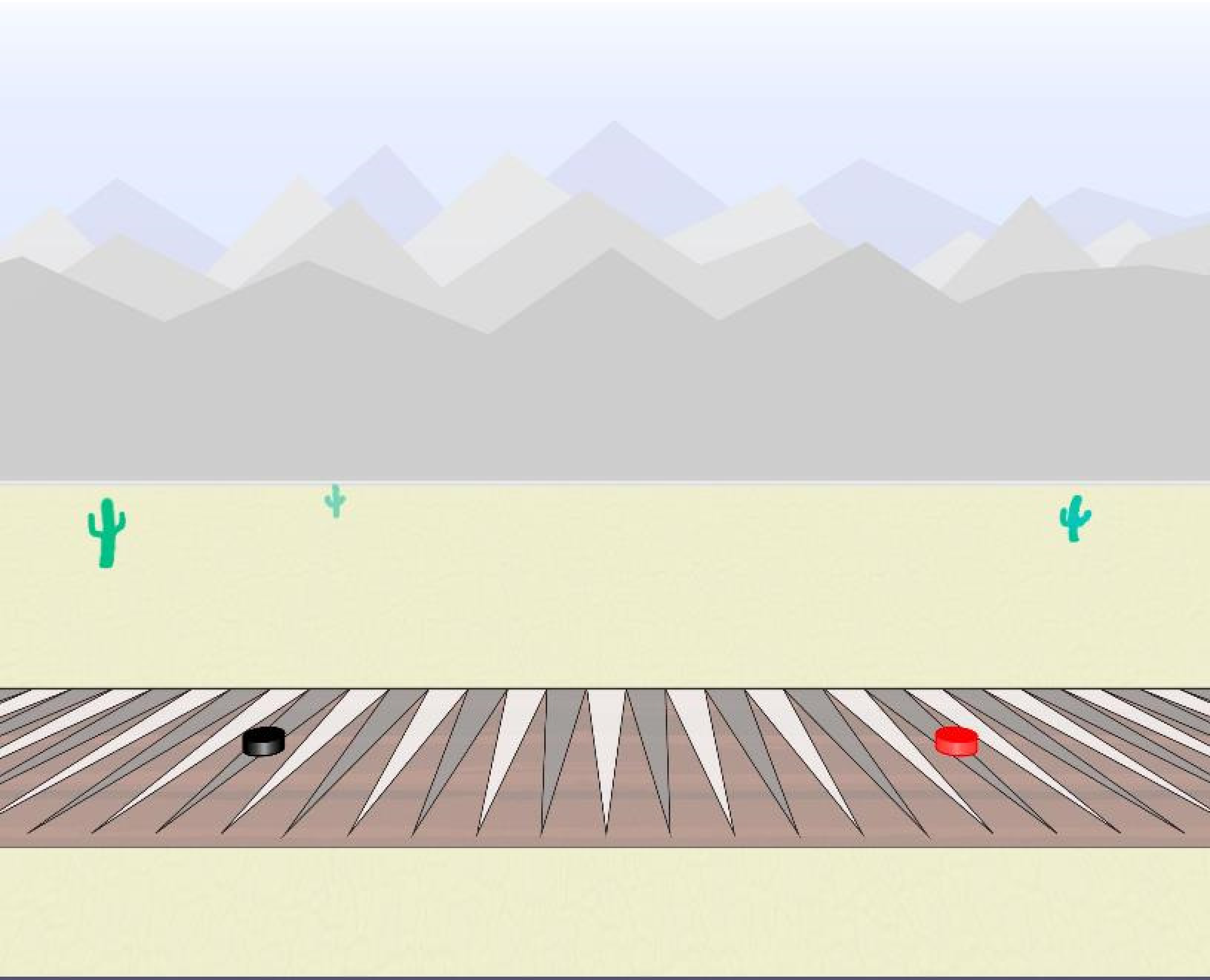
Girls' Angle Bulletin

June/July 2016 • Volume 9 • Number 5

To Foster and Nurture Girls' Interest in Mathematics



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From the Founder

This summer, Girls' Angle welcomes two summer interns, Traci Arnold and Sandy Pelkowsky. We're thrilled that they've joined the team and are excited about what they can contribute. This would not have been possible without the generous help of Dana Albert, who has volunteered to be their direct supervisor.

- Ken Fan, President and Founder

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Girls' Angle welcomes submissions that pertain to mathematics.

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Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Backgammon Duel* by C. Kenneth Fan. See page 19.

An Interview with Dana Randall

Dana Randall is the ADVANCE Professor of Computing, Director of the Algorithms and Randomness Center and Adjunct Professor of Mathematics at the Georgia Institute of Technology.

Working out small cases of the problem is often a very good start to understanding something complex.

Ken: What fascinates you about algorithms?

Dana: Everything we do is an algorithm! Certainly everything a computer does is. It isn't always clear what the best algorithm is for a problem, and theoretical computer scientists try to understand what we can do, how quickly, and what are the provable limits to how quickly you can possibly do it.

Ken: Could you please describe your personal history as it relates to your becoming a mathematician and computer scientist? Was it an effortless process, or were there great challenges? Were there times when you had second thoughts?

Dana: I was fortunate to have the opportunity to study math at Harvard as an undergraduate. It was extremely difficult and the other students were ridiculously smart, but it gave me an incredibly rich foundation, even when I didn't pick up everything. During a summer internship at Bell Labs I discovered my love for discrete mathematics thanks to a very generous mentor who taught me bits of many topics throughout the summer. When I returned to college, I found most of the discrete mathematics was in the computer science department, so I started taking some CS courses, and ended up going on to graduate school in theoretical computer science at UC Berkeley. There were many, many times before finishing my PhD when I had second thoughts! I think everyone does. First, there were many other things I was interested in doing, and second, I wasn't sure I would be very successful with a career as a mathematician. It is very likely that I would have quit academia if the right problems and opportunities hadn't come along when they did. At a point when I was most unsure, a friend and former teacher told me that "there are many different ways to be a mathematician," and those words have kept me going whenever I started to question whether things were going to work out. After graduate school as more things fell into place, and as I became more confident as I had more successes with research, I questioned my choices less and less.

Ken: Could you please tell us about what you think is one of the coolest math or computer science ideas? Why do you think it is such a wonderful idea?

Dana: Randomness is fascinating! There was a famous experiment where a visitor divided a room of strangers into two groups. The members of the group on the left were asked to each flip a coin 100 times and record the sequence of heads and tails. The members of the group on the right were asked to pretend they flipped the coins by writing down what they felt looked like a random sequence of 100 heads or tails, as if they really did the coin flips. The papers were collected from the two groups, mixed up, and given back to the visitor who was able to determine which group each list came from. How could the visitor tell? In a true series of 100 coin flips, it is very likely that you will have a run of 5 heads or 5 tails at some point. But people almost never include a run that long when they make up the sequence. The point is that

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We're committed to producing quality math educational content and make every effort to provide this content to you for free.

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For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Dana Randall and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

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Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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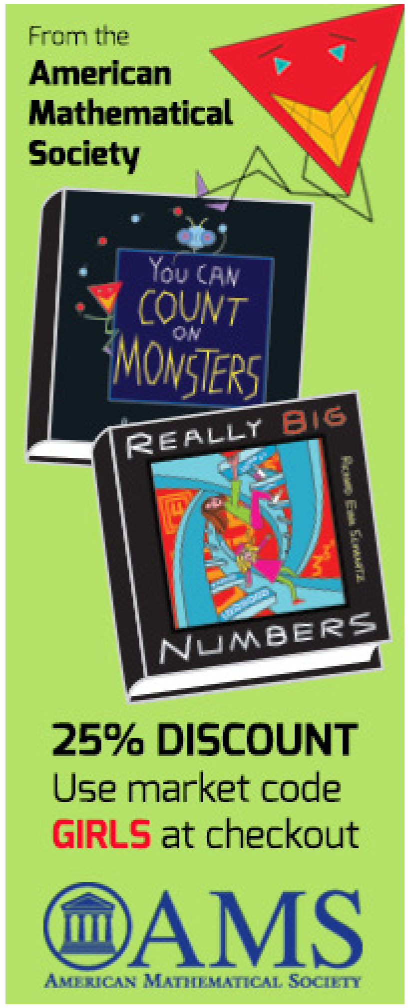

The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

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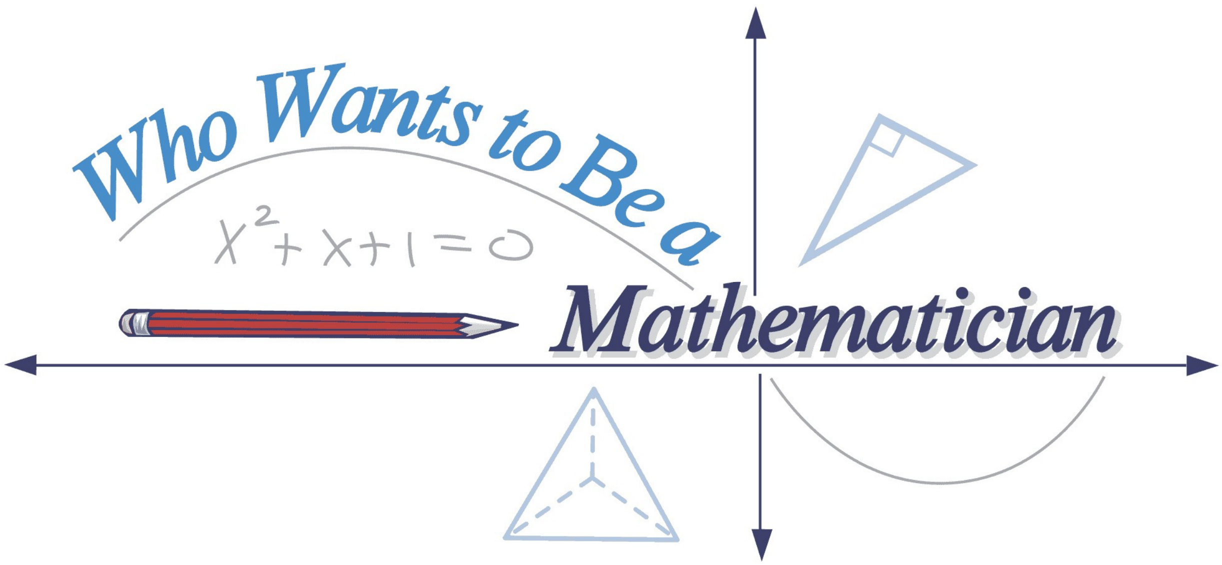

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Math to the Rescue, Part 2[1]

by Heidi Hurst[2]
edited by Jennifer Silva

In Part 1, we looked at determining travel time and social vulnerability for the people who might need to access a Disaster Recovery Center (DRC) following a natural disaster. Now we want to combine the social vulnerability index with drive time.

To do this, we convert both of these data sets into **rasters**. A raster is a giant picture in which the color of each pixel or square of the image represents a specific value (e.g., red = high vulnerability, yellow = medium vulnerability, green = low vulnerability). By cleverly combining the values in each pixel of both the drive time raster and the vulnerability raster, we can combine all of the information in a single raster (see Figure 4).

The example below shows how you can combine a distance raster (let's call it *A*, below in gray) with a social vulnerability raster (let's call it *B*, below middle) to get a raster containing all of the information (we'll call this one *C*, multicolored, at right). Can you figure out a mathematical formula for how we combined the two?

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2	2	2	2	3	2	2	2	3	3
2	2	3	3	1	2	2	3	3	3
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2	2	2	2	3	3	2	3	3	3
3	2	2	2	2	2	2	2	1	2
3	3	3	3	2	2	1	1	1	1
1	2	3	3	2	1	1	1	2	1

601	601	601	602	601	601	601	601	601	603
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Figure 3. An example of adding rasters.

If the rasters above are *A*, *B*, and *C*, we might relate them to each other by the equation $10A + B = C$. Now we have a map that shows regions color-coded by how far away *and* how vulnerable each region is. Pretty cool, huh? And all it took was clever addition (and a little multiplication)!

Step 3: Determining Affected Population

Unfortunately, knowing the various regions computed above doesn't give us enough information to place DRCs. After all, they just show places on a map where someone *could*

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[1] This content was supported in part by a grant from MathWorks. Figures printed with permission of the author.
[2] This project was funded and made possible through the DHS HS-STEM Summer Internship Program administered by Oak Ridge Associated Universities.

drive to a DRC in under 20 minutes – that doesn't mean that there *is* someone there who needs to drive to a DRC. In order to make use of the fancy combined raster from the end of Step 2, we need information about where people actually live.

I used a data set called "LandScan." It uses a special algorithm to estimate where people live. This data is also presented as a raster. A dark pixel shows a place where many people live, while a light pixel shows a place where no one lives.

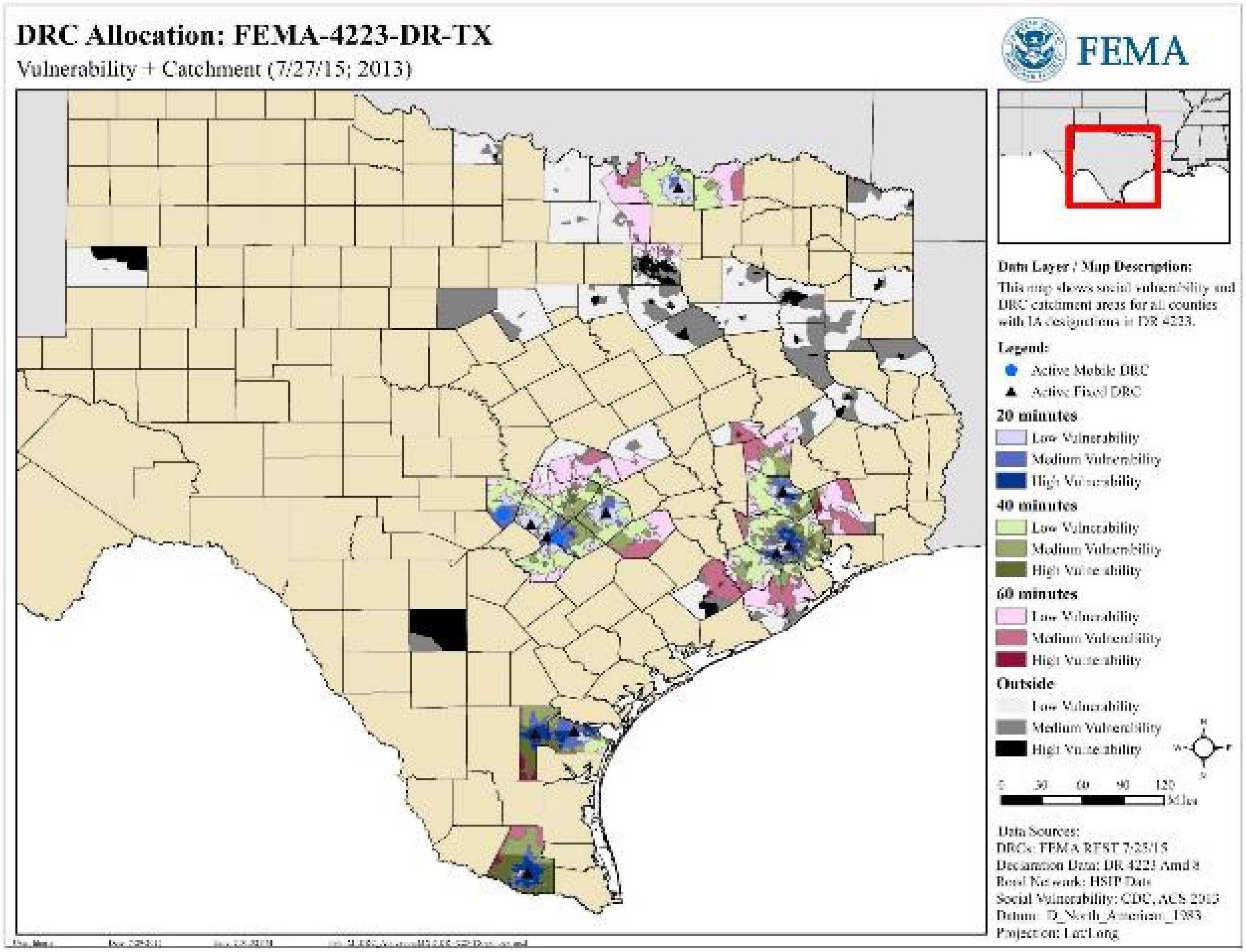


Figure 4. A map created in ArcGIS that combines drive times and social vulnerability.

Step 4: Putting It Together

Steps 1 to 3 give us all the tools we need for our analysis. The final step is to put everything together to decide if a particular DRC is in a good location.

To figure out how many people live in one of the regions from Part 2, we add up the population of every pixel in the LandScan raster inside a specific service area. Thanks to ArcGIS, we don't have to this by hand.

gridcode	DRC_ID	DR_TIME	EVAL_CRIT	TOT_POP
201	0	20	1	901915

The table at left shows just one line of the data set that ArcGIS creates by accounting for the data in Steps 1 to 3. It corresponds to the region with a blue boundary in the combined raster example of Figure 3. The first two boxes are just for recordkeeping (you can see that the "gridcode" is what we called the combined raster value). We can read the last three boxes as follows: "There are approximately 901,915 people living within a 20-minute drive of a DRC that are low vulnerability."

Once the computer does all of this number crunching for us, we can take the results and use pie charts to tell a story about the quality of our DRC locations.

Step 5: Analyzing Results

There is one pie chart for each vulnerability category (low, medium, high, and all combined). Each graph shows how many people are within 20, 40, or 60 minutes (the black slices show people outside of a 60-minute drive).

We can see that more than half of the high vulnerability people (light red slice in the bottom right) are within a 20-minute drive of a DRC. That's pretty good!

Different choices of where to put a DRC will give different pie charts. By comparing pie charts for multiple potential locations, we can decide which one is better.

Can you think of other ways to present and analyze this data? What else could you use this data for? What other variables might be valuable to incorporate?

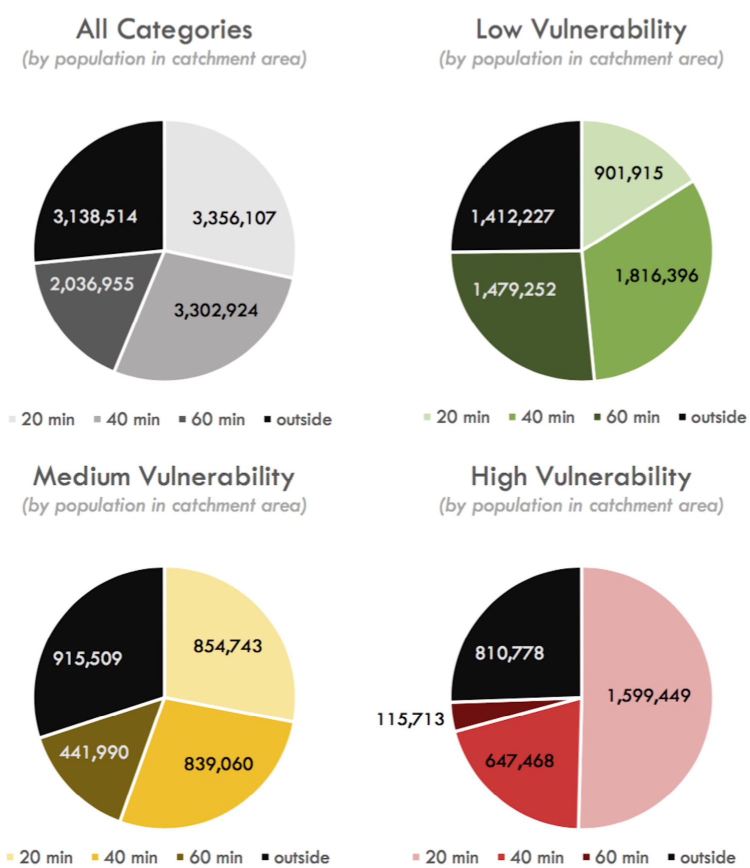


Figure 5. Pie charts summarizing information about a DRC location.

Conclusion

The steps above were written in a coding language called Python so they could be re-run for any disaster in any place. By figuring out the steps ahead of time, we can set up DRCs faster to get people the help they need.

While this approach is new for FEMA, it is just the first step in developing analytical solutions for disaster recovery. There are a number of open questions to tackle in the coming years. This analysis assesses the quality of one DRC location. Could you imagine working the problem from the other direction, i.e., starting with population and vulnerability data and working backwards to suggest optimal locations?

How might you modify the problem to account for the number of people a DRC can serve at one time? Could you estimate how traffic flow to an individual DRC might differ over time? Could you suggest which DRC to close if traffic flow drops off?

My hope is that analyses like this one, and the open problems above, can become more common so we can make disaster relief better for everyone affected. Math to the rescue!

In Search of Nice Triangles, Part 6

by Ken Fan | edited by Jennifer Silva

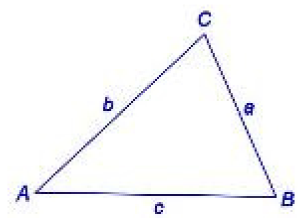
Emily and Jasmine continue their investigation into nice triangles. They've been using "nice" to denote angles that measure a rational multiple of π radians.

Jasmine: I'm eager to move on to the search for triangles that have 3 nice angles and 2 sides of integer length.

Emily: Me too!

Jasmine: I think this could be the last interesting case to consider; if we require only 1 side of integer length, then any triangle can be enlarged while preserving its angles until one of its sides has integer length. And if we want 2 sides of integer length but only 1 nice angle, we can always place the nice angle at the apex of an isosceles triangle and give its equal sides length 1.

Emily: Wow, then we're finally nearing the end of our journey!



Jasmine: So suppose we have a triangle with 3 nice angles, but only 2 sides of integer length. I'll label the vertices and side lengths in the usual way. Let's say that a and b are the integer side lengths. What can we say?

Emily: Well, the law of cosines doesn't seem to be of much help because we have no control over the length of c; and since c would appear in any application of the law of cosines to this triangle, we wouldn't be able to say anything definite about the cosines of the triangle's angles.

Jasmine: How about the law of sines, then?

Emily: Let's try it! The law of sines says that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,$$

where R is the radius of the circumscribing circle.

Jasmine: The first equality tells us that if a and b are integers, then the ratio $\sin A / \sin B$ must be a rational number, specifically a/b.

Emily: And, conversely, if the ratio of $\sin A$ to $\sin B$ is rational, then we would be able to suitably scale the triangle so that the sides opposite angles A and B both have integer lengths.

Jasmine: Great! That means that our task is equivalent to finding pairs of nice angles A and B for which $\sin A / \sin B$ is rational. Once again, we seek special rational numbers!

Emily: Hmm … How are we going to figure out when the ratio $\sin A / \sin B$ is rational?

Emily and Jasmine think.

Jasmine: Gosh, I don't know. But there is one case that jumps out, and that's when the ratio of the sines is 1, in which case $\sin A = \sin B$. And since A and B are angles in a triangle, both have positive measure and their sum must be less than 180°. We might as well assume that $A \geq B$ by relabeling if necessary. Then $B < 90°$, and the only angles A between 0° and 180° that satisfy $\sin A = \sin B$ are $A = 180° - B$ and $A = B$. But since $A + B < 180°$, only $A = B$ is acceptable.

Emily: It's true! Any nice angle between 0° and 90° can serve as the base angle of an isosceles triangle, which will necessarily have 3 nice angles, and any isosceles triangle can be scaled up until its 2 equal sides have the same integer length.

Jasmine: And let's not forget the triangle that launched this whole mathematical journey in the first place: the 30-60-90 triangle!

Emily: Ha ha! I forgot about the 30-60-90 triangle. Indeed, $\sin 90° / \sin 30°$ is equal to the rational number 2 since $\sin 90°$ is 1 and $\sin 30°$ is 1/2.

Emily and Jasmine think some more.

Jasmine: No other pairs of nice angles with rational sine ratios jump out at me.

Emily: Me neither.

Jasmine: But surely the 30-60-90 triangle and the isosceles triangles are not the only examples of triangles that can have 2 integer side lengths and 3 nice angles!

Emily: Let's see. We seek nice angles A and B such that $\sin A / \sin B$ is rational. Maybe we can try to do what we did to discover the nice angles X such that $\cos X$ is rational. There, we found a polynomial with integer coefficients that had $2 \cos X$ as a root. We then used the rational root theorem to deduce that the only possible rational values of $\cos X$ are 0, ±1/2, and ±1. Perhaps we can find a polynomial $p(x)$ with integer coefficients such that

$$p(\sin A / \sin B) = 0.$$

Jasmine: That's a good plan, but it still looks hard. Can we even find such a polynomial in the case where B is a right angle? When $B = 90°$, $\sin B = 1$, so we'd need to find a polynomial with integer coefficients such that $p(\sin A) = 0$.

Emily: Maybe we can approach it the same way that we approached the cosine. Since A is a rational multiple of π, we can write $A = k\pi/n$, for some integers k and n. Then we know that $\sin(nA) = \sin(k\pi) = 0$. When we were looking at the cosine function, we found the appropriate polynomial by expressing $\cos(nx)$ as a polynomial in $\cos x$. Let's try to do that for sine!

Jasmine: Unfortunately, the sine case doesn't appear to be as straightforward as the cosine case. For instance, $\sin 2x = 2 \sin x \cdot \cos x$, so not even $\sin 2x$ is a polynomial in $\sin x$.

Emily: Oh dear. Too bad we're dealing with sines instead of cosines!

Jasmine: Say, maybe we *can* work with cosines instead of sines. After all, the cosine function and the sine function are practically the same function. I mean, $\sin x = \cos(\pi/2 - x)$.

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Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues thinking about irreducible polynomials over the finite field with 2 elements.

I bet the result I got last time works over the finite field with p elements.

p prime

Conjecture 1. The elements of the finite field with p^n elements are exactly the roots of all irreducible polynomials over $\mathbb{F}_p$ of degrees that divide n.

Just to make sure, I'm going to go through everything in the more general setting.

Conjecture 2. Different irreducibles over $\mathbb{F}_p$ have completely different roots.

Proof of Conjecture 2. Let $p(t)$, $q(t)$ be distinct irreducible polynomials

If $t-r \mid p(t)$ and $t-r \mid q(t)$, then

$$t-r \mid (p(t), q(t))$$

But $(p(t), q(t)) = 1$ since $p(t)$ and $q(t)$ are irreducible.

Conjecture 2 goes through without a hitch.

Based on what I did last time, I think it makes sense to organize some key facts as lemmas.

Lemma. Suppose $k \mid n$.

Then $x^{p^k-1}-1 \mid x^{p^n-1}-1$

Proof. We know $x^{ab}-1 = (x^a-1)(x^{ab-a}+x^{ab-2a}+x^{ab-3a}+\dots+x^{2a}+x^a+1)$

Substituting $x=p$, $a=k$, $b=\frac{n}{k}$ shows that

$$p^k-1 \mid p^n-1.$$

Substituting $a=p^k-1$, $b=\frac{p^n-1}{p^k-1}$ show that

$$x^{p^k-1}-1 \mid x^{p^n-1}-1.$$

It's so amusing that this lemma follows by applying the same algebraic identity twice!

Lemma. Suppose $k \mid n$. Then $\mathbb{F}_{p^n}$ contains $\mathbb{F}_{p^k}$.

(Technically, $\mathbb{F}_{p^n}$ contains an isomorphic image of $\mathbb{F}_{p^k}$.)

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Proof. We know $x^{p^n-1}-1=0$ for all $x\in\mathbb{F}_{p^n}^*$.

So $\mathbb{F}_{p^n}^*$ is the set of roots of $t^{p^n-1}-1$.

From the previous lemma, we know $t^{p^k-1}-1 \mid t^{p^n-1}-1$.

Let $F=\{x\in\mathbb{F}_{p^n}^* \mid x^{p^k-1}-1=0\}\cup\{0\}$

$=\{x\in\mathbb{F}_{p^n} \mid x^{p^k}-x=0\}$.

F has p^k elements. Since $1^{p^k}-1=0$, $1\in F$.

If $x,y\in F$, then

$(x+y)^{p^k}=x^{p^k}+y^{p^k}=x+y \Rightarrow x+y\in F$.

$(xy)^{p^k}=x^{p^k}y^{p^k}=xy \Rightarrow xy\in F$.

Hence F is a field.

Exploiting the fact that $p=0$ in these finite fields...

This lemma goes through without a hitch too!

Proof of Conjecture 1.

Will show that, if $f(t)$ is irreducible over $\mathbb{F}_p$, then the roots of $f(t)$ are in $\mathbb{F}_{p^n}$ iff $\deg f \mid n$.

Suppose $f(r)=0$ where $r\in\mathbb{F}_{p^n}$.

Then $\mathbb{F}_p[r]\subset\mathbb{F}_{p^n}$.

Also $\mathbb{F}_p[r]=\mathbb{F}_p[x]/f(x)$, so $\#\mathbb{F}_p[r]=p^{\deg f}$

Since $\mathbb{F}_{p^n}$ is a vector space over $\mathbb{F}_p[r]$ we must have

$p^n=(p^{\deg f})^k$ for some integer $k\geq 0$.

$\Rightarrow \deg f \mid n$.

This direction goes through no problem, just as before!

Suppose $\deg f \mid n$. Then $\mathbb{F}_p[x]/f(x)$ has $p^{\deg f}$ elements.

By the lemma $\mathbb{F}_p[x]/f(x)$ is isomorphic to a subfield of $\mathbb{F}_{p^n}$.

Therefore, $\exists r\in\mathbb{F}_{p^n}$ s.t. $f(r)=0$.

Hm. I think my reasoning for this part was a little sketchy. I really only showed that for each root r of $f(x)$, there is an isomorphic image of $F_p[x]/f(x)$ in F_{p^n}. But I really want to show that F_{p^n} contains simultaneously all roots of $f(x)$.

$\Rightarrow f(x) \mid x^{p^n-1}-1$

Oh I see a way to do that. I can show that $f(x)$ is a factor of this polynomial, whose roots are all nonzero elements in F_{p^n}.

(If not $x^{p^n-1}-1$ and $f(x)$ have a nontrivial common factor with degree $<\deg f$, which contradicts irreducibility of f.)

$\Rightarrow$ all roots of f are in $\mathbb{F}_{p^n}$.

(or, rather, $\mathbb{F}_{p^n}$ contains a complete set of roots of $f(x)$.)

Corollary. $p^n=\sum_{d\mid n} d N_d$

where N_d = number of irreducible polynomials of degree d over $\mathbb{F}_p$.

It all works! So this fact is true in general too!

This corollary was known to Carl Friedrich Gauss.

ABB 6/29/16

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Summer Fun!

The best way to learn math is to do math, so here are the 2016 Summer Fun problem sets.

We invite all members and subscribers to the Bulletin to send any questions and solutions to girlsangle@gmail.com. We'll give you feedback and might put your solutions in the Bulletin!

The goal may be the lake, but who knows what wonders you'll discover along the way?

In the August issue, we will provide some solutions. You could wait until the August issue to see the answers, but you will learn a lot more if you try to solve these problems on your own.

Some problems are quite a challenge and could take several weeks to solve, so please don't approach these problems with the idea that you must solve them all. Our main goal is to give you some interesting things to think about.

If you get stuck, try to formulate a related question that you can see a way to actively explore to get your mind moving and your pencil writing. If you don't understand a question, email us.

If you're used to solving problems fast, it can feel frustrating to work on problems that take weeks to solve. But there are things about the journey that are enjoyable. It's like hiking up a mountain. Getting to the top rewards one with a spectacular view, but during the journey, there's a lot to see and experience. So here's a meta-problem for those of you who feel frustrated when doing these problems: see if you can dissolve that frustration and replace it with a relaxed, optimistic sense of adventure!

This is Summer Fun, not Summer Torture!

Summer Fun!

Backgammon Strategy

by Aaron Levy

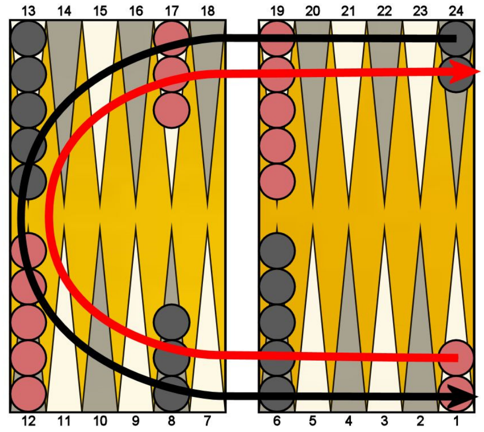


Figure 1. A backgammon board.

Backgammon[1] is one of the oldest board games known. It is a two player game where checkers are moved according to the roll of two dice, and a player wins by removing all of her checkers from the board before her opponent. Players alternate turns, each of which begins with a throw of the dice indicating the possible moves that a player can make. For example, if the black player (see Fig. 1) begins with a 3-1, meaning one of his dice shows a 3 and the other shows a 1, then she must move one of the black checkers 3 spaces forward and another (or the same) checker 1 space forward, where forward is indicated in Fig. 1. She might move, say, the black checkers on the space marked "24" to the spaces marked "23" and "21," or she might move one of the checkers on the 24th space 23rd space, and from there, to the 20th space.

If a player rolls two of the same number, called "doubles," that player must play each die twice. For example, a roll of 5-5 allows the player to make up to four moves of five spaces each.

The goal of the black player is to move *all* of her checkers into the lower right quadrant of the board (which is called her "home board" and consists of spaces marked "1" through "6") at which time she may then begin removing checkers from the board. Since the players alternate rolls and move in opposing directions, there is often interaction between the black checkers and the red checkers. Here are the rules governing those interactions: in the course of a move, a checker may land on any space that is unoccupied or is occupied by one or more of the player's own checkers. It may also land on a space occupied by exactly one opposing checker, or "blot." In this case, the blot has been "hit," and is placed in the middle of the board on the bar that divides the two sides of the playing surface. A checker may never land on a space occupied by two or more opposing checkers; thus, no space is ever occupied by checkers from both players simultaneously. There is no limit to the number of checkers that can occupy a space.

Checkers placed on the bar after being hit must re-enter the game through the opponent's home board (this means that a hit black checker must re-enter on spaces 19 through 24, whereas a hit red checker must re-enter on spaces 1 through 6) before any other move can be made. A roll of 1 allows a black checker to enter on the 24th space, a roll of 2 on the 23rd space, and so forth, up to a roll of 6 allowing entry on the 19th space. Checkers may not enter on a space occupied by two or more opposing checkers.

A player may remove her own checkers from the board only when all her active checkers are in her home board. For black, a roll of 1 permits removal of a checker in the 1st space, a roll of 2 permits removal of a checker in the 2nd space, etc. Black may not use a die roll to remove a checker from a lower space unless there are no black checkers in higher spaces.

Backgammon involves a combination of strategy and luck (from rolling dice). While the dice may determine the outcome of a single game, over a series of many games, the better player will use statistical advantages to accumulate the better record, somewhat like poker. Thus, records of matches between players are good indicators of relative skill.

There are more rules, but those above are sufficient to allow us to develop some skill.

Summer Fun!

[1] This background information is adapted from en.wikipedia.org/wiki/Backgammon.

1. What is the probability of rolling a 6-2? That is, what is the probability that a player's roll shows a 6 on one die and a 2 on the other?

To mentally compute this as well as more difficult calculations for more complicated strategies, it can help to visualize using a 6×6 grid, as in Fig. 2 below. The grid on the left shows an empty grid representing all possible rolls, with the columns representing the roll of one die and the rows representing the other die. The filled-in grid on the right shows the possible 6-2 rolls.

Figure 2. Grids that help us compute the probabilities of various outcomes of rolling two six-sided dice.

Next, what is the probability that a player rolls a 6-2 or a 1-4? How many cells does this correspond to on the grid?

Try using a visualization like this grid for the problems below so as to help quickly see strategies in real time.

2. Supposing all moves are possible, what is the probability that a throw of the dice allows a player to advance a checker by exactly 3 spaces? (This is important information if an opponent's blot is three spaces away.)

3. Suppose the black player has removed all of her pieces from the game board, except for two checkers on the 5th space as shown in Fig. 3a. If she rolls a 3-1, what is her best play, Fig. 3b or Fig. 3c?

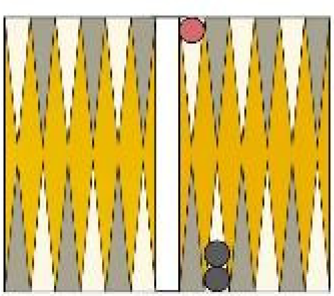


Figure 3a.

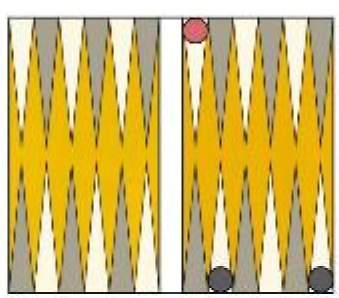


Figure 3b.

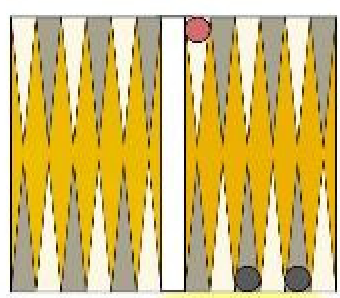
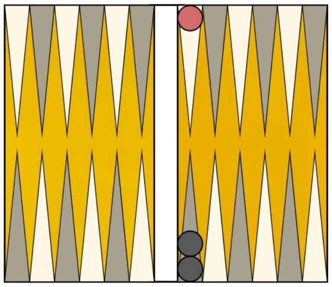


Figure 3c.

4. Suppose black's two checkers are on the 6th space instead of the 5th (as shown at right). Now what is her best play?



Setup for Problem 4.

There is another feature to backgammon that makes the game deep. In match play, opponents play many games, with the stake of each game equal to one point. To speed up match play and to provide an added dimension for strategy, a "doubling cube" is often used. The doubling cube is not a die to be rolled but rather a marker with the numbers 2, 4, 8, 16, 32, and 64 inscribed on its sides (see below, left) to denote the current stake.

At the start of each game, the doubling cube is placed on the bar with the number 64 showing; the cube is then said to be "centered, on 1". When the cube is centered, the player about to roll may propose that the game be played for twice the current stakes. Her opponent must either accept ("take") the doubled stakes or resign ("drop") the game immediately. Whenever a player accepts doubled stakes, the cube is placed on their side of the board with the corresponding power of two facing upward, to indicate that the right to re-double belongs exclusively to the player who last accepted a double. If the opponent drops the doubled stakes, she loses the game at the current value of the doubling cube. For instance, if the cube showed the number 2 and a player wanted to redouble the stakes to put it at 4, the opponent choosing to drop the redouble would lose two, or twice the original stake.

5. Suppose the game is as in Fig. 5 with black on roll. Black has one checker on the 5th space and one on the 2nd, and red has one checker left on the 24th space. Either black will win the game on this roll, or she will lose it. In her position, how many rolls will lose the game? How many will win it? Should she offer to double the stakes?[2]

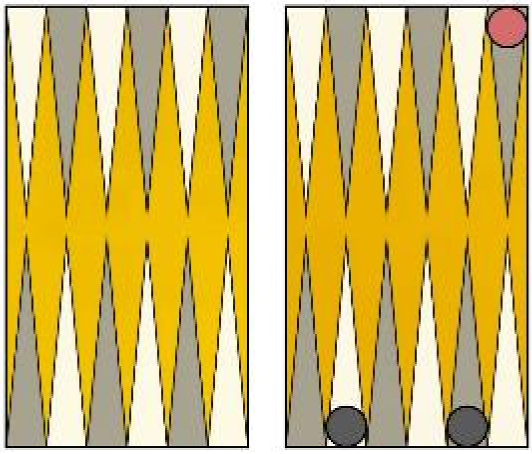


Figure 5.

6. Suppose black doubles. Should red take?

7. Suppose black's probability of winning is p and red's probability of taking is q. Explain in terms of p and q when black should offer a double.

8. On the cover, the black checker heads to the right and the red checker heads to the left. If black rolls first, what is the probability that, eventually, red will be hit? What is the probability that at some point black will be hit? What is the probability that the two checkers will pass by each other in peace?

Summer Fun!

[2] This is adapted from www.bkgm.com/articles/Youngerman/DoublingCubeStrategy.html.

Greedy Algorithms

by Zachary Sethna

1. Giving change.

A cashier must return \$1.57 to a customer using the following coins: pennies (\$0.01), nickels (\$0.05), dimes (\$0.10), quarters (\$0.25), and dollar coins (\$1.00). What is the minimum number of coins the cashier needs in order to make change for the customer? How did you approach finding the minimum number of coins?

A **greedy algorithm** is an algorithm used to solve an optimization problem where, at each stage, one makes the best choice in the moment. For instance, in the previous problem a 'greedy algorithmic' approach would be to pick the largest denomination coin which is less than or equal to the amount still owed. Here's what would happen if we apply this greedy algorithm to our change-giving problem:

Coins Picked	Remaining Change	Greedy Choice
-	\$1.57	Dollar coin
1×\$1	\$0.57	Quarter
1×\$1, 1×\$0.25	\$0.32	Quarter
1×\$1, 2×\$0.25	\$0.07	Nickel
1×\$1, 2×\$0.25, 1×\$0.05	\$0.02	Penny
1×\$1, 2×\$0.25, 1×\$0.05, 1×\$0.01	\$0.01	Penny
1×\$1, 2×\$0.25, 1×\$0.05, 2×\$0.01	\$0.00	-

Greedy algorithms tend to be very useful because they are easy to come up with and fast to use. However, they do not always produce an optimal solution. Let's examine such a case where a greedy algorithm fails to produce an optimal solution.

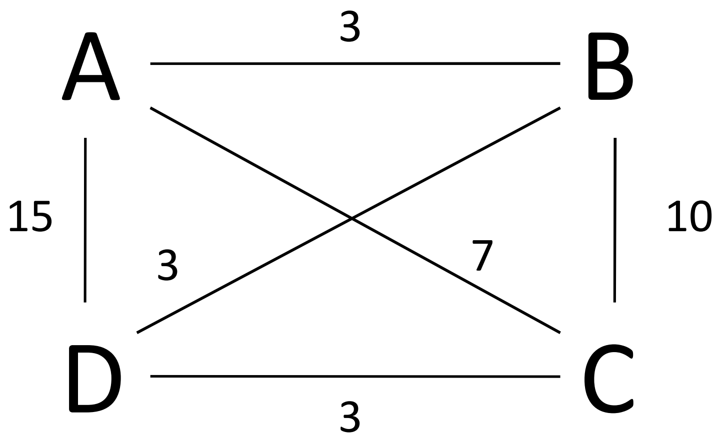
We have a new cashier that has to give a customer \$1.16 in change, but this time, the cashier has the following coin denominations: \$1.00, \$0.50, \$0.10, \$0.07, and \$0.01. What coins will the greedy algorithm choose to make change for the \$1.16? What is the actual minimum number of coins required to produce \$1.16 in change? Why does the greedy algorithm fail this time?

2. Travelling Salesman.

A classic problem in computer science is known as the **travelling salesman**. A common way to state the problem is thus: A salesman starts at a given city (which we will call city A) and must travel to each of the other cities exactly once before returning to the starting city. The salesman, being very busy, is interested in taking the path that costs the least amount of time.

Let's look at a specific example involving 4 cities, A, B, C, and D.

Summer Fun!



A schematic diagram of the 4 cities indicating the time it takes to travel between every pair.

What is the most efficient path? How long does it take? Is the greedy path (constructed by always choosing the nearest unvisited city) the most efficient path?

This problem is a classic computer science problem where the best known algorithms take exponentially longer as the number of cities increases. Since the greedy algorithm doesn't take exponentially longer as the number of cities increases, the greedy algorithm will not always work. Can you construct some examples where the greedy algorithm fails? Can you even construct an example where the greedy algorithm produces the *longest* possible path?

3. Egyptian Fractions.

The ancient Egyptians had a peculiar way of expressing rational numbers. There were explicit symbols for the integers and their associated reciprocals. So, for example, there were symbols for the numbers 3 and 1/3, but no symbol for 5/6. Numbers, like 5/6, for which there was no direct symbol would be expressed as a sum of numbers with symbols. Furthermore, no symbol would be repeated in the sum. Thus, 5/6 could be written as 1/2 + 1/3, but not as

$$1/6 + 1/6 + 1/6 + 1/6 + 1/6.$$

Can all rational numbers be expressed as a finite sum of such symbols with no repeats? We will want a proof or counter-proof!

Hint: Start by examining a few examples, such as

$$\frac{5}{6}=\frac{1}{2}+\frac{1}{3} \qquad \frac{6}{7}=\frac{1}{2}+\frac{1}{3}+\frac{1}{42} \qquad \frac{4}{5}=\frac{1}{?}+\frac{1}{?}+\frac{1}{?} \qquad \frac{12}{13}=?$$

Try some other examples to see if any patterns emerge!

Probability Puzzles with Bags of Coins

by Lauren McGough

1. You're holding a bag with exactly 1 gold coin and 5 silver coins all mixed up.

A. You reach into the bag and pull out a coin. What's the probability that it is gold?

B. Now, suppose you pull out two coins, **with replacement** (that is, pull out a coin, look at it, put it back, mix up the bag well, then pull out the second coin). What's the probability of pulling out the gold coin twice? What's the probability of pulling out the gold coin at least once?

C. What if you pull out two coins **without replacement** (that is, take out a coin, leave it out of the bag, and then take out another coin from the five remaining)? What's the probability of pulling out the gold coin as the first or second coin? What's the probability of pulling two gold coins? What's the probability of pulling two silver coins?

D. With replacement, how many coin pulls are necessary to make it at least 50% probable that you will pull the gold coin at least once? What about without replacement?

E. What if the bag contains 3 gold coins and 3 silver coins? What if the bag contains exactly 3 gold coins and 7 silver coins? Repeat parts A-D for each of these scenarios.

F. Can you answer parts A-D for a bag that has N total coins, exactly g of which are gold?

2. (Bertrand's Box Paradox) Suppose you are holding three unlabeled bags, each of which contains two coins. One contains two silver coins, one contains one gold coin and one silver coin, and the last contains two gold coins. You want to give me the bag with two gold coins as a gift, but you can't remember which bag it was!

A. You reach out and hand me a bag. "Here, try this one," you say. What is the probability that the bag you handed me contains a gold coin?

B. I take out a coin and it's gold. What are the possible bag(s) I could be holding, given this information?

C. For each of the bags I could be holding now: how many gold coins did the bag contain before I peeked at one coin?

D. For each gold coin, what is the probability that the gold coin I took out of my bag was *that* gold coin?

E. Given that the coin I removed from my bag was gold, what is the probability that the other coin in the bag is also gold?

3. I come to visit you again, and you again have a collection of silver and gold coins, this time arranged in four unlabeled cubbies. Each cubby has two drawers, a top and a bottom, and each drawer has one coin. One cubby has a silver coin in both the top and bottom drawers. One has a gold coin in both the top and bottom drawers. One has a silver coin in the top drawer and a gold coin in the bottom drawer. The remaining cubby has a gold coin in the top drawer and a silver coin in the bottom drawer.

Again, you want to give me one cubby as a gift, but you forgot which cubby contains the two gold coins! You tell me to peek inside one of the top drawers. I choose a cubby, peek in the top drawer, and see a gold coin. What's the probability that the coin in the bottom drawer is gold?

4. I come to visit you, but instead I run into your sister, Sal. She comes to me with a big bag of coins, and says, "This bag contains 1,000 coins. Some are gold coins worth \$1 apiece. The other types of coins in it are worthless. I'll sell you the whole bag for \$501! The only catch is that I won't tell you how many gold coins it contains. It could be one, could be six hundred, could be zero. You are only to look at one, single, randomly selected coin in order to make your decision." Assume that the different possible numbers of gold coins in the bag, from 0 to 1000, are equally likely. You pull a coin and it's gold. Is it worth it to buy the bag?

5. (The Monty Hall problem) The next day, I see Sal again. This time, she's holding three bags, and she says, "One of my bags holds a gold coin, and two hold worthless pieces of cardboard!"

A. She mixes the bags around so they aren't in any order, and tells me to pick one. Then, she opens one of the bags I didn't pick. It contains a piece of cardboard! Now, she gets a glint in her eye, and asks, "Do you want to switch to take to the other bag instead?" If I point to the correct bag, I get the prize, but otherwise I get nothing. On first instinct: should I switch or not?

B. Let's analyze the situation. Label the bags 1, 2, and 3, such that the prize is in bag 1. If I pick bag…

... 1, which bag(s) could she open to reveal cardboard? Should I switch?
... 2, which bag(s) could she open to reveal cardboard? Should I switch?
... 3, which bag(s) could she open to reveal cardboard? Should I switch?

C. How many times did switching help versus not help? Does switching improve my chances of getting a gold coin?

D. Sal now presents us with N bags. "There's only one prize among all these bags!" I point to a bag, and of the $N - 1$ other bags, she opens all but one, revealing $N - 2$ bags that the prize is *not* inside. "Do you want to switch now?" What is the probability I picked the prize on the first pick? If I picked the prize, would switching cause me to win or lose? What is the probability I did not pick the prize on the first pick? If I did not pick the prize, would switching cause me to win or lose? What is the relationship between the probability that I don't pick the prize on the first pick, and the probability that switching causes me to win?

Summer Fun!

The Thirty Birds, and other problems with integer constraints

by Matthew de Courcy-Ireland

Mersenne Primes

1. A Mersenne prime is a prime number of the form $2^m - 1$, where m is an integer. Try to formulate a conjecture about values of m for which $2^m - 1$ is or isn't prime. Can you prove it?

2. What can you say about primes of the form $2^m + 1$?

Lattice Points on a Graph

3. What is the graph of the equation $x^2 - y^2 = 1$ in the xy-coordinate plane? In general, what is the graph of $x^2 - y^2 = N$ for $N = 2, 3, 4$, etc.?

4. A lattice point is a point whose coordinates are both integers. How many lattice points are there on the graph of $x^2 - y^2 = 1$?

5. How many lattice points are there on the graph of $x^2 - y^2 = N$, where N is a fixed integer.

The Problem of Thirty Birds

This problem comes from *Liber Abaci* by Fibonacci. Bernadette buys 30 birds using 30 silver coins. There are three types of birds: a partridge costs 3 coins, a dove costs 2 coins, and a sparrow costs half a coin (meaning two sparrows for a silver – half-coins are not in circulation). We will address the question, "How many birds must Bernadette have bought?"

6. Suppose you tried to solve the problem by writing down every possible way of purchasing 30 birds, then computing how much each way cost, and locating those where the cost turns out to be 30 silver coins. How many cases would you have to examine? (Remember: there are no half-coins in circulation, so you cannot purchase an odd number of sparrows.)

7. How far can you narrow the search in Problem 6 using some good reasoning?

8. Does it turn out that the answer is unique?

9. Play around with the numerical values given in the problem. What if instead of using 30 silver coins to buy 30 birds costing 3, 2, or ½ coins depending on the type of bird, Bernadette bought B birds using C coins with prices p, d, and s? Can you find a relationship between B, C, p, d, and s that will guarantee a unique solution to the problem?

10. What can you say if there are more than three types of birds?

Summer Fun!

Similar Tiles

by Long Nguyen and Ken Fan

Two geometric shapes are **similar** if they have the same shape, though they may be of different sizes. If you have two similar figures, all ratios between a length in the second to the corresponding length in the first will be the same, and this ratio is called the **scale factor**.

In this Summer Fun problem set, we will call a tiling of a given shape **special** if all tiles are similar to each other and to the given shape. We shall call a special tiling **proper** if the tiles are smaller than the given shape.

1. Show that every rectangle has a proper special tiling.

2. For any positive integer N, exhibit a rectangle with a special tiling consisting of N tiles.

3. Construct a special tiling of a square that has 6 tiles.

4. For what N does there exist a special tiling of a square consisting of N tiles?

5. Show that all triangles admit proper special tilings.

6. What quadrilaterals admit proper special tilings?

7. Determine all triangles that have a special tiling consisting of just 2 tiles.

8. For each positive integer N, exhibit a triangle with a special tiling consisting of N tiles.

9. Can you come up with a 6-sided shape that has a proper special tiling?

10. Can you come up with an 8-sided shape that has a proper special tiling?

11. Are there arbitrarily large n-sided shapes that have a proper special tiling?

Let's call a tiling **super special** if all the tiles are not only similar to each other, but also the same size as each other (though not the same size as the given shape).

12. For what positive integers N are there proper super special tilings of a square with N tiles?

13. Can you come up with a 3D shape that enjoys a proper special tiling that does not look like a brick?

14. Can you come up with a 3D shape that enjoys a proper special tiling that does not look like a prism?

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 18 - Meet 12	Mentors:	Bridget Bassi, Karia Dibert, Anna Ellison,
May 5, 2016		Neslly Estrada, Jennifer Matthews, Jane Wang (Head)

We held our traditional end-of-session Math Collaboration!

Before getting to the Math Collaboration, I'd like to thank the Princeton University Press for giving us a deep discount on our treasure chest treasures: each member received a signed copy of Professor Anna Frebel's book *Searching for the Oldest Stars*. I would also like to thank Anna Frebel for personally signing each copy, making this the most precious treasure in Girls' Angle history!

I highly recommend *Searching for the Oldest Stars*. It is a science book that can be read cover to cover and explains the young field of stellar archeology. The idea of stellar archeology is to learn about the early composition of the universe by performing spectral analyses of stars. After the Big Bang, it is thought that the only elements in the universe were the first 3 elements in the periodic table: hydrogen, helium, and lithium. Elements higher up the periodic table were synthesized inside stars. In the core of stars, nuclear fusion builds elements all the way up to iron. For higher elements, other processes are required.

Because the oldest stars were formed before very many higher elements were created, they will be "metal poor" (according to Frebel, Astronomers regard anything on the periodic table with atomic number greater than 2 as a "metal"). Their chemical compositions gives us a snapshot of the chemical composition of the early universe. Frebel and her team have set the record for discovering the most metal poor stars.

In her book, Frebel describes all these matters and paints a rich picture of what it is like to be a modern astronomer, including details such as how astronomers manage to stay awake through the night while operating the world's largest telescopes.

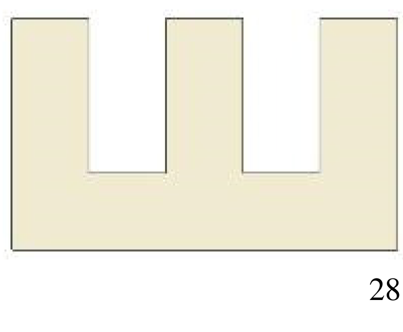
There are new, more powerful telescopes being constructed today, and, if you aren't already excited about that, this book will get you excited about them for the amazing discovery potential that they bring.

The girls solved this session's Math Collaboration in record time an hour ahead of schedule! Here are some problems from the event. Can you solve them?

Make a perspective drawing of a staircase that has one 90-degree turn in it.

How many different integer-sided triangles are there with a perimeter of 20 units?

The top view of a prison looks like a capital letter E (see image at right). Guards (who can see infinitely far) must be placed around the outside of the building so that at any given moment, every part of every wall is being observed by some guard. What is the minimum number of guards required to fulfill this requirement?



Calendar

Session 18: (all dates in 2016)

January	28	Start of the eighteenth session!
February	4	Anna Frebel, Department of Astronomy, MIT
	11	
	18	No meet
	25	
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Session 19: (all dates in 2016) This calendar is tentative.

September	15	Start of the nineteenth session!
	22	
	29	
October	6	
	13	
	20	
	27	
November	3	
	10	
	17	
	24	Thanksgiving - No meet
December	1	
	8	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

We wish NASA good luck as the Juno spacecraft inserts itself into orbit about Jupiter. We can't wait to see the closest images of Jupiter in history!

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- □ Enclosed is a check for $36 for a 1-year Girls' Angle Membership.
- □ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $36 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ______________________________________