Girls' Angle Bulletin

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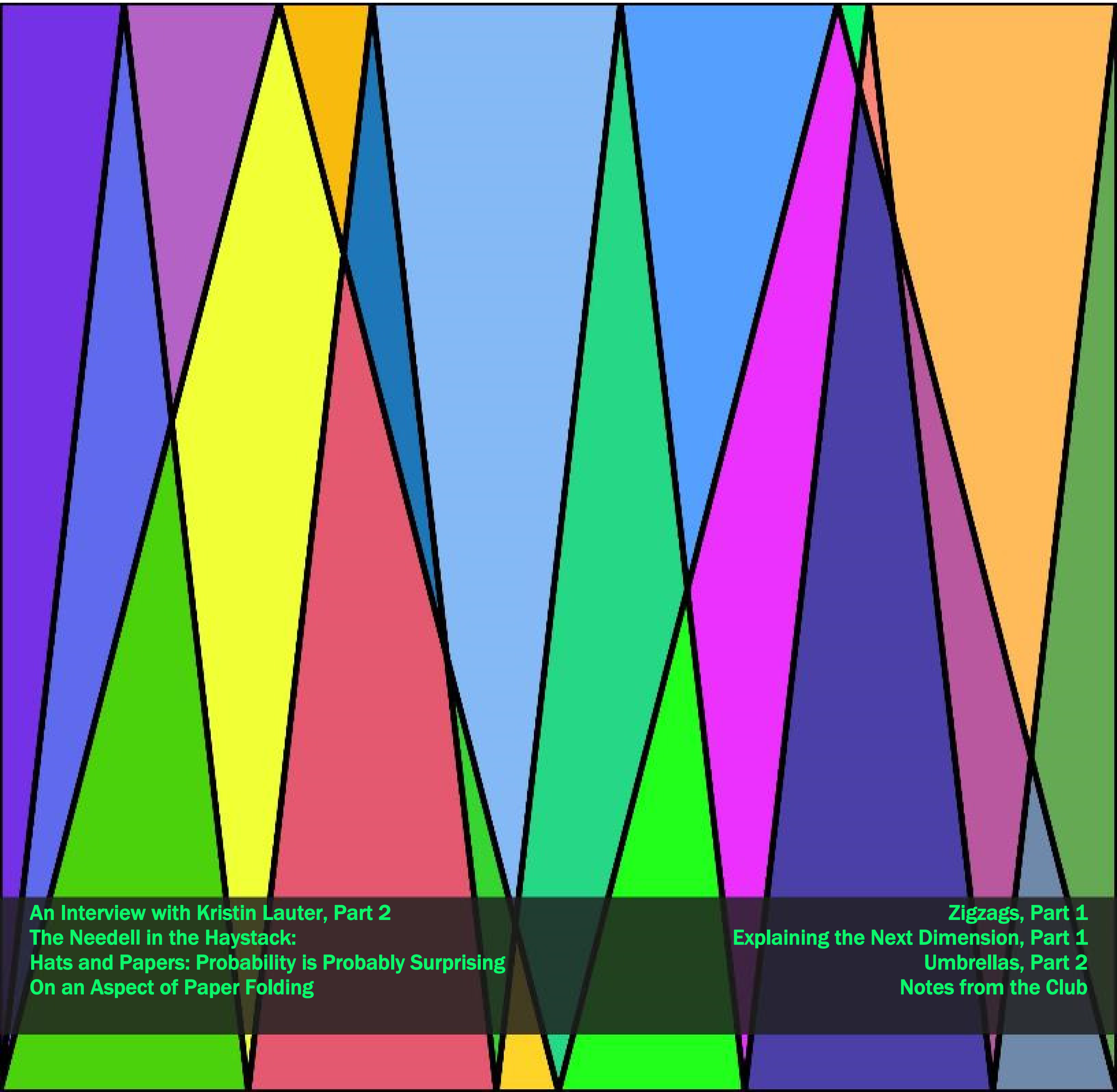
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

I hope that as you read these pages, you feel provoked to do math. And if you do, don't suppress that feeling! Take that scratch paper that you should always have with you whenever you're reading math and start figuring! And if you can't figure it out, email us. - Ken Fan, President and Founder

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Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Crisscrossing Zigzags I by C. Kenneth Fan. See *Zigzags Part 1* on page 15.

An Interview with Kristin Lauter, Part 2

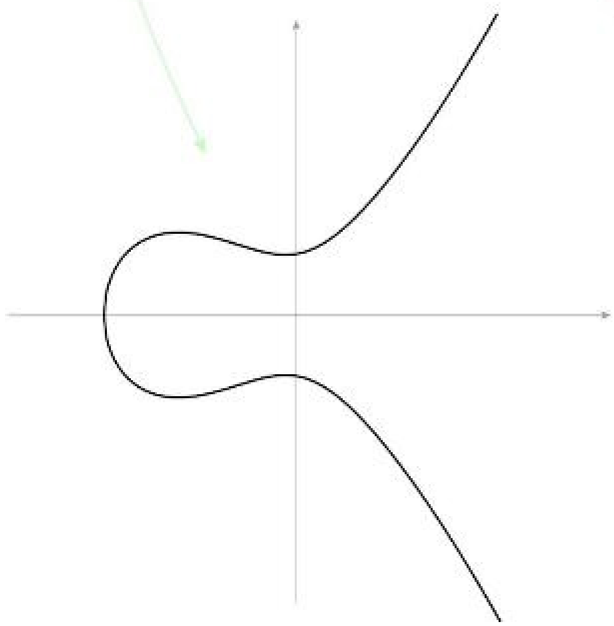
This is part 2 of a multi-part interview with mathematician Kristin Lauter, which was conducted by Ke Huang at the University of Washington in April.

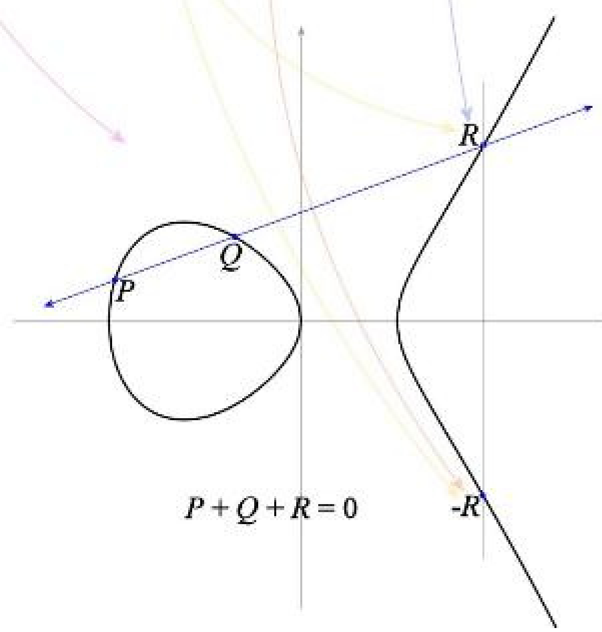
Ke Huang: How does geometry play into [cryptography]?

Kristin: The algebraic geometry comes in in a different way. The simplest example is elliptic curve cryptography. Elliptic curves[1] are beautiful objects because they have an algebraic structure and a geometric structure. And the algebraic structure is what's called a group law. Let me explain. If you draw a picture of an elliptic curve over the real numbers, it might look something like this, or it could look like something like this.

Ke Huang: I used to say they were goldfish.

Kristin: Yeah, exactly. The idea is that you can define a commutative group law on the points of an elliptic curve. You take two points P and Q on the curve and draw a line through the two points, and it'll intersect the curve at another point R. And the idea coming from algebraic geometry is that the sum of these three points that are on a line should add up to zero. If P plus Q plus R is supposed to be zero, then P plus Q should be $-R$. But what does $-R$ mean? Well, if you reflect over the x-axis, at least if you're not in characteristic 2 or 3, you'll obtain $-R$, in the sense that these two points add up to the identity in the group (which can be thought of as zero), which is the point at infinity. Thinking about the point at infinity requires a little bit of imagination, but in terms of the actual algebraic geometry, we usually think of it in terms of projective space. But I'm drawing this picture in affine space. There's a point off at infinity in projective space that is the identity element for the group. So, these two points add up to the identity element. So, this one is minus this. All in all, that's a group law on an elliptic curve, which allows you to take points and add them together and get another point.





[1] An elliptic curve is a curve defined by a simplified equation of the form $y^2 = x^3 + ax + b$, where the cubic $x^3 + ax + b$ does not have multiple roots.

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We're committed to producing quality math educational content and make every effort to provide this content to you for free.

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For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Kristin Lauter and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

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Thank you and best wishes,
Ken Fan
President and Founder
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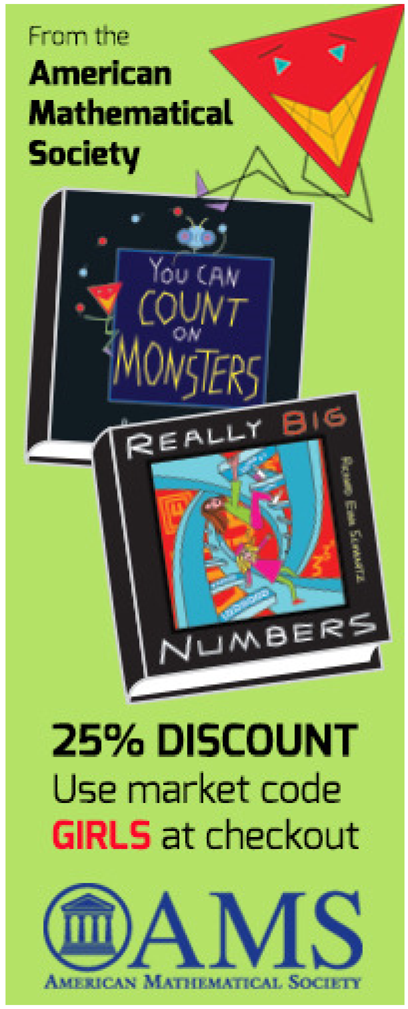

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America's Greatest Math Game: Who Wants to Be a Mathematician.

The Needell in the Haystack

Hats and Papers: Probability is Probably Surprising[1]

by Deanna Needell | edited by Jennifer Silva

As I write this article, it's raining here in Los Angeles (actual rain, water literally coming from the sky)! The weather here is typically moderate and sunny, so LA turns into quite a frenzy with this highly improbable weather. This leads to the question, what does **improbable** really mean? Although almost everyone has some intuition and basic understanding of probability, it turns out to be a rich, beautiful mathematical concept full of surprises. In this article, we will explore some fun problems in probability that will hopefully leave you with some intriguing surprises to ponder into the new year.

To a non-mathematician, probability can be described simply as the *likelihood* or *chance* that an event occurs. For example, we all know that if we roll a fair six-sided die, we expect it to land on 1 one-sixth of the time, meaning that the probability of that event is 1/6. Mathematically speaking, probability is an example of a **measure**, or a function, P, which takes as input an **event** and returns a number between 0 and 1, inclusive. This number is the probability that the event occurs. Formally, an event is a subset of a **sample space**, which is a set that contains all of the possible outcomes of some random experiment. We will typically use the symbol Ω for the sample space. So P is a function whose domain is some collection of subsets[2] of Ω, and whose range is the interval $[0, 1]$.

Let's use die rolls as an example. The experiment is the actual die roll; since a die roll results in a number between 1 and 6, inclusive, the sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$. We can write things like $P(\{1\}) = 1/6$ to mean the probability of rolling a 1 is 1/6. Here, we can let an event be *any* subset of Ω. For example, the event that we roll an even number is denoted by the set $E = \{2, 4, 6\}$, so we can write $P(E) = 1/2$ (since for sample spaces with finitely many equally likely outcomes, the probability of an event is equal to the number of outcomes in that event divided by the total number of possible outcomes – in this case, 3/6).

To be a mathematical probability measure, the function P must obey some basic laws. First, the probability that nothing occurs must be zero: $P(\emptyset) = 0$. The probability that *something* occurs must be one: $P(\Omega) = 1$. And, finally, there is a law that explains how the probability of disjoint events relates to the probabilities of each event. Since we'll only be dealing with finite collections of events,[3] we will say that if two events A and B are disjoint (have empty intersection), the probability that an outcome in A or B occurs must equal the sum of the individual probabilities: $P(A \cup B) = P(A) + P(B)$. We'll call this last law the "union law."

Let's briefly discuss a few probabilistic pitfalls. First, there is the question of whether an event having zero probability means that its occurrence is impossible. Namely, if $P(A) = 0$, does that mean that the event A will never occur? The answer is, in short, no. Let's see why. It is easiest to see this by considering experiments for which the sample space is **continuous**. For

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[1] This content supported in part by a grant from MathWorks.

[2] We're not being precise about what collections of subsets can be used as domains for a probability measure because that involves some subtle math that we will not get into here. Suffice it to say that, in general, it is not possible to take the domain to be the set of all subsets of Ω (although if Ω is a finite or countable set, one can.).

[3] In general, we may have to work with a countably infinite collection of events; so the final law would be the so-called **countable additivity** law, which says that if E_i is a countable collection of pairwise disjoint sets, then $P(\bigcup_i E_i) = \sum_i P(E_i)$.

instance, suppose an experiment generates a random number in the interval $\Omega = [0, 1]$ so that each number (or equivalently, each subinterval of the same size) is equally likely to occur. We'll call this randomly generated number X. This yields the so-called **uniform** distribution, and for any two real numbers $0 \le a < b \le 1$, we have that the probability that X lies between a and b is equal to $b - a$; that is, $P([a, b]) = b - a$. This should match intuition. For example, we can visualize such an experiment by considering a very large number line on the ground spanning from 0 to 1, and generating X by randomly throwing a small pebble on the line and seeing where it lands (of course, for logistical reasons this doesn't exactly give us the uniform distribution,[4] but it serves as a good visual). Then, the probability that the pebble falls between, say, 0.25 and 0.5, is the width of that interval, namely 0.25. So what is the probability that X is *exactly* some number, say 0.5? Or exactly $\pi/4$? The answer must be 0. To see this, instead suppose that $P(\{a\}) = \varepsilon > 0$ for some number $0 \le a \le 1$. Since by definition each number in the interval [0, 1] has the same probability of occurring, it must be that $P(\{b\}) = \varepsilon$ for *any* number $0 \le b \le 1$. Because there are infinitely many numbers in the interval [0, 1], we can consider an event E that consists of more than $1/\varepsilon$ numbers. By using the union law over and over, we would find that $P(E) > 1$, which is a contradiction to the definition of a probability measure. So, we have just witnessed an experiment for which *any specific outcome* has probability zero. But clearly the pebble will always land somewhere; it's just that the specific event wherever it does land had probability zero of occurring. In more catchy terms, the impossible is possible – and even certain!

By the way, there are many other useful continuous distributions. For example, there's the **Gaussian distribution**, which generates a random number X so that the probability that X lies in an interval $[a, b]$ of the real numbers is given by the integral

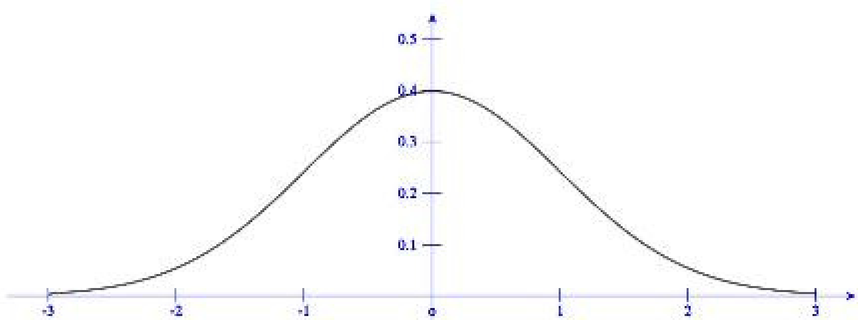

Figure 1. The Gaussian distribution.

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx .$$

Geometrically, this means the probability that X is in some set A is the area under the curve

$$y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

over the range A. This curve is the "bell curve" shown in Figure 1, which has total area 1 but yields non-zero probabilities for any non-empty interval of the real numbers (i.e., the "tails" of the curve get closer and closer to the x-axis but never touch it).

[4] We would need an infinitely small pebble and a very accurate throwing arm!

The second pitfall revolves around the idea of **independence**. First, consider rolling two (fair six-sided) dice, and write the outcome as (X, Y) where X denotes the first die's roll and Y the second's. Note that the sample space has 36 possible outcomes, and one can compute things such as the probability that the dice sum to 7 as

$$P(\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}) = 6/36 = 1/6.$$

Two events A and B are **independent** if knowledge about one event occurring gives no new knowledge about the other one occurring. The event that the first die lands on 1 and the event that the second die lands on 1 are independent events. So if I told you the first die landed on 1, that would give you no new information about the likelihood that the second would also land on 1. Mathematically, we say that two events A and B are independent if $P(A \cap B) = P(A)P(B)$. It is often easier to verify independence mathematically than it is to intuit it. For example, in the two-dice experiment, the event that the first die roll is a 1 is *not* independent from the event that the sum of the dice is 8, since it's impossible to roll a sum of 8 if one of the dice comes up 1. On the other hand, the event that the first die roll is a 1 *is* independent from the event that the sum of the dice is 7. The reader can readily verify these statements mathematically and – perhaps slightly less readily – intuitively.

With these ideas under our belt, let's explore some fun problems. There are now many "classical" probability surprises like the Monty Hall problem and the birthday problem; the former gives an unintuitive optimal strategy for a popular game show problem, and the latter computes the probability that two people in a room full of people have the same birthday. Here we will explore some probably[5] less widely-known problems that are also motivated by simple games.

First, consider a simple setup where three players are given a red or blue hat in such a way that each player gets a red or blue hat with equal probability, independently from the other players. One can imagine the sample space consisting of the eight equally likely outcomes

$$\Omega = \{(R, R, R), (R, R, B), (R, B, R), (B, R, R), (B, B, R), (B, R, B), (R, B, B), (B, B, B)\}.$$

The players can see each other's hats but not their own. They can discuss strategy before the game starts, but cannot communicate once the game begins and hats are drawn. After players see each other's hats, they must all simultaneously guess the color of their own hats or pass. The group wins the game if at least one person guesses her own hat color correctly and no player guesses incorrectly. What is the probability that the group wins if they each simply guess red? We can compute this probability to be $P(\{(R, R, R)\}) = 1/8$; not very high. So perhaps the group decides to have one person guess red and the rest pass. This improves the probability to 1/2. Is there a better group strategy? Spoiler Alert: You should stop reading here if you want to think about it! The answer is yes, there is a much better group strategy: each player should look at the other two players' hats, and if she sees two hats of the same color she should guess the opposite color. (If she sees two different colors, she should pass.) We compute the probability of winning in this case as

$$P(\{(R, R, B), (R, B, R), (B, R, R), (B, B, R), (B, R, B), (R, B, B)\}) = 6/8 = 3/4.$$

Much improved!

[5] Pun intended.

Now let's generalize this problem so that each player places a bet on her hat being a certain color. The player chooses the amount she wants to bet; she wins that amount if she is right, and loses that amount if she is wrong. The group as a whole wins if the total winnings exceed the total loss. What betting strategy maximizes the probability that the group wins, and what is this maximal probability? Do you think it gets easier or harder to win with more players?

Let's first consider the same game but with only two players. To obtain a complete betting strategy, we need only to come up with four real numbers, x_1^R, x_1^B, x_2^R, and x_2^B, where x_i^j corresponds to the bet made on red (if a player wants to bet on blue, we can denote this by a negative bet on red) by player *i* when she sees the other player with a *j*-colored hat. We then want to select these four numbers so that the total winnings are positive for the most possible outcomes. In the two-player case there are only four possible hat outcomes, namely *BB*, *RB*, *BR*, and *RR*. Can you come up with four betting numbers so that all outcomes except, say, *BB*, yield positive earnings? After trying some configurations, see Figure 2 for an example of such a strategy. Thus, the probability the group wins is 3/4. When you were constructing this betting configuration, your strategy may have been to "load" up the risk (highest potential loss) in one outcome, as in the *BB* outcome of Figure 2.

Outcome	Player 1 wins	Player 2 wins	Earnings	Result
BB	-1	-2	-3	Lose
RB	1	-0.5	0.5	Win
BR	-1	2	1	Win
RR	1	-0.5	0.5	Win

Bet on red	Strategies	
View	Player 1	Player 2
R	1	0.5
B	1	2

Figure 2.

Can you find a strategy for three players that yields a winning probability of 7/8? It turns out you can use this technique to come up with a strategy for *n* players for any *n* so that the winning probability is $1 - 1/2^n$. Thus, the more players you have, the higher the probability of winning!

For our last problem, consider a game in which someone writes two numbers on two different sheets of paper; let's call them *X* and *Y*. These are then placed facedown on the table. The only thing you as the player know is that the probability that these two numbers are equal is zero: $P(X = Y) = 0$. You then get to select one paper to flip over (let's assume you flip a fair coin to determine which paper to flip) and view the number written on it, which we'll call *W*. You now have to guess whether the number on the remaining facedown paper, which we'll call *Z*, is larger or smaller than the one in your hand. You win if your guess is correct. What is your strategy for such a game?

First, let's consider a simple strategy where you always guess that the number in your hand is larger. Thus with our notation, you win when $W > Z$. Let's compute this probability. Note that there are only two ways to win: the event that $W = X$ and $X > Y$, or the event that $W = Y$ and $Y > X$. By using the union law and independence, we see that

$$\begin{aligned}
P(\text{win}) &= P(W > Z) \\
&= P(W = X \text{ and } X > Y) + P(W = Y \text{ and } Y > X) \\
&= P(W = X)P(X > Y) + P(W = Y)P(Y > X) \\
&= \frac{1}{2}P(X > Y) + \frac{1}{2}P(Y > X) \\
&= \frac{1}{2}(P(X > Y) + P(Y > X))
\end{aligned}$$

$$= \frac{1}{2} P(X \neq Y)$$

$$= \frac{1}{2}(1) = \frac{1}{2}.$$

Hence, with this strategy you will win with probability 1/2. Can you do any better? Recall that you know nothing about the distribution of the numbers X and Y other than that they are not equal. So how could you possibly do any better than a 50-50 chance? Surprisingly, you can! There is a strategy that gives you a probability of winning that is strictly (and perhaps only very slightly) larger than 1/2. Hint: you may generate continuous random numbers in your strategy if you wish to use them.

Spoiler Alert: Don't read further if you want to think about this yourself. Solution below!

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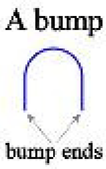
On an Aspect of Paper Folding

by Alana Axelrod-Freed, Milena Harned, and Miriam Rittenberg | edited by Amanda Galtman

Suppose we have n “bumps” in a row, i.e., n ∩-shaped curves.

Two bumps Four bumps

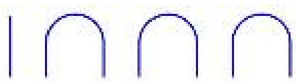


Each bump has two “ends.” We are interested in tying together these bumps by attaching their ends to connecting curves that remain entirely at or below the level of the bump ends. Throughout this paper, when we speak of attaching two ends, we mean to join the two ends with a curve that stays entirely at or below the level of the ends.

Two examples of tying bumps together

How many ways are there to connect n bumps to form a single connected curve, leaving two bump ends unattached, without any of the connecting curves crossing?

In this paper, we will answer this question. But first, we need the solution to a slightly different problem. In order to state this other problem, we introduce another shape: the “loose end.” By “loose end,” we mean a vertical line segment whose bottom is considered an end that can be attached to another end, but whose top is not considered an end.



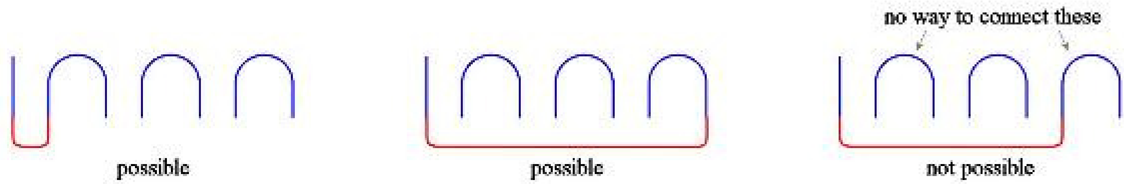
Three bumps and a loose end

Lemma 1. There are 2^n ways to connect one loose end followed by n bumps such that exactly one end remains unattached.

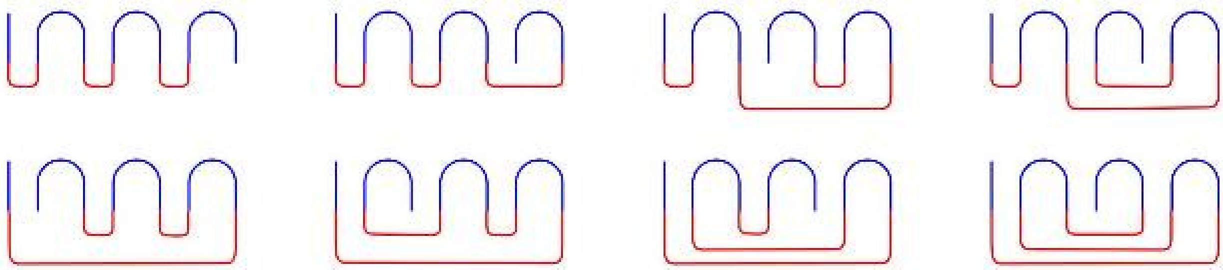
Proof. Let a_n be the number of ways to connect one loose end and n bumps. The proof will be by induction on n.

If there are 0 bumps, there is 1 way to connect everything, namely, by doing nothing. There is nothing to connect. This establishes the base case.

Now suppose there are n bumps and the lemma is true for fewer than n bumps. To tie all the pieces together, the loose end must be attached to one of the ends of some bump. Furthermore, the loose end must attach to either the nearest bump end or the furthest bump end, because attaching to any other bump end makes it impossible to connect the leftmost bump to the

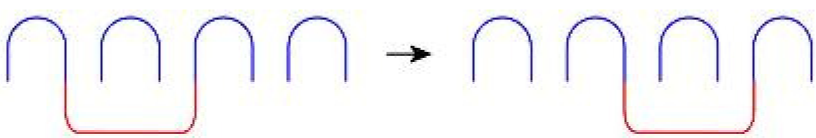


rightmost bump. If the loose end is attached to the nearest bump end, the situation is equivalent to having 1 loose end and $n - 1$ bumps, because the loose end and the leftmost bump effectively become a single loose end. If the loose end is attached to the furthest bump end, we are again left in a situation equivalent to having 1 loose end and $n - 1$ bumps. Therefore, $a_n = 2a_{n-1}$. This recursion relation plus the starting condition of $a_0 = 1$ implies that $a_n = 2^n$. □

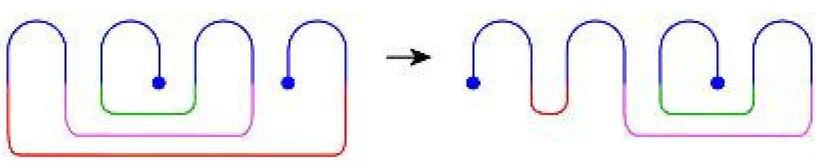


The 8 ways to tie together 1 loose end and 3 bumps.

We also need a second result. To explain it, we must first explain what we mean by "rotating" the connections by a bump. Suppose we have tied together some bumps. We can "rotate" all the connections to the right by a bump by severing each connection and reattaching to the bump end two bump ends to the right of the original connection, with wraparound. For example, the following figure illustrates rotating a single connection by one bump to the right.



The next figure shows what happens when we rotate all the connections to the right by one bump in a valid way of tying together 4 bumps.



Rotating to the right by one bump can also be thought of as moving the rightmost bump to the other side so that it becomes the leftmost bump.

Lemma 2. Suppose we have tied together a bunch of bumps. If we rotate all the connections to the right by a bump, the result is still a valid way of tying together all the bumps.

Proof. First, note that if there is only one bump, then there is no connecting curve and there is nothing to prove. Also, if there are only two bumps, then there is one connecting curve that connects the two bumps. A rotation right by one bump results in the connection to the first bump becoming a connection to the second bump, and vice versa. Therefore, after a rotation to the right by one bump, the two bumps remain connected.

Now assume that there are more than two bumps (and, hence, at least two attached pairs of bump ends). If a and b are bump ends and a is to the left of b, we write $a < b$. Let a and b be an attached pair of bump ends, and let c and d be another attached pair of bump ends. Without loss of generality, we may assume that a and c are the leftmost bump ends of their attached pair, and, furthermore, $a < c$. Since the connecting curves do not cross, we must have either

$$a < b < c < d \text{ or } a < c < d < b.$$

Suppose after rotating to the right by one bump, a, b, c, and d move to a', b', c', and d'. When the bumps are rotated right by one bump, if neither d nor b is part of the last bump, the resulting bump connections will preserve this ordering and not cross. If d is attached to the rightmost bump and $a < b < c < d$, note that c cannot also be attached to the rightmost bump; if it were, then the rightmost bump would be cut off from the other bumps. Therefore, we have $d' < a' < b' < c'$, which does not involve a cross. On the other hand, if d is attached to the rightmost bump and $a < c < d < b$, then, in fact, both d and b are attached to the rightmost bump and we must have $d' < b' < a' < c'$, which also does not involve a cross. Finally, if b is attached to the rightmost bump, but d is not, then we must have $b' < a' < c' < d'$, which does not involve a cross. Since no pair of connections becomes crossed after the rotation, the new, rotated, way of connecting remains valid. □

Now, we turn to the main result of this paper.

Proposition. The number of ways to tie together n bumps so that two bump ends remain unattached is $n2^{n-1}$.

Proof. Consider n bumps. Suppose the first bump end is one of the two unattached bump ends. By thinking of the first bump as a loose end, we see that this situation is equivalent to the condition required by Lemma 1, so there are 2^{n-1} ways to connect the remaining ends. By Lemma 2, there are the same number of ways to connect if we pick any of the other ends to remain unattached. (If we pick an end that is a right bump end, we may rotate by a bump as necessary until that end is the rightmost bump end, then reflect everything in order to directly apply Lemma 2.) There are $2n$ bump ends, so adding the possible ways to connect for each end gives $n2^n$ ways in total. Since each way to connect leaves two ends unattached, we divide by 2 to avoid double counting, and we conclude that there are $n2^{n-1}$ ways to tie together n bumps. □

Incidentally, the reason why "paper folding" is in our title is because our result relates to paper folding in the following way. Suppose you have a rectangular strip of paper with dimensions 1 by $2n$. How many ways are there to fold it into a stack of $2n$ squares? Our result answers this question if we restrict to folding schemes where the squares in positions $2k - 1$ and $2k$ are next to each other in the stack, for all integers k from 1 to n, inclusive. The general problem, also known as the "stamp folding problem," is unsolved.

Zigzags, Part 1

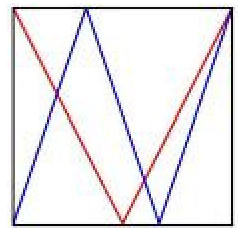
by Ken Fan | edited by Jennifer Silva

Emily notices Jasmine at a table in the school library and goes to join her. As she approaches, she sees that Jasmine is working on a geometry problem.

Jasmine: Hi Emily!

Emily: Hey. Is that a cake-cutting problem?

Jasmine: Not really. It's a problem from a Girls' Angle meet. Essentially, the problem is to take a square, then to draw two zigzags across it like this [see figure at right]. The zigzags are regular, meaning that horizontal coordinates of the points where they hit the top and bottom sides are evenly spaced.



Emily: So let me guess, you want to know what the area of each piece is?

Jasmine: Actually, the problem asked for the fraction representing the area of the smallest piece.

Emily: Fraction? Oh, I see. Since scaling preserves ratios of lengths and areas, with regular zigzags, the fractions representing the areas of the pieces won't change if the figure is scaled. In fact, that's true even if we only scale in one direction; the fraction of the total area that each piece comprises would be the same if we drew regular zigzags across a rectangle of any aspect ratio.

Jasmine: Right. In any case, I'm up for computing the fraction of the area for all of the pieces, not just the smallest one. So let's do that.

Emily: Most of those shapes look like triangles. Let's see. There are six triangles with at least one side that is horizontal or vertical. There's one triangle without any horizontal or vertical sides. And there's a quadrilateral.

Jasmine: Since the scale doesn't matter, let's assume this is a unit square. That way, finding the fraction of the area becomes the same as finding the area. We can use analytic geometry to figure out the various lengths and the coordinates of any points of intersection.

Emily: Sounds like a good plan. We might as well put the square in the plane so that its vertices are at (0, 0), (1, 0), (1, 1), and (0, 1).

Jasmine: The red zigzag goes from the upper left corner to the midpoint of the lower side, then to the upper right corner, so it touches the lower side at (1/2, 0).

Emily: The blue zigzag hits the top side at (1/3, 1) and the bottom side at (2/3, 0).

Jasmine: For the coordinates of those two intersection points inside the square, we can write down the equations for the lines that each intersection is on, then solve them simultaneously.

Girls!

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Explaining the Next Dimension, Part 1

by C. Kenneth Fan | edited by Jennifer Silva

If you're having trouble getting a handle on the concept of a fourth spatial dimension, here are two side-by-side dialogues which we hope will help. In the left dialogue, you are explaining to a flatlander[1] named Skate what a cube is. In the parallel right dialogue, Hyperia is explaining to you what a hypercube is. Skate is a two-dimensional being confined to a two-dimensional world (i.e, a plane). Skate has no experiential knowledge of a third spatial dimension, just as we three-dimensional creatures have no experiential knowledge of a fourth spatial dimension.[2] If you can empathize with Skate's conceptual difficulties and see how Skate can overcome them, you are also seeing how you can overcome your difficulties with the fourth dimension.

Skate: What is a cube?	**You**: What is a hypercube?
You: A cube is a three-dimensional version of a square.	**Hyperia**: A hypercube is a four-dimensional version of a cube.
Skate: What do you mean? What's this third dimension?	**You**: What do you mean? What's this fourth dimension?
You: You know how you can walk forward and backward as well as left and right?	**Hyperia**: You know how you can move forward and backward, left and right, and up and down?
Skate: Sure!	**You**: Of course!
You: Well, the third dimension is just another direction you can move, which we call up and down. When you move up or down, you are moving in a direction perpendicular to both forward/backward and left/right.	**Hyperia**: Well, the fourth dimension is just another direction you can move, which we'll call **ana** and **kata**.[3] When you move ana or kata, you are moving in a direction perpendicular to forward/backward, left/right, and up/down.
Skate: Up and down? There are no such directions.	**You**: Ana and kata? There are no such directions.
You: There are, though you unfortunately don't have access to them, confined as you are to a 2D world.	**Hyperia**: There are, though you unfortunately don't have access to them, confined as you are to a 3D world.
Skate: Can you point me in that direction?	**You**: Can you point me in that direction?

[1] Flatland is a 2D world created by Edwin Abbott in his book *Flatland: A Romance of Many Dimensions*.
[2] According to Einstein's relativity theory, we live in a 4D combination of space and time called spacetime. But in this article, we're not concerned with time. We're thinking of the fourth dimension as a fourth spatial dimension entirely analogous to left/right, back/forth, and up/down.
[3] "Ana" and "Kata" were coined for exactly this purpose by Charles Howard Hinton in his book *A New Era of Thought*.

You: Unfortunately, I can't. You're confined to the flat page, and the direction is perpendicular to the page. Imagine trying to explain forward and backward to a one-dimensional creature who was confined to a line that only allowed movements left and right.

Skate: Hmm. Well, I'll take your word for it for now because I want to get back to that cube. What is it?

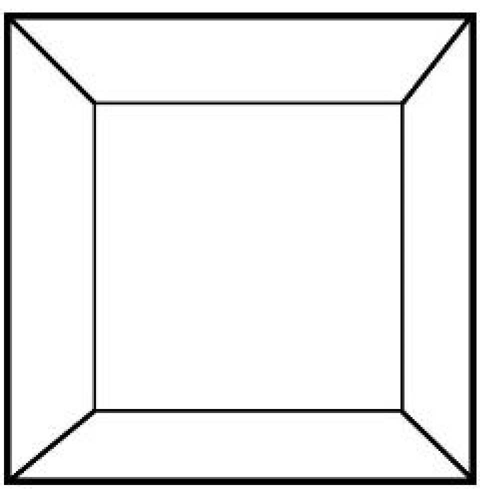
You: You can make a cube by taking a square, then moving that square in the up/down direction for a distance equal to one of its side lengths. The points that the square sweeps through in 3D are the points of a cube. The resulting solid has 6 square faces, 12 edges, and 8 vertices.

Skate: I can't imagine squares becoming faces. Can you show me a cube?

You: Unfortunately, I can't, because you're confined to 2D. But I can show you a projection of a cube to your 2D world.

Skate: Please do!

You: Happy to – here it is:



Hyperia: Unfortunately, I can't. You're confined to your space, and the direction is perpendicular to your space. Imagine trying to explain up and down to a two-dimensional creature who was confined to a plane that only allowed movements left, right, forward, and backward.

You: Hmm. Well, I'll take your word for it for now because I want to get back to that hypercube. What is it?

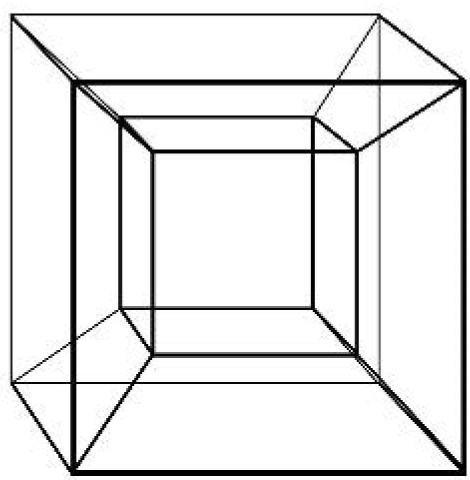
Hyperia: You can make a hypercube by taking a cube, then moving that cube in the ana/kata direction for a distance equal to one of its side lengths. The points that the cube sweeps through in 4D are the points of a hypercube. The resulting hypersolid has 8 cubic faces, 24 square faces, 32 edges, and 16 vertices.

You: I can't imagine cubes becoming faces. Can you show me a hypercube?

Hyperia: Unfortunately, I can't, because you're confined to 3D. But I can show you a projection of a hypercube to your 3D world.

You: Please do!

Hyperia: Happy to – here it is:[4]



[4] Technically, Hyperia is showing us a projection of a hypercube onto a 2D plane. Your picture of a cube may look 3D to you because you are used to interpreting drawings three-dimensionally, but it is indeed a flat drawing. For the purposes of this dialogue, you are supposed to interpret Hyperia's drawing as that of a 3D object.

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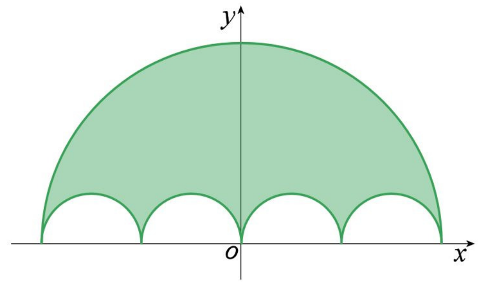
Umbrellas, Part 2

by Ken Fan, Milena Harned, and Miriam Rittenberg
edited by Amanda Galtman



U_4

Recall that U_n was defined to be the set of all points in the coordinate plane that can be reached in n steps starting from the origin, where a step has length one unit and a nonnegative vertical displacement. We also defined S_n by

$$S_n \equiv \{ (x, y) \mid y \geq 0, x^2 + y^2 \leq n^2, \text{ and } (x - (n + 1 - 2k))^2 + y^2 \geq 1 \text{ for } k = 1, 2, 3, \ldots, n \}.$$

We shall prove by induction on n that $U_n = S_n$. We established the cases $n = 1$ and $n = 2$ in part 1, so assume $n > 2$ and $U_k = S_k$ for all $k < n$.

Throughout, we will think of points in the plane as tips of position vectors with tails at the origin.

Our strategy will be to first show that U_n is a subset of S_n and then show the reverse, namely, that S_n is a subset of U_n.

Suppose P is in U_n. We can write $P = Q + (\cos\theta, \sin\theta)$, where Q is in U_{n-1} and $0 \leq \theta \leq \pi$. Since both Q and $(\cos\theta, \sin\theta)$ have nonnegative vertical coordinates, so does P. By the triangle inequality, $| Q + (\cos\theta, \sin\theta) | \leq |Q| + |(\cos\theta, \sin\theta)| = |Q| + 1 \leq (n - 1) + 1 = n$.

We denote by C_j the interior of the unit circle centered at $(j, 0)$, i.e., the points (x, y) that satisfy the inequality $(x - j)^2 + y^2 < 1$. We will show that P is not in C_{n+1-2k} for any integer k. If the vertical coordinate of Q is greater than or equal to 1, then the vertical coordinate of P is also greater than or equal to 1, so P is not in any C_j. Alternatively, assume the vertical coordinate of Q is less than 1. Then Q must be between two circles C_{s-1} and C_{s+1}, where s is an integer with the opposite parity of n. Showing that P is not in C_{n+1-2k}, for any integer k, is equivalent to showing that P is not in C_j for any j having the same parity as s.

To show this, we horizontally translate everything by $-s$. Let Q' and P' be the horizontal translates of Q and P, respectively. Note that $P' = Q' + (\cos\theta, \sin\theta)$. Suppose $Q' = (a, b)$. Then $0 \leq b < 1$ and $-1 < a < 1$. Also, Q' is in neither C_{-1} nor C_1, i.e., $(a + 1)^2 + b^2 \geq 1$ and $(a - 1)^2 + b^2 \geq 1$. These inequalities are equivalent to $a^2 + b^2 \geq \pm 2a$, or, $a^2 + b^2 \geq |2a|$. We must show that P' is not contained in any C_j, where j is an even integer.

Observe that

$$(a + \cos\theta)^2 + (b + \sin\theta)^2 = 1 + a^2 + b^2 + 2a\cos\theta + 2b\sin\theta \geq 1 + |2a| + 2a\cos\theta + 2b\sin\theta \geq 1,$$

where the last inequality follows since $|2a| + 2a\cos\theta \geq 0$ and $2b\sin\theta \geq 0$. Therefore, P' is not in C_0.

Now let $j \leq -2$. We show that P' cannot be in C_j. Notice that the horizontal coordinate of P', which is $a + \cos\theta$, satisfies $a + \cos\theta \geq a - 1 > -2 \geq j$. Therefore, the point $(a - 1, b)$ is closer than P' to the center of circle C_j. We can move the point $(a - 1, b)$ even closer to the center of circle C_j by minimizing a. The minimum occurs when Q' is on the boundary of C_{-1}, i.e., when $Q' = (-1 + \cos\varphi, \sin\varphi)$, where $0 \le \varphi \le \pi/2$. (We know that $0 \le \varphi \le \pi/2$ because Q' is between C_{-1} and C_1.) We compute the square of the distance of the point $(-1 + \cos\varphi, \sin\varphi) + (-1, 0)$ from the center of circle C_j (which is located at $(j, 0)$):

$$(-1 + \cos\varphi - 1 - j)^2 + \sin^2\varphi = 1 + (j + 2)^2 - 2(j + 2)\cos\varphi \ge 1,$$

where the inequality follows because $j \le -2$, so $-2(j + 2)\cos\varphi \ge 0$.

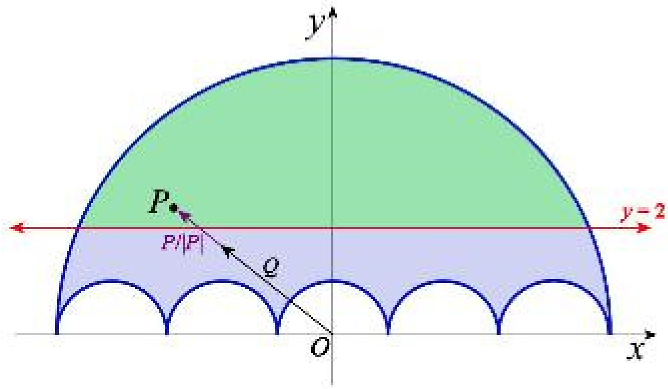
By symmetry, P' cannot be contained in C_j for any $j \ge 2$.

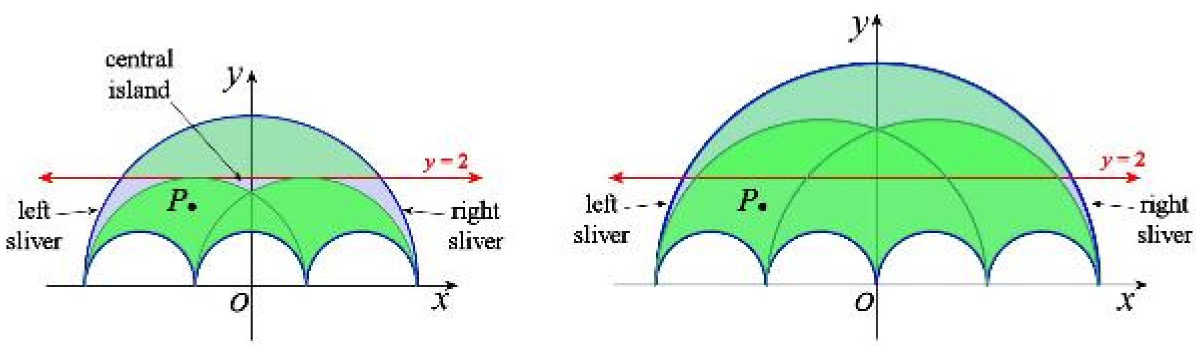
We conclude that P is in S_n.

We now wish to show that S_n is a subset of U_n. Our argument will be to show that more and more points of S_n are in U_n, until we've shown that all points of S_n are in U_n.

Suppose $P = (a, b)$ is in S_n.

First, note that if $b \ge 2$, then $(|P| - 1)P/|P|$ has vertical coordinate greater than or equal to 1 and distance from O less than or equal to $n - 1$, hence is in $S_{n-1} = U_{n-1}$. This shows that $P = Q + P/|P|$, where Q is in U_{n-1}, and hence P is in U_n. Assume from now on that $b < 2$.

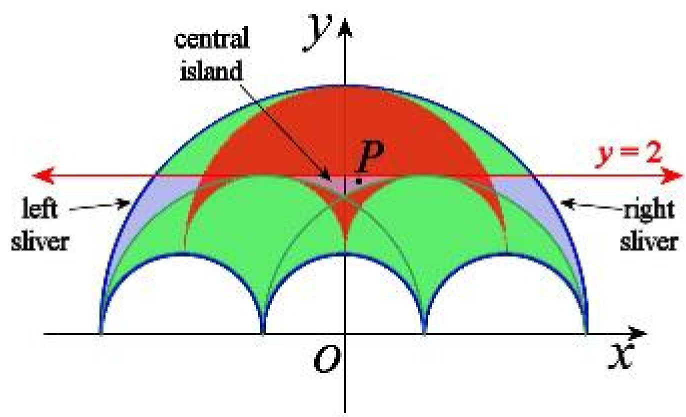


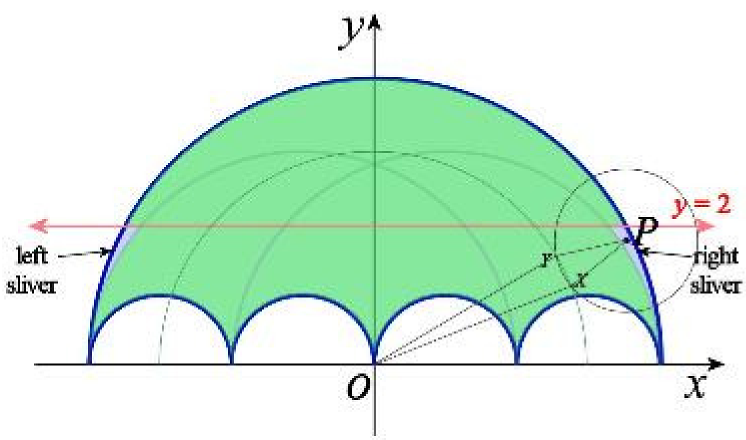


$n = 3$ $\qquad$ $n = 4$

For this paragraph, refer to the figure above. If P is in one of the circles of radius $n - 1$ centered at $(-1, 0)$ or $(1, 0)$, then either $P + (1, 0)$ or $P - (1, 0)$ is contained in $S_{n-1} = U_{n-1}$, and, hence, P is in U_n. The boundaries of these two circles of radius $n - 1$ intersect at $(0, \sqrt{n(n-2)})$. Note that $\sqrt{n(n-2)} > 2$ for $n \ge 4$. (Please check this.) This means that for $n \ge 4$, we have shown that P is in U_n, except that we have not yet addressed points in two narrow slivers. The slivers are regions for which $0 \le b < 2$, P is inside the circle of radius n centered at the origin, and P is either to the left of the circle of radius $n - 1$ centered at $(-1, 0)$ or to the right of the circle of radius $n - 1$ centered at $(1, 0)$. We'll refer to these narrow slivers as the left sliver and the right sliver. In the case $n = 3$, not only do we have to address those two narrow slivers, but we also have to address a central island of points in the upper half plane below the horizontal line $y = 2$ and between, but outside, the circles of radius 2 centered at $(-1, 0)$ and $(1, 0)$.

Let's address points in the central island in the $n = 3$ case. Refer to the figure at right. For these points, we will show that $P - (0, 1)$ is in $S_2 = U_2$, from which it follows that P is in U_3. If we vertically translate S_2 by $(0, 1)$ (the red region in the figure), its lower boundary (which consists of two semicircles of radius 1) rises so that the two peaks of the semicircles coincide with the left and right extremes of the central island. Since circles of radius 1 internally tangent to circles of radius 2 are completely contained inside the circles of radius 2, the vertical translate of S_2 by $(1, 0)$ will completely and conveniently contain the central island.





Next, we will address points P in the left and right slivers. By symmetry, it suffices to show that points P in the right sliver are in U_n. If P is on the circle of radius n, centered at the origin, then the expression $P = nP/|P|$ shows that P is in U_n. Now assume P is in the right sliver but not on the circle of radius n centered at the origin. Then $|P| < n$. Also, we know that $|P - (1, 0)| > n - 1$. Since the horizontal coordinate of P must exceed 1, we see that $|P| > n - 1$. Therefore, the circle of radius 1 centered at P must intersect the circle of radius $n - 1$ centered at the origin in two points, X and Y, which we label so that the vertical component of X does not exceed that of Y. We claim that the vertical coordinate of X is between 0 and b. The points X and Y are simultaneous solutions to the equations

$$(x - a)^2 + (y - b)^2 = 1$$

$$x^2 + y^2 = (n - 1)^2.$$

If we subtract the second from the first, we find

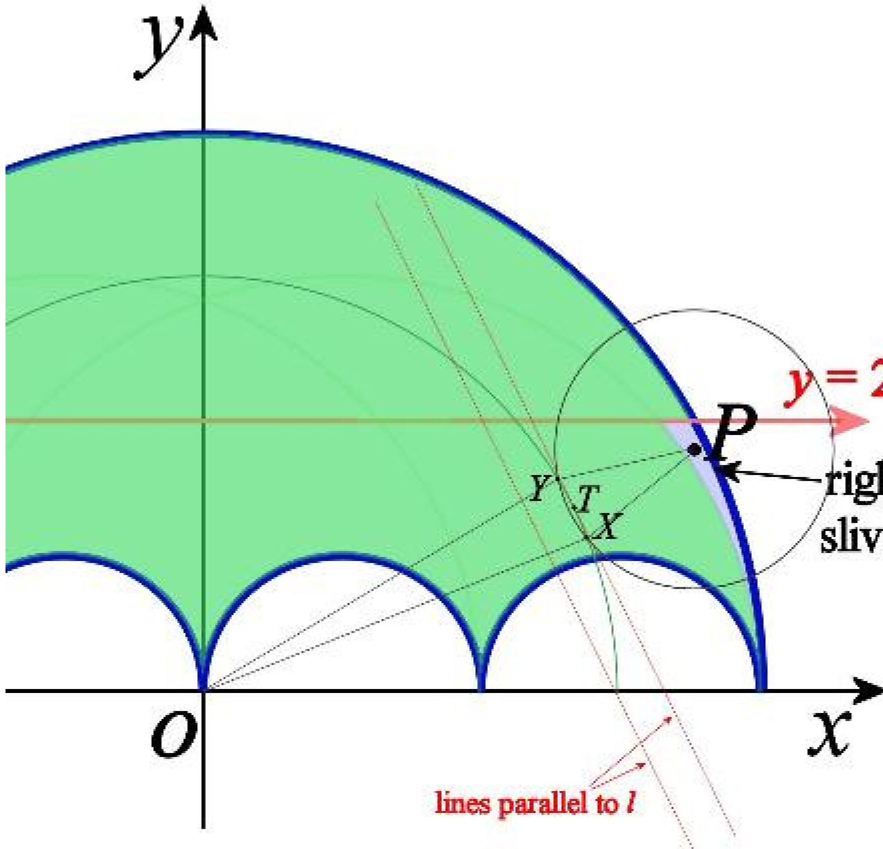
$$a^2 - 2ax + b^2 - 2by = 1 - (n - 1)^2,$$

or

$$2ax + 2by = a^2 + b^2 + (n - 1)^2 - 1.$$

This is the equation of the line that passes through the points X and Y. Let us call it line l. It is perpendicular to the line segment connecting the origin to P. Since $b > 0$, the line segment connecting the origin to P intersects the circle of radius $n - 1$ centered at the origin at a point T with positive y-coordinate.

Imagine a line parallel to l moving from right to left across the circle of radius $n - 1$ centered at the origin. It first touches the circle at the point T. As it moves to the left, the line will intersect the circle in two distinct points symmetric about T. Both points will have positive vertical coordinates until the line contains the point $(n - 1, 0)$. From this point on, at least one of the points of intersection between the line and the circle will have a negative vertical coordinate. (Both intersection points will have negative vertical coordinates when the line passes the intersection with $(-(n - 1), 0)$ and before it contains the point $-T$.)



Since the unit circle centered at $(n - 1, 0)$ is entirely contained within the circle of radius $n - 1$ centered at $(1, 0)$, the distance between P and $(n - 1, 0)$ must exceed 1. Since X and Y are both exactly 1 unit away from P, the line l must be between the tangent line at T to the circle of radius $n - 1$ centered at the origin, and the parallel line that passes through the point $(n - 1, 0)$. This shows that X and Y have positive vertical coordinates.

On the other hand, since P is to the right of the circle of radius $n - 1$ centered at $(1, 0)$, the point $P - (1, 0)$ is to the right of the circle of radius $n - 1$ centered at the origin. This shows that X and Y must have vertical coordinates less than that of P.

Therefore, $P - X$ is a unit vector with nonnegative vertical component. Since X is on the circle of radius $n - 1$ centered at the origin, X is in $S_{n-1} = U_{n-1}$. Since $P = X + (P - X)$, we see that P is a sum of a vector in U_{n-1} and a vector in U_1, and hence, P is in U_n, as desired. Since we have explored all possibilities, S_n is a subset of U_n.

Since U_n is a subset of S_n and S_n is a subset of U_n, we must have $U_n = S_n$, and the proof is done.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 23 - Meet 8 Mentors: Grace Bryant, Neslly Estrada, Katie Gravel,
November 1, 2018 Adeline Hillier, Claire Lazar, Elise McCormack,
Charity Midenyo, Kate Pearce, Jane Wang

Explore the triangle as a tile.

Session 23 - Meet 9 Mentors: Grace Bryant, Neslly Estrada, Jacqueline Garrahan,
November 8, 2018 Adeline Hillier, Claire Lazar, Jenni Matthews,
Elise McCormack, Kate Pearce, Laura Pierson,
Gisela Redondo, Jane Wang

Solve the cryptarithm GIRLS + ANGLE = HEAVEN, given that V is 5?

Session 23 - Meet 10 Mentors: Grace Bryant, Neslly Estrada, Katie Gravel,
November 15, 2018 Adeline Hillier, Claire Lazar, Kate Pearce, Laura Pierson,
Gisela Redondo, Jane Wang, Jasmine Zou

How can a sum of infinitely many positive numbers be finite?

Session 23 - Meet 11 Mentors: Grace Bryant, Jacqueline Garrahan, Adeline Hillier,
November 29, 2018 Claire Lazar, Elise McCormack, Charity Midenyo,
Kate Pearce, Laura Pierson, Gisela Redondo,
Shohini Stout, Savannah Tynan, Jasmine Zou

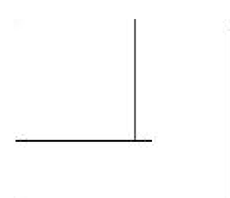
Design an interesting network of streets and give each other path counting problems.

Session 23 - Meet 12 Mentors: Grace Bryant, Jacqueline Garrahan, Adeline Hillier,
December 6, 2018 Claire Lazar, Elise McCormack, Kate Pearce,
Laura Pierson, Savannah Tynan, Jane Wang, Jasmine Zou

We held our traditional end-of-session Math Collaboration. This session's was designed by Elise McCormack who brilliantly marshalled the collaborative nature of jigsaw puzzles for the core of her beautifully designed enigma. Try your hand at a few of the math problems:

How many solutions are there in positive integers x and y to the equation $1/x + 1/y = 1/10$?

The figure at right shows a rectangle split into a bunch of squares. It turns out that the length and width of the rectangle are positive integers with a greatest common denominator of 2. What is the perimeter of the rectangle?



What is the eighth smallest positive integer that has exactly 4 factors?

Calendar

Session 23: (all dates in 2018)

September	13	Start of the twenty-third session!
	20	
	27	
October	4	
	11	
	18	
	25	
November	1	
	8	
	15	
	22	Thanksgiving - No meet
	29	
December	6	

Session 24: (all dates in 2019)

January	31	Start of the twenty-fourth session!
February	7	
	14	
	21	No meet
	28	
March	7	
	14	
	21	
	28	No meet
April	4	
	11	
	18	No meet
	25	
May	2	
	9	

SUMIT 2019 is scheduled for April 6 and 7, 2019. Registration will open sometime in February. Visit http://girlsangle.org/page/SUMIT/SUMIT.html for updates.

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minute walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____