Girls' Angle Bulletin

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To Foster and Nurture Girls' Interest in Mathematics

From the Founder

At Girls' Angle, mentors are directed to get members to do math through inspiration, not assignment. So it's a great feeling when a member is eager to show us some math they did between meets. The most recent time that happened was April 14, when one of our 11-year-old members brought in a marvelous recurrence relation for a sequence she had been investigating. For details, turn to this issue's *Member's Thoughts* on page 27.

- Ken Fan, President and Founder

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Girls' Angle welcomes submissions that pertain to mathematics.

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Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

On the cover: *The Mountain Clock* by Ken Fan and Julia Zimmerman. To understand this illustration, see *The Mountain Clock* on page 10.

An Interview with Susan Jane Colley

Susan Colley is the Andrew and Pauline Delaney Professor of Mathematics at Oberlin. She serves as Editor-Elect of *The American Mathematical Monthly*. She received her doctoral degree from the Massachusetts Institute of Technology.

Ken: What do you love about math?

Susan: Lots of things: its precision, its economy, to name just two characteristics. I admire its expressive power: how mathematics adds to our language and ability to communicate.

I also find that the best and most compelling mathematics has strong aesthetic qualities; the best arguments and proofs have a wonderful inevitability about them.

Ken: When did you realize that you wanted to become a mathematician?

Susan: When I was quite young, actually, around sixth grade. It was at that time that I began reading popular books about mathematics, such as Kasner and Newman's *Mathematics and the Imagination* and Newman's compilation *The World of Mathematics*. Those books—and others—made it clear to me that mathematics was full of great ideas.

Ken: Could you please describe a mathematical idea that caught your fancy from before college?

Susan: Two things: One was my informal understanding about topology (of the "rubber sheet" variety). I really liked the idea of how one could come up with a way to call two spaces "the same" when they were not the same in one way (i.e., not identical), but shared properties and characteristics that really did make them the same in a more fundamental way. The other idea that fascinated me was that one could prove something to be impossible (like trisecting an angle). It's the undeniable logic of the argument, I suppose.

Ken: Could you please describe your personal history as it relates to your becoming a mathematician? Was it an effortless process, or were there great challenges? Were there times when you had second thoughts?

I had to become better organized in order to give myself the time and tools to think about things adequately.

Susan: For the most part, I never had second thoughts. I've always felt that mathematics is the most intriguing, exciting, and appropriate subject for me and I have always wanted to share my love for it. I went to MIT as an undergraduate (and remained to pursue my Ph.D.), so I was surrounded by science and engineering. That was good for me, as prior to college I attended an all-girls school that did not place special emphasis on science. At the same time, I knew that laboratory work was not going to be my forte; I was not especially adept with the physical objects, nor especially patient with the notion that theory and reality didn't always agree. I suppose most people would have headed toward practical applications, but I was pushed toward pure mathematics!

In graduate school there was a difficult period for me. One particular faculty member was quite discouraging and I am not completely clear just why. At the time he predicted that I would not become a "research star" and therefore recommended that I should not plan to have an academic career in mathematics. In retrospect, I think that he held a narrow view of the career path for (academic) mathematicians: that it should minimize the teaching aspect, and that research should be of the very highest quality or not pursued at all. I had recently married and perhaps that may have caused this person to view me as therefore not sufficiently dedicated to mathematics. Fortunately, there were other faculty at MIT who were very supportive of me and helped to restore my self-confidence, although at times self-doubt still lingers. The best antidote for that, I've found, is simply to forge ahead.

Ken: What advice do you have for a student who aims to become mathematician? How should they best apportion their "math time"? What is the best way to study math?

Susan: The most important advice is to be patient—patient with the mathematics and with yourself—and to develop a tolerance for the frustration that inevitably occurs when attempting to solve a difficult problem.

I suppose that it goes without saying (although I'll say it anyway), but different people will study mathematics differently. I've always liked learning the literature surrounding a problem or area of mathematics, perhaps because even if I don't come up with a solution, at least I will have added to my own personal knowledge. I know that some others may find too much study to be constraining, however, and that it stifles creativity.

How you work on mathematics depends on you, of course, but having blocks of time to concentrate would seem to be essential for everyone. I don't know that means full eight-hour workdays without interruption, however. I find that I begin to get stale in my thinking after three or four hours, at which point I need to clear my head with some different type of task, at least for a while.

One thing that has been very important in sustaining me mathematically is to have a good collaborator. I have been fortunate to have worked with several people, but especially Gary Kennedy of the Ohio State University. Having some responsibility to a person as well as to the mathematical problem helps to keep me on task. It has also enhanced the social aspect of mathematics for me, something that one is not aware of when beginning one's career.

Here's a nifty observation due to Prof. Colley:

A cylinder's volume is $\pi r^2 h$, where r is the radius of the base, and h is the height. How much does this volume change if we change the radius by Δr and the height by Δh? Answer:

$$\pi(r + \Delta r)^2(h + \Delta h) - \pi r^2 h.$$

This expression is equivalent to

$$\pi(2rh\Delta r + r^2\Delta h + 2r\Delta r\Delta h + h(\Delta r)^2 + (\Delta r)^2\Delta h).$$

When Δr and Δh are small, this expression is closely approximated by $\pi(2rh\Delta r + r^2\Delta h)$. (Why?) A can of soda is roughly a cylinder with base radius 1" and height 5". At these dimensions, notice that the volume is 10 times more sensitive to changes in r than changes in h. That means that if you make the base circle a little bit smaller, to maintain the same volume, you'd have to increase the height quite a bit! Such a modification might fool people into thinking that the taller can is much bigger, when it's actually the same size. Nevertheless, do you think a soda company could get away with charging more for the taller can?

Prof. Colley published this observation in "Calculus in the Brewery," *The College Mathematics Journal*, **25** (1994), no. 3, 226-227. We urge you to read her article because she uses multivariable calculus and it has a nice illustration!

Ken: What is algebraic geometry to you? What are your goals in algebraic geometry?

Susan: Broadly speaking, algebraic geometry is the study of solutions to systems of polynomial equations. Since polynomials are mathematical objects that are both amenable to computation and can be used to approximate other functions, algebraic geometry can give an effective window into all geometry, and other parts of mathematics, too.

The subject of algebraic geometry is really quite vast and full of many subareas and I don't pretend to have mastery of the entire subject. (Few people, if any, do.) The particular area that I

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For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Susan Colley and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

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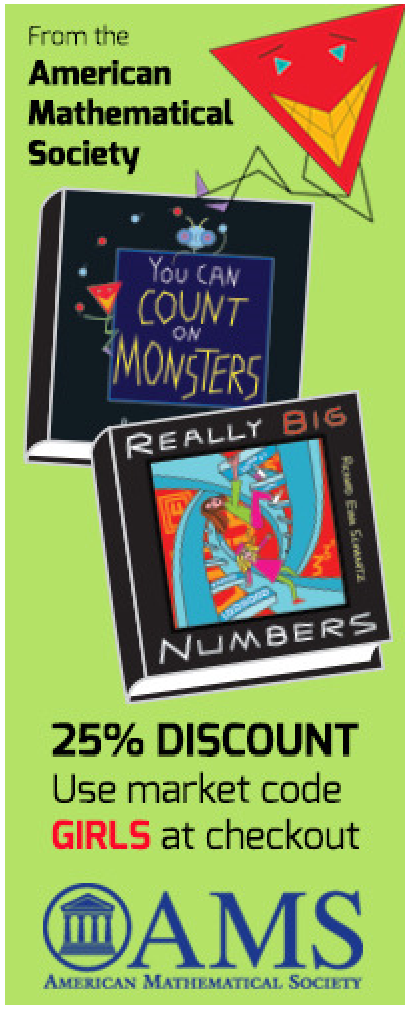


The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

Meditate$^{\text{Math}}$

Miquel's Theorem

The geometric diagram on the next page illustrates Miquel's Theorem, which is named after Auguste Miquel, a French mathematician who lived in the first half of the 19$^{\text{th}}$ century.

Here's the setup: Draw a triangle *ABC*. Now pick 3 points: *A*' on side *BC*, *B*' on side *CA*, and *C*' on side *AB*. You may pick any points on those sides that you wish. Next, draw the unique circle the passes through *A*, *B*', and *C*'. Similarly, draw the unique circle through *B*, *C*', and *A*' and the unique circle through *C*, *A*', and *B*'.

Does it look like the three circles have a common point of intersection?

Miquel's theorem states that they do!

Can you prove it?

Find a quiet place to meditate upon this diagram. Ignore time. Just observe. What do you see? What can you explain? Record your observations. Jot down any explanations.

If you'd like a little help to get going, here is a list of some things to consider.

1. The red and blue circles intersect in two points, *B*' and a point we'll label *M*. What kind of quadrilateral is quadrilateral *AB*'*MC*'? What kind of quadrilateral is quadrilateral *CA*'*MB*'?

2. Consider a quadrilateral inscribed in a circle (a so-called **cyclic quadrilateral**). What is the relationship between the measures of opposite angles in the quadrilateral?

3. Can you express the measure of angle *B*'*MC*' in terms of the angles of triangle *ABC*?

4. How do the three highlighted angles relate to each other?

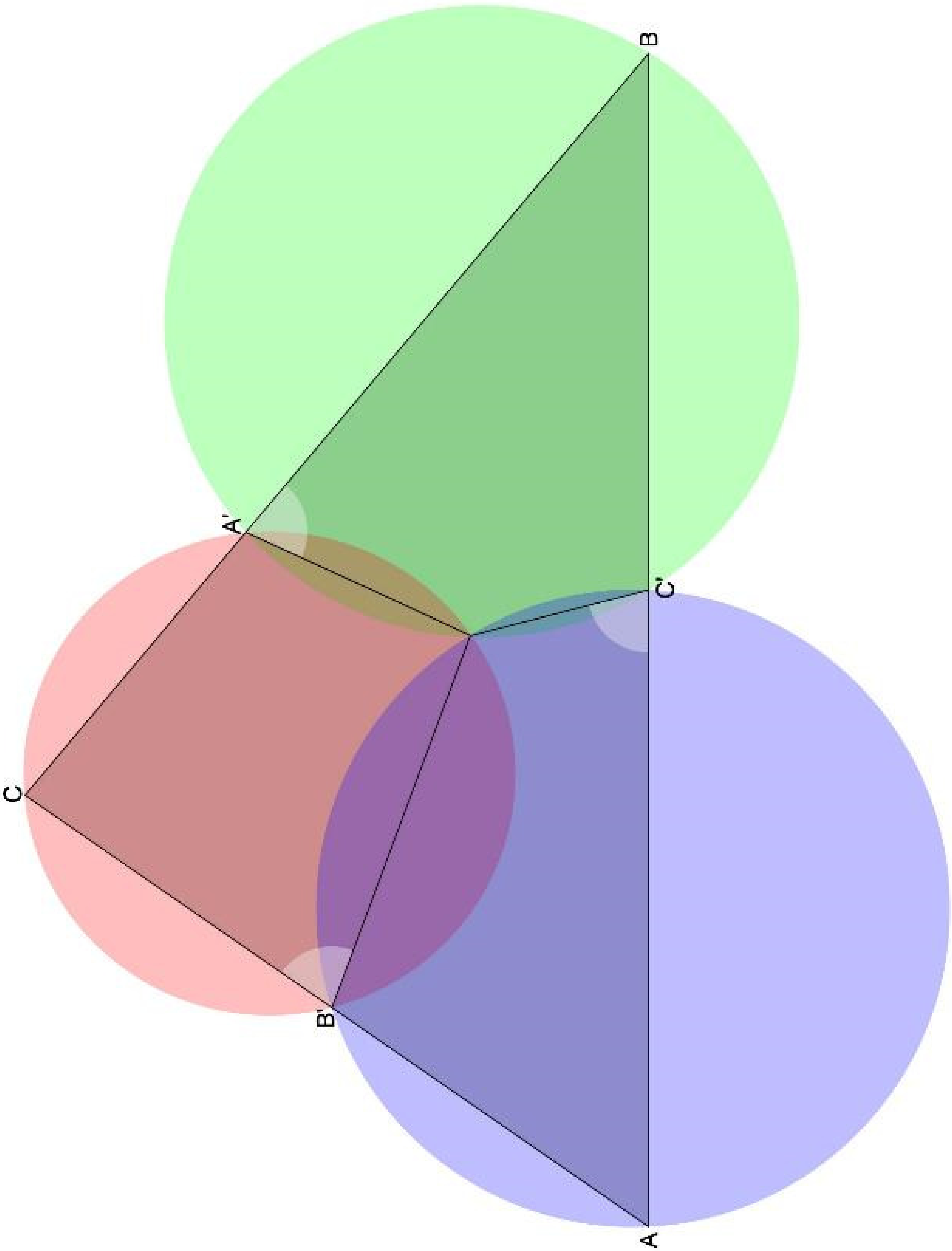
5. The common point of intersection of the 3 circles is called the **Miquel point**. What points can be Miquel points? That is, what is the locus of points *P* for which there exists points *A*', *B*', *C*' on different sides of triangle *ABC* so that *P* is the associated Miquel point?

6. Suppose that the Miquel point is the center of the circumscribed circle of triangle *ABC*. In this case, where are *A*', *B*', and *C*'?

7. What happens in the limit as *A*' approaches *C*, *B*' approaches *A*, and *C*' approaches *B*?

8. Can you see that the Miquel point sits on the circumcircle of triangle *ABC* if and only if the points *A*', *B*', and *C*' are collinear? (In this case, one of the points *A*', *B*', or *C*' will lie outside of the triangle.)

Miquel's Theorem



The Mountain Clock, Part 2

by Ken Fan | edited by Jennifer Silva

The Dean of the Royal Academy clambered to the top of Academy Tower, the highest building on campus. He had just been tasked by the King to solve the city's time woes. There could not be a more important problem!

At the top of the tower were the four Academy Broadcasters, standing at the ready. The broadcasters listened intently as the Dean dictated a message to them. The broadcasters then took their positions facing out onto the academy grounds, one for each compass direction. They inhaled deeply, then bellowed out the following message in unison:

> *Illustrious professors of the Royal Academy! By order of the Dean, report to the Main Hall immediately. No less than the chronological fate of our entire city is at stake. All classes are cancelled until further notice. Make haste!*

Even before the echoes of the Dean's message made their last bounces, the academy grounds erupted in raucous commotion as professors took leave of their students, committees, lectures, consultations, researches, experiments, and writings, and hurried over to the Main Hall.

There, the Dean recounted all of the trials and tragedies that the city had suffered since the implementation of the 32-hour day and the monumental Mountain Clock. Foremost among these were the spurious clock readings due to poor synchrony between the Royal Time Keepers and the mass confusion that befell city dwellers whenever a time keeper team became somehow incapacitated. The Dean concluded, "As the intellectual hub of the city, it is up to us to find a solution. Your ideas! Let's hear them now!"

"Obviously, the synchronization problem is a direct and predictable consequence of poor communication betwixt Royal Time Keeper squads. We espouse the development of an inter-peak communication network," proposed the Professor of Communication.

"The problem is not one of communication. No doubt, the problem is the variation in skill between the different time keeper teams, as, evidently, uniform time keeper performance translates to uniform Mountain Clock performance," asserted the Professor of Fitness.

"I beg to differ," interjected the Professor of Diversity. "For no matter how practiced the time keepers are, there will still be variation among Royal Time Keeper teams, thus rendering spurious clock readings an inevitability."

"Even if good communication and uniform quality is implemented, the Royal Time Keepers, strong and fit as they are, are, nevertheless, still prone to illness. Clearly, we would be wise to promote the redundancy of backup teams," explained the Professor of Mistakes.

And on it continued, well into the 31^{st} hour. It was the largest multidisciplinary collaboration the city had ever seen. Every professor contributed ideas. Well, everybody but one, the Professor of Mathematics, as the Dean couldn't help but notice.

"You haven't spoken," said the Dean in the direction of the mathematician. "Have you no thoughts?"

A hush came over the gathering as the professors all turned to face the silent one.

"Not yet, sir!" came the reply. "I'm thinking."

"Very well. It's late," announced the Dean, just as all of the time statues were being retired. "Let us adjourn. Work these ideas. We reconvene here in one week, when the King will want a solution!"

The following days were packed with activity. Professors of the Royal Academy oversaw a myriad of projects designed to test various proposals and procedures. Students of the Royal Academy assisted in building models, running errands and experiments, and playing the role of time keepers for critical simulations. Like an orchestra conductor, the Dean directed the entire affair.

Throughout this bustle, the mathematician sat in a meditative state, thinking and glancing intently forward at nothing in particular. Some even resented what they misinterpreted as laziness on the part of the mathematician. They'd see her lying about, apparently doing nothing. In truth, though, her mind was actively searching. And after days of contemplation, inspiration struck. She finally had something to tell the Dean! She found him standing before a life-size replica of the Frogfish in the middle of the quad, whose once lush lawn was now mostly dirt, well-trodden and strewn with all manner of makeshift contraptions. Sweat dripped from the Dean's brows as he issued orders left and right.

"Bring in the dogs!" he commanded. Suddenly, a pack of ravenous dogs appeared and began sniffing around the Frogfish.

"Yes, fifty pounds of dog food," he confirmed to an assistant. "Oh, and a barrel of water and some fire sticks!"

"Where are those dog trainers?" he wondered aloud, then turned to another, "Find the trainers!"

"Hey, hey!" he yelled off into the distance. "That red cloth doesn't go there, put it over there! Yes, there, by the Dragon!"

"Excuse me, Dean," said the mathematician during an apparent hiatus in the Dean's parade of orders. "I've had an idea. May I share it with you?"

"Not now! The dogs are hungry, and I can't find the trainers! The King comes tomorrow. We don't need ideas now, we need action! Go help find the dog trainers!"

"But sir," the mathematician tried again, but it was no use. The dog trainers had arrived and the Dean descended upon them, gesturing further instructions.

The next day, all gathered back in the Main Hall. Seated in front was the King.

"Have you found a solution?" asked the King.

"I am proud to say that after a week of intense effort, we are ready to present you with a solution to the city's time problems," responded the Dean.

"Wonderful! Let's hear it!" bade the King.

The Dean cleared his throat.

"To avoid spurious readings," declared the Dean, "the solution is to hide the time statues while the Royal Time Keepers move them into place!"

"And how are you going to hide these giant time statues?" asked the King.

"Ah! We encircle each time statue with a colossal curtain, in Royal Red, of course! The curtains shall be closed while Royal Time Keepers do their moving."

"But how will the Royal Time Keepers know when to reopen the curtains?" inquired the King.

"An excellent question, to which we have an answer! Upon each peak, we build five towers, taller than the curtains that hide the statues. When the time statue has been moved to its proper place, a Royal Time Keeper climbs the tower and lights a torch at the top of the tower. When all five torches are ablaze, the Royal Time Keepers open their curtains!"

The King furrowed his brow and rubbed his chin. "And what if a time keeper team becomes incapacitated?"

"Ah, yes! Every time keeper team will have a backup team."

"What?" exclaimed the King. "We have to double the number of Royal Time Keepers?"

"Oh, no! We anticipated the magnitude of that cost, and I think Your Highness will find our solution most elegant. There will be no increase in the number of Royal Time Keepers!"

"But how will you create backup teams out of nothing?"

"In fact, we split each current time keeper team in half. For instead of moving the statues by hand, time keepers will employ trained woolly mammoths to move them!"

"Woolly mammoths?" responded the King skeptically.

"*Trained* woolly mammoths, four per peak – three to move the statue and one as backup. We tried other animals, but only the woolly mammoth could do the job."

"How, Dean, are you going to get four woolly mammoths up each mountain peak?"

"No problem, sir! They will be carried up just as the time statues were. It only has to be done once because the woolly mammoths will live atop the peak in woolly mammoth stables. Each time Royal Time Keepers ascend a peak, they'll bring food and water for the woolly mammoths."

"Let me see if I get this," said a doubtful King. "You propose to perch several woolly mammoths atop each mountain peak along with five signaling towers, each taller than the Academy Tower and topped with a fire torch, and encase each time statue in a colossal curtain of Royal Red? Is that your solution?"

"Isn't it amazing how it all works?" affirmed the Dean, proudness in his voice.

But the king was not pleased. "That is not just costly, it is crazy! Is there no other solution?" An uncomfortable silence fell over the Main Hall as the King sat for several moments looking glum.

Finally, the silence was broken by a quiet, but assured, voice from the crowd.

"There is," said the voice.

"Who said that?" asked the King, turning around in his seat.

"That's our Professor of Mathematics," replied the Dean. "She had no ideas. You know mathematicians, their heads are always on peak six."

"Apologies, sir," said the mathematician. "I did try to convey my idea to you yesterday, but you were too busy to hear it. Admittedly, I got the idea rather late in the process."

"No matter," said the King. "Tell us your idea."

"Your highness might not like it," said the mathematician, "because it will mean change. But it might have qualities that please."

"Tell it!" demanded the King. "I care only that it entails fewer changes than the current proposal, and, please, no woolly mammoths."

"Well," began the mathematician, "a major source for all our troubles are the spurious clock readings, which result when more than one time statue must be moved to show the new hour, such as when the hour changes from all statues on to all statues off."

"That is a rough transition time," agreed the King.

"Yes," said the mathematician, "so I asked myself, 'is it possible to cycle through the 32 binary digit patterns in such a way that from hour to hour, only 1 time statue is moved?'"

The crowd murmured.

The mathematician continued. "I decided to try it, starting at 00000. The first difficulty that arose was in determining which of the 5 digits should be changed to a 1. All seemed reasonable choices. To break the tie, I created a rule: change the rightmost digit that yields a new pattern until all 32 patterns have appeared. I began writing down the unique sequence generated by this rule, not sure whether I'd get stuck. But it worked! I've written down the entire sequence to prove it."

The mathematician produced a scroll showing a full cycle of 5-digit binary patterns. "If this scheme is adopted, there will no longer be spurious readings, because only one time statue is moved from hour to hour. What's more, since only one time statue is moved, there'd be no need to synchronize time statue moves between teams."

"But a team might still be disabled somehow," pointed out the Professor of Mistakes. "Are you going to have a backup team?"

"Yes," responded the mathematician. "But because only one statue needs be moved each hour, only one time keeper team and one backup team would be necessary at any given time. This means that only 160 time keepers would be needed instead of the current 400. And, instead of time keeper teams sitting around the same time statue for 8-hour shifts, they'd get to spend their time hiking from peak to peak to stay fit. Each team would handle just about equal work."

"That's brilliant!" said the Professor of Fitness. "I heartily approve!"

The King studied the scroll. He muttered, "My beloved binaries – actually my bane."

"If I may, sir," continued the mathematician. "The choice of binary counting was a fine one, but it *is* a convention. This is just an alternative convention with the advantage of hourly transitions effected by the movement of a single time statue. People will quickly adjust to associating these new patterns with the various hours of the day."

"True, indeed!" perked up the King. "And no woolly mammoths toiling their lives away as prisoners atop a mountain peak. I hereby pronounce your idea a beautiful one, worthy of our great city. And as King of this great city, I hereby order its immediate implementation!"

And that was the moment that the city of the Mountain Clock overcame its time troubles and began its ascent to becoming the most envied city of the ancient world. Sadly, over the eons, people got lazy and the 32-hour day was eventually abandoned for longer hours of leisure. All that remains of the great Mountain Clock is the effaced statue of the Frogfish, which, to the untrained eye, appears as just another large boulder.

What beautiful idea will you contribute to your city?

The End.

The sequence invented by the mathematician has been rediscovered several times throughout history. Today, they are often referred to as "Gray codes" after Frank Gray who, himself, called them "reflected binary codes" in a 1947 patent application while working at Bell Labs.

New Mountain Clock Schedule

Hour	Statue Code				
	Frogfish	**Apple Bearer**	**Bird Lady**	**Dragon**	**Lion**
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	1	1
3	0	0	0	1	0
4	0	0	1	1	0
5	0	0	1	1	1
6	0	0	1	0	1
7	0	0	1	0	0
8	0	1	1	0	0
9	0	1	1	0	1
10	0	1	1	1	1
11	0	1	1	1	0
12	0	1	0	1	0
13	0	1	0	1	1
14	0	1	0	0	1
15	0	1	0	0	0
16	1	1	0	0	0
17	1	1	0	0	1
18	1	1	0	1	1
19	1	1	0	1	0
20	1	1	1	1	0
21	1	1	1	1	1
22	1	1	1	0	1
23	1	1	1	0	0
24	1	0	1	0	0
25	1	0	1	0	1
26	1	0	1	1	1
27	1	0	1	1	0
28	1	0	0	1	0
29	1	0	0	1	1
30	1	0	0	0	1
31	1	0	0	0	0

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues thinking about irreducible polynomials over the finite field with 2 elements.

Now about the first conjecture…

Conjecture 1. The elements of the finite field with 2^n elements are exactly the roots of all irreducible polynomials over $\mathbb{F}_2$ of degrees that divide n.

$\Updownarrow$

This would be equivalent to showing that an irreducible polynomial either has all its roots or none of is roots in the field depending on whether its degree divides n.

Let $f(x)$ be irreducible over $\mathbb{F}_2$.

Then the roots of f are in $\mathbb{F}_{2^n}$ iff $\deg f \mid n$.

Suppose f has a root in $\mathbb{F}_{2^n}$, say $f(r) = 0$ where $r \in \mathbb{F}_{2^n}$.

$\mathbb{F}_2[r]$ is the smallest field that contains $\mathbb{F}_2$ and r.

Then $\mathbb{F}_2[r] \subset \mathbb{F}_{2^n}$.

Also $\mathbb{F}_2[r] \cong \mathbb{F}_2[x]/f(x)$, so $\# \mathbb{F}_2[r] = 2^{\deg f}$.

But $\mathbb{F}_{2^n}$ is a vector space over $\mathbb{F}_2[r]$, so

$\# \mathbb{F}_{2^n} = (2^{\deg f})^k$ for some integer $k \geq 0$.

$\Rightarrow 2^n = 2^{k \deg f} \Rightarrow \deg f \mid n$.

This gets one direction!

Now suppose $\deg f \mid n$.

Must show that $\mathbb{F}_{2^n}$ contains all roots of f.

Since all finite fields of the same size are isomorphic, I think all I have to show is that $\mathbb{F}_{2^n}$ contains $\mathbb{F}_{2^k}$ whenever k divides n.

Does $\mathbb{F}_{2^n}$ contain $\mathbb{F}_{2^k}$ for all $k \mid n$?

F^* denotes the set of invertible elements in F. In a field, that's all nonzero elements.

I know that the nonzero elements in a finite field form a cyclic group, so I know that this polynomial has roots exactly the nonzero elements of the field.

$\mathbb{F}_{2^n}^*$ is cyclic so $x^{2^n-1} - 1 = 0$ for all $x \in \mathbb{F}_{2^n}^*$.

Suppose $k \mid n$. Does $x^{2^k-1} - 1 \mid x^{2^n-1} - 1$? ☆

If so, then take $F = \{x \in \mathbb{F}_{2^n} \mid x^{2^k-1} - 1 = 0\} \cup \{0\}$

$= \{x \in \mathbb{F}_{2^n} \mid x^{2^k} - x = 0\}$

For proofs that finite fields of the same size are isomorphic and the nonzero elements of a finite field form a cyclic group, see any book on abstract algebra.

Key:
Anna's thoughts
Anna's afterthoughts
Editor's comments

Claim: F is a field with 2^k elements in $\mathbb{F}_{2^n}$.

$0, 1 \in F$.

If $x, y \in F$ then

$$(x+y)^{2^k} = x^{2^k} + y^{2^k} = x + y$$

$\Rightarrow x+y \in F$.

$$(xy)^{2^k} = x^{2^k} y^{2^k} = xy$$

$\Rightarrow xy \in F$.

So this means that all I have to do is show that ☆ is true.

But I know this identity from algebra, so I just have to check if $2^k - 1$ divides $2^n - 1$.

$$(x^{ab} - 1) = (x^a - 1)(x^{ab-a} + x^{ab-2a} + x^{ab-3a} + \cdots + x^{2a} + x^a + 1) \quad (\star\star)$$

Does $2^k - 1 \mid 2^n - 1$?

This is ☆☆ with $x = 2$, $a = k$, $ab = n$!

It does!

$\mathbb{F}_{2^n}$ does contain a field isomorphic to $\mathbb{F}_{2^k}$ iff $k \mid n$.

The conjecture is true!

<u>Theorem</u> Let N_d be the number of irreducible polynomials over $\mathbb{F}_2$ of degree d.

Then $\mathbb{F}_{2^n}$ consists exactly of the roots of all irreducibles over $\mathbb{F}_2$ whose degree divides n and

$$2^n = \sum_{d \mid n} d N_d.$$

I wonder if I really used that the characteristic is 2 or if this theorem is true for any nonzero characteristic. Hm.

Is 2 special or does all this generalize to $\mathbb{F}_{p^n}$? for any prime p

---

Just for fun, I feel like finding out how many irreducible polynomials there are over F_2 with degree p^k, where p is a prime number.

What is N_{p^k} where p is prime?

$$N_1 = 2$$

$$N_p = \frac{2^p - 2}{p}$$

$$N_{p^2} = \frac{2^{p^2} - (2^p - 2) - 2}{p^2} = \frac{2^{p^2} - 2^p}{p^2}$$

$$N_{p^3} = \frac{2^{p^3} - (2^{p^2} - 2^p) - (2^p - 2) - 2}{p^3} = \frac{2^{p^3} - 2^{p^2}}{p^3}$$

$$2^{p^k} = \sum_{m=0}^{k} p^m N_{p^m}$$

$$N_{p^k} = \frac{2^{p^k} - \sum_{m=0}^{k-1} p^m N_{p^m}}{p^k}$$

$$N_{p^k} = \frac{2^{p^k} - 2^{p^{k-1}}}{p^k}$$

Amusing...

ABB 4/24/16

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Math to the Rescue, Part 1[1]

by Heidi Hurst[2]
edited by Jennifer Silva

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

Nearly three years ago, flood waters in Colorado threatened to destroy my house. Heavy snowmelt and torrential rains caused the rivers to swell, and my dad packed up the car and drove to higher ground. Fortunately, my house and my family were safe in the end. However, many people lost their homes in the flood, and some even lost their lives.

After natural disasters like the 2013 Colorado Flood, it takes a lot of resources and hard work to get a community back on its feet. A government agency called the Federal Emergency Management Agency (FEMA) helps communities in times like this. One way FEMA assists survivors of disasters is by setting up Disaster Recovery Centers (DRCs). These temporary centers provide all sorts of resources to a community for up to three months after a disaster, such as money to fix broken roofs, mini-fridges for perishable medicine, and crisis counseling.

The location of a DRC is very important to ensure that people can get the help they need. After all, if you're just a few minutes' walk to a DRC but the only way to get there is a bridge that got washed away by the storm, it won't be useful at all. Making sure that FEMA opens the DRC in the best location can play a big role in helping a community to recover quickly.

It might seem a bit daunting to figure out where to put DRCs, but math can help provide an answer! For an entire summer, that's exactly what I did: use math to figure out where to put the DRCs. I'll walk you through the tool I developed.

Heidi Hurst is a senior at Harvard University.

I broke this question into three tasks which, together, show a good overall picture of the problem at hand. First, we determine how long it takes for people to get to a DRC from their houses. Second, we determine how vulnerable these people are by estimating how likely they are to be able to get help and how much help they might need (we call this combination of traits **social vulnerability**). Third, we determine how many people are affected, and where those people live. Finally, we combine all three of these to determine a good DRC location choice, then create graphs and charts to explain this to others.

Step 1: Determining Travel Time

We want to know how long it takes for people to reach a DRC from their homes. We could poll everyone who lives in the area, then take the average of all of those times. However, that would be very time-consuming!

---

[1] This content was supported in part by a grant from MathWorks. Figures printed with permission of the author.
[2] This project was funded and made possible through the DHS HS-STEM Summer Internship Program administered by Oak Ridge Associated Universities.

In order to get an accurate estimate, we'd need to ask a lot of people right after a disaster, which might be hard if phones are down. Then, if we wanted to run the analysis for a different DRC, we'd have to do the whole process all over again! We must make these assessments as quickly as possible so people can get the help they need; we have to be fast *and* efficient.

Instead of asking people how long it takes and taking the average, there is a computer program known as ArcGIS that we can use to estimate travel time. ArcGIS creates a map that typically looks like a bullseye with really ragged edges. The DRC is in the center, and each ring around it shows areas where you can get to a DRC in under 20 minutes, under 40 minutes, or under 60 minutes.

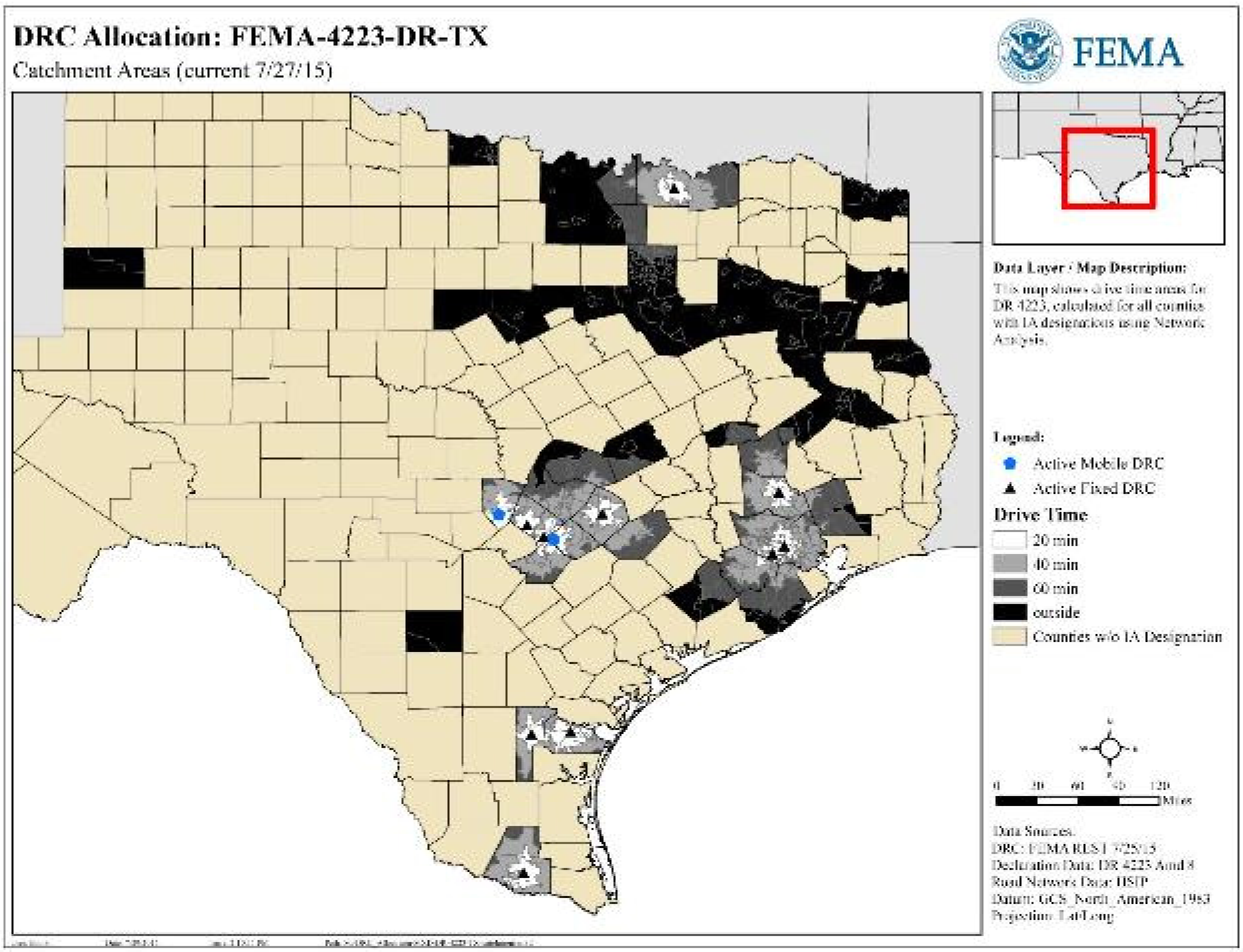


Figure 1. A map created in ArcGIS that shows drive times to DRCs.

How does the computer calculate these areas? They aren't just circles. Instead, the software uses information about the roads to calculate how far from a DRC someone could drive in 20 minutes. Each piece of road is stored in the data set with information about the speed limit and the length of the road. Because time equals distance divided by speed, we can then calculate the time it takes to drive that piece of road.

The software also allows us to note if some pieces of road are impassible (something that often happens in the case of a natural disaster). After we enter information about damaged roads and bridges, ArcGIS gives us an accurate drawing of a map showing where a person can drive to a DRC in under 20 minutes, under 40 minutes, or under 60 minutes.

Step 2: Determining Vulnerability

The second thing we need to figure out is how vulnerable people are. Social vulnerability uses American Community Survey data to estimate "a community's ability to prevent human suffering and financial loss in the event of disaster."[3] In general, communities that are more socially vulnerable will need more help after a disaster.

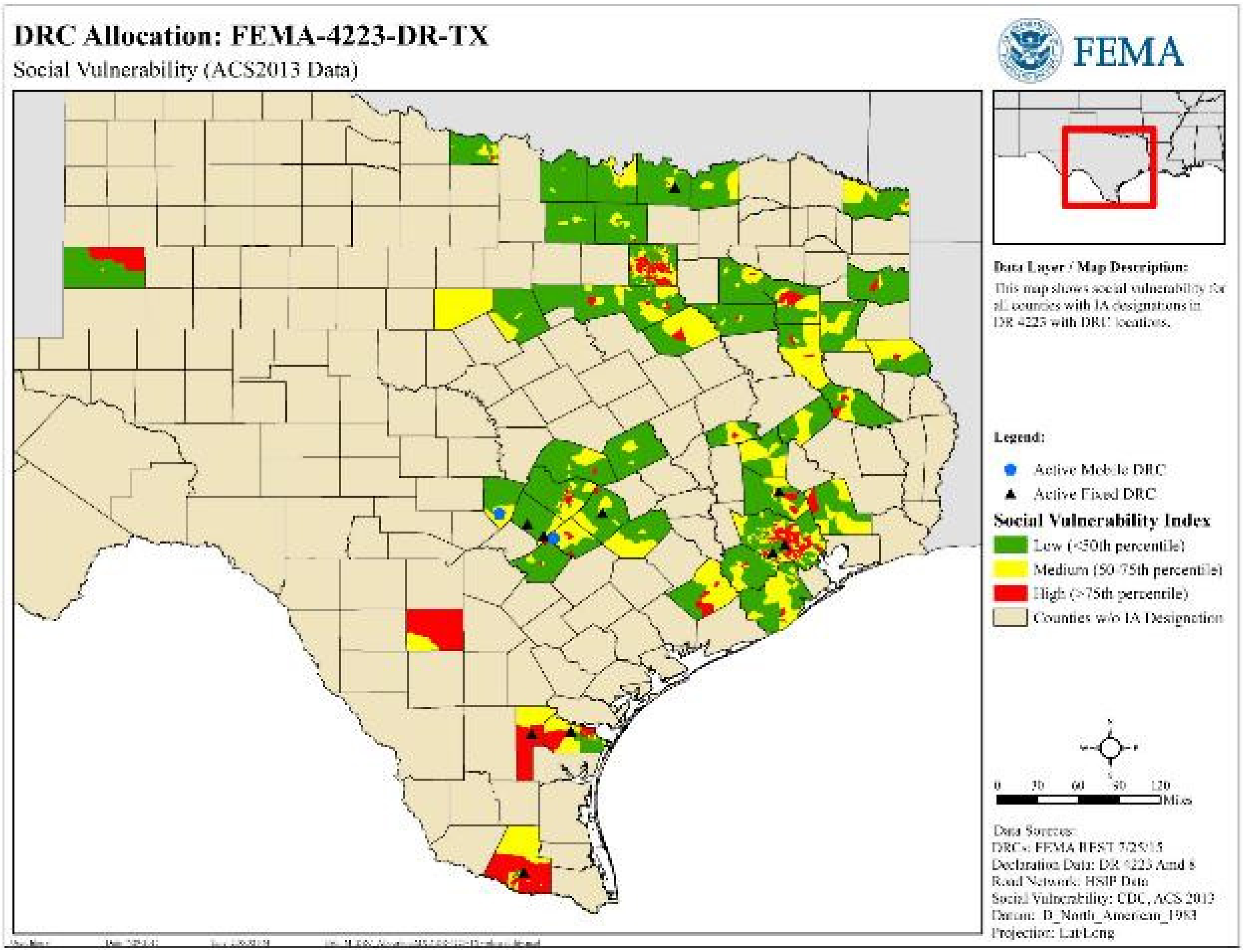


Figure 2. A map created in ArcGIS showing social vulnerability.

We want to make sure that the people who need the most help are able to get to a DRC quickly. To do this, we can look at a map of social vulnerability (in - you guessed it! - ArcGIS). We break social vulnerability into three groups: low, medium, and high. High vulnerability folks, for instance, encompass a wide variety of characteristics: the elderly, people with disabilities, people without cars, people who don't speak English, and many others who have a harder time getting help.

We would like to find a useful way to combine the social vulnerability index with drive time (**Step 3**). Try to think of a way to do this. Next time, we'll explain how we did this and are able to create maps that show regions that reflect both social vulnerability and drive time.

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[3] svi.cdc.gov/Documents/FactSheet/SVIFactSheet.pdf

In Search of Nice Triangles, Part 5

by Ken Fan | edited by Jennifer Silva

Emily and Jasmine continue their investigation into nice triangles.

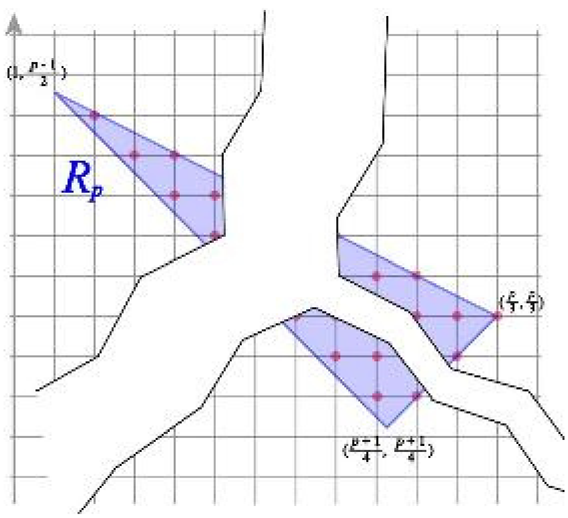
Jasmine: How'd your piano lesson go?

Emily: I'm not sure. I'm working on the E flat major fugue from book 2 of the Well-Tempered Clavier. My teacher thinks that I play it too fast, but I like it fast. It feels upbeat to me. Anyway, I've been looking forward to counting more lattice points!

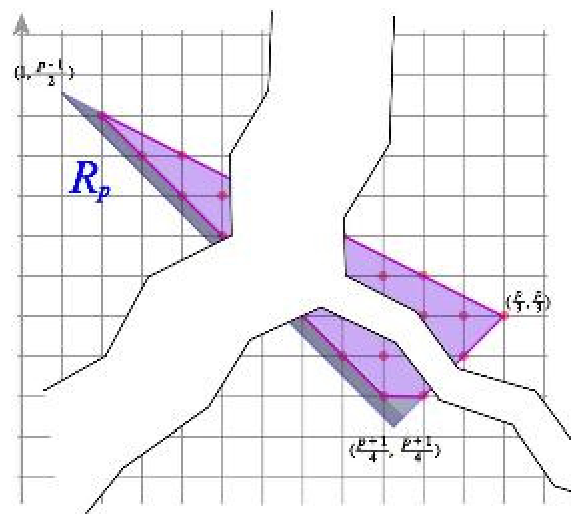
Jasmine: I wish we could think of a way to count the number of lattice points in R_p without breaking into several cases that depend on the remainder we'd get after dividing p by 12.

Emily: That'd be amazing, but I like your idea of shrinking R_p as necessary to become a lattice polygon, then using Pick's theorem to figure out the number of lattice points. Why don't we go ahead and try that? We've done the case where p is 3 mod 12, so we might as well do the case where p is divisible by 12.

Jasmine: All right! I'll draw a picture.



The case where p is divisible by 12. The white "river" flowing between vertices of R_p indicates that the actual triangle is much larger and the vertices are far apart.



Snip away the gray areas to get the purple lattice polygon which contains all the lattice points in R_p.

Emily: It looks like we can shrink R_p by lifting the edge with slope -1 up a bit, then snipping off a little isosceles right triangle from the right-angled corner.

Jasmine: That looks right to me. It's as if the boundary of R_p is a stretched rubber band and the lattice points are pegs, and the purple region is what's grabbed by the rubber band when it's let go.

Emily: That's a neat way to think about it.

Jasmine: In this case, when the rubber band hugs the contained pegs, it becomes a quadrilateral.

Emily: We already computed that the area of R_p is $(p - 3)^2/48$. From R_p, we remove a trapezoid and an isosceles right triangle to get our quadrilateral.

Jasmine: I get $(2p - 9)/16$ for the area of the trapezoid.

Emily: And the area of the little isosceles right triangle is 1/4 – it's a quarter of a unit square.

Jasmine: So the area of the quadrilateral is $(p - 3)^2/48 - (2p - 9)/16 - 1/4$.

Emily: Now we have to count lattice points on the boundary of the quadrilateral.

Jasmine: The side with slope -1 has $p/4 - 1$ lattice points.

Emily: And the side with slope 1 has $p/3 - p/4$ lattice points.

Jasmine: The two lattice points on the short flat side are already accounted for, so all we have to do is count lattice points on the side with slope -1/2.

Emily: I get $p/2 - p/3$ lattice points on that side, but the lattice points at the endpoints have already been counted.

Jasmine: Right, so that means that the total number of lattice points on the boundary of the quadrilateral is $(p/4 - 1) + (p/3 - p/4) + (p/2 - p/3) - 2$, which simplifies to $p/2 - 3$.

Emily: Pick's theorem says that

$$(p - 3)^2/48 - (2p - 9)/16 - 1/4 = I + (p/2 - 3)/2 - 1,$$

where I is the number of lattice points inside the quadrilateral.

Jasmine: We actually want to know the total number of lattice points in or on the boundary of the quadrilateral, which is $I + (p/2 - 3)$. We can get that on the right-hand side of Pick's equation by adding $(p/2 - 3)/2 + 1$ to both sides:

$$(p - 3)^2/48 - (2p - 9)/16 - 1/4 + (p/2 - 3)/2 + 1 = I + (p/2 - 3).$$

The left-hand side of this equation simplifies to $p^2/48$.

Emily: I can't believe how much cancels! Nifty formula. And since p is divisible by 12, we know that p^2 is divisible by 12^2, or 144; and since 48 divides evenly into 144, this formula does produce whole numbers.

Jasmine: That's a good check. According to this formula, there should be 12 integer-sided triangles with perimeter 24. Is that so?

Emily and Jasmine work out from scratch all integer-sided triangles with a perimeter of 24.

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 18 - Meet 5 March 3, 2016	Mentors:	Asra Ali, Bridget Bassi, Anna Ellison, Neslly Estrada, Hannah Larson, Jane Wang (Head)

Codes, origami, mathematical doodling, perspective drawing, probability, and proofs were topics of the day. For the perspective drawing, we drew the room we were in, line by line. Each line was an exercise in understanding principles of perspective drawing.

Session 18 - Meet 6 March 10, 2016	Mentors:	Karia Dibert, Neslly Estrada, Lydia Goldberg, Hannah Larson, Jane Wang (Head)

The Fujimoto Hydrangea origami model consists only of folds that are parallel to the sides of the origami square or run at 45° angles to the sides. A number of such "precreases" are made, and then the paper is collapsed, bringing four squares in the corners of the origami square together to meet at the center. Look up this model on the internet and try to understand how the collapse works. Also, how does the model enable one to iterate, in principle, forever?

Session 18 - Meet 7 March 17, 2016	Mentors:	Asra Ali, Bridget Bassi, Karia Dibert, Hannah Larson, Jane Wang (Head)

Some of our members were having trouble determining the numerical order of numbers expressed as fractions and decimals. To address this, we played several rounds of Number Hot Potato (see page 24 of Volume 8, Number 4 of this Bulletin) to provide a fun way to practice that skill. For example, in one round, the person with the hot potato had to say a number not said before, could not be equal to 1, but had to be closer to 1 than the previously spoken number.

Can you fold a perfect equilateral triangle starting with a square sheet of paper? What is the largest such triangle you can fold? What is the triangle with the largest area that you can fold? What is the triangle with the largest perimeter that you can fold? (Assume the original square is 1 unit on a side.)

Session 18 - Meet 8 March 31, 2016	Mentors:	Asra Ali, Bridget Bassi, Karia Dibert, Neslly Estrada, Hannah Larson, Jane Wang (Head)

We met Debbie Seidell's newborn!

Here's a quick ordering true/false question: Are the numbers 12/13, 13/14, and 14/15 written in increasing numerical order? Can you figure this out without having to do any computation?

Proof writing has become an ongoing theme of this session. Mathematical writing requires precision and clear thinking. Some of our members have been able to deduce interesting results, but are having difficulty constructing a good proof. To develop this ability, here are two types of exercises that we have been using at the club:

1. Scrambled proof problems. Take a well-written proof and split the proof into lots of pieces. Scramble up these pieces. The problem is to unscramble the pieces so that the result is a coherent proof. The goal of this exercise is to give students exposure to well-written proofs and to force them to think carefully about every statement in the proof and how they relate to each other.

2. Articulation practice. The basic concept for these exercises is to present the student with some mathematical situation which they must then describe in writing. The goal is to describe the situation clearly, concisely, and accurately. For example, one might present an interesting set of numbers, such as {-1024, -256, -64, -16, -4, -1, 2, 8, 32, 128, 512}. How succinctly can you describe the contents of this set?

For more on proofs, check out Timothy Chow's article *What is a Proof?* on page 3 of Volume 1, Number 5 of this Bulletin.

Session 18 - Meet 9 April 7, 2016	Mentors:	Karia Dibert, Hannah Larson, Jennifer Matthews, Jane Wang (Head)

We played more Number Hot Potato, but this time, many of the rules were invented by the players. It turns out that this really pushed participants mathematical limits as they tried to invent rules that would make the round a big challenge for everyone else.

Other topics of the day: patterns in Pascal's triangle, Pythagorean triples, probability, and the geometry of circles.

Session 18 - Meet 10 April 14, 2016	Mentors:	Karia Dibert, Neslly Estrada, Jennifer Matthews, Jane Wang (Head)

Girls' Angle member $\boldsymbol{\pi}$ has been investigating a sequence of numbers. She presented a beautiful recurrence relation that matched the 4 terms of the sequence she had found so far. Her recurrence relationship is so wonderful to behold that we're dedicating this issue's *Member's Thought* column to it. See page 27.

Session 18 - Meet 11 April 28, 2016	Mentors:	Bridget Bassi, Anna Ellison, Neslly Estrada, Anuhya Vajapeyajula, Jane Wang (Head)

Some members worked on the famous problem that Ramanujan solved while stir-frying vegetables: A man lives on a street where the house numbers run consecutively from 1 to n, where $50 < n < 500$. One day he notices that the sum of the house numbers to his left and the sum of the house numbers to his right are equal. How many houses were on the street and in what house did he live?

Coincidentally, the movie *The Man Who Knew Infinity* about Ramanujan opens in theaters on April 30. Check it out!

A Marvelous Recurrence

by Ken Fan

This session, ***π*** created a remarkable recurrence relation while investigating a special sequence of numbers. She had computed the first four terms of the sequence to be 2, 6, 12, and 18. Instead of computing more terms, she decided to try to come up with a rule that tells how to generate more numbers. Calling her sequence p_m, she came up with

$$p_m = \frac{p_{m-1}^2 - (p_m - p_{m-1})p_{m-2}}{p_{m-2}}, \text{ for } m > 2.$$

Let's take a close look at ***π***'s recurrence relation and unlock some of its secrets.

Isolating the next term

The first thing we might want to do with a recurrence relation is generate more terms of the sequence. If we imagine going through the process of computing successive terms, we will notice that the new term that we are trying to compute appears on both sides of ***π***'s recurrence relation. To find the new term, we would then have to solve an equation.

It would be more efficient if, instead of having to solve an equation for each new term, we simply had to evaluate a formula. This suggests trying to isolate p_m in the recurrence relation:

$$p_m = \frac{p_{m-1}^2 - (p_m - p_{m-1})p_{m-2}}{p_{m-2}} \qquad (\pi\text{'s original recurrence relation})$$

$$p_m = \frac{p_{m-1}^2}{p_{m-2}} - (p_m - p_{m-1}) \qquad (\text{divide through by } p_{m-2})$$

$$2p_m = \frac{p_{m-1}^2}{p_{m-2}} + p_{m-1} \qquad (\text{add } p_m \text{ to both sides})$$

$$p_m = \frac{1}{2}\left(\frac{p_{m-1}^2}{p_{m-2}} + p_{m-1}\right) \qquad (\text{divide both sides by 2})$$

Now, to generate new terms of the sequence, we just substitute the previous two terms into the right-hand side of the last equation and evaluate the resulting expression. For example, if we substitute 2 for p_{m-2} and 6 for p_{m-1}, we compute $\frac{1}{2}(6^2/2 + 6) = 12$, which should come as no surprise since ***π*** designed her recurrence relation to match the initial sequence 2, 6, 12, 18.

Finding a formula for *p_m*

Notice that both terms in the parenthetic expression of the last equation share a factor of p_{m-1}. Let's factor it out:

$$p_m = \frac{p_{m-1}}{2}\left(\frac{p_{m-1}}{p_{m-2}} + 1\right).$$

Inside the parenthesis, the fraction is the ratio of consecutive terms of the sequence. Notice that if we divide throughout by p_{m-1}, we will obtain a ratio of consecutive terms on the left-hand side of the equation too:

$$\frac{p_m}{p_{m-1}} = \frac{1}{2}\left(\frac{p_{m-1}}{p_{m-2}} + 1\right).$$

In order to accentuate the fact that terms of the sequence that appear in this equation always appear as part of a ratio of consecutive terms, we define a new sequence $r_m = p_m/p_{m-1}$, for $m > 1$. With this definition, $\boldsymbol{\pi}$'s recurrence relation becomes

$$r_m = (r_{m-1} + 1)/2.$$

This recurrence relation is slightly more complicated than the recurrence relation $s_m = s_{m-1}/2$, which would be the recurrence relation for a **geometric sequence** with common ratio 1/2. Notice that the equation can be rewritten as

$$2r_m - r_{m-1} = 1.$$

So we have a whole sequence of equations (down the left column below):

$$\begin{array}{rcl|c|rcl}
2r_m - r_{m-1} & = & 1 & (\times 2^{m-3}) \rightarrow & 2^{m-2}r_m - 2^{m-3}r_{m-1} & = & 2^{m-3} \\
2r_{m-1} - r_{m-2} & = & 1 & (\times 2^{m-4}) \rightarrow & 2^{m-3}r_{m-1} - 2^{m-4}r_{m-2} & = & 2^{m-4} \\
 & \vdots & & \vdots & & \vdots & \\
2r_5 - r_4 & = & 1 & (\times 2^2) \rightarrow & 2^3r_5 - 2^2r_4 & = & 2^2 \\
2r_4 - r_3 & = & 1 & (\times 2^1) \rightarrow & 2^2r_4 - 2^1r_3 & = & 2^1 \\
2r_3 - r_2 & = & 1 & (\times 2^0) \rightarrow & 2^1r_3 - r_2 & = & 1
\end{array}$$

Look what happens when we multiply each equation by a higher and higher power of 2 as we go up, starting with $2^0 = 1$ at the bottom, $2^1 = 2$ at the second from bottom, and so forth, all the way to 2^{m-3} at the very top, then add up all the resulting equations. A lot cancels:

$$2^{m-2}r_m - r_2 = 2^{m-3} + 2^{m-2} + \ldots + 2^2 + 2 + 1 = 2^{m-2} - 1.$$

Since we know $r_2 = p_2/p_1 = 6/2 = 3$, we find that $r_m = 1 + 1/2^{m-3}$.

Finally, since $p_m = p_1r_2r_3r_4 \cdots r_{m-1}r_m$, we conclude that

$$p_m = 2\left(1+\frac{1}{2^{-1}}\right)\left(1+\frac{1}{2^0}\right)\left(1+\frac{1}{2^1}\right)\cdots\left(1+\frac{1}{2^{m-3}}\right) = 2(1+2)(1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{2^{m-3}}\right).$$

Since each successive term is obtained by multiplying the previous by something a little bigger than 1, the sequence is an increasing one.

Can you show that the sequence is bounded above? Also, if you know about generating functions, how does this relate to the generating function of partitions into distinct parts?

Calendar

Session 18: (all dates in 2016)

January	28	Start of the eighteenth session!
February	4	Anna Frebel, Department of Astronomy, MIT
	11	
	18	No meet
	25	
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Session 19: (all dates in 2016) This calendar is tentative.

September	15	Start of the seventeenth session!
	22	
	29	
October	6	
	13	
	20	
	27	
November	3	
	10	
	17	
	24	Thanksgiving - No meet
December	1	
	8	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is \$20/meet for members and \$30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: _______________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ___________________________________