Girls' Angle Bulletin

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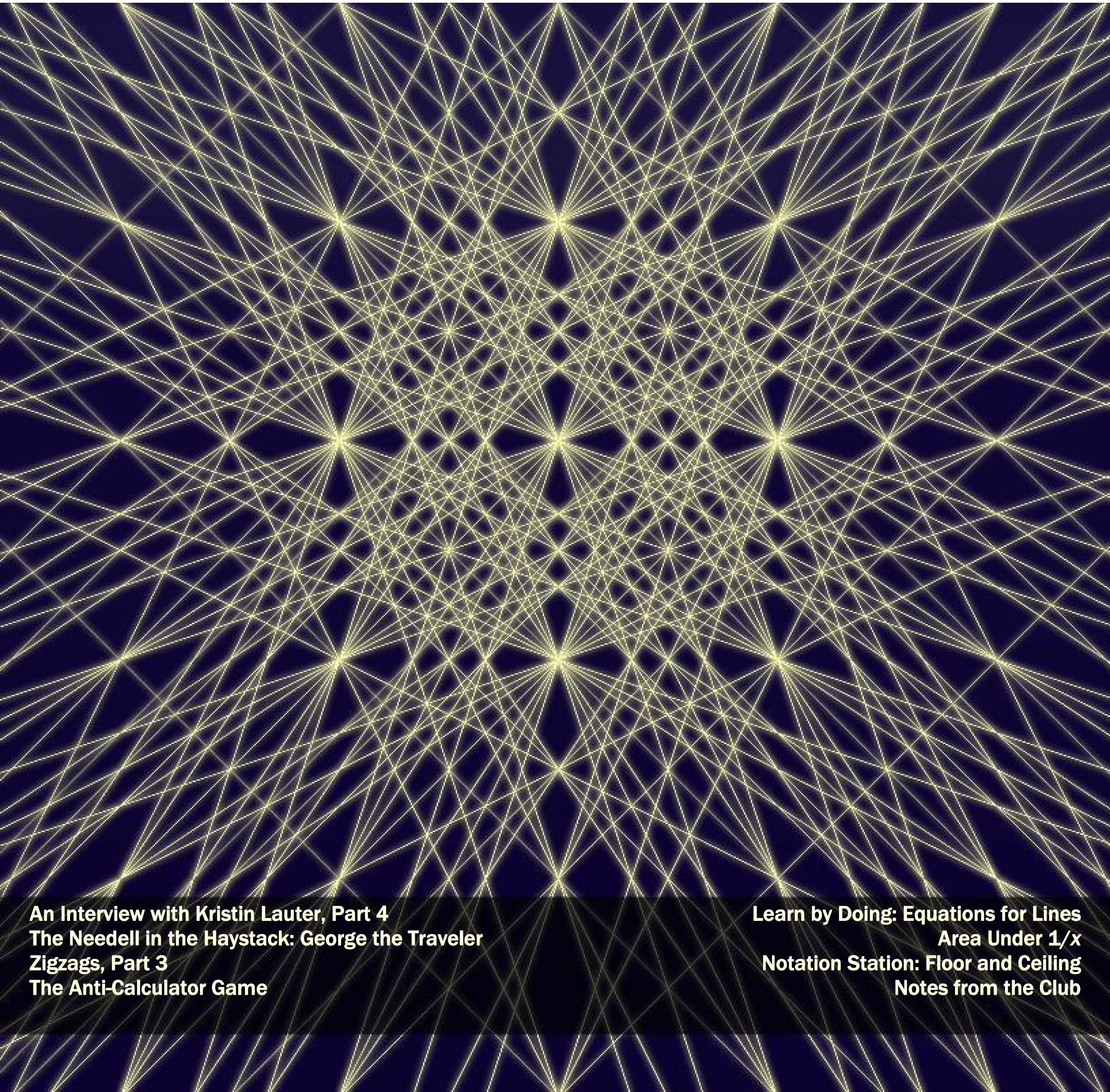
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

When you're expecting ice cream, but get a mouthful of mashed potato, you go "Blech!" even if it's the best mashed potato ever. If you balk at math, try approaching it from a completely different angle. You might find that it's actually quite a delicious topic! - Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

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Executive Editor: C. Kenneth Fan

Girls' Angle: A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *A Few Standard Lines* by C. Kenneth Fan. For more, see *Learn by Doing* on page 19.

An Interview with Kristin Lauter, Part 4

I got good advice from my advisors when I was in graduate school: do what you love! You have to have a real passion for what you work on. So if you're doing mathematics or computer science because you love it, then you should stick with it and persist, and don't let anybody dissuade you! It's possible to have a really wonderful career in mathematics or computer science, and it's worth it. It's worth all the hard work.

This is the fourth and final part of our interview with mathematician Kristin Lauter, which was conducted by Ke Huang at the University of Washington in April of 2018.

Ke Huang: Could you please give us an example of something in math from your youth that you thought was really beautiful?

Kristin: I remember in college studying quantum mechanics, and I thought that it was crazy that it's all described in terms of Hilbert spaces, when Hilbert spaces were envisioned and constructed and defined independently without regard to any applications. I find it really interesting that mathematicians are often motivated by curiosity and beauty to define objects and find relationships and prove things, where you are inventing something in your mind and working on it, and then that ends up reflecting some existing processes in nature. That's what I find most amazing.

Ke Huang: I completely agree. It's crazy how it all works. So, you are the cofounder of the Women In Numbers network. Could you tell us about Women In Numbers and how you founded it?

Kristin: Well, I'd be happy to, because it's probably the initiative and the activity in my career that I was most inspired by, and most proud of! We started WIN because, after roughly 10 years of working as a research mathematician after I got my PhD, I still found it very frustrating to attend number theory conferences where I was the only woman speaker! I had also gone through a difficult period after I had children. I have twins, and for quite a while after they were born, I wasn't invited to any conferences, and I was extremely frustrated by that. I felt isolated and excluded from the research community. And so I wanted to try to build a community for women in number theory and in research, because I knew that there were plenty of women out there doing number theory. I could think of dozens, I mean, more than dozens. I was at a conference in 2006 with two close colleagues, Renate Scheidler and Rachel Pries[1], where we were discussing this problem, and we just sat down over lunch and made a list of 75 women that we knew of that were number theorists at the postdoc level or beyond, wondering why weren't any of these people being invited to any of these conferences? And so we decided to create our own conference, Women In Numbers. The name was my husband's idea, WIN, and everybody likes the WIN name.

[1] An interview with Rachel Pries appears in Volume 11, Number 6 of the Girls' Angle Bulletin.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Kristin Lauter and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
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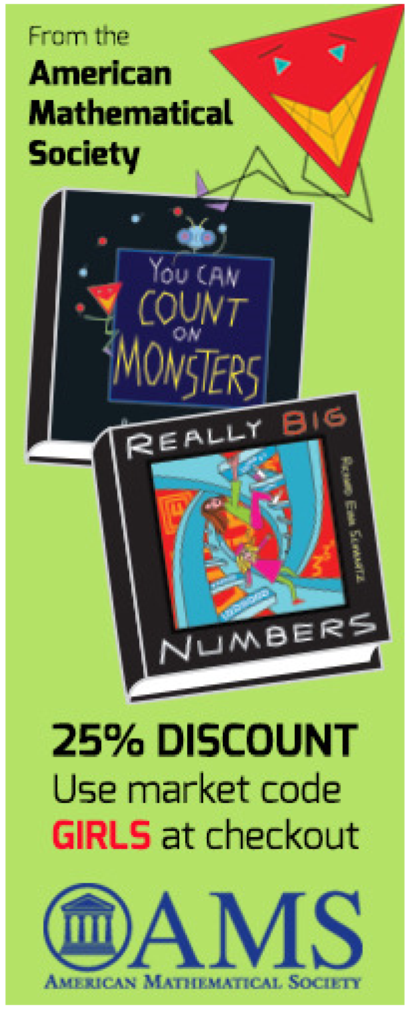

The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

dissuade you! It's possible to have a really wonderful career in mathematics or computer science, and it's worth it. It's worth all the hard work.

Ke Huang: Definitely true. Do you have any advice that you would give your high school self, I guess, looking back?

Kristin: Well, I was very young in high school… I was also very naïve and I'm not even sure… The problem is that even if I had told my naïve self all the things I've learned about the real world, I certainly wouldn't have been happy to hear those things at that time, and I might not have believed them. So I'm not sure that I would give my high school self any particular advice. But one thing very concrete that I would say to certainly undergraduates, and, these days, even high school students, is to do internships during the summer in industry or government, because that gives you an idea of what's out there, what mathematics is used, and what would be expected of you. You can figure out what the gaps are in your skill set and fill those in, and you can figure out whether you like certain types of work or not and certain types of environments. So I think doing an internship is the number one piece of advice.

Ke Huang: Thank you so much for this interview!

Kristin: Oh, you're welcome, it was my pleasure!

Printed with permission from Kristin Lauter.

Participants at Women in Numbers 4, which took place at Banff, Alberta from August 13-18, 2017. Dr. Lauter is at right. The conference was organized by Jennifer Balakrishnan, Chantal David, Michelle Manes, and Bianca Viray. For more information, visit womeninnumbertheory.org.

Dr. Balakrishnan mentored at Girls' Angle during her last year of graduate school at MIT and her first year as an NSF Postdoctoral Fellow at Harvard. Dr. Viray is a Girls' Angle Director.

The Needell in the Haystack[1]

George the Traveler

by Deanna Needell | edited by Jennifer Silva

I write this article in dedication to George, my amazing father and forever globetrotter. He has been my best friend, traveling buddy, adviser, listener, motivator and my constant support. He has been my *reason*, in both senses of the word.

When traveling the globe, a typical quandary is how to most efficiently visit all of the places you want to see. For example, if George is in Munich, he may want to visit the Hofbräuhaus, Starbucks, the Hard Rock Cafe, and several other hot city sites. But he may wish to see all of these in some sort of optimal way – for example, by walking the shortest distance possible or driving for the least amount of time. Alternatively, George may be jetsetting around the globe with plans to visit Barcelona, Rome, Paris, Singapore, Machu Picchu, and Mumbai, and wants to visit all of those cities while using the least amount of airline miles. These goals are solutions to the so-called "traveling salesperson problem."

The traveling salesperson problem is one of many problems that can most easily be formulated by using tools from **graph theory**. In this context, a **graph** is not the plotting tool one uses with an x-axis and a y-axis; rather, it is a discrete object consisting of a set of **vertices** V and a set of **edges** E. The vertex set V is simply a set of objects. In the example above, each city in the tour would correspond to a vertex:

$$V = \{\text{Barcelona, Rome, Paris, Singapore, Machu Picchu, Mumbai}\}.$$

Each edge in E would then correspond to a connection between two cities. In an **undirected graph**, these edges are simply pairs of cities, with no direction information. For example, there might be a connection between Rome and Paris indicating that there are flights between the two cities. In a **directed graph**, these edges contain directional information, so can be written as ordered pairs. The edge (Paris, Rome), for example, would then indicate that there is a one-way flight from Paris to Rome, whereas the edge (Rome, Paris) would indicate a one-way flight from Rome to Paris. In a **weighted** graph (which can be directed or undirected), there is also a numeric value associated with each edge; for example, the edge (Paris, Rome, 4000) might indicate there is a flight from Paris to Rome that costs 4000 airline miles. While a graph can be defined by listing its vertex and edge sets, it is often easier to visualize by drawing a diagram where vertices are drawn as circles and edges as arcs between them. [See Figure 1 on the next page.] We call any traversal of edges in the graph a **path**. In Figure 1, for instance, there is a path from Mumbai to Machu Picchu – via the edges with weights 14000, 9000, and 10000 – that takes you through the vertices Paris and Singapore before ending in Machu Picchu. We will call any path that begins and ends at the same vertex a **cycle**.

Before trying to find an optimal route, we may ask a simpler question: for every pair of cities, is there a path that connects them? In other words, for every pair of vertices $u, v \in V$, is there a path from u to v? If the answer is yes, we say that the graph is **connected**. The graph shown in Figure 1 is connected, as we can verify by inspection. A related question is whether there exists a path in the graph that visits each vertex exactly once. Such a path is called a **Hamiltonian path**. Similarly, a **Hamiltonian cycle** is a cycle that visits each vertex exactly once. Does the graph in Figure 1 contain a Hamiltonian path? Once you've answered that

[1] This content supported in part by a grant from MathWorks.

question, try removing edges (as few as possible, to make it more fun) so that the resulting graph no longer does.

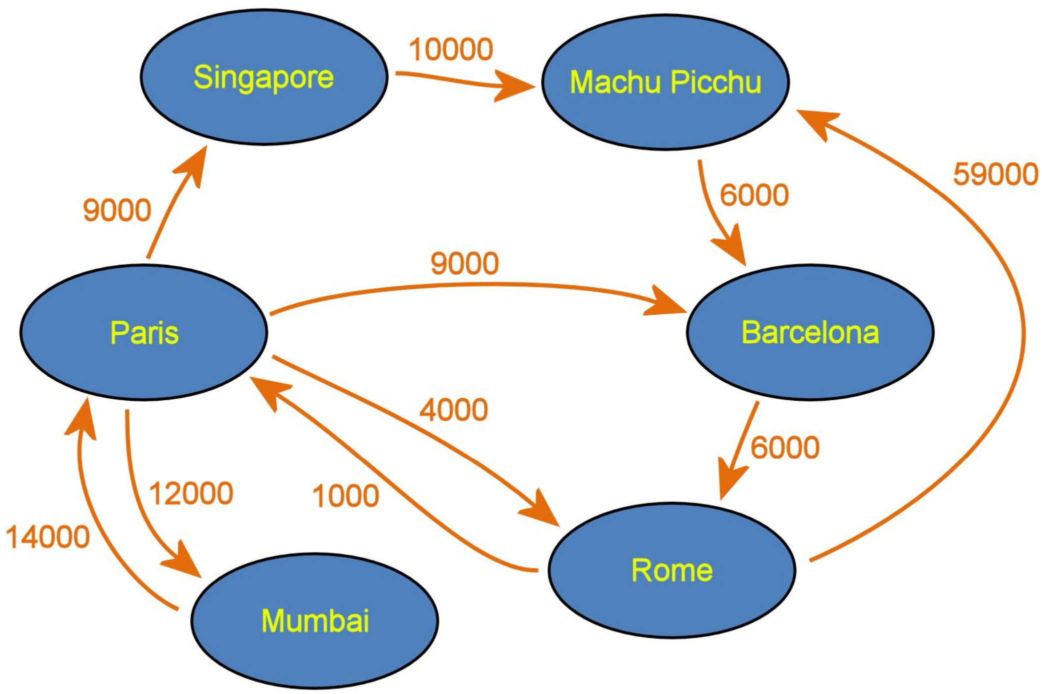

FIGURE 1. Example of a graph with cities as vertices and weighted directed connections as edges.

The traveling salesperson problem goes a step further: it asks for a Hamiltonian path such that the sum of the edge weights used in the path is as small as possible. Stare at the graph in Figure 1 until you become convinced that the optimal path costs 35,000 miles. George would be pretty happy with that, as it would leave him plenty of spare change for a pumpkin spice latte. How did you find this path? How hard was it for you to find on this graph with six vertices and ten edges? According to the International Civil Aviation Organization, there are approximately 43,983 airports in the world. Imagine how hard it would be to look for optimal paths in such a large graph!

These questions about "hardness" bring up some interesting questions themselves. What do we mean by "hard"? Is what is "hard" different for a human than for a computer? It might be hard for a human to multiply two 10-digit numbers in her head, but for a computer this may be "easy." How can we measure the difficulty of such tasks, say, if we focus only on how computers can solve them?[2] For a given algorithm (i.e., set of instructions to solve a problem), we can compute its **computational cost** by adding up the total number of elementary operations (such as addition and subtraction – operations that take a constant amount of time on a given machine). We can then compute this cost as a function of the input size to get an idea of how the cost grows as the input size grows. If a graph has $m = |V|$ vertices and $n = |E|$ edges, for instance, we may compute the cost of an algorithm as a function of m and n. For example, a simple algorithm that adds the weights of all of the edges would have a cost of n since it requires n elementary addition operations.

[2] And who is programming the computers? Humans, of course! But we will clarify what we mean momentarily.

When we begin to consider what kinds of problems are feasible computationally, we start to realize that the function that describes the computational cost becomes extremely important. For example, even for a reasonable problem size such as when $n = 90$, a computational cost of n^2 means 8100 operations, which can be executed on a basic machine in well under one second. But a computational cost of, say, 2^n on the same input size would require 2^{90} operations (which is approximately 1.23×10^{27} operations), which could take over 37 years even on today's fastest computers![3] George is a kind and patient person, but he would likely not be willing to wait 37 years just to plan a trip! In general, then, it seems that polynomial computational costs are far more reasonable than exponential computational costs. We thus define a class of problems,[4] denoted simply by P, which consists of all problems that can be solved (deterministically) by a machine in time that is polynomial in the size of the input. Given a problem, we then would like to know whether that problem is in the class P. Notice that in some cases it may be easy to show that a problem is in P, as you need only construct such an algorithm and show that its computational cost is polynomial. But how can one know whether a problem is *not* in P? This innocuous question is what has led to a large branch of computer science called **complexity theory**. By dabbling a bit in this theory, we will come to one of the most sought after "Millennium Prize Problems" – problems whose solutions result in a prize of $1,000,000![5]

After defining the class P, the next important class is called NP. A word of caution: NP does *not* stand for non-polynomial, a common misconception. Rather, the class NP consists of all problems that can be *verified* in polynomial time. In other words, given a solution to the problem, can one write an algorithm that can confirm it is a solution, in polynomial time? If yes, the problem is in the class NP. It is naturally the case that $P \subset NP$ by definition, since any problem that can be solved in polynomial time can also be verified in that time. Consider again the problem of finding paths in a graph. Finding whether there is a path from one vertex to another, such as whether one can fly from Singapore to Rome, turns out to be in the class P, thus also NP.[6] For the Hamiltonian path problem, on the other hand, it is not yet known whether it belongs to P or not. It does, however, belong to NP, since given a proposed solution, the computer need only traverse the solution and verify each vertex was indeed visited. The "million-dollar question" is whether P = NP. That is, in fact, a Millennium Prize Problem; the first person to prove that either $P \neq NP$ or $P = NP$ will receive a nice check for $1,000,000! Put less succinctly, the question is whether there exist problems whose solutions can be verified in polynomial time, but cannot actually be solved in polynomial time. Before the reader makes up her mind about whether she believes $P = NP$ or $P \neq NP$, let's delve in a bit further to what is known.

To study this question, another class of problems has been defined. This class is called NP-Complete. It consists of a set of NP problems, each of which can be reduced in polynomial time to any other problem in NP. In other words, for problem X to be in NP-Complete, any other NP problem Y can be transformed in such a way that being able to solve X quickly means one can also solve Y quickly. In that sense, NP-Complete problems are the "tough" problems in NP, since solving them leads to solving the others. There are now many problems proven to be in NP-Complete; for example, the Hamiltonian path problem mentioned above is in this prestigious class. In fact, even a variant of the traveling salesperson problem is in NP-Complete; given a particular budget amount b, the problem of asking whether there is a tour of all of the vertices

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[3] As of this writing, the fastest supercomputer can perform about 10^{17} floating point operations per second.

[4] We briefly remark here that technically, this discussion applies to *decision* problems and not, e.g., to optimization or other general problems. However, we will omit this detail for the sake of intuitive discussion.

[5] The Millennium Prize Problems are a collection of 7 problems announced by the Clay Mathematics Institute in 2000. As of this writing, only one has been solved (the Poincaré conjecture by Grigori Perelman in 2003).

[6] We invite the reader to come up with an algorithm that solves this problem in polynomial time.

with a sum of weights at most b is an NP-Complete problem. Let's call this latter variant the "budgeted traveling salesperson problem."[7] It is easy to see that this version is in NP, since one need only traverse a tour and add up the weights to verify that a proposed solution is under the specified budget.

The benefit of defining this class (and others) is that studying these problems now has extra motivation. Indeed, if one can design a polynomial-time algorithm that solves one of the problems known to be in NP-Complete, then one has just proved that $P = NP$! And if we are interested in solving a problem even in a non-polynomial way, the relationship to this set of "core" problems is extremely useful. It's like that saying, "Why reinvent the wheel?" If we have algorithms to solve NP-Complete problems, then (at least in theory, and often in practice), we also have algorithms to solve many others. There are hundreds of concrete problems now known to be in NP-Complete, and lists are constantly maintained and updated.

To see how the list of NP-Complete problems might be updated, let's consider a simple example, of transforming the Hamiltonian path problem to the budgeted traveling salesperson problem. So, let's consider a graph G with vertices V and edges E. For now, let's think about the undirected version, meaning that edges do not contain direction (this just makes things simpler to draw; the idea for directed graphs is the same). Suppose you wish to solve the Hamiltonian path problem on this graph (i.e., determine whether there is a Hamiltonian path, and if so, find it) and you have at your disposal a "black box" that solves the budgeted traveling salesperson problem. In order to transform (often called "reduce") the Hamiltonian path problem into the budgeted traveling salesperson problem, we have to come up with a polynomial-time method that will allow us to solve the former problem using only the black box that solves the latter. In this case, there are many strategies to do this. Let's mention just one.

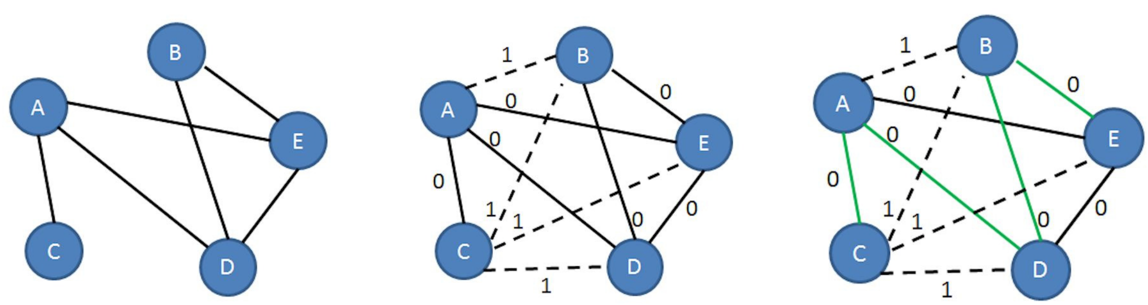

FIGURE 2. Reduction of the Hamiltonian cycle problem to the budgeted traveling salesperson problem.

For visualization, suppose we are given the graph shown in the left of Figure 2. Next, we will add all of the possible edges, until we reach a graph that has an edge between every pair of vertices; such graphs are called **complete**. For the edges we added, let's assign a weight of 1, and for the original, let's assign a weight of 0. In this example, the graph will now look like the center graph of Figure 2. Now, we ask for a solution to our budgeted traveling salesperson problem with a budget of 0. We readily observe that a tour with a budget $b \leq 0$ exists if and only if a Hamiltonian path from the original graph G exists. Indeed, if the traveling salesperson black-box algorithm finds a tour with a budget of 0 (or less, technically), then that means it must only use edges that have weights 0, namely, those edges from the original graph G. An example of a Hamiltonian path is shown in the right of Figure 2. Thus, we have reduced the Hamiltonian path problem to the budgeted traveling salesperson problem.

[7] Contrast this to the traveling salesperson problem, where you have to check that there is no more optimal Hamiltonian path to verify that a proposed solution is a solution.

Reductions from one "decision" problem to another are thus quite useful; they allow us to port algorithms from one problem to the next, as well as to study large classes of problems at a time. But let us once again return to the original formulation of our traveling salesperson problem. What would your first attempt at an algorithm for this problem be? Perhaps you would consider all permutations (orderings) of the n vertices and attempt a tour on the vertices in each order, adding up the weights as you go along. Doing this for all permutations would allow you to then identify the cheapest tour, and you will have solved the problem. However, this forces you to check all $n!$ permutations! The factorial function is far from polynomial (in fact, one can use **Stirling's approximation**[8] to see that it grows even faster than exponential), so this algorithm is not feasible even for moderate sizes of the input; remember that George does not want to wait 37 years for a solution! More sophisticated approaches have been obtained that have computational cost more like $n^2 2^n$, but this is still exponential. So does there exist a polynomial time algorithm for finding Hamiltonian paths? Does $P = NP$? In a poll in 2012, researchers were asked whether they believed $P = NP$. The results: 83% said no, 9% said yes, and 8% said they didn't know or believed it was impossible to answer. The reader should ponder this and make her own opinion … and perhaps one day she will be the one to answer this challenge and take home the prize!

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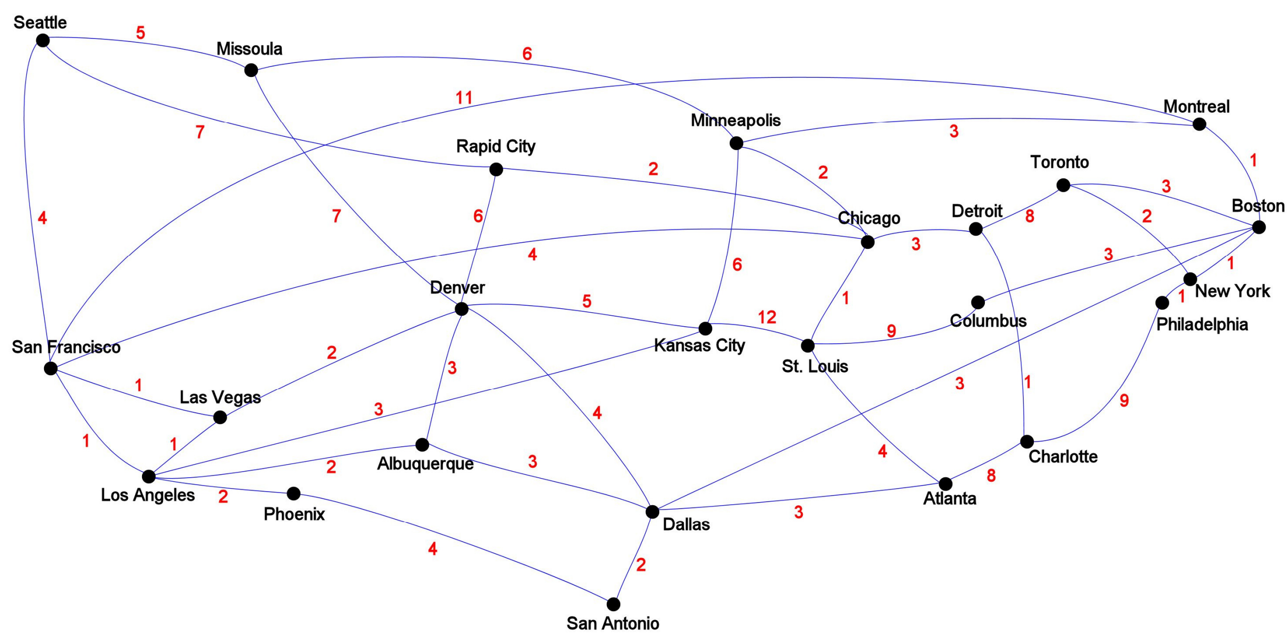
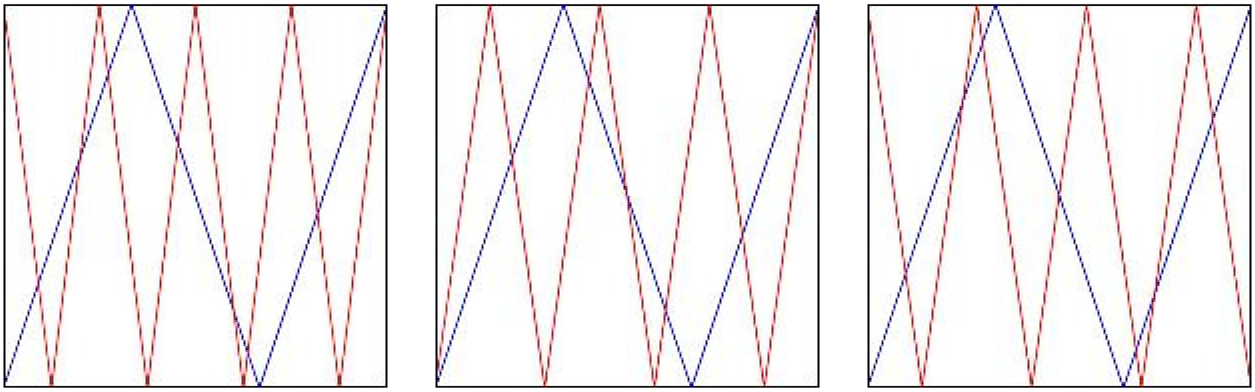
[8] Stirling's approximation for $n!$ is $\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.



This is a fictitious map of airline routes between US cities. The numbers represent the cost to take the associated route. Is there a Hamiltonian cycle through the cities? Is there a Hamiltonian path? What are the optimal routes?

Zigzags, Part 3

by Ken Fan | edited by Jennifer Silva



Zigzags. At left, the case $(n, m) = (8, 3)$. Middle, the case $(n, m) = (7, 3)$ with zigzags starting and ending in the same corner. Right, the case $(n, m) = (7, 3)$ with zigzags starting and ending in different corners.

Jasmine: There's so much regularity in these zigzags. I think we should be able to say a lot more about the triangles and quadrilaterals that are formed when two zigzags split the square. For example, I think a lot of these triangles will have to be similar to each other since their corresponding sides will often be parallel, in which case their angles will be congruent.

Emily: That makes a lot of sense. There aren't that many ways in which these two zigzags can cut out triangles. Let's see. The n-zigzag creates two right triangles at the left and right ends of the square. And if the square is a unit square, both have legs of length 1 and $1/n$, so they are congruent to each other. If the m-zigzag leaves one or both uncut, we'll get one or two such right triangles, and those are the only possible right triangles we can get.

Recall that Emily and Jasmine are assuming that m and n are relatively prime with $m < n$.

If you're having trouble following this dialogue, try drawing pictures of the situation as you read along.

Jasmine: Or, one of those right triangles will be split in two by the m-zigzag. If it is, then one of the resulting triangles will have a vertical side of the square as one of its sides, and it will have sides of slope n and $-m$, or $-n$ and m, depending on whether the m-zigzag starts in the upper corner or the lower corner. As far as the shape of the triangle goes, these two possibilities are congruent to each other.

Emily: And the other triangle will be one with a horizontal side and sides of slope n and $-m$, or $-n$ and m, again, depending on whether the m-zigzag emanates from the upper corner or the lower corner. Once again, both possibilities are congruent.

Jasmine: Actually, many of the triangles that arise will be geometrically similar to this triangle with a horizontal side. Whenever the m-zigzag bounces off of the base of one of the isosceles triangles produced by the n-zigzag, we will get two triangles that are both similar to this triangle with a horizontal side. The m-zigzag makes $m - 1$ such bounces, so we'll get $2(m - 1)$ more triangles similar to the triangle with a horizontal side and sides with slope n and $-m$.

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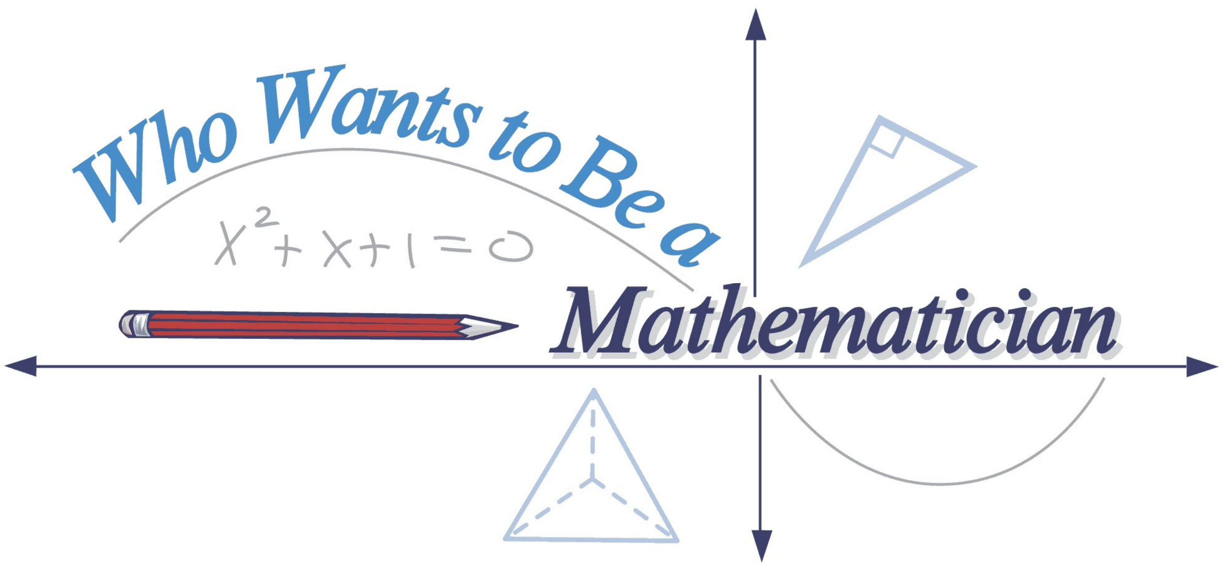

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The Anti-Calculator Game

by Lightning Factorial

Resist that temptation to reach for your calculator for every computation. Giving in to this temptation can hinder development of your conceptual understanding of mathematics, especially if you are quick to see computationally intensive ways to find solutions. To develop your ability to think in higher level concepts, you should refrain from computing and, instead, see if you can think your way around computations.

To practice, try to do these problems with as little computation as possible. Each can be solved in a conceptual way. You might find that you can solve them all in your head.

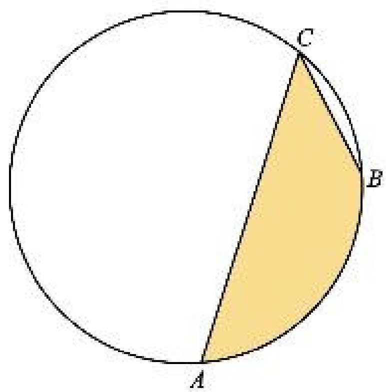
1. What is the average of all the odd numbers between 0 and 100?

2. What is 28/99 as a decimal? What is 3/11 as a decimal? What is 1/225 as a decimal?

3. What is 27 times 33? What is 85^2? What is $111{,}111^2$? What is $12{,}345{,}679 \times 81$?

4. What is the remainder if you divide 119,638,226,448 by 13? Or if you divide 2^{2019} by 11?

5. The radius of the circle is 10 units. The arcs AB and BC measure 90° and 45°, respectively. What is the area of the orange region?



6. Which is biggest: 49/61, 17/29, or 25/37? Which is biggest: $\sqrt{99}+\sqrt{101}$ or $\sqrt{98}+\sqrt{102}$?

7. What is 1/2 + 1/6 + 1/12 + 1/20 + 1/30 + 1/42 + 1/56 + 1/72 + 1/90?

8. What is 1 – 2 + 3 – 4 + 5 – 6 + 7 – 8 + 9 – 10 + 11 - . . . + 99 – 100?

9. A line passes through the points (1000, 2000) and (1100, 2050). Where does it cross the vertical axis?

10. Let n be the product of the first 100 prime numbers. What is the greatest common factor of n and 24?

11. How many ways can you make change for ten dollars using only dimes and nickels?

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Learn by Doing

Equations for Lines

by Addie Summer | edited by Amanda Galtman

We will work in the xy-coordinate plane. What's amazing about introducing a coordinate system is that, suddenly, we can use numbers to describe geometric objects precisely.

In this Learn by Doing, we'll explore equations that describe lines. We'll assume you've seen the so-called **slope-intercept** form of an equation for a line, namely $y = mx + b$, where m and b are constants. If you've never seen the slope-intercept form, take a moment to consider the points (x, y) such that $y = mx + b$, and convince yourself that those points constitute a line. If you're very comfortable with slope-intercept form, you might prefer to skip the first few problems, which explore the meaning of m and b in depth.

1. Show that the line described by the equation $y = mx + b$ intersects the y-axis at the point $(0, b)$. (That is, show that substituting 0 for x and b for y in the equation $y = mx + b$ results in a true equation.) Furthermore, show that $(0, b)$ is the *only* point on the y-axis whose x- and y-coordinates satisfy the equation $y = mx + b$.

In other words, the line described by the equation $y = mx + b$ intersects the y-axis in exactly one point, whose y-coordinate is b. For this reason, b is called the ***y*-intercept** of the line.

2. Suppose $y_1 = mx_1 + b$ and $y_2 = mx_2 + b$, that is, both (x_1, y_1) and (x_2, y_2) are points on the line described by the equation $y = mx + b$. Assume that $x_1 \neq x_2$. Show that $m = (y_2 - y_1)/(x_2 - x_1)$.

In other words, if we pick any two points on the line, the ratio of their vertical displacement to their horizontal displacement is equal to m. Let's pick three distinct points P, Q, and R on the line. Let's walk from P to Q along a dogleg path by going horizontally to a point Q' that is vertically aligned with Q, turning through a right angle, and then climbing vertically to Q. Problem 2 tells us that $QQ'/PQ' = m$. Similarly, let's walk another dogleg path to get from P to R by first walking horizontally to a point R' that is vertically aligned with R, turning through a right angle, and then climbing vertically to R. (Draw a picture!) Then $RR'/PR' = m$ is also true. Since both QQ'/PQ' and RR'/PR' are equal to m, they are equal to each other, and that means that the two right triangles $PQ'Q$ and $PR'R$ are similar to each other. Since corresponding angles in similar triangles have the same measure, we see that the angle that PQ makes with the horizontal has the same measure as the angle that PR makes with the horizontal. In other words, you can get from P to any other point on the line by walking backward or forward in the same direction. Let's call the angle that PQ (or PR) makes with the horizontal a. If m is positive, as it gets bigger and bigger, the angle a gets closer and closer to 90°. So m is a measure of how quickly the line rises or falls as you go from left to right. For this reason, m is called the **slope** of the line.

3. If a line makes a 45° angle with the horizontal, rising as you go from left to right, what is the slope of the line?

The vertical and horizontal displacements between two points on a line are also called the **rise** and the **run**, respectively. So the slope is the ratio of the rise to the run between any two points on the line.

4. If $m = 0$, convince yourself that the line is horizontal (i.e., parallel to the x-axis).

5. If $m < 0$, convince yourself that if you walk from left to right along the line, you will be descending (i.e., your y-coordinate will be decreasing). On the other hand, if $m > 0$, you'll be ascending as you walk along the line from left to right.

6. Suppose b_1 and b_2 are constants with $b_1 \neq b_2$. Convince yourself that the equations $y = mx + b_1$ and $y = mx + b_2$ describe lines that are parallel to each other. That is, show that no point sits on both lines.

7. Convince yourself that the lines described by the equations $y = mx + b$ and $y = -mx - b$ are reflections of each other in the x-axis.

8. Suppose $m \neq 0$. Convince yourself that the lines described by $y = mx$ and $y = x/m$ are reflections of each other in the line $y = x$.[8]

9. Let R_1 be the right triangle whose vertices are (0, 0), (1, 0), and (1, m). Let R_2 be the right triangle whose vertices are (0, 0), (0, 1), and ($-m$, 1). Show that R_1 and R_2 have the same side lengths and, hence, are congruent. Show that their hypotenuses are perpendicular to each other. Conclude that the line of slope $-1/m$ is perpendicular to the line of slope m.

In Problem 9, you showed that lines of slope m and $-1/m$ are perpendicular to each other. Notice that if $m = 0$, that is, if one of the lines is horizontal, then the slope of the perpendicular line, which should be $-1/m$, is actually undefined. This is one of the drawbacks of slope-intercept form: it cannot be used to describe vertical lines.

10. Suppose l is a vertical line, i.e., all of its points have the same x-coordinate. Show that the ratio of the rise to the run between any two points is undefined.

Since a vertical line consists of points with the same x-coordinate, we can describe vertical lines using equations of the form $x = c$, where c is a constant. Notice that there is no way to rearrange the equation $x = c$ so that it is in slope-intercept form. We cannot eliminate the variable y from an equation in slope-intercept form. This suggests modifying the slope-intercept form of the line by introducing a coefficient in front of y to make an equation of the form:

$$cy = mx + b.$$

11. If $c \neq 0$, show that $cy = mx + b$ describes a line with slope m/c and y-intercept b/c.

12. If $c = 0$ and $m \neq 0$, show that $cy = mx + b$ describes the vertical line consisting of all points whose x-coordinate is equal to $-b/m$.

13. If both $c = 0$ and $m = 0$, show that $cy = mx + b$ describes either the empty set (if $b \neq 0$) or the entire plane (if $b = 0$).

For this reason, if we introduce a coefficient in front of y for the purpose of describing lines, we must insist that the coefficients of x and y not be both equal to zero.

[8] This fact gains importance when you study the derivative of inverse functions in calculus.

We can reassign the constants in the equation $cy = mx + b$ and rearrange the terms so that the equation looks like $Ax + By = C$, where A, B, and C are constants. This equation is called the **standard form** of a line. In the standard form of a line, we insist that A and B not be both equal to zero.

14. In the standard form $Ax + By = C$, convince yourself that if $A = 0$, the line described is horizontal, and if $B = 0$, the line described is vertical.

15. In the standard form $Ax + By = C$, where $B \neq 0$, show that the slope is $-A/B$ and the y-intercept is C/B.

16. Suppose $A' = kA$, $B' = kB$, and $C' = kC$, where k is a nonzero constant. Show that the same points satisfy the two equations $Ax + By = C$ and $A'x + B'y = C'$. In other words, the two equations $Ax + By = C$ and $A'x + B'y = C'$ describe the same line.

17. Conversely to Problem 16, show that if $Ax + By = C$ and $A'x + B'y = C'$ describe the same line, then there exists a constant k such that $A' = kA$, $B' = kB$, and $C' = kC$.

Problems 16 and 17 alert us to the fact that lines do not have a unique description in standard form. Equations that describe lines in standard form are unique only up to a nonzero constant multiple. By contrast, any nonvertical line is described by a unique equation in slope-intercept form.

Next, we will explore the geometric meaning of the constants A, B, and C in the standard form of an equation for a line. Remember that we are assuming that $A \neq B$.

18. Suppose $P = (x_1, y_1)$ is a point on the line described by $Ax + By = C$. Let $Q = (x_1 + B, y_1 - A)$. Show that Q is also on the line $Ax + By = C$.

19. In the setup of Problem 18, let $R = (x_1 + A, y_1 + B)$. Use the converse of the Pythagorean theorem to show that PQR is a right triangle with right angle at P.

In other words, from any point on the line described by $Ax + By = C$ (indeed, any point in the plane), if we move directly to the point that is situated A units horizontally right and B units vertically up (interpreting negative numbers as a move backward) from the point, we will be going in a direction perpendicular to the line. One of the beautiful features of the standard form of a line is that we can read this perpendicular direction straight from the coefficients of x and y.[9]

20. Suppose C_1 and C_2 are distinct constants. Show that the lines described by $Ax + By = C_1$ and $Ax + By = C_2$ are parallel.

21. In the setup of Problems 18 and 19, show that the line described by $Ax + By = C + A^2 + B^2$ is parallel to the line $Ax + By = C$ and contains the point R.

Since PR is perpendicular to both lines $Ax + By = C$ and $Ax + By = C + A^2 + B^2$, the length of PR is the distance between the two parallel lines. By the Pythagorean theorem, the length of PR is

[9] If you know about the dot product, this fact can be made manifest by writing the equation $Ax + By = C$ in the form $(A, B) \cdot ((x, y) - (x', y')) = 0$, where (x', y') is any point on the line, i.e. $Ax' + By' = C$.

$\sqrt{A^2+B^2}$. If we imagine changing the value of C at a steady rate, the lines $Ax + By = C$ will steadily sweep through a family of parallel lines. Every time C increases by $A^2 + B^2$, the line shifts by a distance of $\sqrt{A^2+B^2}$. In other words, we can find the distance between the two parallel lines, $Ax + By = C$ and $Ax + By = C'$, by dividing $|C' - C|$ by $\sqrt{A^2+B^2}$.

22. Show that the distance of the point (x', y') from the line $Ax + By = C$ is $\left|\frac{Ax'+By'-C}{\sqrt{A^2+B^2}}\right|$.

If $A^2 + B^2 = 1$, then the distance between the lines $Ax + By = C$ and $Ax + By = C'$ is just $|C - C'|$, and the distance of the points (x', y') from the line $Ax + By = C$ is just $|Ax' + By' - C|$. In other words, if (A, B) is a point on the unit circle centered at the origin, changes in the constant term of the equation in standard form directly correspond to distance. Since any point on the unit circle centered at the origin can be written as $(\cos\theta, \sin\theta)$, we can describe any line l by an equation of the form $(\cos\theta)x + (\sin\theta)y = C$, where θ is the angle between the horizontal and the line perpendicular to l, and C is a constant. This form of equation for a line becomes unique if we require that $0 \le \theta < \pi$.

23. Let $A_1x + B_1y = C_1$ and $A_2x + B_2y = C_2$ be the equations for two lines in standard form. Show that these two lines are perpendicular if and only if $A_1A_2 + B_1B_2 = 0$.

For the remaining problems, let $A_1x + B_1y = C_1$, $A_2x + B_2y = C_2$, and $A_3x + B_3y = C_3$ be the equations for three lines in standard form.

24. Assuming that no two lines are parallel, show that the three lines are concurrent if and only if

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0.$$

Problem 24 can be deduced from the following formula for the area of the triangle bounded by the three lines $A_1x + B_1y = C_1$, $A_2x + B_2y = C_2$, and $A_3x + B_3y = C_3$. The formula appears on page 33 of the book *A Treatise on Conic Sections Containing an Account of Some of the Most Important Modern Algebraic and Geometric Methods* by George Salmon, published in 1863.

25. If the three lines bound a triangle, then its area, up to sign, is given by the formula

$$\frac{1}{2}\frac{\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}^2}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}\begin{vmatrix} A_2 & B_2 \\ A_3 & B_3 \end{vmatrix}\begin{vmatrix} A_3 & B_3 \\ A_1 & B_1 \end{vmatrix}}.$$

By definition, $\begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc$ and $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = aei + dhc + gbf - gec - ahf - dbi.$

These are examples of **determinants**, which you can study in any course on linear algebra.

Area Under $1/x$

by Ken Fan | edited by Amanda Galtman

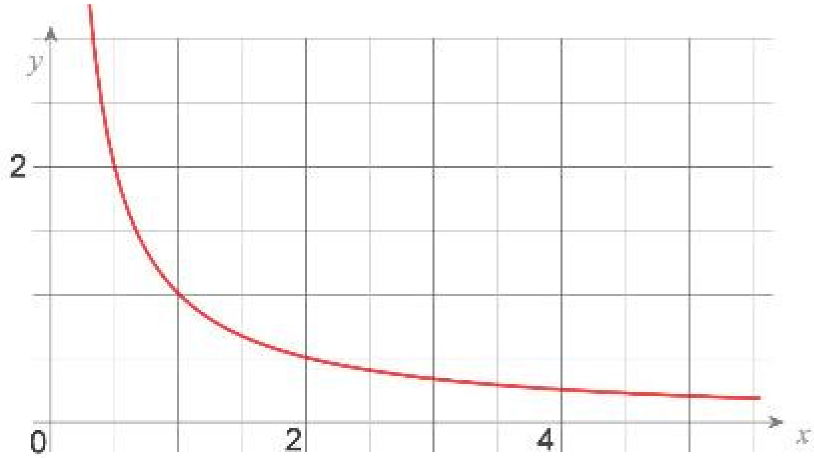

Figure 1. The graph of $f(x) = 1/x$ for some positive values of x.

Above is shown part of the graph of $f(x) = 1/x$, for positive values of x.

Let t be a positive real number, and define a function, $A(t)$, as follows: Consider the region bounded by the x-axis, the vertical lines $x = 1$ and $x = t$, and the graph of $y = 1/x$, as illustrated below. If $t \geq 1$, we define $A(t)$ to be the area of this region. If $0 < t < 1$, we define $A(t)$ to be the negative of the area of this region. The reason for this choice will become clear near the end of this article.

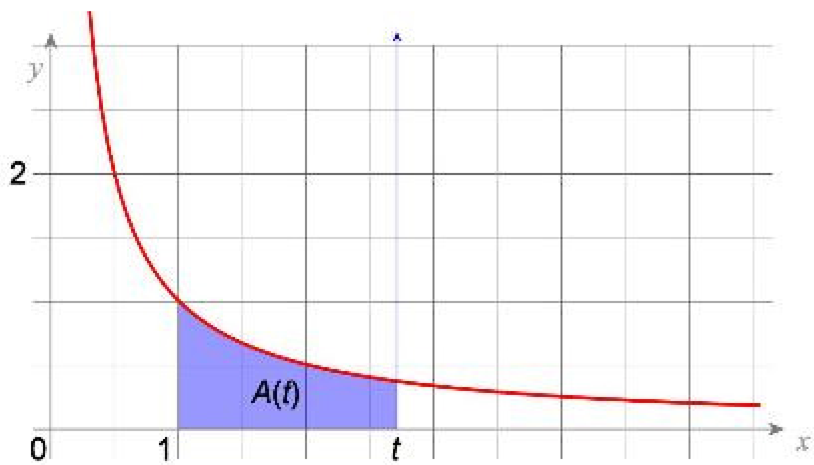

Figure 2. An illustration of the definition of $A(t)$.

Note that $A(1) = 0$.

What can we say about $A(t)$? It turns out that we can say quite a bit, because the graph of $f(x)$ behaves nicely with respect to scaling.

Let's assume $t > 1$. Imagine stretching the coordinate plane horizontally by a scale factor $s > 1$ so that the point (x, y) moves to the point (sx, y). The region that defines $A(t)$ stretches as shown in Figure 3.

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Notation

by Girls' Angle Staff

Good notation facilitates communication. To learn notation, use it. Practice makes perfect!

Floor and Ceiling

There are 100 pieces of origami paper in a package. You've got a group of 8 paper folders and want to hand out the origami paper fairly. How many sheets should each folder get? We divide 100 by 8 and get 12.5. Well, you can't really give someone half a sheet of origami paper, so we round down and give everyone 12 sheets.

The need to round down a number x to the nearest whole number arises often enough in mathematics that there is special notation for it. It's called the **floor** of x and is denoted $\lfloor x \rfloor$. More precisely, $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Using floor notation, we can say that the answer to the origami paper distribution question is $\lfloor 100/8 \rfloor$. More generally, if you have n sheets of origami paper and f paper folders, and you want to give each the same number of sheets, then each can be given $\lfloor n/f \rfloor$ sheets.

The floor notation uses square brackets without the top horizontal bits. In fact, it used to be that the floor of x *was* denoted by using square brackets. The reason why the top horizontal bits were nixed is because we often need to round *up* instead of down, and so it would be nice to have a way to denote rounding up too. For example, imagine you want to make a square box whose length and width are a whole number of inches. You want to store a cylindrical cake inside. The circular base of the cake has a circumference of c inches. What is the smallest possible width of the box?[1] By eliminating the top horizontal bits of the square brackets to denote floor, it becomes very suggestive to denote rounding up by taking square brackets and removing the bottom horizontal bits, like this: $\lceil x \rceil$. The notation $\lceil x \rceil$ is called the **ceiling** of x. More precisely, $\lceil x \rceil$ is the least integer greater than or equal to x.

The best way to get used to new notation is to use it. Here are some problems for you to think about. Use the notation $\lfloor x \rfloor$ and $\lceil x \rceil$ as you work on them.

1. Show that $\lfloor x \rfloor \le x \le \lceil x \rceil$, with equality if and only if x is an integer.

2. Show that $\lceil x \rceil = -\lfloor -x \rfloor$.

3. Evaluate $\sum_{n=1}^{100} \left(\lceil \sqrt{n} \rceil - \lfloor \sqrt{n} \rfloor \right)$.

$$\lfloor x \rfloor$$

The floor of x.

$$\lceil x \rceil$$

The ceiling of x.

[1] The answer is c/π inches, rounded up.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 24 - Meet 5 March 7, 2019	Mentors:	Talia Blum, Grace Bryant, Anna Ellison, Amy Fang, Adeline Hillier, Claire Lazar, Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo, Melissa Sherman-Bennett, Shohini stout, Jane Wang, Rebecca Whitman, Jasmine Zou

If you're particularly quick at seeing ways to compute answers, or you have a habit of using your calculator or computer for mathematical computations, you might be preventing yourself from developing more conceptual ways of thinking about math. There's no doubt that sometimes, you do have to buckle down and work through a laborious computation. But there is also a conceptual side to mathematics, and when you are strong on the conceptual side you can often spare yourself entirely from slogging through tedious computations. Also, computing all the time can obscure patterns. To help develop this conceptual side, we created a series of "anti-calculator" problems, and can try a batch of them on page 17.

Session 24 - Meet 6 March 14, 2019	Mentors:	Talia Blum, Grace Bryant, Kelly Chen, Anna Ellison, Amy Fang, Katie Gravel, Adeline Hillier, Claire Lazar, Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo, Melissa Sherman-Bennett, Savannah Tynan, Jane Wang, Rebecca Whitman, Josephine Yu, Jasmine Zou

We had an epic game of cliffhanger pitting a team of members against our director and moderated by mentors Gisela and Savannah. In cliffhanger, players try to hit a target number by forming mathematical expressions using randomly generated numbers. Each random number must be used exactly once somewhere in the expression. The difference between the value of the expression and the target number is how far a token representing that player is moved toward the edge of a cliff. The goal is to be the last person to fall over the cliff. Can you beat what the players came up with for these rounds? We had 30 seconds to come up with written expressions.

Round	**Random Numbers**	**Target**	**Best Difference Achieved**
1	1, 1, 1, 2	14	2
2	2, 2, 4, 5	10	1
3	1, 2, 4, 5	18	0
4	2, 5, 5, 5	15	0
5	2, 5, 6	12	1

(Hint: You can definitely do better in rounds 2 and 5.)

Session 24 - Meet 7 March 21, 2019	Mentors:	Talia Blum, Kelly Chen, Anna Ellison, Amy Fang, Katie Gravel, Adeline Hillier, Rebecca Nelson, Kate Pearce, Gisela Redondo, Savannah Tynan, Jane Wang, Rebecca Whitman

After many year's absence, the Fujimoto hydrangea reappeared at the club! The Fujimoto hydrangea is a particularly geometric origami model designed by Shuzo Fujimoto. There's a lot of mathematics in paper folding. For example, suppose you have a dual-colored sheet of origami paper that is lilac on one side and sunflower yellow on the other. Can you describe all possible ways to make a single fold so that the resulting model has one side that shows equal amounts of lilac and yellow? What if your dual-colored paper is of a different shape, like a circle, an equilateral triangle, a regular hexagon, or a 1 by 2 rectangle?

Session 24 - Meet 8 April 4, 2019	Mentors:	Talia Blum, Anna Ellison, Amy Fang, Katie Gravel, Adeline Hillier, Claire Lazar, Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo, Melissa Sherman-Bennett, Shohini Stout, Jane Wang, Rebecca Whitman, Jasmine Zou

Some members began exploring powers modulo n and discrete logarithms. In the discrete logarithm problem, you are given two whole numbers a and g, and you must find x such that $g^x = a \pmod{p}$, for some fixed prime number p. We assume that g is a generator modulo p, that is, the remainders of the powers of g upon division by p eventually run through all possible remainders from 1 to $p - 1$. (Note that if p divides g, then p divides all powers of g and the only remainder you will get is 0.) Some cryptographic systems are based on there being no known fast algorithm for solving the discrete logarithm problem in general.

One way to motivate thinking about the discrete logarithm is to encode the letters of the alphabet as the numbers 1 through 26, and let 27 represent a space and 28 represent a period. Write down a word or two and encode it using this scheme. Then, for each number a in the sequence, replace it with the remainder obtained by dividing 15^a by 29. Pass this new sequence of numbers to a friend and see if they can decode it. (The message has to be short so that frequency clues cannot be used to decode the message.)

Session 24 - Meet 9 April 11, 2019	Mentors:	Grace Bryant, Kelly Chen, Katie Gravel, Adeline Hillier, Rebecca Nelson, Kate Pearce, Laura Pierson, Melissa Sherman-Bennett, Jane Wang, Rebecca Whitman

Some members have been inventing axioms that describe what sets are and what can be done with them. In other words, they've been trying to come up with their own version of set theory. When they've explored this to their satisfaction, we'll show them the Zermelo-Fraenkel set theory axioms to compare and contrast with what they came up with.

Session 24 - Meet 10 April 25, 2019	Mentors:	Anna Ellison, Adeline Hillier, Claire Lazar, Rebecca Nelson, Kate Pearce, Laura Pierson, Melissa Sherman-Bennett, Savannah Tynan, Rebecca Whitman

Some members counted coin fountains. For more on this topic, check out page 27 of Volume 5, Number 5 of this Bulletin.

Calendar

Session 23: (all dates in 2018)

September	13	Start of the twenty-third session!
	20	
	27	
October	4	
	11	
	18	
	25	
November	1	
	8	
	15	
	22	Thanksgiving - No meet
	29	
December	6	

Session 24: (all dates in 2019)

January	31	Start of the twenty-fourth session!
February	7	
	14	
	21	No meet
	28	
March	7	
	14	
	21	
	28	No meet
April	4	
	11	
	18	No meet
	25	
May	2	
	9	

Session 25: To be announced…

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

A heartfelt Thank You to Monica Concepcion, Rachel Gesserman, and all employees at the Broad Institute for giving Girls' Angle a marvelous, inspiring home for the past three years!

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, professor of mathematics, Boston College
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton University
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, University of Utah School of Medicine
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Liz Simon, graduate student, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, associate professor, University of Washington
Karen Willcox, Director, Institute for Computational Engineering and Sciences, UT Austin
Lauren Williams, professor of mathematics, Harvard University

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____