Girls' Angle Bulletin

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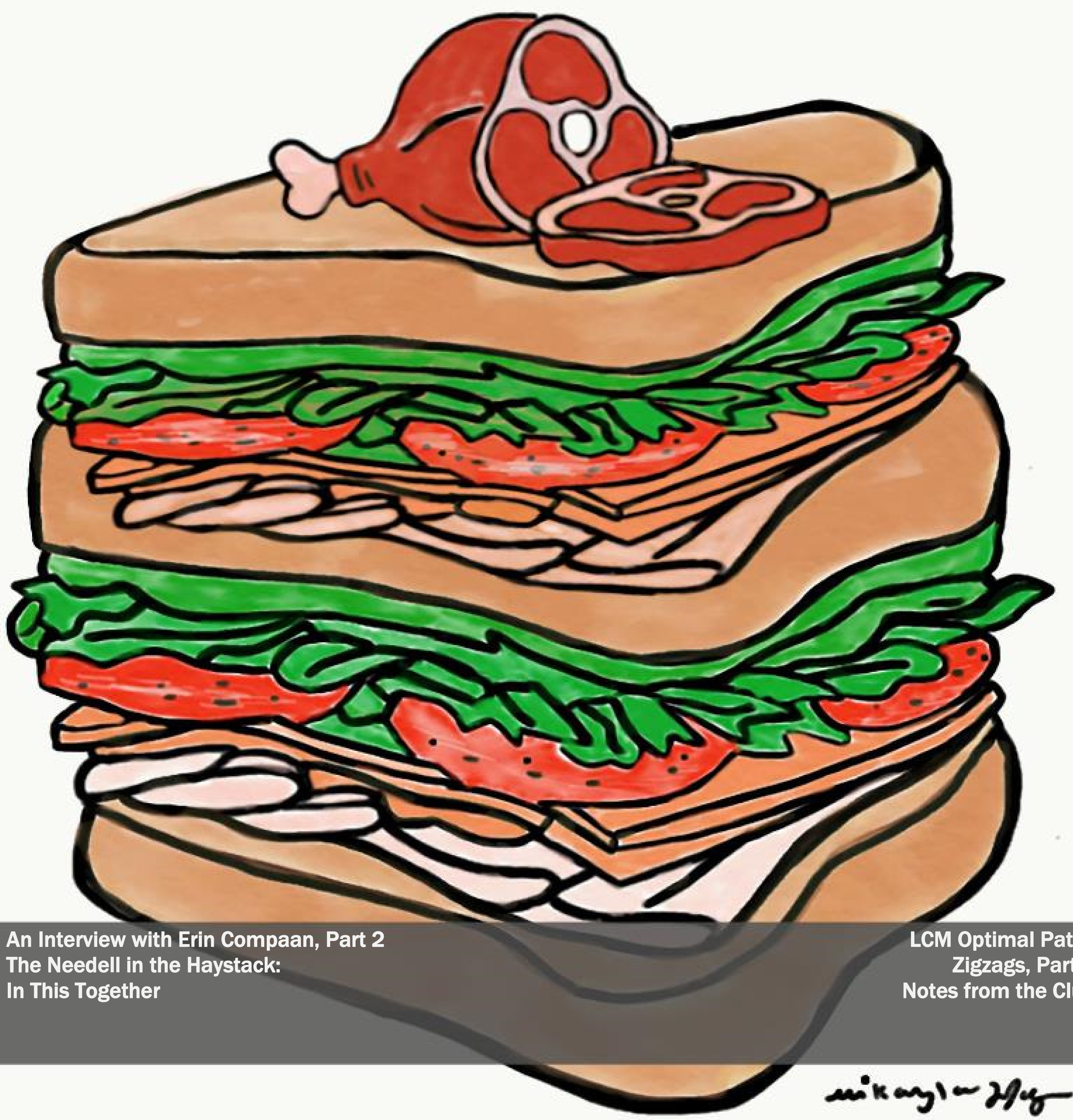
To Foster and Nurture Girls' Interest in Mathematics



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From the Founder

This is our 3rd issue in a row with student work, and we'd love to have more! If you're looking for a research project, check out "LCM Optimal Paths" and see if you can extend the 8th graders work. If you can, consider writing it up for publication here. - Ken Fan, President and Founder

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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

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Editors: Amanda Galtman, Jennifer Sidney Silva
Executive Editor: C. Kenneth Fan

Girls' Angle: A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Optimal Sandwich* by Mikayla Higgins. For the meaning of this cover and its connection to mathematics, see "LCM Optimal Paths" on page 13.

An Interview with Erin Compaan, Part 2

This is the concluding half of our interview with Draper Labs mathematician Erin Compaan. At the end of the first half, Erin gave us the following math problem: For which α do we have, for every sequence $\{a_n\}_{n = \ldots, -2, -1, 0, 1, 2, \ldots}$, the inequality

$$\int_0^{2\pi} \max_{t \in [0, 2/\pi]} \left| \sum_{n \in \mathbb{Z}} a_n e^{inx - in^2 t} \right|^2 dx \le C \sum_{n \in \mathbb{Z}} |n|^{2\alpha} |a_n|^2 ,$$

where C is a constant that does not depend on the a_n. Here, she resumes by interpreting this math problem.

Erin: The idea is if you take some coefficients a_n and you sum up a complex modulation and try to figure out how big it can be, can you bound it (up to a constant independent of the a_n) by a sum of $|a_n|^2$, weighted by $|n|^{2\alpha}$, and, if so, how big does α have to be for this to work? This particular problem is a one-dimensional version of a more general question. A version of this on the full line instead of an interval was solved in the 60s and 70s. But the higher dimensional Euclidean version has only been resolved in the last few years. The version over the integers is still open.

Ken: After you earned your doctoral degree you became a National Science Foundation Postdoctoral Fellow at MIT. Is that correct?

Erin: Yes.

Ken: Was Gigliola Staffilani[1]…

Erin: Yeah, she was my mentor.

Ken: How did that come about?

Erin: My doctoral advisor knew her and thought it would be a good place for me to go. He asked her on my behalf if she would be willing to be a mentor and she agreed to do that. It was very kind as it's a fair bit of work for the mentor to even go through with an application, much less actually mentor someone.

Ken: How did that experience go?

Erin: It was good. I enjoyed working with Gigliola. I think I decided in the years that I was there that I didn't really want to stay on the academic path, which has nothing to do with Gigliola or MIT. It was just a matter of enjoying some of the aspects of more applied math that I'd seen in internships and things, and wanting perhaps a bit more control over my career than the academic life might offer.

Ken: That's interesting. Could you please explain what you mean by more control and what it is about the kinds of problems you encounter in industry that attract you?

Erin: Regarding having control, for me, a lot of that is geography. In academia, I feel it is harder to control where in the world you will live. But more important to me is being able to work on problems which have a more direct impact on helping others to do their own work. So I interned at several places during grad school and I got to see a variety of problems. Most of these projects are confidential, so I can't get into details, but I enjoyed finding interesting phenomenon that helped further other people's work as well as my own.

[1] Gigliola Staffilani is Abby Rockefeller Mauze Professor of Mathematics at the Massachusetts Institute of Technology. She also serves on Girls' Angle's Advisory Board.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Dr. Erin Compaan and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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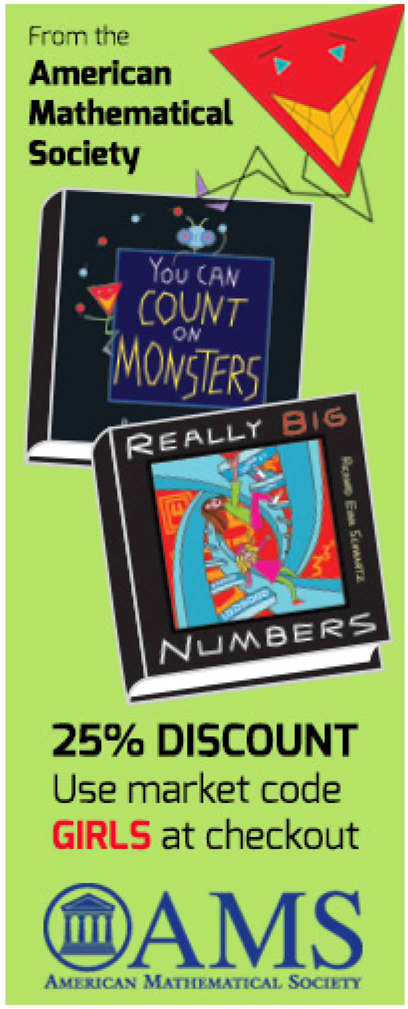

The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

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The Needell in the Haystack[1]

In This Together

by Deanna Needell | edited by Jennifer Sidney Silva

You're no doubt reading this from home, living in a world very different than the world from only months ago. For this reason, I felt it appropriate to discuss how mathematics and data science can help fight the current Covid-19 crisis.

I dedicate this article to all who are working hard to keep our communities going. I cannot give an exhaustive list of such heroes, but I would like to thank each and every one of them: the healthcare workers and first responders at the forefront, delivery personnel bringing essentials to our homes and the grocery store employees stocking shelves, the scientists and lab technicians diligently working on vaccines and understanding the virus, the many other essential workers aiding with our daily needs, and even the folks simply staying home to protect others. This crisis has brought together communities – both locally and globally – in ways most of us have never seen. The situation is scary and grim, yet at the same time has generated an abundance of support and kindness. With these things in mind, let's talk math.

I'll begin with my own story of how I got involved in the projects I am about to discuss. I started by asking who in my research group at UCLA would be interested in working on Covid-19 projects after I saw the call to action issued by the National Science Foundation and the White House; the first one I received was titled "Call to Action to the Tech Community on New Machine Readable COVID-19 Dataset." Although I consider much of my own work important, it is certainly not every day that something like this arrives in my inbox. I got many positive responses, and word quickly spread that I was doing this. Many other folks in my department and outside collaborations wrote to me that they wanted to be involved. The group got so large so quickly that it seemed I needed a better way to organize everything. I quickly created a "seminar course" on the topic and recruited volunteers to serve as group leaders on several projects. The outpouring of support was supremely touching, and the team continues to grow.

So what can mathematicians do to help, and what projects in particular are we currently working on? The answer to the first question is *many things*, and the examples I give here are certainly only a very small taste. I will mention some websites at the end of this article for folks interested in getting more involved. Please note that I will discuss some methods and projects without including results, as these are all works in progress. This article, more than ever, is "hot off the press."

The first thing people ask me when I say that I am working on Covid-19-related mathematical research is whether we will be able to predict the spread of the virus. In fact, this type of modeling is precisely what many mathematicians and epidemiologists are actively working on. Understanding how the virus will and has spread will help us choose good strategies going forward in order to mitigate its spread and reduce infections. The classic model is called an **SIR model**, which connects the number of susceptible people, $S(t)$, infected people, $I(t)$, and recovered people, $R(t)$, as a function of time, t (hence the term "SIR model" for "Susceptible," "Infected," and "Recovered"). An example of this type of model would be a system of three differential equations that relates these three functions, such as the following

[1] This content supported in part by a grant from MathWorks.

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LCM Optimal Paths

by Antonella Catanzaro, Jaemin Feldman, Mika Higgins, Bradford Kimball,
Henry Kirk, Ana Chrysa Maravelias, and Darius Sinha[1] | edited by Amanda Galtman

Dedicated to C. Kenneth Fan for his invaluable contributions and mentorship throughout this project.

Definitions

Here are some terms we define for this paper.

Bread. A synonym for city 1.

City. Any number from 1 through n, where n is the number of cities.

Meat. Any number greater than one.

Path. An integer sequence that comprises integers 1 through n.

Complete path. A path in which each number 1 through n appears at least once.

Sandwich. A sequence consisting of two pieces of bread surrounding a sequence of meat.

Meat content of a sandwich. The numbers within a sandwich excluding the bread.

Open-faced sandwich. A sequence with exactly one piece of bread that occurs either at the beginning or the end.

Prime sandwich. A sequence where a prime number is sandwiched directly between two ones.

Introduction

Fix a positive integer n. Our objective is to find a minimal cost route between n cities labeled 1 through n, beginning at city 1, where the cost of traveling from city a to b is the least common multiple (LCM) of a and b. This problem is similar to the "Traveling Salesman" problem, which is to find a loop that runs through each city exactly once and minimizes the travel distance. In this problem, however, we only require that every city be visited (possibly multiple times), starting at city 1, and using a number theoretic cost instead of a physical one. To the best of our knowledge, this problem is new. In fact, the sequence of least costs (as a function of n) is not yet in the Online Encyclopedia of Integer Sequences.

First Attempts

After finding the optimal paths for one, two, and three cities with simple diagrams, our first attempt to solve the problem with a search tree began with four cities. We started by drawing a

[1] All authors are students at the Buckingham, Browne, and Nichols Middle School in Cambridge, Massachusetts.

square with each vertex labeled a different integer from 1 to 4. We then observed different pathways and tried to determine the one that was the most economical, simply by guessing and checking. The optimal path we found had a cost of 12. To check our work, we used the search tree depicted in Figure 1. This search tree allowed us to make our pursuit of an optimal path systematic. We started from city 1, or the root of the tree (as defined in the problem), and branched off to cities 2, 3, and 4. (Note that it is never economical to travel from a city x to itself, as doing so would unnecessarily increase the cost by x.) This branching continued, and as we went along, we kept track of the cost of each path as it developed. We sustained this until the cost 12 was exceeded (as we had already found a path that had a cost of 12), or until a cost of 12 was almost reached, but the cost of visiting the remaining cities would necessarily bring the cost above 12. (Note that the cost of visiting a set of cities is at least equal to the sum of the cities.) We continued this exhaustive search and discovered that we had, in fact, found the unique optimal path through 4 cities:

1, 3, 1, 2, 4.

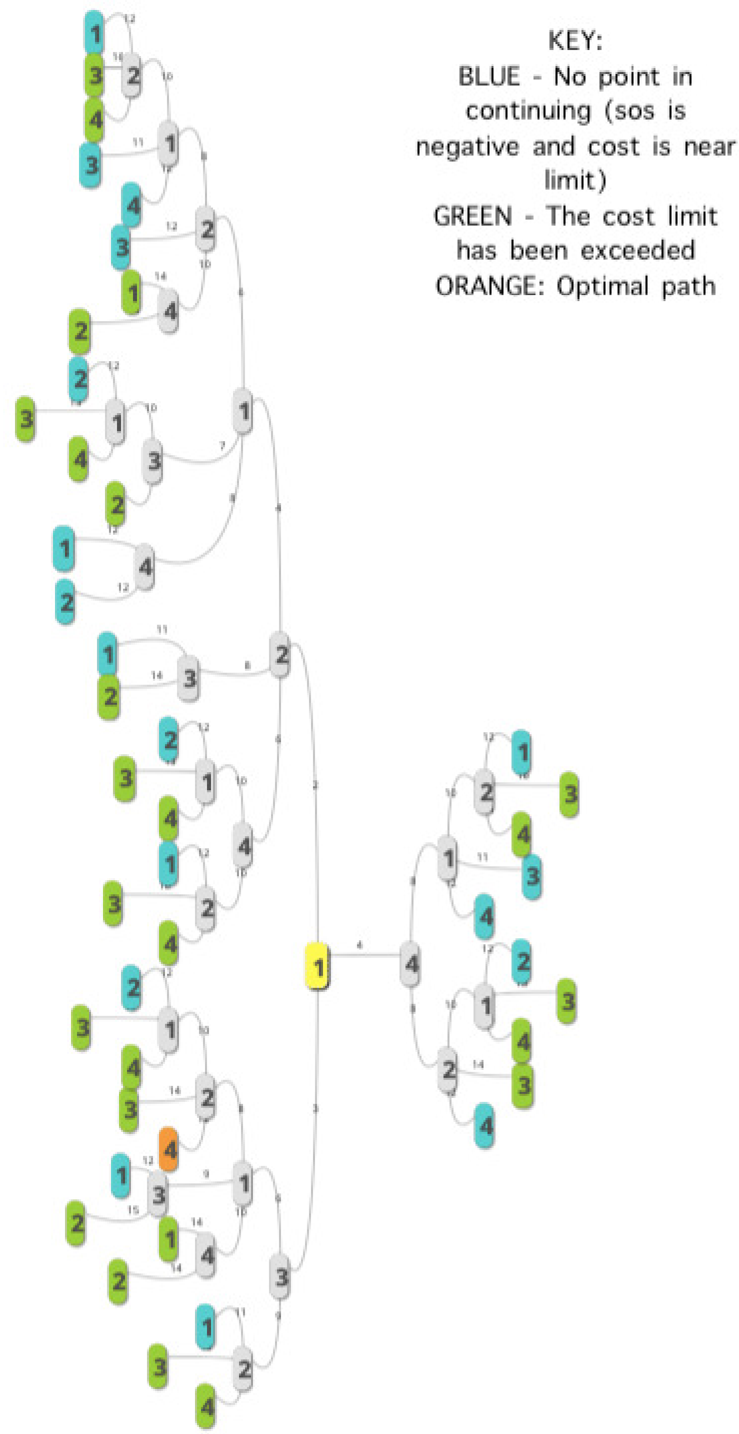

Figure 1. Our tree for the case $n = 4$. (Here, "sos" is an internal piece of lingo we used to flag when all cities had been visited.)

In the figure, the last city visited in that path is highlighted in orange. The optimal path for four cities is 1, 3, 1, 2, 4, and its cost is 12.

Propositions and Proofs

Method of Finding our Paths

After attempting to calculate the optimal paths for six cities by hand, we realized that this process was too tedious; we needed an efficient method to compute the optimal paths of all cities up to 20. Therefore, we developed a program using Python. However, for even modestly large n, the program took impractically long to compute complete optimal paths, and we were compelled to look for patterns in previous optimal paths in order to reduce the size of the search tree. This led us to observe the following propositions.

Proposition 1 (LCM Rules). Let a and b be positive integers. Then $\mathrm{LCM}(a, b) < a + b$ if and only if a divides b or b divides a.

Proof. Let d be the greatest common factor of a and b. Define m and n by $a = md$ and $b = nd$. Note that m and n are relatively prime, by definition of d. Since $\mathrm{LCM}(a, b) = mnd$, our inequality can be rewritten as $mnd < md + nd$, or $mn < m + n$. Now $mn < m + n$ is equivalent to $(m - 1)(n - 1) = mn - m - n + 1 < 1$. Since m and n are positive integers, this last inequality holds if and only if $m = 1$ or $n = 1$. Because m and n are relatively prime, a divides b if and only if $m = 1$, and b divides a if and only if $n = 1$. □

Corollary 1.1. In a complete optimal path, city a must be directly followed or preceded by a city b where b is either a factor or multiple of a, other than a.

Proof. To prove this corollary, we note that $\mathrm{LCM}(a, b)$ is never equal to $a + b$, for if two numbers are neither multiples nor factors of each other, then their LCM must be at least two times the larger of the two numbers. If their LCM is twice the larger number, it is bigger than their sum. Now observe that it never makes sense to travel from a city directly to itself, because elimination of the revisit does not affect the number of cities visited but reduces the cost. If b is not a factor or multiple of a, then the cost of the sequence a, 1, b, which is $a + b$, is less than the cost of the sequence a, b, which is $\mathrm{LCM}(a, b)$. □

Corollary 1.1 effectively reduces our "street map" to roads connecting numbers to their proper factors and multiples.

Proposition 2 (Alternate Paths Between Two Cities). Let c be a factor of both a and b. Then the cost of the path a, c, b is the same as the cost of the path a, 1, b.

Proof. Since c divides both a and b, we have $\mathrm{LCM}(a, c) = a$ and $\mathrm{LCM}(c, b) = b$, hence the cost of the path a, c, b is $a + b$. This is true for any c that divides both a and b, and hence it is true in particular for $c = 1$. □

Corollary 2.1. There exists a complete optimal path for n cities that visits all meat exactly once.

Proof. Consider an optimal complete path in which meat c is visited twice. If the second c is the last visited city, we can eliminate it and obtain a cheaper complete path, which is a contradiction. Therefore, the second occurrence of c is sandwiched between two cities, say a and b, so a, c, b is a subpath. By Proposition 1, we may assume that a divides c or c divides a, and also that b divides c or c divides b. If c divides both a and b, by Proposition 2, we can replace the subpath

a, c, b with the subpath a, 1, b without affecting the cost. If a divides c and c divides b, or if b divides c and c divides a, we can remove this second occurrence of c and obtain a cheaper complete path, contradicting optimality. Finally, if a and b both divide c, then the subpath a, c, b costs $2c$, whereas the subpath a, 1, b costs $a + b < 2c$. In this case, we can replace c with 1 to obtain a cheaper complete path, contradicting optimality. Thus, any further occurrences of a meat city c can be replaced with bread. □

Corollary 2.1 considerably reduces the size of our search tree and tells us that there are complete optimal paths whose structure is an open-face sandwich composed of unique pieces of meat.

Proposition 3 (Permutation and Reversal of Sandwich Meat Content). The meat content of a sandwich can be reversed without affecting the cost. The meat content of two sandwiches in a path can be swapped without affecting the cost.

Proof. Because the order of addition does not affect a sum, swapping the meat content of two sandwiches does not affect the total cost, and because $\mathrm{LCM}(a, b) = \mathrm{LCM}(b, a)$, reversing the order of the meat content of a sandwich does not change the sandwich's cost. □

Proposition 3 allows us to prune the search tree whenever we come upon two paths that differ by permutation and reversal of sandwich meat content; only one of the paths needs to be followed further. Also, see the section "standard form" below.

Proposition 4 (Prime Sandwiches). Let p be a prime number where $n < 2p \leq 2n$. Then an optimal sequence for n cities contains the subsequence 1, p, 1, with the possible exception of the case where p is the largest prime less than or equal to n. In that exceptional case, an optimal sequence might end with 1, p instead of containing the subsequence 1, p, 1.

Proof. Note that the only factors or multiples of p less than or equal to n are 1 and p. By Proposition 1, in an optimal path, city p can only be preceded or followed by bread. Since all paths start at 1, any such prime p must appear within an optimal sequence sandwiched by bread or the optimal sequence must end with 1, p. Let P be the largest prime less than or equal to n. Observe that the cost of the sequence 1, …, 1, P, 1, …, 1, p, compared to the sequence obtained by swapping P and p, is greater by $(2P + p) - (2p + P) = P - p > 0$. Thus, in any complete optimal path, any prime $p < P$ in the hypothesis, must be directly sandwiched between bread. □

Proposition 4 tells us how cities corresponding to prime numbers in its hypothesis must appear in an optimal sequence, which further reduces our search.

Proposition 5. If n is prime, all complete optimal paths end in city n.

Proof. Consider a complete optimal path of cost c. A complete optimal path is a succession of sandwiches, followed by an open-faced sandwich that ends in city X. If a 1 is added to the end of that path, it becomes a non-optimal closed sandwich with a cost of $c + X$ and the meat content of sandwiches within the path can be flipped and permuted without affecting the cost. Suppose there is meat content within the path that begins or ends with a city $Y > X$. The meat content can be permuted so that the path ends with Y, 1. When that 1 is removed, a complete optimal path with a cost of $c + X - Y$ is revealed. If $Y > X$, then $c + X - Y < c$, contradicting that path's optimality. Therefore, in a complete optimal path, the last number must be bigger than all the pieces of meat that are next to pieces of bread.

Let n be a prime number. Suppose that a complete optimal path does not end in city n but rather in city X, where $X < n$. Then n, being prime and greater than $n/2$, must be sandwiched between two pieces of bread. Since n is next to a piece of bread, the last city must be greater than n. However, $X < n$, by hypothesis, which is a contradiction. Therefore, n must be the last city. □

Proposition 5 allows us to condense our search tree by ensuring that all complete optimal paths end with 1, n, when n is prime.

Proposition 6 (1, p, $2p$ Path). Let p be a prime that satisfies $2p \le n < 3p$. Then in a complete optimal path, p is always sandwiched between 1 and $2p$.

Proof. By Bertrand's postulate, there are primes between $n/2$ and n, so by the conclusion of the first paragraph of the proof of Proposition 5, p cannot be the last city.

The only factors and multiples of p less than or equal to n are 1, p, and $2p$, while the only factors and multiples of $2p$ less than or equal to n are 1, 2, p, and $2p$. By Corollary 1.1, city p can be preceded only by 1 or $2p$. Suppose p is preceded by $2p$. From the subpath $2p$, p, we can travel on to 1 or $2p$. If 1, we have the subpath $2p$, p, 1 as desired. So assume we have the subpath $2p$, p, $2p$. Note that 1, $2p$, p, $2p$ costs more than 1, p, $2p$, the path 2, $2p$, p, $2p$ costs more than 2, 1, p, $2p$, and p, $2p$, p, $2p$ costs more than p, $2p$. Hence we may assume that p is preceded by 1. By Corollary 1.1, the subpath 1, p can be extended to either 1, p, 1 or 1, p, $2p$. If extended to 1, p, $2p$, we are done, so assume the path continues 1, p, 1. City $2p$ must occur somewhere, and can be preceded by either 1, 2, or p. Let us consider each of these cases separately.

Case 1. City 1 precedes $2p$. If $2p$ is preceded by 1, by applying Proposition 3, we can rearrange the path so that it has a subpath 1, p, 1, $2p$, which costs more than 1, p, $2p$, a contradiction.

Case 2. City 2 precedes $2p$. First, we claim $2p$ cannot be the last city, for if it were, our path would look like …1, p, 1, …, 2, $2p$, which costs the same as the path obtained by eliminating the p, 1 and tacking p at the end. But we already know that no complete optimal path ends in p. So city $2p$ is followed by 1, 2, or p. If followed by 1, we can apply Proposition 3 to rearrange the sequence to obtain the subpath 2, $2p$, 1, p, 1, and, as in case 1, we can eliminate the first 1 in this subpath to obtain a less expensive complete path. If followed by 2, we can replace this 2 with 1 without affecting cost or completeness and proceed as in the previous sentence to obtain a contradiction. Finally, if followed by p, the path must then continue to either 1 or to $2p$, i.e., we have the subpaths 2, $2p$, p, 1 or 2, $2p$, p, $2p$. In the first case, we are done, as p is sandwiched between 1 and $2p$. In the second, we can eliminate the middle two cities $2p$ and p to obtain a less expensive path that is still complete (since we know p occurs elsewhere), a contradiction.

Case 3. City p precedes $2p$. If city $2p$ is preceded by p, then that p must be preceded by either 1 or $2p$. That is, we have the subpaths 1, p, $2p$, as desired, or $2p$, p, $2p$. However, we can reduce $2p$, p, $2p$ to just $2p$ without changing the completeness of the path (since we know p occurs elsewhere), while reducing cost, contradicting optimality.

Since all possibilities lead either to a contradiction of our assumptions or to p being sandwiched between 1 and $2p$, the proposition follows. □

Proposition 6 allows us to condense our search tree because we must ensure that p and $2p$ are next to each other.

Standard Form for Paths

A path is said to be in **standard form** if and only if all of the following are true:

- no meat occurs more than once
- in any sandwich, the first meat is less than or equal to the last meat
- the sandwiches are in ascending order according to their first piece of meat

By Proposition 3, any path where no meat occurs more than once can be put in standard form without changing its cost. Also, there always exists complete optimal paths in standard form.

Future Work

We are also interested in the following unresolved questions:

- What if the requirement of starting at city 1 is removed?
- Is there a practical way to find complete optimal paths without a computer?
- Do all complete optimal paths for a given n have the same number of pieces of bread?
- What do the patterns and complete optimal paths look like for larger n?

We also make the following conjecture:

Conjecture. For any n, all complete optimal paths end in the same city.

Proposition 5 proves this in the case where n is prime, but we were unable to prove it in general.

Data

We list the complete optimal paths that are in standard form and their cost for all n up to 20.

n	Cost	Standard Form Complete Optimal Path(s)
1	0	1
2	2	1, 2
3	7	1, 2, 1, 3
4	12	1, 3, 1, 2, 4
5	21	1, 3, 1, 2, 4, 1, 5
6	28	1, 3, 6, 2, 4, 1, 5
7	40	1, 3, 6, 2, 4, 1, 5, 1, 7
8	51	1, 5, 1, 7, 1, 3, 6, 2, 4, 8
9	65	1, 5, 1, 7, 1, 4, 8, 2, 6, 3, 9 1, 5, 1, 7, 1, 8, 4, 2, 6, 3, 9

n	**Cost**	**Standard Form Complete Optimal Path(s)**
10	79	1, 4, 8, 2, 6, 3, 9, 1, 7, 1, 5, 10 1, 7, 1, 8, 4, 2, 6, 3, 9, 1, 5, 10
11	100	1, 4, 8, 2, 6, 3, 9, 1, 5, 10, 1, 7, 1, 11 1, 4, 8, 1, 5, 10, 2, 6, 3, 9, 1, 7, 1, 11 1, 5, 10, 1, 7, 1, 8, 4, 2, 6, 3, 9, 1, 11 1, 4, 8, 2, 10, 5, 1, 6, 3, 9, 1, 7, 1, 11 1, 5, 10, 2, 4, 8, 1, 6, 3, 9, 1, 7, 1, 11
12	114	1, 5, 10, 2, 8, 4, 12, 6, 3, 9, 1, 7, 1, 11
13	138	1, 5, 10, 2, 8, 4, 12, 6, 3, 9, 1, 7, 1, 11, 1, 13
14	158	1, 5, 10, 2, 8, 4, 12, 6, 3, 9, 1, 11, 1, 13, 1, 7, 14
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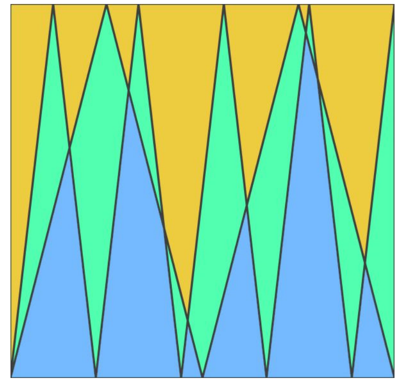
19	295	1, 7, 14, 2, 8, 16, 4, 12, 6, 18, 9, 3, 15, 5, 10, 1, 11, 1, 13, 1, 17, 1, 19 1, 7, 14, 2, 10, 5, 15, 3, 9, 18, 6, 12, 4, 8, 16, 1, 11, 1, 13, 1, 17, 1, 19 1, 7, 14, 2, 10, 5, 15, 3, 9, 18, 6, 12, 4, 16, 8, 1, 11, 1, 13, 1, 17, 1, 19 1, 7, 14, 2, 16, 8, 4, 12, 6, 18, 9, 3, 15, 5, 10, 1, 11, 1, 13, 1, 17, 1, 19
20	324	1, 11, 1, 13, 1, 17, 1, 19, 1, 7, 14, 2, 8, 16, 4, 12, 6, 18, 9, 3, 15, 5, 10, 20 1, 11, 1, 13, 1, 17, 1, 19, 1, 7, 14, 2, 16, 8, 4, 12, 6, 18, 9, 3, 15, 5, 10, 20

Zigzags, Part 8

by Ken Fan | edited by Jennifer Sidney Silva
with special guest Noam Elkies
and cameo by Alison Miller



This is the final installment of Emily and Jasmine's dance with zigzags. For further details and their proof of the zigzags theorem, please refer to earlier installments.

In the last installment, Emily and Jasmine proved

The ZigZags Theorem. *Let* n *and* m *be distinct positive integers. Let a rectangle be crisscrossed by an* n-*zigzag and an* m-*zigzag, each bouncing back and forth between the top and bottom edges. Then the region of the rectangle below both zigzags, the region above both zigzags, and the region between the two zigzags split the rectangle exactly in thirds.*

They were astonished that their proofs involved elaborate rational polynomials, which – when combined – simplified all the way down to 1/3. They felt that there must be a reason for the dramatic simplification. So they searched for a more conceptual proof. For example, they tried to rearrange the pieces to form shapes that could easily be seen to have area one-third that of the rectangle. They tried to relate the areas to a 3D volume. Nothing worked. But they couldn't devote much time to it because they had a lot of schoolwork to catch up on.

Emily is on her way to Jasmine's house,[1] hoping that they'll be able to think more about zigzags, when her phone buzzes.

"Noam D. Elkies?" She doesn't recognize the name of the message sender. "Must be spam."

Then her phone rings. It's Jasmine.

Jasmine: Emily!

Emily: Hey, Jaz! I'm almost there; I can see your door.

Jasmine: Great, hurry! Did you see the email?

Emily: What email?

Jasmine: The one from Noam Elkies.

Emily: Oh – you got that, too? I thought it was spam. What's it about?

Jasmine: He's a mathematician at Harvard. He claims to have found a conceptual proof!

Emily breaks into a sprint. Just as she runs up the plain concrete path to Jasmine's front door, the door swings wide open and Jasmine appears.

[1] This visit occurred long before the implementation of Covid-19-related social distancing.

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Jasmine: That does get the result for any r-zigzag and s-zigzag with r not s, except that both zigzags must have a common endpoint because both $\|rx\|$ and $\|sx\|$ start off at 0 when x is 0. But the zigzags theorem is true even when the zigzags don't have a common endpoint, which occurs when r and s have the same parity and the r- and s-zigzags begin and end in different corners of the rectangle.

Emily: Look at Noam's last comment. He says that the result still holds when the zigzags are independently shifted left or right.

Jasmine: Yes. In his formulation, that's clear because his fundamental functions have period 1; so shifting them doesn't change their average value over a line, provided that the four forbidden slopes 0, 1, -1, and infinity are avoided. Another way of putting it is that we can average over lines (of allowable slope) that do not pass through the origin. They can have any y-intercept.

Emily: I think that flexibility enables us to get the case where the zigzags do not have a common endpoint. We just have to shift one of them by 1/2. For example, if s and r are both odd, we can get the case where the zigzags have no common endpoint by using Noam's ideas on the function $m(rx, s(x + 1/2))$ over the line that connects (0, 0) with (r, s), together with mirror symmetry. Then we can get the case where s and r are both even by deducing it from the case of s/d and r/d, where d is the greatest common factor of r and s.

Jasmine: Nice… so Noam's observations really do prove our entire zigzags theorem, and in a much more conceptual way. His argument can be summed up by saying that the zigzags theorem is explained by the identity

$$m(x, y) = b(x) + b(y) - (b(x + y) + b(x - y))/2.$$

Not only does this eliminate a lot of algebraic manipulation of rational polynomials, it even eliminates the need to study the various shapes that the zigzags create! Incredible!

Emily: What a beautiful proof. I still like our analysis of those shapes, though, and I think it's neat that sums of squares appeared in our way of proving it. But this does provide an insightful explanation for the zigzags theorem.

Jasmine: It's exciting that our result attracted the attention of a professional mathematician!

Emily: Let's reply with a thank you email and answer his question about how we found it!

And so concludes Emily and Jasmine's adventure with zigzags. Special thanks to Noam Elkies for permission to use his email in this story and to Alison Miller for her permission to use her name. This series is a fictionalized account of the discovery of the zigzags theorem; its purpose is to convey the process of doing math with the hope that some reader will be inspired to embark upon her own adventure of mathematical discovery.

Here's the nonfiction: The author discovered and proved the zigzags theorem, in a manner closely resembling this fictionalized account, while vacationing at the Trapp Family Lodge in the summer of 2017. Astonished by the simplification of the rational polynomials, he told a number of mathematician friends, including Noam Elkies, and challenged them to prove it. Within hours, Prof. Elkies responded with the email on page 21, whose mathematical content is exactly as written by Prof. Elkies, but with modifications to the first and last non-mathematical parts for purposes of integration into this story.

- Ken Fan

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 26 - Meet 5 March 5, 2020	Mentors:	Adeline Hillier, Jenny Kaufman, Tina Lu, Rebecca Nelson, Kate Pearce, Laura Pierson, Emma Wang, Karissa Wenger, Hanna Yang, Jasmine Zou

You have 10 colored pencils, all with different lengths. You arrange them in a row, from shortest to longest. You want to switch their order to longest to shortest. On each move, you are allowed to pick any group of 3 consecutive pencils (consecutive in the row, not according to their height) and move the third of the group back two places. What is the minimum number of moves needed? Is it even possible? What if you had N colored pencils? For which N is it possible to reverse the order and what's the minimum number of moves needed?

Think of other types of moves and other goal configurations for the pencils. What can you dream up?

Session 26 - Meet 6 March 19, 2020 Online	This was our first virtual club meet.

Can you figure out the cube root of a large perfect cube without using a calculator? See if you can figure out the cube roots of these perfect cubes:

74,088 1,860,867 134,217,728

What strategies did you use to determine the answer?

How about these: What is the 6th root of the perfect 6th power 2,565,726,409? What is the 9th root of the perfect 9th power

1,009,036,084,126,126,084,036,009,001?

Session 26 - Meet 7 March 26, 2020 Online	Mentors:	Rebecca Nelson, Kate Pearce, Laura Pierson, Rebecca Whitman, Jasmine Zou

Many guessing games are predicated on the use of yes/no questions. But suppose you play a guessing game where the person who knows what you are trying to guess may answer with one of three words instead of two? What three words would you want those possible responses to be, and how would you modify your interrogation strategy to suit?

Session 26 - Meet 8 April 2, 2020 Online	Mentors:	Jenny Kaufman, Shradha Kaushal, Elise McCormack, Rebecca Nelson, Laura Pierson, Gisela Redondo, Savannah Tynan, Rebecca Whitman, Hanna Yang, Annie Yun, Jasmine Zou

Tiling problems offer an endless source of mathematical questions. How compactly can you fit copies of a shape together without overlap? Tilings occur when the fit is absolutely snug, with no gaps between. Take your favorite shapes. Do you they tile the plane? Do they tile a rectangle? How many ways can they tile a rectangle?

Session 26 - Meet 9 April 9, 2020 Online	Mentors:	Shradha Kaushal, Tina Lu, Elise McCormack, Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo, Savannah Tynan, Emma Wang, Rebecca Whitman, Hanna Yang, Annie Yun, Jasmine Zou

Here's a classic game you can play and challenge others with. Write the numbers 1-10 in a row

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10.$$

The goal is to get a row of all 5s,

$$5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5,$$

by picking any two numbers and adding 1 to one and subtracting 1 from the other and repeating this process and many times as needed.

Can you think of variations of this game?

Session 26 - Meet 10 April 16, 2020 Online	Mentors:	Rebecca Nelson, Kate Pearce, Laura Pierson, Gisela Redondo Emma Wang, Rebecca Whitman, Hanna Yang, Jasmine Zou

We've entered the "big data" era. What ways can you think of to display this data meaningfully?

Over a week collect some data and figure out a useful way to display it. For example, you could track how many steps you took in a day or how much time you spent on various activities, like sleeping, eating, or doing math.

Look for trends in your data, does it change over the weekend? Does it look like one activity has an effect on another activity? If you think so, can you think of a way to quantify the trend or effect?

Session 26 - Meet 11 April 23, 2020	Mentors:	Jenny Kaufman, Tina Lu, Elise McCormack, Kate Pearce, Laura Pierson, Gisela Redondo, Savannah Tynan, Emma Wang, Rebecca Whitman, Hanna Yang, Jasmine Zou

There are N people and each person knows a specific piece of information. At each step the N people can pair up and exchange the information they know. What is the minimum number of steps required for all N people to learn all the pieces of information?

Calendar

Session 26: (all dates in 2020)

January	30	Start of the twenty-sixth session!
February	6	
	13	
	20	No meet
	27	
March	5	
	12	Meet cancelled
	19	Girls' Angle club meets are now conducted online
	26	
April	2	
	9	
	16	
	23	
	30	
May	7	

Session 27: (all dates in 2020)

September	10	Start of the twenty-seventh session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Due to current circumstances, Girls' Angle club meets are being conducted online until further notice. We look forward to when we can return to our in-person club.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- □ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- □ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: _________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ______________________________________