Girls' Angle Bulletin

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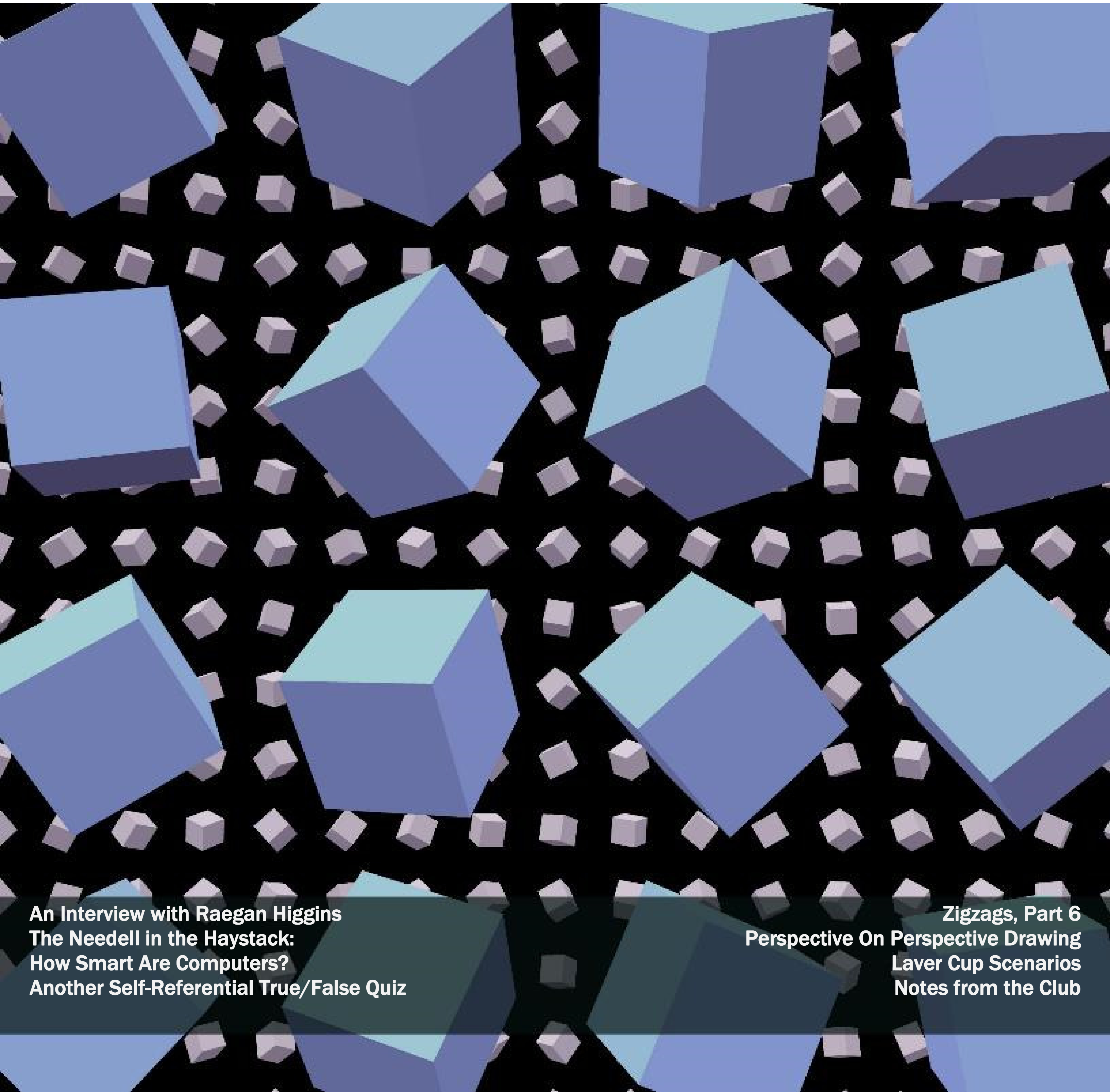
To Foster and Nurture Girls' Interest in Mathematics



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From the Founder

This fall, one of our members pleaded with us to carry out an intricate mathematical derivation "just for fun." To all our members: Absolutely! Please do! And the more fun, the better! - Ken Fan, President and Founder

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Executive Editor: C. Kenneth Fan

Girls' Angle: A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

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On the cover: *Cubes In Perspective* by C. Kenneth Fan. For more on perspective drawing, see page 20.

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An Interview with Raegan Higgins

> *…the idea of one size fits all must be let go of.*

Raegan Higgins is an Associate Professor of Mathematics at Texas Tech University. She earned a Bachelor of Science from Xavier University of Louisiana and a master's degree and a doctor of philosophy in math from the University of Nebraska.

Ken: In an interview you had with Mathematically Gifted and Black, you told a remarkable story of when you were in middle school. You spoke about how you barely passed the Algebra entrance exam and began struggling within weeks. Your teacher, Mrs. Gwendolyn Scott, gave you extra work and insisted that your parents sign off on all your graded assignments. And now, here you are, a professor of mathematics! I think that's amazing because under those circumstances, I think a lot of students would have felt so bad that they would have given up. Can you remember how you were feeling at that time and what it is about yourself that enabled you to rise to the occasion and discover an interest in math?

Raegan: That was a long time ago. ☺. I do remember feeling uncomfortable because I had never done so poorly or needed extra help in the past. I also was relieved because I had help and a plan that I could work. Yes, that may seem contradictory. My parents, especially mother, insisted that I do my best always. So, it was already in me to strive for my best. From that experience, I learned that I won't always know things initially, but I can figure it out. This happened with "math" as the thing.

Ken: Can you remember what wasn't making sense to you at the time and how you figured it out?

Raegan: I was like most students who see letters in math for the first time; letters in math just didn't belong to me. It really helped when I figured out that I had been doing Algebra for a long time. For example, if we're having a family dinner and we need to figure out how many adult tables and kids table we needed, I was using Algebra. I knew an adult table generally sat 8 and a kids' table sat 4; we generally sat at the card table. ☺. Once I realized that I used it every day, it wasn't as daunting.

Ken: What was it exactly that caused you to discover that you like math? Was it a particular problem?

Raegan: There was no particular problem. I came to like math because it was the topic/subject that showed me that I had ability to do well at something new. If I had a Mrs. Scott in science, I may have come to love it. I really think it was the support that I found that caused me to like math.

Ken: When did you decide that you wanted to become a mathematician? What led you to that decision?

Raegan: After having such a positive experience in Algebra, I wanted to change students' lives like Mrs. Scott changed mine. So, my plan was to be a math teacher. Once I got to college, my advisor, Dr. Vlajko Kocic, encouraged me to explore my options outside of teaching. He talked to me about graduate school. I had some idea of what graduate school was because my father, who is a retired high school teacher, has a master's degree in Industrial Arts Education. I recall him going to class and doing

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homework. So graduate school was not a new idea. The idea of earning a doctorate was. Dr. Kocic **told me** that I was going to graduate school and I worked to achieve that. At Xavier, there were upper class female math majors who were going to graduate school and who were doing well. So, his suggestion of graduate school seemed reasonable. Honestly, it was the norm, the expectation, the rule, the standard. Call it what you want; most Xavier math majors went to graduate school.

Ken: Could you please describe one of the greatest challenges you faced on the way to becoming a mathematician after you had decided to become a mathematician and how you overcame it?

Raegan: The greatest challenge was qualifying exams. At the University of Nebraska- Lincoln, each student who wanted to pursue a doctorate had to "qualify." Of the 3 exams, the analysis exam presented the most challenge. Each time, I failed the exam. However, I was improving. During studying, my colleagues and I found that we needed more time. Before my last attempt, a colleague asked if we could have another hour. The extra time was granted, and I finally passed. I must say that exam was harder than the previous ones so that made me happy. I knew the material and showed that I did.

Ken: You are one of the first two African-American women to earn a doctoral degree in mathematics from the University of Nebraska-Lincoln. We had the good fortune of interviewing the other, Prof. Christina Eubanks-Turner.[1] Was it important to you to have a colleague who was also African American and a woman going through graduate school with you?

Raegan: Christina and I agreed to go to grad school together after leaving Xavier. The agreement was based on the fact we knew how each other thought and learned and we knew that would help us. I don't think we were thinking about gender and race; we needed someone that could speak to our way of learning and understanding. Now, once we were in the thick of graduate school, I appreciated having her there for reasons outside of school. I needed the feelings of home and Christina and her family provided that. I needed the ability to just be and I could do that with her.

Ken: You're an expert in time scale calculus. Could you please explain what that is?

Raegan: Calculus is an area of mathematics that helps us understand the changes in values that are related by a function. There are two types of calculus: differential and integral. Differential calculus focuses on breaking things into small (different) pieces and tells how these pieces change from one point in time the next. On the other hand, integral calculus joins (integrates) those small pieces and tells how much of something is made as a result of the changes. These happen on the real line.

Time scale calculus allows us to differentiate (break apart) and integrate (put together) on *any* non-empty closed subset of the real numbers $\mathbb{R}$. We call this set a time scale. When we read "subset of the real numbers," think "some part of the number line." So, you do not have to use the number line; you can use any part of it. For example, you could use just the natural numbers – 1, 2, 3, etc. To say a set is closed means the set has all its limit points. Now what is a limit point? Informally, think of a limit point as a point that some sequence of

[1] Our interview with Prof. Christina Eubanks-Turner is in the previous issue, Volume 13, Number 1.

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Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Raegan Higgins and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
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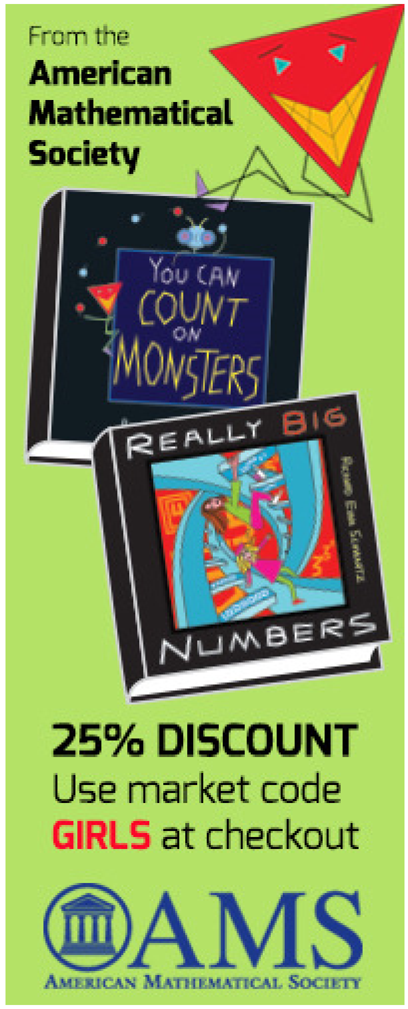

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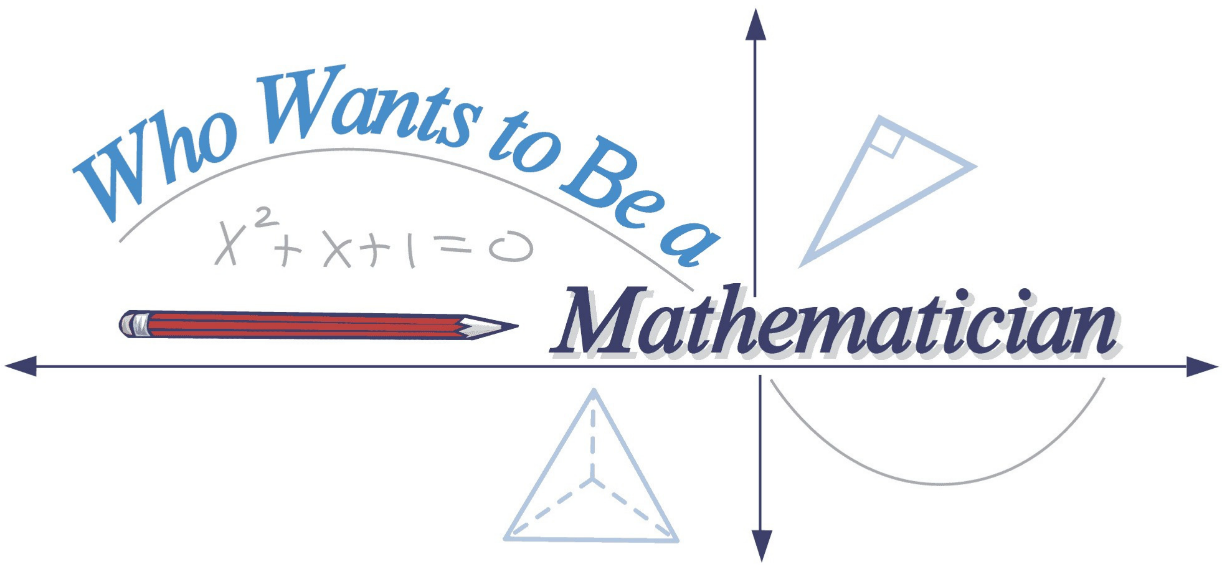

America's Greatest Math Game: Who Wants to Be a Mathematician.

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The Needell in the Haystack[1]

How Smart Are Computers?

by Deanna Needell | edited by Jennifer Sidney Silva

My father bought me my first cell phone after I graduated high school in 1998. It was a Nokia phone that was basically the shape of a brick, and it weighed almost as much as one, too. But it did the job. Mind you, back then, the job of a cell phone was literally just to make phone calls. Fast forward 21 years and now we have smart phones that fit in our pockets and communicate information from anywhere in the world in a split second.

In fact, not only are our phones "smart," but so are our other devices. From your favorite at-home assistant to smart kitchen appliances and smart watches, more and more of our daily lives are filled with smart computers. Now when I compose an email or a text message, the computer completes my own sentences for me. In fact, computers are even able to create real-looking but entirely synthetic human faces and have full conversations that sound nearly human. Jexi, a movie that is out in theaters as I write this article, is about a smart phone that begins to give unsolicited advice to its owner. This is not the first of such movies, and it may not be that unrealistic, either. So this begs the question, how are computers able to be so smart?

In this article, we'll dip our toes into this world and explore some of the mathematical ideas that make this kind of learning possible. We will focus on one concrete application of image reconstruction, but the main building blocks are also used in text generation and countless other smart applications. All of this falls into the category of **machine learning**, a branch of computer science and mathematics that has replaced the ironically-misnamed **artificial intelligence** (you, the reader, may decide at the end of this article whether it is appropriate to call computers intelligent).

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[1] This content supported in part by a grant from MathWorks.

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Another Self-Referential True/False Quiz

by Michelle Chen

Inspired by self-referential tests such as the one in Volume 10, Number 3 of this Bulletin, Michelle created one of her own. Can you get a perfect score?

1. Statement 5 is the first true statement.	T	F
2. There are an even number of true statements.	T	F
3. There are 3 consecutive false statements in a row.	T	F
4. There are an even number of false statements.	T	F
5. If the answer to this statement is changed, then another answer will become wrong.	T	F
6. The statement below is false.	T	F
7. The statement below is false.	T	F
8. The next two statements are true.	T	F
9. Statement 14 is the last false statement.	T	F
10. At least one of statement 11 or statement 12 is true.	T	F
11. Both statement 1 and 15 are true.	T	F
12. If you swap the answers to Statements 10 and 12, no other answer become wrong.	T	F
13. There are 4 false statements after Statement 9, not including Statement 9.	T	F
14. There are more false statements than true statements	T	F
15. Exactly one of Statement 1 or Statement 5 is true.	T	F

If you like this sort of thing, check out the self-referential multiple choice test created by **Ghost Inthehouse**, **HolAnnherKat**, **Katnis Everdeen**, and **Shark Inthepool** on pages 20-21 of Volume 11 Number 2 of this Bulletin, and the one that inspired theirs, which was written by James Propp and can be found on the internet.

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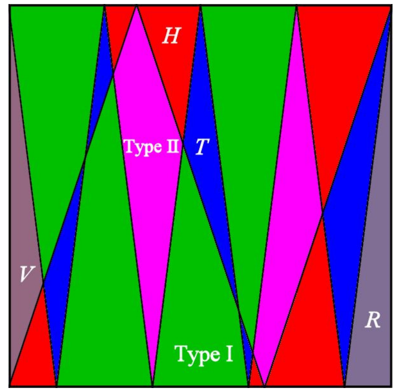
Zigzags, Part 6

by Ken Fan | edited by Jennifer Sidney Silva



For your convenience, here are the different types of triangles and quadrilaterals that Emily and Jasmine defined in "Zigzags, Part 3." Also, recall that they are assuming that n and m are relatively prime and $n > m$. They found that there are $2m$ triangles of type H and $n - m$ triangles of type T. They also determined the horizontal side lengths of all triangles of type H.

Jasmine inhales the delicious aroma of Cake Country and makes a beeline for the booth in the back where she and Emily love doing math. That booth is the most sheltered, and has a nice spacious table to spread out all of their papers.

As she scoots in, she realizes that she and Emily have neglected to compute the area of any of the triangles of type H. They'd figured out the horizontal side lengths of all of the ones that occur in the zigzag pattern, but none of their areas. She decides to compute those areas while waiting for Emily.

Given that all triangles of type H are similar to each other, she knows that she needs to compute the area of just one. The areas of the others can then be found by multiplying by the square of the scale factor. Since they had computed the lengths of the horizontal sides, she decides to compute the area of the triangle of type H that has a unit horizontal side length.

Mr. ChemCake: Where's Emily?

Jasmine: Piano. She's coming after her lesson.

Mr. ChemCake: Ah, I see. Can I get you anything while you wait?

Jasmine: Yes! I've been craving one of your brownie bonanzas. May I please have two, one for me and the other for Emily?

Mr. ChemCake: Two brownie bonanzas coming right up!

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Perspective On Perspective Drawing

by Addie Summer | edited by Amanda Galtman

Let's come to understand the motivation and reason for mathematics in perspective drawing.

Please get up. Bring this article with you. Find the window in your home with your favorite view. Set up a chair before that window and take a seat so you can enjoy that view in comfort.

What is needed to make a painting of that view?

Mind you, I don't mean a rough sketch. I mean a painting so convincing that if you momentarily close your eyes, and your friend sneaks over and places the painting over the window, when you reopen your eyes, you think nothing has changed. You feel like you are still sitting before a window staring out upon your favorite view. Is it possible to make a painting that realistic?

In truth, it's not really possible, but you can get startlingly close. For one thing, if you alternately look through your window with your left and right eyes, you'll notice that the view shifts. In fact, there will be things that you can see with your left eye near the right edge of the window that are blocked from your right eye. Try it and see!

Why does that happen? We see something when light from that object travels into our eyes and gets registered. For the most part, we can assume that the light travels in a straight line. (It actually doesn't. For example, it bends when it enters the glass of the window pane. But to gain understanding, it helps to make simplifying assumptions and consider the more complicated truth later.) Our eye sees an object through the window if the line segment that connects object to eye, which represents the path that the light takes from object to eye, passes through the window. Since your eyes are in different places, the line segment connecting the object to your left eye is different from the line segment connecting the object to your right eye. For this reason, it is possible for the window frame to get in the way of one of these line segments but not the other.

This fact creates a problem for the painter.[1] Suppose there's a pretty little hummingbird hovering in the view through the window. Where on the canvas should the painter paint the bird? The painter imagines the canvas placed over the window. The drawing of the bird should be placed exactly where the line segment connecting the actual bird to the eye passes through the canvas. But which eye? The left or the right?

Do you see the painter's problem? Because of it, when you create a perspective drawing, you have to design it for a specific eye, with a fixed position relative to the canvas. Once you've decided which eye location to draw for, you can figure out where objects must be painted. This also implies, however, that when you make a perspective drawing, it will be best viewed by a single eye located at a specific point relative to the canvas. If the observing eye shifts from that optimal point a little bit, the perspective illusion will still be strong, but if it strays too far, everything in the drawing will look awry.

Anyway, you decide from where exactly you want your painting to be viewed and press on.

---

[1] Having two eyes presents problems for the painter, but it enables us to see 3D. One way to exploit our two eyes is as a distance estimator. For more information, see *Math In Your World* in Volume 6, Number 5 of this Bulletin, pages 12-13, subtitled *Eyeballing the Distance*.

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To make the painting, you *could* construct line segments (with, say, a laser pointer) from the chosen eye location to the different objects, note where each segment intersects the canvas, and put there a little dab of paint of the appropriate color. This approach would work because it fully captures the concept of perspective drawing. However, the approach would be tedious and inconvenient—not to mention that when we paint, it's not often possible to position the canvas in such a way that we can directly examine how lines of sight intersect with it.

What can be done to make the job less daunting and more doable? Over and over, we learn the value of considering the simplest situations, where understanding is easier to gain and can provide us with guidance for more complex situations. Perhaps you can postpone the drawing of your favorite view and start by drawing a simpler scene, or simple parts of a complex scene. You might, for example, notice an unadorned building, a road, or a tiled sidewalk. Perhaps there's a flock of geese flying in a "V" formation that doesn't actually contain any lines but suggests lines that can be used as a guide to the placement of each goose.

It is reasonable—indeed, very mathematician-like—to suspect that drawings of basic geometric objects will enjoy nice properties that can be used to assist us in drawing them. For example, do you think it is true that a drawing of a line is a line? It is! So the edges of a blocky building are drawn as a collection of line segments. Slowly but surely, math emerges. And if we find that drawings of basic geometric objects have nice properties that we can use to help us draw, we can apply this knowledge to drawing complex scenes by looking for basic geometric shapes and relationships that the scene suggests. For example, there may be a cluster of trees whose trunks suggest a group of parallel lines, or perhaps you observe that a subject's left eye together with the tips of their left ear and chin form the vertices of an isosceles right triangle. If this intrigues you enough to start analyzing how to draw basic shapes, your mathematical journey has begun!

Perhaps your journey will begin with a study of how to draw points, lines, and planes. Then, after you've understood how to draw those, perhaps blocks, cubes, pyramids, and circles come next, followed by spirals, multiple objects, and even tessellations on curved surfaces like the vault in Giovanni Paolo Panini's painting at right. Or, perhaps you'd prefer to develop along different lines. Whatever path you choose, it's generally good advice to start simple and build from there. You're sure to concoct a few theorems along the way!

For more, check out the Summer Fun problem set on perspective in Volume 6, Number 5 of this Bulletin.

Courtesy National Gallery of Art, Washington

Interior of Saint Peter's, Rome by Giovanni Paolo Panini. Panini was, among other things, a professor of perspective.

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Laver Cup Scenarios

by Lightning Factorial | edited by Amanda Galtman

I love a great tennis tournament. For decades, I've looked forward to the four slams, the Australian Open, the French Open, Wimbledon, and the US Open. For the last three years, two weeks after a US Open champion has been crowned, tennis fans have been treated to some scintillating tennis at the great new tournament aptly named the Laver Cup.

Created in honor of "Rocket" Rod Laver, the Laver Cup assembles a dozen of the top male tennis players into two teams of six: Team Europe and Team World. Each team is captained by a tennis legend: Bjorn Borg for Team Europe and John McEnroe for Team World.

The tournament spans three days, with one doubles and three singles matches scheduled for each day. The first day's matches are each worth 1 point. The second day's matches are each worth 2 points. And the final day's matches are each worth 3 points. All told, there are $4 \times 1 + 4 \times 2 + 4 \times 3 = 24$ points up for grabs. Naturally, the first team to earn more than 12 points wins the Laver Cup.

Yet, even more than a great tennis tournament, I love a great tennis match. Few tennis matches are as weighty and exciting to watch as one that is winner-take-all. I couldn't help but wonder, "How likely is it that the Laver Cup come down to a final, winner-take-all match?"

My first thought was that the answer surely depends on the relative skill of the two teams. If one team were much weaker, losing every match, the stronger team would win all 4 points available the first day, win all 8 points available the second day, and then win the cup with the first match on the third day – there'd be no chance for a winner-take-all match. But we cannot know how much stronger one team may be. Rather than trying to model a situation where one team is stronger than the other, I decided to assume that the teams are evenly matched, that is, each team has a 50-50 chance of winning any given game. In this way, I would get an indication of how much excitement the tournament can produce based on its intrinsic design.

To find the answer, let's first understand exactly when and how a winner-take-all match can occur. A winner-take-all match is one where the winner wins the Laver Cup. That means that both teams have to be within a few points of the magic number of 13 points needed to take the title. Since the first eight matches are worth a total of 12 points, no team can win the cup before the third day. If there is a winner-take-all match, it can occur only on the third day, when matches are worth 3 points. That means that if there is a winner-take-all match, both teams must have at least 10 points. Therefore, the total number of points the two teams must have prior to a winner-take-all match is at least 20. After the first match of the third day, 15 points will have been awarded, and after the second, 18 points will have been awarded – still not enough for both teams to have 20. In order for both teams to have at least 20 points, three matches on the third day must be played, which means that the only match that could possibly be a winner-take-all match is the 12^{th} match of the tournament.

Since the total value of the first 11 matches is 21 points, there are only two ways for the teams to each have at least 10 points: Team World has 10 points and Team Europe has 11, or vice versa. Thus, our problem is equivalent to finding the probability that one of the teams has 10 or 11 points going into the last match.

Notice that we only have to keep track of one of the team's scores. At each stage, we know how many points have been awarded, so knowing one team's score allows us to compute the other's. Since Girls' Angle is based in Cambridge, Massachusetts, we'll track Team World. Thus, we ask: What is the probability that Team World has 10 or 11 points after the 11^{th} match?

We'll work our way to the 11^{th} match, one match at a time.

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At the beginning of the tournament, no points have been awarded and Team World has 0 points. After the first match is played, the winning team receives 1 point. Since we are assuming a 50-50 chance of either team winning, after the first match, there is a 50-50 chance that Team World has 1 point and a 50-50 chance that Team World remains at 0 points. In other words, Team World has one way to get to 1 point and one way to remain at 0 points, with both ways being equally likely.

After the second match, which is also worth 1 point, Team World could have 0, 1, or 2 points, but the number of ways these point totals could happen aren't the same. To have 0 points, Team World must lose both matches. To have 2 points, Team World must win both matches. To have 1 point, Team World could either win the first match and lose the second, or lose the first match and win the second. Both possibilities are equally likely, and both are just as likely as losing or winning both matches, since we are assuming that winning and losing any given match have equal probability.

We record this information in the table below.

Points / Matches	0	1	2	3
Match 1	1	1	0	0
Match 2	1	2	1	0
Match 3				

Table 1. Number of ways to achieve various point totals for Team World.

How do we fill out the third row of the table? The third match is worth 1 point, so Team World either gains a point or not, with equal probability. Each number in the row above this row tells us how many equally likely ways there are for Team World to come into the third match with a certain number of points: There's one way for Team World to arrive at the third match with 0 points, two ways with 1 point, and one way with 2 points. The one way that Team World can arrive at the third match with 0 points spawns one way for Team World to have 0 points after the third match and one way to have 1 point after the third match. Each of the two ways Team World can arrive at the third match with 1 point spawns one way to have 1 point after the third match and one way to have 2 points after the third match. Finally, the one way Team World can arrive at the third match with 2 points spawns one way to have 2 points after the third match and one way to have 3 points after the third match. The blue arrows in the table illustrate how the numbers in the second row contribute to the numbers in the third row.

Let's look at the situation from the point of view of the third row. By examining the blue arrows, we see that the number of ways Team World can emerge from the third match with, say, 1 point, is the sum of the number of ways Team World can have 0 points or 1 point after the second match. In this way, we can compute the entries in the third row to be 1, 3, 3, and 1, respectively. Each number tells how many ways Team World can achieve the corresponding point total, and all the ways are equally likely. Table 2 shows the results of these computations through the first day of competition.

To compute the fifth row, we use the same technique. We just have to account for the fact that the fifth match is worth 2 points instead of 1. For example, to find the number of ways Team World can have 3 points after the fifth match, we add together the number of ways Team World can have 1 or 3 points after the fourth match. That is because the only way to have 3

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points after the fifth match is to either have 1 point after the fourth match and win the fifth match, or to have 3 points after the fourth match and lose the fifth match. This is illustrated by the blue arrows in Table 2. The same pattern of arrows holds throughout the second day of competition.

Matches \ Points	0	1	2	3	4	5	6
Match 1	1	1	0	0	0	0	0
Match 2	1	2	1	0	0	0	0
Match 3	1	3	3	1	0	0	0
Match 4	1	4	6	4	1	0	0
Match 5							

Table 2. Number of ways to achieve various point totals for Team World for the first day of competition.

The third day requires *two* adjustments. One adjustment reflects that games on the third day are worth 3 points. Another adjustment recognizes the possibility of a match ending the tournament and not contributing to further possibilities. For example, having 0 points after the ninth match is a tournament-ending scenario, because Team Europe reached 15 points and claimed the Laver Cup. No 10th match would be played, so the one way of having 0 points after the ninth match does not contribute one way for Team World to have 3 points after 10 matches. Making these adjustments, we arrive at the following table:

Laver Cup Point Possibilities For Team World

Matches \ Points	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Row Total
1	**1**	**1**															**2**
2	**1**	**2**	**1**														**4**
3	**1**	**3**	**3**	**1**													**8**
4	**1**	**4**	**6**	**4**	**1**												**16**
5	**1**	**4**	**7**	**8**	**7**	**4**	**1**										**32**
6	**1**	**4**	**8**	**12**	**14**	**12**	**8**	**4**	**1**								**64**
7	**1**	**4**	**9**	**16**	**22**	**24**	**22**	**16**	**9**	**4**	**1**						**128**
8	**1**	**4**	**10**	**20**	**31**	**40**	**44**	**40**	**31**	**20**	**10**	**4**	**1**				**256**
9	**1**	**4**	**10**	**21**	**35**	**50**	**64**	**71**	**71**	**64**	**50**	**35**	**21**	**10**	**4**	**1**	**512**
10				**21**	**35**	**50**	**85**	**106**	**121**	**128**	**121**	**106**	**85**	**50**	**35**	**21**	**964**
11							**85**	**106**	**121**	**213**	**227**	**227**	**213**	**121**	**106**	**85**	**1504**
12										**213**	**227**	**227**	**426**	**227**	**227**	**213**	**1760**

Table 3. Number of ways to achieve various point totals for Team World for the Laver Cup tournament. Shaded cells correspond to situations where the tournament ends.

Let's compute probabilities!

For example, what is the probability that Team World will have 5 points after the sixth match? We focus on the sixth row of the table and see that there are 64 equally likely ways for the tournament to unfold through the sixth match. Of these, 12 lead to Team World having 5

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points. Therefore, the probability that Team World has 5 points after the sixth match is 12/64. (You may simplify these fractions if you wish, but we'll write them so that their numerator and denominator correspond to meaningful numbers.)

Another example: What is the probability that spectators will get to see only one match on the third day? We look at the ninth row and see that of the 512 equally likely ways the tournament can unfold through the ninth match, 15 see Team World being crowned champion and 15 see Team Europe win the cup (if Team World has fewer than 3 points after the ninth match, they've lost the cup). Therefore, the probability that we get to watch only one match on the third day is 30/512 – fortunately, not too likely for evenly matched teams.

This example is trickier: what is the probability of the tournament wrapping up when the 10^{th} match ends? The 10^{th} row shows 964 equally likely ways in which the tournament can unfold through the 10^{th} match. We also count 21 + 35 + 50 = 106 ways for Team Europe to win the cup and another 106 ways for Team World to win the cup, for a total of 212 ways that the tournament ends right after the 10^{th} match. However, we have to be very careful when we compute the probability: we cannot simply divide 212 by 964. The reason is that to compute probability, we must determine all possible outcomes, and the 964 only accounts for the scenarios where the tournament reaches a 10^{th} match. The number does not include the ways in which the tournament can end prior to the 10^{th} match. Before the tournament commences, ending before the 10^{th} match is a possibility and, as we computed, occurs with probability 30/512. Dividing 212 by 964 represents the probability of the tournament ending right after the 10^{th} match *given that the tournament made it to a* 10^{th} *match*. The probability of the tournament ending right after the 10^{th} match is the probability that the teams make it to a 10^{th} match (which is 1 – 30/512), multiplied by the probability that a 10^{th} match is played and the outcome decides the Cup (212/964). This comes out to 212/1024.

In other words, when you settle into your cozy chair for the opening ceremonies, there's a 212/1024 chance that you'll see the award ceremony right after the 10^{th} match. If your friend joins you during the 10^{th} match without knowing the team score, then your friend will think the probability is 212/964. Arriving in the middle of the 10^{th} match, your friend can disregard the ways the tournament could have ended after the ninth match. Probabilities adjust with new information. (If, for example, unlike your friend, you knew that the team score was 8-7, then you would know that, actually, there is no chance of the tournament ending after the 10^{th} match.)

So, what is the probability of the Laver Cup ending in a winner-take-all match? As we saw, it is the probability that Team World has 10 or 11 points after the 11^{th} match. From the table, there are 1504 equally likely ways the tournament can unfold through the 11^{th} match, of which 227 + 227 = 454 see Team World with 10 or 11 points. However, we know that the probability is *not* 454/1504 – that's the probability that there will be a winner-take-all match *given that the tournament made it to an* 11^{th} *match*. At the outset of the tournament, we have no idea if the tournament will make it that far. Fortunately, we can compute the probability that it does: The probability that the tournament makes it to a 10^{th} match is 1 – 30/512, and the probability that the tournament makes it to an 11^{th} match given that it made it to a 10^{th} match is 1 – 212/964. So, the probability of a winner-take-all match comes out to (1 – 30/512)(1 – 212/964)(454/1504), which is 454/2048.

By the way, it's possible that after all 12 matches, the two teams end in a draw with 12 points each. In that event, a single set with tiebreak determines the winning team. A winner-take-all set would be pretty exciting, too. Can you show that the probability of the tournament ending in a winner-take-all *set* is equal to 426/4096? (The answer is not 426/1760. That is the probability of the tournament ending in a draw *given that a* 12^{th} *match is played*.)

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More Questions

Unless otherwise specified, assume that the teams are evenly matched.

1. Verify the entries in the table below, which give the probabilities that the teams are tied after each match of the tournament:

Match	1	2	3	4	5	6	7	8	9	10	11	12
P(tied)	0	1/2	0	3/8	1/4	7/32	3/16	11/64	0	1/8	0	213/2048

What is the easiest way for you to see why certain entries are 0?

2. What is the probability that a team enters the third day of competition having won no points at all, yet succeeds in winning the Laver Cup? (Assume that the teams are equally likely to win the tiebreaker set at the end.)

3. Instead of ramping up the point value of the matches each day, suppose the tournament designers awarded a single point for every match. What would the probabilities that we calculated become?

(Partial Spoiler Alert!) In this scenario, the probability of a winner-take-all match occurring becomes zero. Many thanks to Roger Federer and his team for avoiding this design flaw! What other advantages do you see in ramping up the match values each day?

Philosophical note: While a winner-take-all match is likely regarded as desirable, most tennis aficionados would snicker at the winner-take-all point or game because of the server's tremendous advantage.

4. If you wanted to extend the tournament by a day but still make all four matches on the fourth day have equal value, what point value would you give each match? Why?

5. Suppose you wanted to create a tournament lasting D days. Day d has four matches, each worth $p(d)$ points. Assuming that $p(1) = 1$ (i.e., the matches on the first day are worth 1 point each), what is the minimum point value for matches on the following days so that you can guarantee that no team wins the tournament before the Dth day? What should $p(d)$ be if you want the total points awarded on day d to equal all the points awarded on the first $d - 1$ days?

6. Did you notice that to compute the probability that Team World has p points after match m, you can find the value in row m of the table for p points and divide by 2^m? That is, rather than considering conditional probabilities as we did in the text, you can simply divide the table entry by 2^m. Why?

7. What other ways can you think of to simplify the computation of these probabilities? What other patterns do you see? Can you explain them?

8. How would you redo this analysis to allow for different probabilities of winning each match?

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 25 - Meet 8 October 31, 2019	Mentors:	Emily He, Adeline Hillier, Jenny Kaufmann, Tingyi Lu, Aileen Ma, Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo, Melissa Sherman-Bennett, Karissa Wenger, Rebecca Whitman

Today a few members explored the **pigeonhole principle**. It started off with the seemingly outlandish claim, "I know there are two people in Boston right this moment walking around with the exact same number of hairs on their heads!" Quite the bold claim, right? How could one possibly know this? Here are a couple of related problems:

1. There are 10 pairs of socks of distinct styles in a drawer. What's the minimum number of socks you must pull out of the drawer in the dark to guarantee you have a pair?

2. How many people would have to be in a room in order to guarantee that some two celebrated their birthday on the same day?

These questions can all be solved using the pigeonhole principle, which states that if n pigeons are put into m pigeonholes, then at least one pigeonhole must hold at least $\lceil n/m \rceil$ pigeons, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x and is called the "**ceiling** of x."

Session 25 - Meet 9 November 7, 2019	Mentors:	Adeline Hillier, Jenny Kaufmann, Tingyi Lu, Rebecca Nelson, Kate Pearce, Laura Pierson, Christine Soh, Savannah Tynan, Karissa Wenger, Rebecca Whitman

Do you know the sum of the numbers from 1 to 100? Can you find a general formula for the sum of the numbers 1 to N? If you already know the formula, can you prove it? If you already know how to prove it, can you think of a different way to prove the formula? For example if you know an algebraic proof of the formula can you think of a geometric proof, or vice versa? What is the easiest way for you to see that your formula is correct? How many different proofs for the formula can you think of?

Session 25 - Meet 10 November 14, 2019	Mentors:	Adeline Hillier, Shradha Kaushal, Tingyi Lu, Aileen Ma, Kate Pearce, Laura Pierson, Nehar Poddar, Christine Soh, Emma Wang, Karissa Wenger, Rebecca Whitman, Jasmine Zou

A farmer needs to enclose a rectangular region of a field with a fence. She must use exactly 40 meters of fencing to enclose an area of 96 square meters. What are the dimensions of the rectangular plot? What is the answer if she must use 100 meters of fencing to enclose 500 square meters?

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Here's another problem: Suppose you have a 4 foot by 5 foot blanket. You wish to make a rectangular border around the blanket with equal margins on all sides. What is the maximum margin width you can have if you build the border using material from a 10 foot by 10 foot piece of fabric?

Both of the above problems involve **quadratic** functions, which several members have become quite interested in. A quadratic function has the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants with $a \neq 0$. If you haven't worked with quadratic functions before, graph a few to get a general sense of how they look. If you fix b and c, but let a vary, how is the graph affected? If you fix a and c, but let b vary, how is the graph affected? If you fix a and b, but let c vary, how is the graph affected? Can you give a quadratic function whose graph has a minimum at (2, 3)? If $f(x)$ is a quadratic function, how many solutions in x can there be to the equation $f(x) = y$, where y is a constant?

Session 25 - Meet 11 November 21, 2019	Mentors:	Emily He, Adeline Hillier, Jenny Kaufmann, Shradha Kaushal, Tingyi Lu, Rebecca Nelson, Kate Pearce, Laura Pierson, Nehar Poddar, Melissa Sherman-Bennett, Christine Soh, Emma Wang, Karissa Wenger, Rebecca Whitman

Let ABC be a triangle. Draw its inscribed circle O. Let P be the point where O touches BC, Q be the point where O touches CA, and R be the point where O touches AB. Find a formula for the length BP in terms of the side lengths of the triangle. (Notice that $BP = BR$.)

Can you find formulas for the lengths CQ and AR in terms of the side lengths *without doing any further computation*?

Session 25 - Meet 12 December 5, 2019	Mentors:	Adeline Hillier, Jenny Kaufmann, Tingyi Lu, Rebecca Nelson, Kate Pearce, Laura Pierson, Nehar Poddar, Rebecca Vessenes, Emma Wang, Karissa Wenger, Rebecca Whitman

Our traditional end-of-session math collaboration was designed and created by mentors Jenny Kaufmann and Laura Pierson. Starting a few years ago, mentors have been creating many of our end-of-session math collaborations and the result has been a series of the most fun and creative math collaborations I've seen in a long while. This one beautifully extends that list with a clever application of Conway's Game of Life. *Congratulations to all members for solving it and for all your wonderful work this session!* Try your hand at solving some of the problems:

If order doesn't matter, how many ways can 24 be written as the product of three positive integers?

How many of the following statements are true?
A) If statement B is true, so is statement C.
B) This statement is true if and only if statement E is true.
C) Statement A is false.
D) An odd number of statements are true.
E) More statements are true than false.

Let r and s be the solutions to $x^2 - 11x + 24 = 0$. Find $(r - 1)(s - 1)$.

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Calendar

Session 25: (all dates in 2019)

September	12	Start of the twenty-fifth session!
	19	
	26	
October	3	
	10	
	17	
	24	Sally Seaver, Biotechnology Consultant
	31	
November	7	
	14	
	21	
	28	Thanksgiving - No meet
December	5	

Session 26: (all dates in 2020)

January	30	Start of the twenty-sixth session!
February	6	
	13	
	20	No meet
	27	
March	5	
	12	
	19	
	26	No meet
April	2	
	9	
	16	
	23	No meet
	30	
May	7	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

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Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

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Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

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Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory
Yaim Cooper, Institute for Advanced Study
Julia Elisenda Grigsby, professor of mathematics, Boston College
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, assistant dean and director teaching & learning, Stanford University
Lauren McGough, postdococtoral fellow, University of Chicago
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, University of Utah School of Medicine
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Liz Simon, graduate student, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, associate professor, University of Washington
Karen Willcox, Director, Institute for Computational Engineering and Sciences, UT Austin
Lauren Williams, professor of mathematics, Harvard University

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

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Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

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Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

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