, and, by symmetry, to 0 mod q too!

This is kind of fun! I should get a nice formula when the exponent is the square of a prime too...

$$2^{p^2} = 1\,N_1 + pN_p + p^2 N_{p^2}$$

$$= 2 + 2^p - 2 + p^2 N_{p^2}$$

$$p^2 N_{p^2} = 2^{p^2} - 2^p$$

Curious. So if the conjectures are true, this should be divisible by p^2. I don't see why that'd be the case... I'll have to think about that more later.

Right now, I feel like trying to prove the conjectures. The second one feels easier... and since it deals with common factors, I bet I can use the Euclidean algorithm to show it.

Conjecture 2: Use Euclidean algorithm.

Let $p(t), q(t)$ be irreducible polynomials.

If $t - r \mid p(t)$ and $t - r \mid q(t)$, then $t - r \mid (p(t), q(t))$.

But $(p(t), q(t)) = 1$ since $p(t)$ and $q(t)$ are irreducible.

Yes, the Euclidean algorithm does imply it. If they have a common root, the highest common factor must exceed 1, which is not possible since both polynomials are irreducible.

ABB 2/24/16

Now for the first conjecture...

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

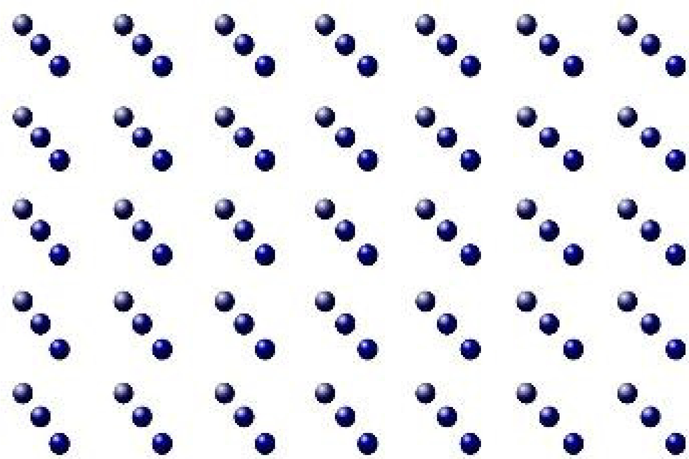
More Thoughts On Multiplication

by Addie Summer

The wonders of multiplication never cease!

To make communication clearer, I will use the symbol "×" for "times".

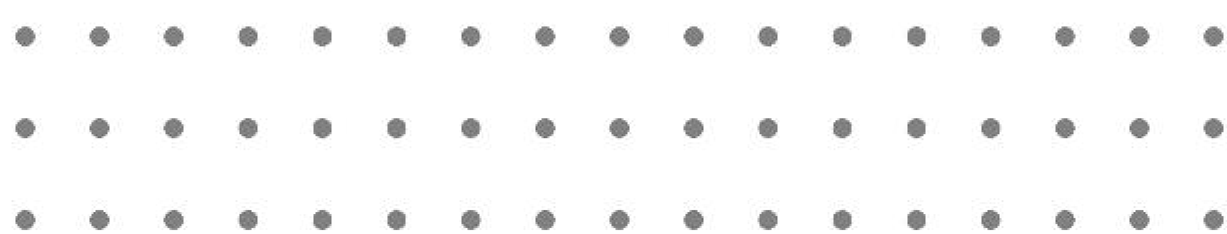
Let's go 3D. Take a look at the 3 by 5 by 7 block of dots shown below. How many dots are there? If we count the total number of dots in this block by adding up groups of 7 dots each in rows that are parallel to the front and top of the block, then we would have to compute how many such rows there are so we know how many 7s to add together. The number of such rows is the number of dots in the left face of the block, and that's a rectangular arrangement of dots which we know has 3×5 dots. So the total number of dots is $(3 \times 5) \times 7$.



On the other hand, we could count the number of dots in the 5 by 7 front face of the block, and then add this number as many times as there are planes of dots parallel to the front face. The front face is a 5 by 7 rectangular arrangement of dots, so there are 5×7 dots in it, and there are 3 planes of dots that are parallel to this front face. Counting this way, we find the total number of dots to be $3 \times (5 \times 7)$.

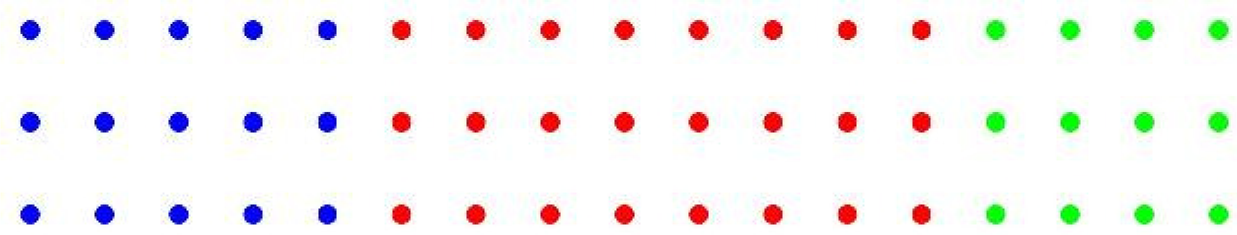
Thus, $(3 \times 5) \times 7 = 3 \times (5 \times 7)$, or, more generally, $(a \times b) \times c = a \times (b \times c)$, for any numbers a, b, and c. This property of multiplication, is called **associativity**. It tells us that if we have a product of several numbers, then the order in which we compute the products does not matter. For this reason, it is perfectly safe to write $3 \times 5 \times 7$ and not specify explicitly which product to compute first, because it doesn't matter. Either way, the result will be 105.

What else can we learn about multiplication by playing with rectangular arrangements of dots? Take a look at this rectangle of dots:



There are 3 rows with 17 dots in each row. We know that the total number of dots in this rectangular array is 3×17.

But now, I'm going to color the dots in this rectangular array.



All I've done is color the dots, so there are still 3 × 17 dots. But the coloring suggests another way to count the dots. We can separately count the numbers of dots of each color, then add these subtotals together. The dots of each color form smaller rectangles, each with a width of 3 dots.

The number of blue dots is 3 × 5.

The number of red dots is 3 × 8.

The number of green dots is 3 × 4.

Therefore, the number of dots is 3 × 5 + 3 × 8 + 3 × 4.

Notice that the number of dots in a row, 17, is also the sum of the numbers of dots in the rows of each colored rectangle. That is, 17 = 5 + 8 + 4. Thus,

$$3 \times 17 = 3 \times (5 + 8 + 4) = 3 \times 5 + 3 \times 8 + 3 \times 4.$$

There was nothing special about the actual width and lengths we used in this discussion, so what this illustrates is an instance of the general law that:

$$a \times (x + y + z) = a \times x + a \times y + a \times z.$$

And there's no particular reason why we split the big rectangle up into 3 smaller rectangles. We could just as well have split the big rectangle into 2, 4, 5, or more rectangles. In general, if you have some numbers $x_1, x_2, x_3, \ldots, x_n$, then

$$a \times (x_1 + x_2 + x_3 + \ldots + x_n) = a \times x_1 + a \times x_2 + a \times x_3 + \ldots + a \times x_n.$$

This remarkable property of multiplication is known as **distributivity**. The above identity, especially when there are only two variables x_1 and x_2, is referred to as the **distributive law**.

The distributive law corresponds to counting the number of dots in a rectangular array by splitting it into smaller rectangles, each with the same width as the original.

Together, the commutative, associative, and distributive laws give us a lot of flexibility when it comes to multiplying numbers. For example, suppose I need to multiply 25 and 21. I can use the distributive law to rewrite 25 × 21 as 25 × 20 + 25 × 1, since 21 = 20 + 1. Computing 25 × 1 is straightforward because that's adding up 25's just once, so that equals 25. And 25 × 20 is 25 × (2 × 10), which, by associativity, is (25 × 2) × 10. Now 25 × 2 is 25 + 25, or 50, so 25 × 20 is the same as 50 × 10, which is 500. Therefore 25 × 21 is 500 + 25, or 525.

In fact, many multiplication algorithms are based on the distributive law and writing one of the factors in so-called "expanded form," as we just did for 21 when we wrote it as 20 + 1. For example, compare the two computations for 281 × 142 shown at the top of the next page. One uses the distributive law explicitly and the other shows one of the standard multiplication algorithms. The standard algorithm merely supplies a template in which to organize an application of the distributive law.

$$
\begin{aligned}
281 \times 142 &= 281 \times (100 + 40 + 2) \\
&= 281 \times 100 + 281 \times 40 + 281 \times 2 \\
&= 28100 + 11240 + 562 \\
&= 39902
\end{aligned}
$$

$$
\begin{array}{r}
281 \\
\times\ 142 \\
\hline
562 \\
1124 \\
281 \\
\hline
39902
\end{array}
$$

In the multiplication algorithm, some zeroes are unwritten.

The patterns of multiplication are easier to see when we use variables instead of specific numbers. When using variables, we generally drop the multiplication symbol "×", partly to avoid confusion with the letter X, which is a commonly used variable.

For example, consider the expression $(a + b)(c + d)$ (i.e., the product of $a + b$ and $c + d$). If we apply the distributive law three times, we can express this product as $ac + ad + bc + bd$:

$$
\begin{aligned}
(a + b)(c + d) &= (a + b)c + (a + b)d \\
&= ac + bc + ad + bd.
\end{aligned}
$$

This algebraic identity shows us two ways to compute the product $(a + b)(c + d)$. Earlier, I explained the distributive law in terms of chopping a rectangle into smaller rectangles. Since the above identity is obtained by using only the distributive law, we should expect that there is also a nice interpretation of it in terms of the chopping up of rectangles. Can you find it?

Here's another beautiful multiplicative pattern: $(a + b)(a - b) = a^2 - b^2$. We can deduce this by applying the distributive law a few times to the left hand side of this equation, or we can reuse the computation we just made, substituting a for c and $-b$ for d. This identity tells us that we can turn the computation of a product into the subtraction of two squares.

We've skipped a discussion of multiplication involving negative numbers. For that, we refer the reader to *Negative Times Negative Is Positive*, on page 19 of Volume 3, Number 3 of this Bulletin.

For example, consider 28×22. If we can express this in the form $(a + b)(a - b)$, we could then compute the product by computing $a^2 - b^2$ instead. We seek a number, a, which yields 28 if you add a certain amount, b, and yields 22 if you subtract the same amount. In other words, on a number line, a has to be exactly halfway between 22 and 28, so $a = 25$. To get 28 from 25, we add 3, therefore $b = 3$, and $28 \times 22 = (25 + 3)(25 - 3)$. Using the identity, this is the same as $25^2 - 3^2$, and if you know the perfect squares well, this might be quicker for you to compute than multiplying 28 and 22 directly.

Can you find an interpretation of the identity $(a + b)(a - b) = a^2 - b^2$ in terms of counting rectangular arrangements of dots?

What patterns in multiplication can you discover?

If you're eager to explore more patterns in multiplication, but aren't sure what to try, here are a few suggestions:

1. Explore powers of $a + b$.
2. Explore triangular arrangements of dots.
3. Find a formula for the sum of all entries in an n by n multiplication table.

Write to us about anything you find: girlsangle@gmail.com!

In Search of Nice Triangles, Part 4

by Ken Fan | edited by Jennifer Silva

Emily and Jasmine continue their investigation into nice triangles.

Emily: This search for nice triangles is quite the journey!

Jasmine: And I feel like we're not even at the halfway point.

Emily: Where should we go next?

Jasmine: So far, we've determined all integer-sided triangles with 3, 2 – which is the same as 3 – or just one nice angle. Maybe we can reexamine triangles with three nice angles, but only two integer side lengths. That case would include some 30-60-90 right triangles.

Emily: Hmm. What you just said makes me want to look at the case of zero nice angles!

Jasmine: None at all?

Emily: Sure, why not? Or maybe the real question to ask is what triples of integers can be the side lengths of a triangle? After all, not all triples of whole numbers work.

Jasmine: Because of the triangle inequality …

Emily: Yes, because of the triangle inequality. Since the shortest path between two points is the straight one, the sum of the lengths of any two sides of a triangle must exceed the length of the third; otherwise, it would be shorter to go from one vertex of a triangle to another by walking through the third vertex.

Jasmine: The converse is also true: if you have three positive numbers and the sum of any two of them exceeds the third, then the three numbers can be realized as the side lengths of a triangle. If we pick any two positive integers, say a and b, can they always appear as side lengths of some integer-sided triangle?

Emily blankly stares forward as she imagines the situation in her mind's eye. She pictures sticks of lengths a and b joined and hinged together at a common endpoint forming an angle. She animates the angle so that it opens and closes, then closes off the angle with a line segment to form a triangle.

Emily: Yes – in fact, there's always an isosceles one if we add a side whose length is the greater of a and b.

Jasmine: Good observation! Let's see if we can determine exactly how many integer-sided triangles have a and b as side lengths.

Emily: In other words, you're asking how many positive integers, c, satisfy the triangle inequalities $a + b > c$, $a + c > b$, and $b + c > a$.

Jasmine: Right, and we might as well assume that $a \le b$ since we'd otherwise be able to just swap their labels. Given that we're fixing a and b, I'd like to isolate c in each of the triangle

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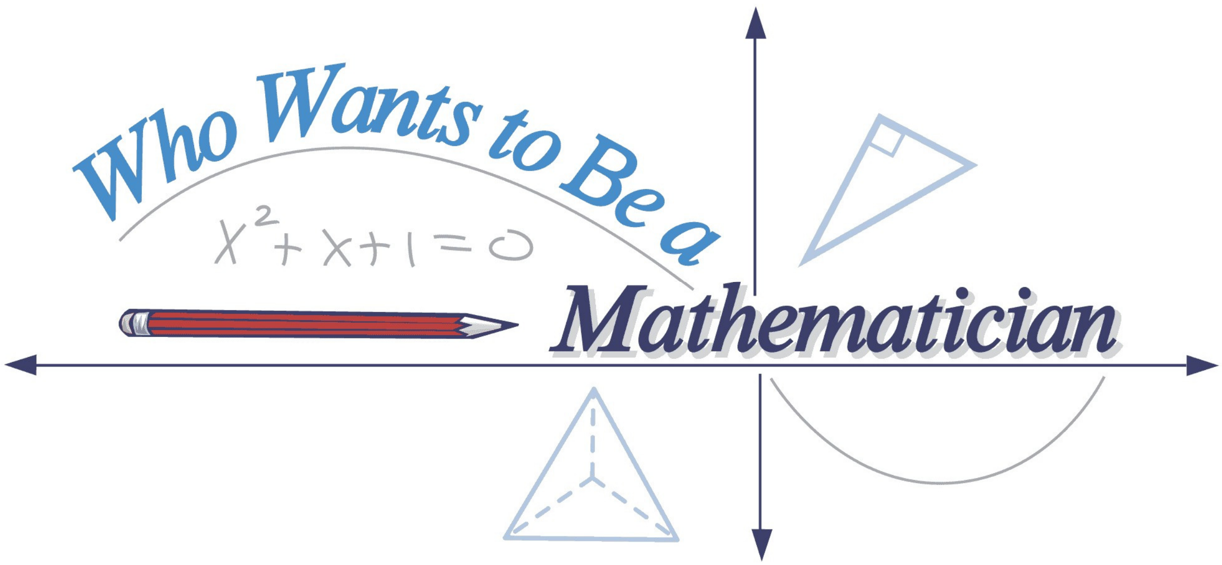

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Learn by Doing

The Crisp Counterexample
by Girls' Angle Staff

Mathematicians guess at the truth, state their guesses in precise statements called **conjectures**, and then attempt to prove these conjectures. In a conjecture, the mathematician provides a precise description of the assumptions being made. These assumptions are known as **hypotheses**. The conjecture concludes with a precise statement about what the mathematician thinks is true under the given hypotheses.

Often, conjectures turn out to be false, and there is no more concise a way to demonstrate that a conjecture is false than to provide a **counterexample**.

A counterexample to a conjecture is an example of something that satisfies all the hypotheses of the conjecture, but for which the conjecture's conclusion is false for that example.

Coming up with counterexamples can be difficult because conjectures aren't generally made unless a mathematician has good reason to believe that the conjecture is true. That means that to find a counterexample, one must observe something that the conjecturer hadn't noticed. Of course, if the conjecture is true, then it becomes impossible to find a counterexample.

Let's look at a couple examples of counterexamples.

<u>Counterexample 1</u>

Conjecture: Let x be a positive integer. Then $x^2 + x + 41$ is a prime number.

If you substitute some small whole numbers for x, say single digit values of x, you'll discover that for all those values, $x^2 + x + 41$ *is* a prime number. But the conjecture is false, and all it takes to show that it is false is one counterexample: $x = 41$. Notice that

$$41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \cdot 43.$$

The value $x = 41$ is a counterexample because the number 41 satisfies the hypothesis (that x be a positive integer), yet the conclusion is false: $41^2 + 41 + 41$ is composite. There are other counterexamples, but to show that the conjecture is false, all you need is one.

Notice that $x^2 + x + 41$ is composite when $x = -41$ too, but $x = -41$ is *not* a counterexample to this particular conjecture because -41 is not a positive integer.

<u>Counterexample 2</u>

Conjecture: Let a and b be real numbers. If $a^2 > b^2$, then $a > b$.

A counterexample is $a = -2$ and $b = -1$.

In this *Learn By Doing*, we will present a collection of falsehoods. Provide confirmation of their falsity by supplying a counterexample.

Falsehood 1. Let n be an integer. Then n is divisible by n.

Falsehood 2. Let p be a (positive) prime number. Then $p + 1$ is composite.

Falsehood 3. Every positive integer can be expressed as a sum of 3 or fewer perfect squares.

Falsehood 4. Let n be a positive integer. If $n! + 1$ is composite, then it is a perfect square.

Falsehood 5. The diagonals of a quadrilateral always intersect. (By "diagonal," we mean a line segment whose endpoints are two vertices of the quadrilateral that are not connected by one of its sides.)

Falsehood 6. Two circles in the plane are **externally tangent** to each other if they are tangent and their interiors do not intersect. The maximum number of mutually externally tangent circles in the plane is 3.

Falsehood 7. Let n be a positive integer. Suppose that n divides evenly into $5^n - 5$. Then n is prime. (Aside: Fermat's little theorem states that $a^p - a$ is divisible by p if p is prime.)

Falsehood 8. Let Q be a convex quadrilateral. (A region is convex if, for any two points in the region, the line segment connecting the two points is also in the region.) Pick a diagonal of Q. This diagonal splits Q into two pieces, both triangles. At most one of these triangles can be obtuse.

Falsehood 9. Let A, B, and C, be subsets of the integers. Suppose that their intersection is empty. Then some pair of them must have an empty intersection.

Falsehood 10. Let N be a positive integer and suppose that N is divisible by 11. If you alternately add and subtract the decimal digits of N, right to left, you always get 0.

In a Cartesian coordinate system, a **lattice point** is a point whose coordinates are integers.

Falsehood 11. In 3D Cartesian space, there does not exist a regular hexagon whose vertices are lattice points.

Aside: In the Cartesian plane, it is known that there does not exist an equilateral triangle whose vertices are lattice points. (For a proof, see, for instance, the Women In Mathematics video featuring Bridget Tenner on the Girls' Angle website.)

Falsehood 12. Let p be a permutation of the numbers 1, 2, 3, ..., n. (That is, p is a one-to-one and onto function from $\{1, 2, 3, ..., n\}$ to itself.) The **order** of p is the smallest positive integer k such that $p^{(k)}(x) = x$. (The notation $p^{(k)}(x)$ means to compose p repeatedly a total of k times and evaluate the composite function at x.) The claim is that the order of p is always less than or equal to n^2.

Falsehood 13. Draw a circle and mark a point on it with a dot. Fix an angle α. If you repeatedly rotate the dot through the angle α, then the dot will eventually return to its starting point.

Falsehood 14. Let a and b be real numbers that cannot be expressed as a finite decimal number. Then $a + b$ also cannot be so expressed.

Falsehood 15. The product of a number with a finite decimal expansion and a number that does not have a finite decimal expansion cannot be expressed as a finite decimal.

Falsehood 16. Let $f(x) = ax^2 + bx + c$. Suppose that $f(n)$ is an integer for all integers n. Then a, b, and c are integers.

Falsehood 17. It is impossible to partition a square into an odd number of squares.

Aside: Interestingly, a result of Paul Monsky states that a square cannot be partitioned into an odd number of triangles that all have the same area.

Falsehood 18. Five people meet. Some shake hands with each other, and some do not. The claim is that you can always find 3 people for whom either no pair of them have shaken hands with each other, or every pair of them has shaken hands with each other.

Falsehood 19. For a positive integer n, let $D(n)$ denote the number of ways that n can be expressed as a sum of distinct positive integers. For example, $D(4) = 2$ since 4 and 1 + 3 are the two ways that 4 can be written as a sum of distinct positive integers. (For this problem, a single number will be regarded as such a sum.) The claim is that for every positive integer m, there exists a positive integer n, such that $m = D(n)$.

Falsehood 20. Let S be a subset of the real numbers such that $0 \le x \le 1$ for all x in S. Then S contains a maximal element. (A maximal element is an element in S that is greater than or equal to every element of S.)

Falsehood 21. Let a_{nm} be a double sequence of real numbers. That is, for each pair of positive integers n and m, a_{nm} is a real number. Then, $\lim_{n\to\infty} \lim_{m\to\infty} a_{nm} = \lim_{m\to\infty} \lim_{n\to\infty} a_{nm}$.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 18 - Meet 1 January 28, 2016	Mentors:	Asra Ali, Bridget Bassi, Lydia Goldberg, Jenni Matthews, Wangui Mbuguiro, Jane Wang (Head)

Sometimes, understanding can be improved by focusing on only a part of the whole picture. A lot of mathematics is about finding an interesting part of the whole and studying just that part. For example, modular arithmetic is a way to study properties of integers by focusing on the remainders they leave after dividing by some fixed number (called the **modulus**). Knowing only the remainder means knowing the original integer only up to a multiple of the modulus. It is like knowing one facet of the original integer. And if you know the remainder with respect to several moduli, you can narrow down the value of the original integer (via the Chinese remainder theorem).

For example, suppose n is a positive integer less than 1000 and leaves remainders of 3 when you divide it by 7, 5 when you divide it by 11, and 6 when you divide it by 13. From this information alone, n is determined. Can you figure out what it is?

Session 18 - Meet 2 February 4, 2016	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Lydia Goldberg, Wangui Mbuguiro, Isabel Vogt, Jane Wang (Head), Sibo Wang
	Visitor:	Anna Frebel, Department of Astronomy, MIT

Anna Frebel is the Silverman (1968) Family Career Development Assistant Professor in the Department of Physics at the Massachusetts Institute of Technology. We were extremely fortunate to enjoy a visit from her. She held an hour-long Q&A with members. Here is a sampling of some of the questions and Anna's answers.

How do you know the age of a star? Light from a star is passed through a prism to reveal its spectrum. This **spectrographic analysis** is used to determine the chemical composition of stars. The oldest stars, which are the ones we see far away, are mostly hydrogen and helium. High abundance of higher elements indicates formation after generations as very high elements are made in supernova explosions. This is also why planets like Earth must come later than the oldest stars because Earth consists of many higher elements such as silver (atomic number 47).

What are the brightest stars? In the night sky, the brightest star is Sirius (which is also known as the dog star). I'm not sure what the second brightest star is, but some other bright stars are Canopus, Vega, and Betelgeuse, which is a red giant in the constellation Orion. Incidentally, the color of a star indicates how hot it burns, and when the hydrogen nuclear fuel is used up, the star burns less hot and turns redder, so Betelgeuse is near the end of its life.

When will the sun become a red giant? The red giant stage happens starting around 90% into the lifetime of a star. The sun is about 4.5 billion years old and has another 5 or 6 billion years left, so roughly in about 5 billion years.

What happens when a black hole swallows a star? This happens fairly often, in fact. The star gets stretched out as it spirals in. Part of its matter become part of the "accretion disc" before much of it is swallowed up. As a rough analogy, think of water going down the drain of a bathtub.

What is outside the universe? There is no inside or outside. The universe is all that there is. Though it is expanding, it is not expanding into something. It is simply expanding and things inside of it are growing further and further apart. This expansion is accelerating due to "dark energy." The current model predicts that eventually we will not be able to see any other galaxy.

How do we know the universe won't end? We don't actually know. We look at the data, we create models that explain what we see, and we make predictions by extrapolating the models into the future. Current models suggest the universe is infinite.

Session 18 - Meet 3 February 11, 2016	Mentors:	Asra Ali, Bridget Bassi, Karia Dibert, Anna Ellison, Neslly Estrada, Anuhya Vajapeyajula, Isabel Vogt, Jane Wang (Head)

Some members traveled to the basement of geometry, asking the most basic questions, such as, "what is a point?" and "what is a line?" How do points and lines relate? What are the most basic axioms of geometry?

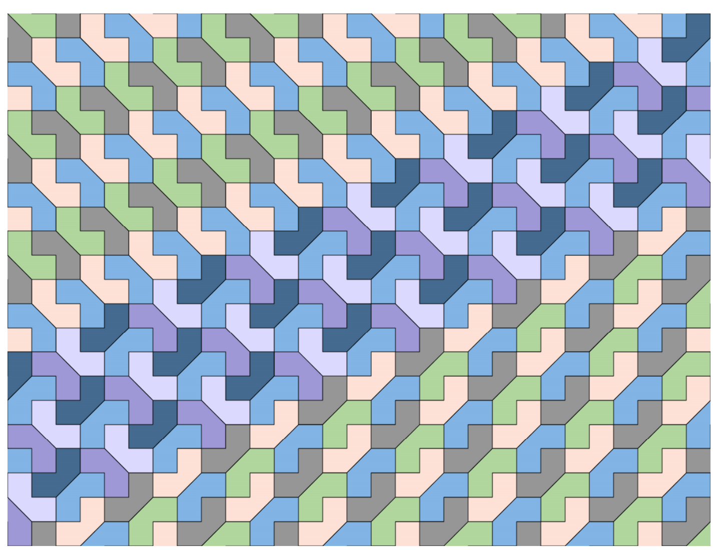
Session 18 - Meet 4 February 25, 2016	Mentors:	Karia Dibert, Anna Ellison, Neslly Estrada Hannah Larson, Wangui Mbuguiro, Isabel Vogt, Jane Wang (Head), Sibo Wang

Crooked politicians, secret codes, guards, and target practice were among the activities of the day. Some also worked on cooking up counterexamples, see the *Learn by Doing* on page 24.

This winter, 27 seventh graders at the Pollard Middle School in Needham, Massachusetts participated in a Girls' Angle Math Collaboration.

As part of the event, participants designed, completely from scratch, a 7-sided tile that can be used to tessellate the plane. They produced the marvelous tile used to form the tiling shown at right and determined its precise shape. As you can see, it's quite a versatile tile!

This was our 4th Math Collaboration at the Pollard Middle School, thanks to math teacher Vincent Marino, former Principal Lisa Chen, and current Principal Tamatha Bibbo. Mr. Marino has recently written a testimonial for Math Collaborations which you can read on the Girls' Angle blog.



Calendar

Session 17: (all dates in 2015)

September	17	Start of the seventeenth session!
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	Jinger Zhao, Two Sigma
	10	

Session 18: (all dates in 2016)

January	28	Start of the eighteenth session!
February	4	Anna Frebel, Department of Astronomy, MIT
	11	
	18	No meet
	25	
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

