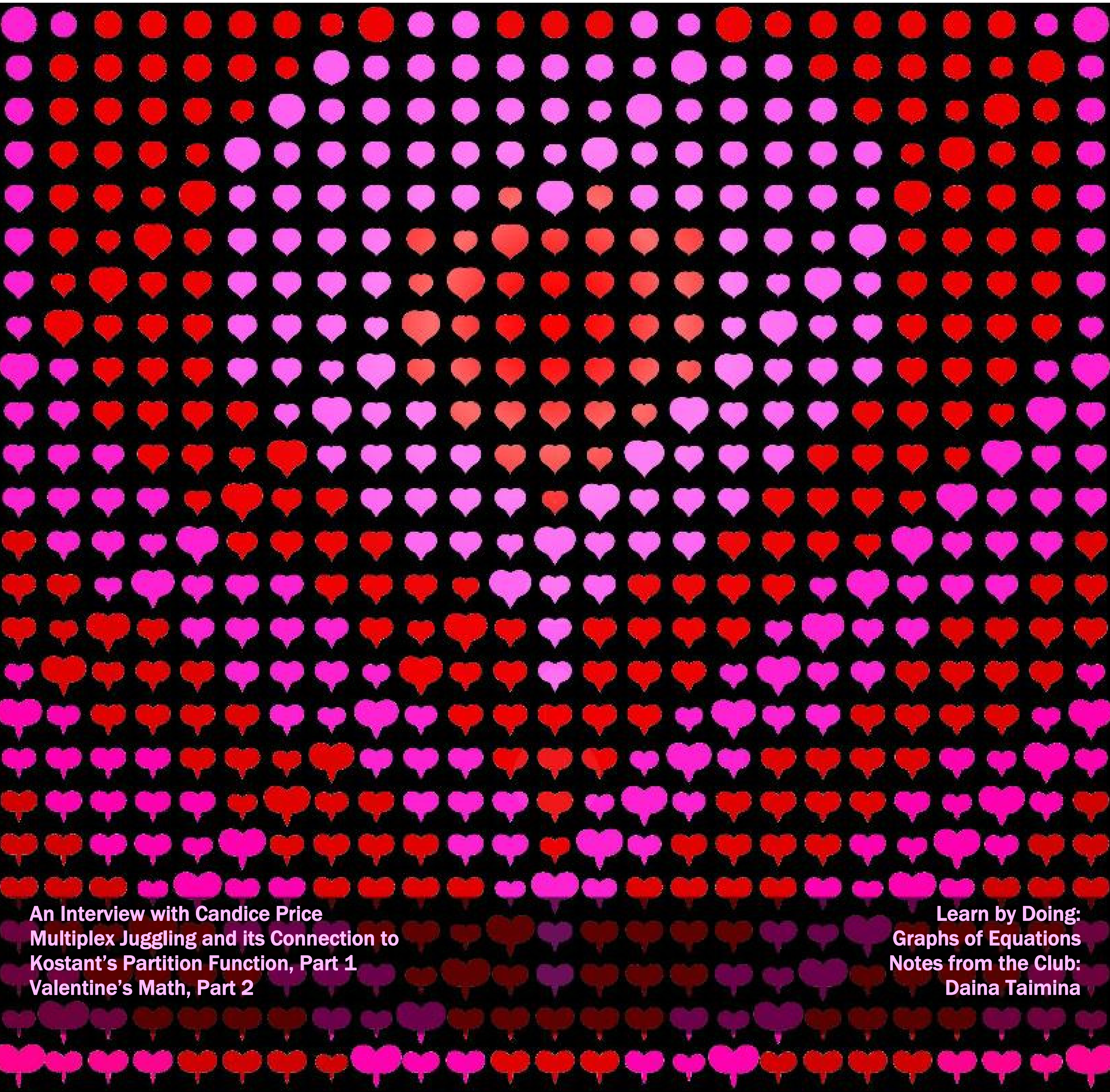
Girls' Angle Bulletin

April/May 2021 • Volume 14 • Number 4

To Foster and Nurture Girls' Interest in Mathematics



From the Founder

To learn math, all you have to do is ask math questions and try to answer them. Before long you'll be understanding all sorts of things you would never have dreamed of when you began. At the club, member's dreams never cease to inspire and amaze! - Ken Fan, President and Founder

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Girls' Angle Bulletin

The official magazine of
Girls' Angle: A Math Club for girls
Electronic Version (ISSN 2151-5743)

Website: www.girlsangle.org
Email: girlsangle@gmail.com

This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

The print version of the Bulletin is printed by the American Mathematical Society.

Editors: Amanda Galtman
Jennifer Sidney Silva
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Have a Heart* created by Juliette Bennett, C. Kenneth Fan, and Violet Freimark, with special thanks to mentors Bridget Li and Kate Pearce.

An Interview with

Candice Price

Candice Price is Assistant Professor of Mathematics & Statistics at Smith College. She earned her bachelor's degree from California State University, Chico, her master's degree from San Francisco State University, and her doctoral degree in mathematics from the University of Iowa under the supervision of Isabel Darcy. She is a cofounder of the organization *Mathematically Gifted and Black*.

Ken: You are involved with so many things, but I'd like to start with how your journey into mathematics began. Could you please retrace your personal history into mathematics, starting, perhaps, with when mathematics first caught your eye?

Candice: My journey in mathematics started in the 3rd grade when I learned how to multiply. This was for two reasons: 1) I loved this new way that numbers can interact, and 2) It wasn't taught to me in a concrete way where we start to understand what multiplication means in a physical space, but in a fun memorizing way with music from *Schoolhouse Rock!*. I fell in love with mathematics that year. My enjoyment of mathematics led to me wanting to discover more about mathematics and I started to "self study"… which really meant watching the show *Square One* on PBS. This was a double edged sword because while I learned a lot of cool mathematics, I started to become bored in my classes when we discussed mathematics. My mother noticed this and asked for my school to give me a "gifted test." My mathematics understanding was measured at the 12th grade level. My school decided to move me up a grade, from 4th to 5th and later advocated for my entry into a Gifted and Talented program in middle school. I started to stray away from mathematics in high school after a few setbacks: I had to take Geometry and Algebra 2 twice. When I started at California State University, Chico in 1998, I had planned to major in psychology. But because I still enjoyed mathematics, I enrolled in a pre-calculus class in my second semester of my first year. My instructor was Lori Holcombe. She was amazing! She was a tough teacher but also supportive. She reached out to me and said that she saw my talent and enjoyment of mathematics and said I should be a mathematics major. I joined the Louis Stokes Alliance for Minority Participation program at Chico and have been on this mathematical journey ever since.

Ken: Fascinating! Would you please tell us more about your circumstances in high school? What happened in Geometry and Algebra 2?

I want to be careful about the word "gifts" here. I work very hard to understand mathematics. It does not just come to me, but I love to learn, to struggle through the various topics.

Candice: One big reason for these setbacks was that I felt extremely visible and invisible at the same time. I was a young Black woman in a class of mostly non-Black students in my Geometry class. I didn't want to stand out too much. So I didn't work very hard in that class. I didn't do my homework and I rarely answered questions in class. I was also at that awkward age of 13-14 and I was younger than everyone else. I retook the course in summer school and excelled. It was my favorite class in high school. In my first Algebra 2 experience, my teacher never called me by the name I preferred. He, a 60+-year-old white man, always called me by a nickname I hate. He never showed any respect for me as a person. He also lectured from his desk and never interacted with us. He also obviously had issues with the Black students in his class. Many other students were able to transfer to other classes, but I was not able to. I hated that experience and it showed in my work. That class failed me. When I took it the second time, I was in a much more welcoming environment and I finally could just focus on the math.

Ken: The Algebra 2 experience is so disturbing. That kind of behavior shouldn't be tolerated. What other ways should math education be handled in high school so that instead of encountering setbacks, your gifts would have been properly recognized and developed?

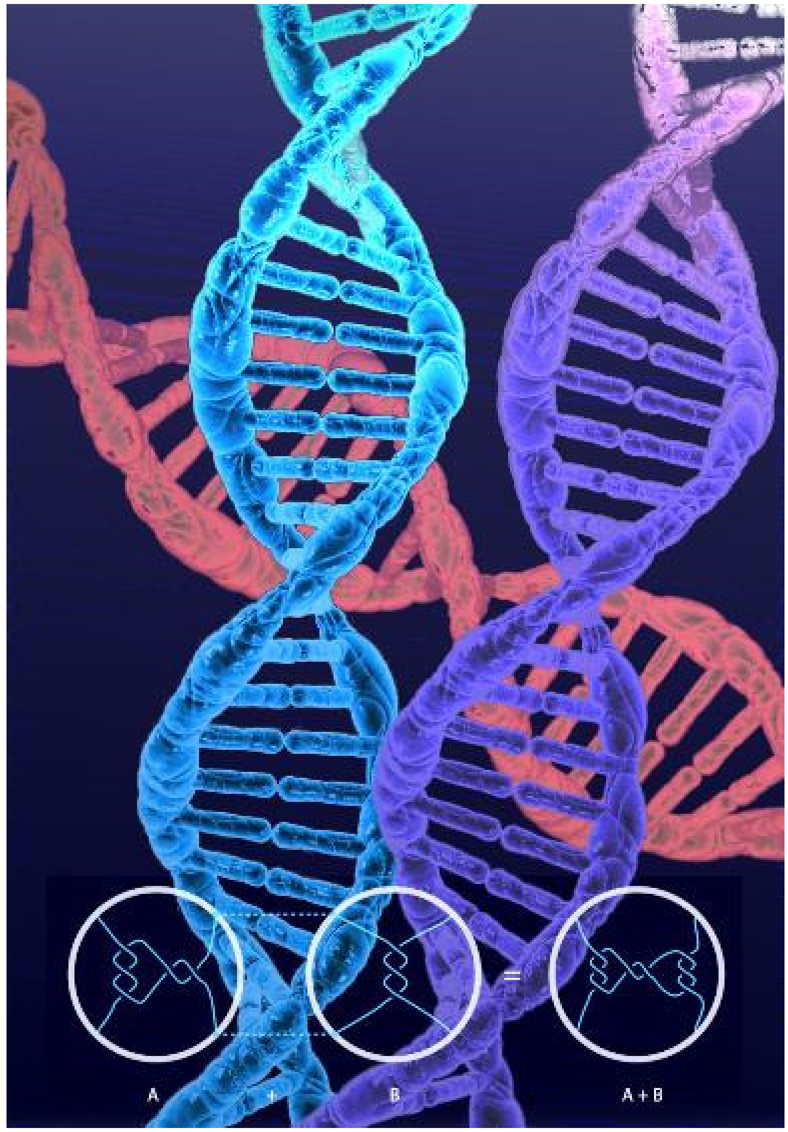
Candice: I want to be careful about the word "gifts" here. I work very hard to understand mathematics. It does not just come to me, but I love to learn, to struggle through the various topics. I love to see math show up in places. That I would say is my gift, the joy of learning math. I think nurturing that joy is what we should do. It is not just with mathematics either. If a student loves to read and write, we should nurture that with more books, trips to book stores and libraries, going to an author reading event. With mathematics, it is important that we find out what it is about mathematics a person likes and feed that part of them.

Ken: That's great advice! When did you start thinking of mathematics as a possible career choice for yourself?

Candice: I think it would definitely be after my class with Lori Holcombe. I knew that I wanted to have a job/career that included mathematics. But it wasn't until I started my master's program that I realized that I wanted to be a mathematics professor. At San Francisco State University, I met Dr. Jamylle Carter and she was the first Black woman in STEM I had ever met. During my academic lifetime, I have only had one professor that was a Black woman. And it was in a humanities course. So I realized that one reason I was always a bit unsure if this was a good career for me was because I could not see myself doing it… because I had no visible examples of Black women in mathematics. I went to an event called the *Infinite Possibilities Conference* and this was the first time I saw Black women in mathematics at different levels, in different careers and full of joy.

Ken: Did you have any role models or champions who were supportive of your mathematical interest when you were growing up?

Candice: Oh yes!! I have many, many champions! Perhaps too many to mention hahah. I would say that it started with my mother who always advocated for me and supported me. She always had high expectations for me and my siblings, but was always there to support us and celebrate our successes.



Ken: What is mathematical modeling? What is the purpose of creating a mathematical model?

Candice: One way that I think about mathematical modeling is using mathematics to describe other areas of interest. I know that may sound a bit vague so I will elaborate a bit. So let's look at the following scenario: I own a fish hatchery. I would like to know how many fish I can sell each month without depleting my population by so much that they are not able to reproduce enough fish to keep up with the pace of the selling. Now I could just think of a number and go with that, but how will I test if that is the right amount to sell? I cannot risk just trying it out for a while, that would be too expensive. Or that amount may not give me the optimal amount of profit. But, if I can use mathematics and model the situation with equations or functions, I can test different values in those functions to see which one optimizes my profit, without ruining my fish population. Now, we can really do this with any problem or issues. It may not be the perfect mathematical model, because those don't exist, but many good models provide some really helpful information on any problem.

Ken: One of your interests is DNA Topology. What is DNA Topology?

Candice: DNA topology is an applied mathematics area where topological objects are used to model the interaction between proteins and DNA. Some of these proteins will change the shape (topology) of DNA's tertiary structure. This can allow for DNA to look knotted, linked, or supercoiled. If we can model these shapes with equations from knot theory, we can discover the mechanisms of the proteins that create the shapes we are studying.

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We're committed to producing quality math educational content and make every effort to provide this content to you for free. We are also committed to surviving as a nonprofit!

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Thank you and best wishes,
Ken Fan
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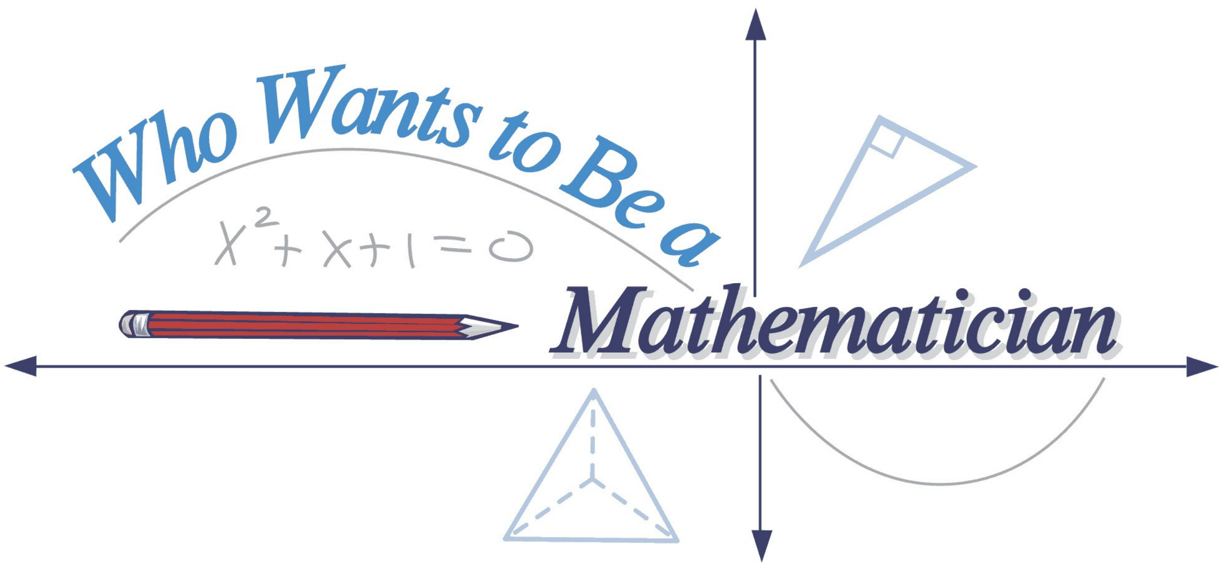

America's Greatest Math Game: Who Wants to Be a Mathematician.

Multiplex Juggling and its Connection to Kostant's Partition Function, Part 1

by Pamela E. Harris and Maria Rodriguez Hertz[1] | edited by Jennifer Sidney Silva

Did you know that juggling has been around for thousands of years? The earliest record of juggling we have is on the wall of Tomb 15 at Beni Hasan in Egypt (see Figure 1). The wall painting was made ca. 2130 BCE - 1991 BCE by the ancient Egyptians [1]. This tomb belonged to Baqet III, a provincial governor. Because the painting is on a tomb wall, we think juggling might have had some sort of ritual or religious significance and that people juggled in order to guarantee a good harvest. It is equally likely, though, that Baqet III just really liked juggling and wanted to take it with him to the afterlife [2]. Either way, that means people have been juggling for over 4000 years!

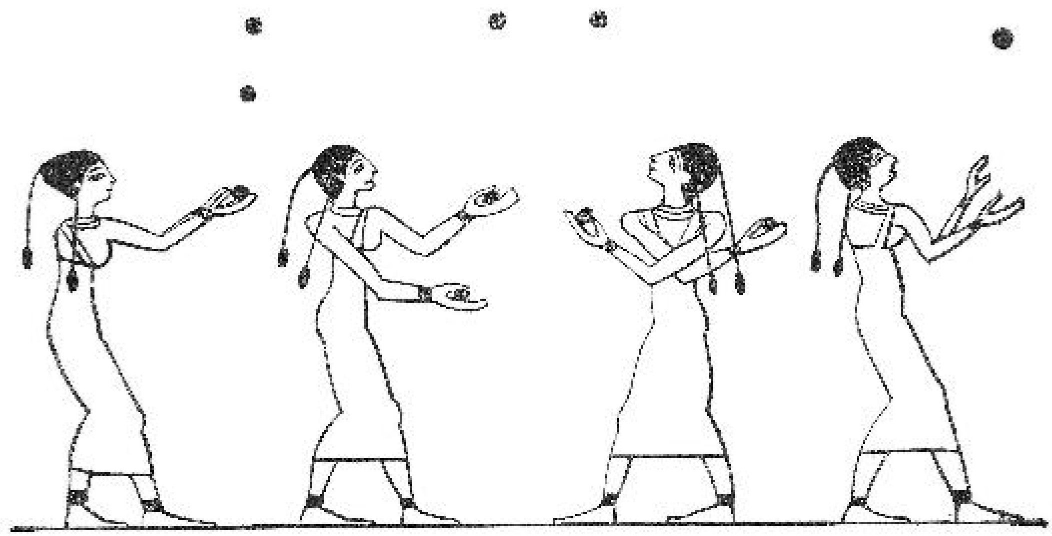

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Figure 1. Four juggling Egyptian women from the wall painting of Tomb 15.

Juggling has not only been around for thousands of years, it has also been very popular all around the world. In Ancient China, juggling was a form of art; Xiong Yiliao, a Chu warrior, juggled nine balls so well during a battle that the other army fled in fear [3]. In Ancient Greece, jugglers were all the rage, performing fabulous tricks for guests at feasts [4]. Juggling and acrobatics appear all over Greek and Etruscan tombs, coins, and vases [11, 12]. In the Western Hemisphere, there are records of juggling among the Naskapi, Eskimo, Achomawi, Bannock, Shoshone, Ute, and Zuni tribes in North America [6]. When Hernán Cortés came to the Americas in the 16th Century, he brought Aztec foot jugglers – which are referred to as antipodists – back to Europe [5]. See Figure 2 for some depictions of juggling.

Given the fascinating skills and entertaining nature of juggling, it is no surprise that, until recently, juggling was part of the International Brotherhood of Magicians. However, in June of 1947, a group of eight jugglers decided that juggling was a very different discipline from magic – so different, in fact, that they met at a restaurant and created the International Jugglers' Association [7].

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[1] This work is supported in part by a grant from MathWorks.

(a) Lanzi, an ancient Chinese man who juggled seven swords.

(b) Girl juggling on an ancient Greek vase.

(c) An Aztec antipodist (foot juggler).

Credits: (a) is from a Ming Dynasty collection of woodcuts originally printed in [3], courtesy of Wikimedia Commons. (b) is a public domain image courtesy of the Metropolitan Museum of Art. (c) is originally from the "Trachtenbuch" of Christoph Weiditz, courtesy of Wikimedia Commons.

Figure 2. Depictions of juggling.

Now imagine you are a juggler and you came up with a brand new juggling pattern or trick. You want to explain it to your juggler friend to see if she has ever seen it before. Your friend is countries away so you can't show her the trick; you have to explain it to her. In this case, you'd need to create a system for explaining juggling motions. To solve this problem, jugglers have created many systems of notation. These have led to what is now called **mathematical juggling**. If you've ever juggled or seen someone juggle, you already have a good idea of how it works.

Say you have some number of balls. Now, imagine juggling with these balls. You throw them up in the air one at a time, and when a ball falls to your hands, you throw it back up. Let's formalize this idea. Let n be the number of balls we have available to juggle. Now imagine all the balls are up in the air. We measure the height from your hands to the balls. To make things simple, we only let the balls be at heights that are whole numbers. That is, the balls can be at heights 0, 1, 2, 3, …, where a ball at height 0 means you have it in hand. We also begin by restricting the type of juggling to one in which we can only catch and throw one ball at a time. This means that at any given point in time, we only have one ball at any single height. Such a description of the balls in midair is what we call a **juggling state**, and an example of this is illustrated in Figure 3.

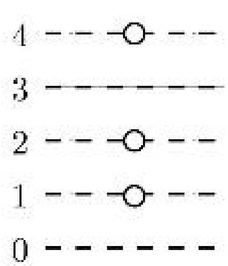

Figure 3. A juggling state with one ball at height 1, one ball at height 2, and one ball at height 4.

To write down a juggling state, we write how many balls (i.e., 0 or 1) are at height 1, followed by how many balls are at height 2, and so on, and we place these numbers inside angle brackets to create a vector. For example, the juggling state in Figure 3 is written $\langle 1, 1, 0, 1\rangle$ because there's 1 ball at height 1, 1 ball at height 2, 0 balls at height 3, and 1 ball at height 4.

But juggling isn't stationary; once you've thrown the balls up in the air, they start falling. To mimic this behavior, we move from one juggling state to the next having the process follow a few rules. A single unit of time passes between one juggling state and the next. And from one time unit to the next, every ball falls 1 unit in height; for example, if a ball is at height 3 at time 2, it will be at height 2 at time 3. If a ball falls to height 0, it falls to your hands and you must immediately throw it up to some height that does not already have a ball. This means you cannot hold a ball in your hand. Let's take the juggling state in Figure 3 and show a next possible step. Because one of the balls falls to height 0, we must immediately throw it up; let us do so to height 4. We illustrate the transition between these juggling states in Figure 4.

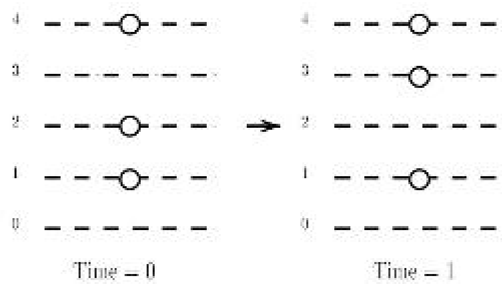

Figure 4. Moving from juggling state $\langle 1, 1, 0, 1\rangle$ to juggling state $\langle 1, 0, 1, 1\rangle$, since a ball reached height 0 and was thrown to height 4.

We can now illustrate these juggling states in what is called **conveyor belt notation**: the images look like a conveyor belt holding buckets at each height above 0, and after each time step the conveyor belt lowers all buckets by one unit in height. In Figure 5, we illustrate Figure 4 by providing the change from one juggling state to the next using this new conveyor belt notation.

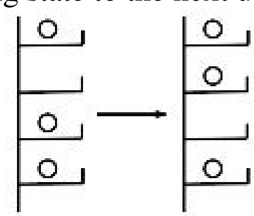

Figure 5. Moving from $\langle 1, 1, 0, 1\rangle$ to $\langle 1, 0, 1, 1\rangle$, depicted in conveyor belt notation.

When we have various juggling states that we can move through by following the rules we explained, we call this a **juggling sequence**. Figure 5 is a juggling sequence with only 1 time step. If a juggling sequence starts and ends with the same juggling state, we call this a **periodic juggling sequence**. Now, let's say we set the start and ending juggling states and we give ourselves a specific amount of time to get from the starting state to the ending state. Then, we ask: in how many ways can we get from the starting state to the ending state? This is our motivating question when studying mathematical juggling.

Let's begin counting juggling sequences.

Example 1. Let's set our starting state to be $\langle 1, 0, 1\rangle$, our ending state to be $\langle 1, 1\rangle$, and our time length to be 3. One possible sequence is that at time 1, when the ball at height 1 falls to height 0, we throw it up to height 4, giving us state $\langle 0, 1, 0, 1\rangle$. At time 2, we can just let both balls fall.

At time 3, we must throw the ball that fell to height 0 to height 1. This juggling sequence is illustrated in Figure 6. We claim there are three other distinct juggling sequences that meet these criteria and encourage you to practice finding them. For the answer, see Figure 8.

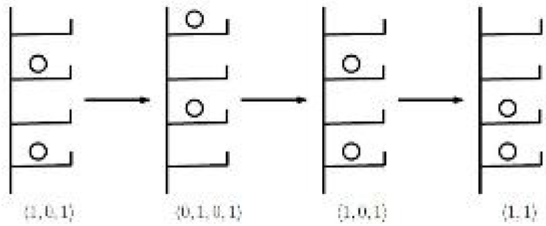

Figure 6. A juggling sequence with starting state ⟨1, 0, 1⟩, ending state ⟨1, 1⟩, and time 3.

We now let JS(s, e, t) denote the set (or collection) of all possible juggling sequences that have s as a starting state, e as an ending state, and have a time length of t. You can think of a set as a bag with things inside of it. The bag labeled JS(s, e, t) would have all the juggling sequences with starting state s, ending state e, and time length t. We use js(s, e, t) to be the number of such juggling sequences, i.e., the number of things in the bag with label JS(s, e, t). For example, Figure 8 shows all the contents of JS(⟨1, 0, 1⟩, ⟨1, 1⟩, 3). Note that js(⟨1, 0, 1⟩, ⟨1, 1⟩, 3) is 4 because there are four juggling sequences in JS(⟨1, 0, 1⟩, ⟨1, 1⟩, 3). We say that the juggling sequence shown in Figure 6 is an element of JS(⟨1, 0, 1⟩, ⟨1, 1⟩, 3).

Now that we can do some traditional juggling, we can make it a bit more fun and challenging. So far, we have only let each conveyor bucket hold one ball; we have assumed that when the balls get to height 0, our juggler can only hold one ball at a time. But we're ambitious jugglers, and we want to practice holding multiple balls at the same time.

So what if we said each of our buckets can hold 2 balls, or 3, 4, or any other number you can imagine? How would this change the number of juggling sequences? For our purposes, let's say that the **capacity** of a bucket is the number of balls it can hold. If we want to refer to our actual hands in this juggling scenario, we can use **hand capacity** to refer to the maximum number of total balls we can hold in our hands when we juggle. Now we can create a new type of juggling sequence called a **multiplex juggling sequence**. This sequence has the same rules as a normal juggling sequence, but now our hand capacity does not need to be 1. Instead, it can be any positive integer we want. As before, we let MJS(s, e, t, m) be the set of all multiplex juggling sequences with starting state s, ending state e, time length t, and hand capacity m, and we use mjs(s, e, t, m) to be the number of such multiplex juggling sequences.

Let's look at a multiplex juggling example next.

Example 2. Set the starting state to ⟨1, 1⟩, ending state to ⟨1, 1⟩ (you might remember that this makes this a **periodic** multiplex juggling sequence), our hand capacity to 2, and our time length to 2. One possible multiplex juggling sequence with these specifications is illustrated in Figure 7. Notice that in that figure at time 1, the ball at height 2 fell to height 1 and the ball at height 1 fell to height 0. We then threw the ball that fell into our hand up to the bucket at height 1. So our juggling state at time step 1 is ⟨2⟩. Then, at time 2, both the balls fell to height 0 and we threw one to height 1 and the other to height 2, which means that we get state ⟨1, 1⟩. We claim that there are two other distinct multiplex juggling sequences and encourage you to practice finding them. After you try on your own, you'll be able to check that your answer matches ours in Figure 4 of Part 2.

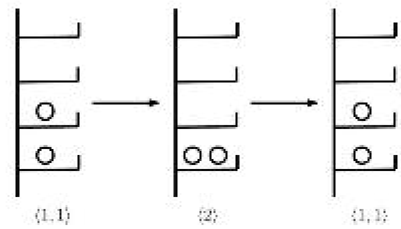

Figure 7. An element of MJS(⟨1, 1⟩, ⟨1, 1⟩, 2, 2).

Exercise 1. Find MJS(⟨1, 1⟩, ⟨2⟩, 3, 2). (We will provide a solution in Figure 5 of Part 2.)

Now that we know how to juggle, we can use these concepts and apply them to other problems. For example, there is a relationship between multiplex juggling sequences and Kostant's partition function. At this point, we highly encourage you to visit the article "Partitions from Mars" [8, 9], where Kostant's partition function is formally defined in the context of what we do next. However, here is a short description: We have currency that cannot be exchanged for different denominations. This currency is denoted α_1, α_2, and α_3, which comprises coins labeled α_1, α_2, α_3, $\alpha_1 + \alpha_2$, $\alpha_2 + \alpha_3$, and $\alpha_1 + \alpha_2 + \alpha_3$. The question of interest is to count the number of ways we could pay for an object with cost $n\alpha_1 + m\alpha_2 + k\alpha_3$ when we are allowed to use those coins. Here n, m, and k are nonnegative integers, and we denote the count by $\wp(n\alpha_1 + m\alpha_2 + k\alpha_3)$. We call each way of adding up to $n\alpha_1 + m\alpha_2 + k\alpha_3$ a **partition** and sometimes refer to the coins as **parts**.

Example 3. Consider an object costing $\alpha_1 + \alpha_2 + \alpha_3$. We now describe the four different ways we could pay for this object given the types of coins available, where we utilize parentheses to specify which coins we use. In this case there are four possible partitions for $\alpha_1 + \alpha_2 + \alpha_3$, which are given by

$$\begin{aligned}&1(\alpha_1) + 1(\alpha_2) + 1(\alpha_3),\\ &1(\alpha_1 + \alpha_2) + 1(\alpha_3),\\ &1(\alpha_1) + 1(\alpha_2 + \alpha_3), \text{ and}\\ &1(\alpha_1 + \alpha_2 + \alpha_3).\end{aligned}$$

Next, we ask you to complete the following exercise:

Exercise 2. Determine all of the multiplex juggling sequences in MJS(⟨1⟩, ⟨1⟩, 3, 1). (We will provide answers in Figures 1 and 2 of Part 2.)

By Exercise 2, there are the same number of partitions of $\alpha_1 + \alpha_2 + \alpha_3$ as there are juggling sequences in MJS(⟨1⟩, ⟨1⟩, 3, 1). This is not a coincidence; in fact, there is a beautiful result underlying these examples! The connection was given by Benedetti, Hanusa, Harris, Morales, and Simpson, who established the following result [10]:

Theorem 1 (Corollary 3.8 of [10]). For any n and r nonnegative integers

$$\mathrm{mjs}(\langle n\rangle, \langle n\rangle, r, n) = \wp(n(\alpha_1 + \alpha_2 + \cdots + \alpha_r)).$$

Can you figure out how to prove this? In Part 2, we will describe the connection.

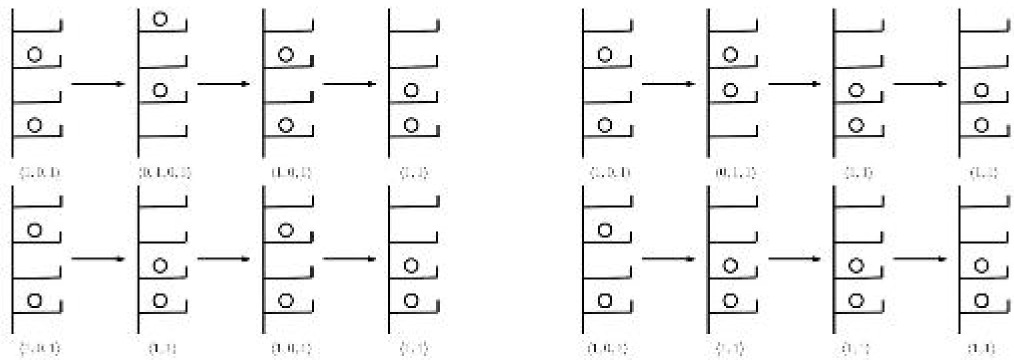

Figure 8. All possible juggling sequences with starting state ⟨1, 0, 1⟩, ending state ⟨1, 1⟩, and time 3. (Solution to the exercise in Example 1.)

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Valentine's Math, Part 2

by Ken Fan | edited by Jennifer Sidney Silva

Emily and Jasmine's first attempt to construct an equation that describes a Valentine heart failed. Their attempts to fix it led to better-looking shapes, but each one suffered from a common malady: bowing out instead of tapering at the bottom. Emily and Jasmine's plan was to describe, in a mathematical formula, the distance of the heart's points from the origin. Their first attempt to model this "distance function" used a basic sine wave. To eliminate the bulge at the bottom, Emily suggested refining the distance function to more accurately describe their sketch.

Jasmine: I still feel hesitant about attempting a more accurate modeling of our sketch.

Emily: Why?

Jasmine: Well, our original graph of the distance function was based on a sample of just a few distances measured directly on our sketch.

Emily: Yes, it was crude – and we ended up with a ram's head instead of a Valentine heart.

Jasmine: But to refine our model, we would have to take many more measurements, and that seems tedious. We'd probably have to take several measurements near the bottom tip, and it's not that easy to measure small angles accurately.

Emily: That's true. Okay, let's try a conceptual approach instead.

Jasmine: I like that better! We know from our first attempts that modeling the distance function with a sinusoidal wave does not produce the tapering of the heart that we'd like to see.

Emily: The top halves of the hearts look okay. It would be a pity to lose that part of our work.

Jasmine: What exactly was our equation?

Emily: It was

$$x^2 + y^2 = \left(1 + \frac{2ay|x|}{x^2 + y^2}\right)^2,$$

where a is the amplitude of the sine wave.

Jasmine: Let's take square roots of both sides so that the left-hand side of the equation is exactly the distance from the origin:

$$\sqrt{x^2 + y^2} = 1 + \frac{2ay|x|}{x^2 + y^2}.$$

Emily: If the right-hand side of the equation were just a plain 1, then we would be looking at the equation of a circle. The part added to the 1 measures the deviation from the circle. The

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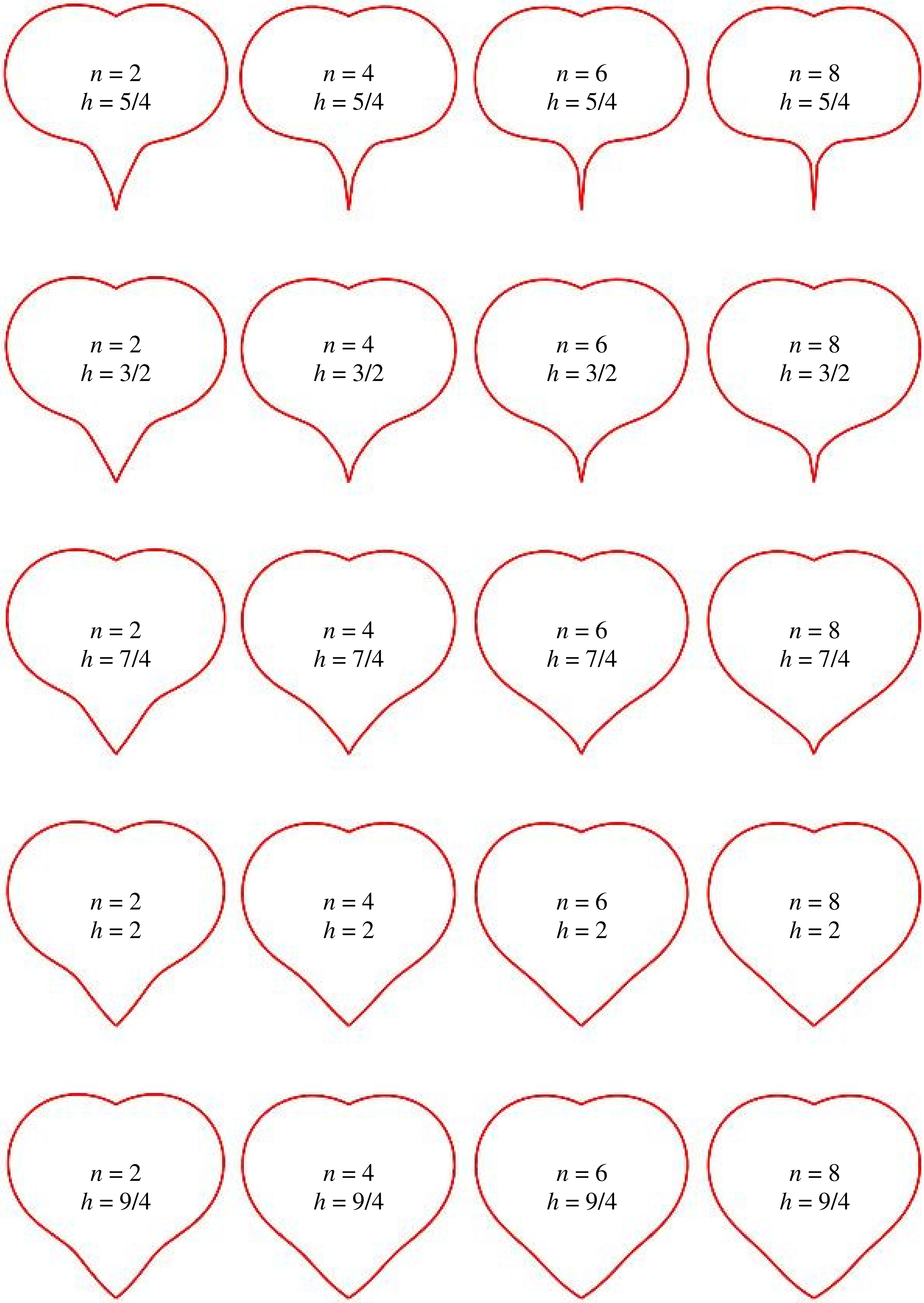


Figure 3. Candidate Valentine Hearts

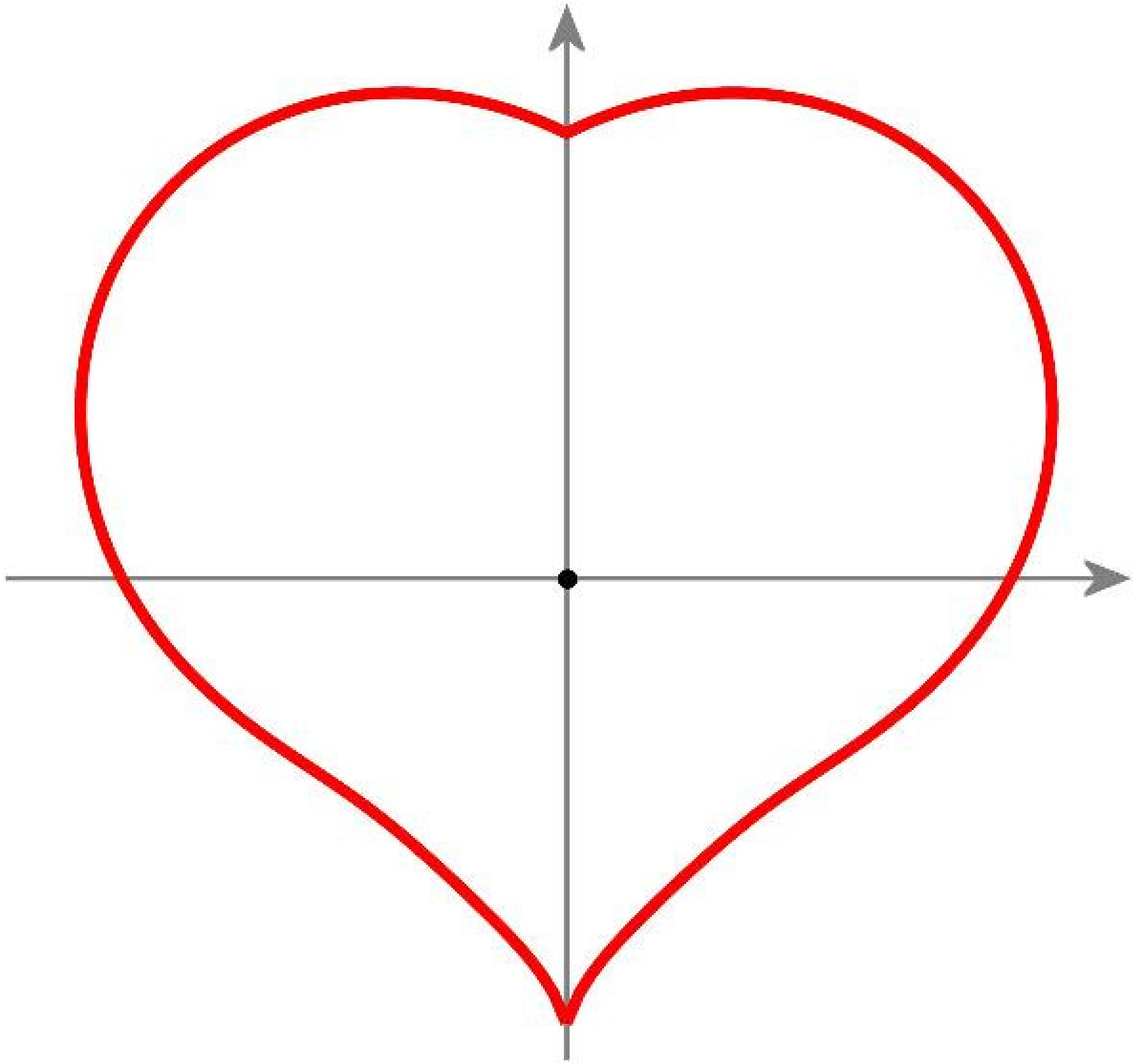


Figure 4. A plot of the solutions to the equation $\sqrt{x^2+y^2} = 1 + \frac{1}{2}\frac{y|x|}{x^2+y^2}(1 + \frac{1}{(7/4+y)^6})$.

Emily: Jasmine, you know what?

Jasmine: What?

Emily: The applet I was playing with earlier gives the equation for a Valentine heart in 3D. We could have taken that equation and looked at a cross section of it to find an equation for a 2D Valentine heart! It wouldn't have been as much fun, but now I'm curious to know what that equation is and how it works.

Jasmine: Let's check!

To be continued…

Learn by Doing

Graphs of Equations
by Addie Summer | edited by Amanda Galtman

In this issue, Emily and Jasmine depicted a Valentine heart as the solutions to a mathematical equation. In this Learn by Doing, we'll explore graphs of equations in more depth.

2D

Let's begin by considering equations that have two unknowns, x and y. If we have such an equation, its **graph** is obtained by finding all the pairs (x, y) that satisfy the equation and plotting these solutions (x, y) in the xy-coordinate plane.

1. Recall that the line with slope m and y-intercept b is given by $y = mx + b$. If we subtract everything on the right-hand side of this equation from both sides, we obtain:

$$y - (mx + b) = 0.$$

This equation is a special instance of the equation $Ax + By + C = 0$. Specifically, it is the equation $Ax + By + C = 0$ when $A = -m$, $B = 1$, and $C = -b$. In fact, show that the graph of the equation

$$Ax + By + C = 0,$$

where A, B, and C are any constants with A and B not both equal to 0, is a straight line.

In Problem 1, notice that when $B = 0$, the corresponding line is vertical. (Even though the equation involves only x when $B = 0$, we still find all solutions (x, y) that satisfy the equation and plot all the solutions (x, y) in the coordinate plane.) This is one way in which the graph of an equation can differ from the graph of a function. The graph of a function never has more than one point on any given vertical line. (Some express this by saying that the graph of a function must "pass the vertical line test.")

2. If the graphs of the equations $Ax + By + C = 0$ and $Dx + Ey + F = 0$ correspond to the same line, must we have $A = D$, $B = E$, and $C = F$?

3. Find an equation whose graph is a circle. If you have trouble with this, do Problem 4.

If you did not have trouble with Problem 3, skip Problem 4.

4. Let r be a positive number and let P be the point (a, b) in the coordinate plane. The circle of radius r and center P is the set of points that are a distance of r from P. An equation is a statement that two things are equal, and the sentence, "The circle of radius r and center P is the set of points that are a distance of r from P," states that a circle is defined by equating r to the distance of a point from P. Use the Pythagorean theorem to find an expression for the distance of a point (x, y) to the point (a, b). Then, set this expression equal to r to obtain an equation whose graph is a circle of radius r centered at P.

If you square both sides of the equation obtained in Problem 4, you get the equation

$$(x-a)^2 + (y-b)^2 = r^2.$$

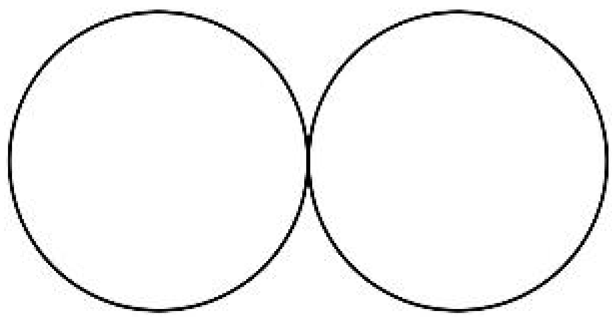
This is known as the **standard form** of the equation of a circle with radius r and center (a, b).

In this article, we'll call a subset of the plane **realizable** if the subset is the graph of an equation. So far, we have seen that all lines and all circles are realizable.

5. Suppose A and B are realizable subsets of the plane. Show that the union of A and B is realizable.

6. Suppose A and B are realizable subsets of the plane. Show that the intersection of A and B is realizable.

7. Find an equation whose graph is a figure eight, like that shown below.



The equations of the form $Ax + By + C = 0$, where A, B, and C are constants with A and B not both equal to zero, constitute the family of **linear** equations in x and y. That is, any polynomial equation in x and y with no term of degree bigger than 1 can be written in this form. (The degree of a term of a polynomial is the sum of the exponents on all the variables in the term. For example, the degrees of the terms x, xy, and x^2y^3 are 1, 2, and 5, respectively. The degree of a polynomial in x and y is the largest degree among all of its terms.)

The standard form of the equation for a circle is a degree-2 polynomial equation, but not every polynomial equation of degree 2 represents a circle. The most general degree-2 polynomial equation in the variables x and y can be put into the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

where A, B, C, D, E, and F are constants and not all three of A, B, and C are zero.

8. What do the graphs of degree-2 polynomial equations in x and y look like?

If you are having trouble with Problem 8, here is a series of hints, with the hints that give the least information coming first.

Hint 1. Work out the graph of several examples. If you find examples hard to graph, simplify the examples by setting some of the coefficients to 0.

Hint 2. If $B \neq 0$, we can change variables with a linear transformation in such a way that the cross term disappears. That is, let $x = aX + bY$ and $y = cX + dY$, where $ad - bc \neq 0$. Choose a, b, c, and d so that after substituting, you obtain a polynomial of degree 2 in X and Y with no cross term, i.e., the coefficient of XY is 0. What can you say about the situation where $B = 0$?

Hint 3. Suppose $B = 0$ but neither A nor C is 0. Notice that we can make a substitution of the form $x = aX$ and $y = Y$ and pick a so that the coefficients of X^2 and Y^2 are the same. What can you say about the situation where $B = 0$ and $A = C$?

Hint 4. This hint suggests a different approach to understanding Problem 8 from the one outlined in Hints 1-3. Write $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in the following way:

$$Ax^2 + Bxy + Cy^2 = -(Dx + Ey + F).$$

This form suggests the following interpretation of the graph of the equation. We add a third dimension and represent it by a z-axis, perpendicular to the xy-coordinate plane. The graph of the function $-(Dx + Ey + F)$, i.e., the graph of the equation $z = -(Dx + Ey + F)$, is a plane. This plane forms a cross-section of the graph of the function $Ax^2 + Bxy + Cy^2$, i.e., the graph of the equation $z = Ax^2 + Bxy + Cy^2$. If we project this cross-section vertically onto the xy-coordinate plane, we obtain the graph of the original equation.

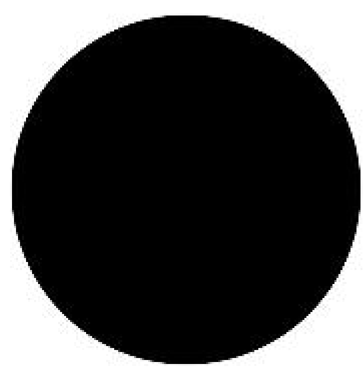
Thus, the problem of understanding the graph of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ becomes essentially the same as the problem of understanding (vertical projections of) planar cross-sections of the graph of $z = Ax^2 + Bxy + Cy^2$.

The graphs of the equations $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ include the **conic sections**. For more on conic sections, see *Math In Your World: Cosmic Conics I* and *Math In Your World: Cosmic Conics II* by Aaron Lee in Volume 6, Number 6 and Volume 7, Number 1 of this Bulletin.

9. Devise an equation, different from Emily and Jasmine's, whose graph is a Valentine heart.

Regions in the Plane

So far, we've looked at graphs of equations that turn out to be points, curves, or unions of points and curves.



10. Produce an equation whose graph consists of all the points of the square with vertices (1, 1), (-1, 1), (-1, -1), and (1, -1). (Hint: You might want to use the absolute value function.)

11. Produce an equation whose graph consists of all the points on and inside the unit circle centered at the origin.

12. Produce an equation whose graph consists of the union of all the vertical lines $x = a$, where a is an integer, and all the horizontal lines $y = b$, where b is an integer.

13. Produce an equation whose graph consists of all the points (a, b), where a and b are integers.

14. Can you produce an equation whose graph makes an infinite checkerboard pattern in the plane?

These questions may make one wonder what functions are "allowed" in creating these equations. After all, if you have any subset R of the plane, you could define a function $f(x, y)$ to be 0 if (x, y) is in R and 1 otherwise. Then the graph of the equation $f(x, y) = 0$ would be R by construction. This consideration suggests the game of restricting the functions available and seeing what kind of graphs are possible. For the problems in this Learn by Doing, we suggest starting by restricting yourself to the functions commonly introduced in school: addition, subtraction, multiplication, division, exponents, logarithms, trigonometric functions, absolute value, and the floor and ceiling functions. Feel free to reduce or expand these restrictions as you wish!

Polynomials

Suppose we restrict ourselves to functions that can be built by only adding and multiplying numbers and variables. The resulting functions are the **polynomials**. In the plane, we are then specifically considering equations of the form $p(x, y) = 0$, where $p(x, y)$ is a polynomial in x and y (with real number coefficients).

15. Revisit Problems 10-14 and, for each, either solve it using only a polynomial equation or prove that it is not possible to do so.

If you had trouble with Problem 15, Problems 16-17 may help.

16. Let $p(x, y)$ be a polynomial. Suppose there is a circle of radius $r > 0$ such that $p(x, y) = 0$ for all points (x, y) inside the circle. Prove that $p(x, y)$ is the zero polynomial, so the graph of the equation $p(x, y) = 0$ is the entire plane.

17. Let $p(x, y)$ be a polynomial. Suppose that there is a line segment of length $d > 0$ such that $p(x, y) = 0$ for all points (x, y) on the line segment. Prove that $p(x, y) = 0$ for all points (x, y) on the line that contains the line segment.

18. By Problem 17, there is no polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is an equilateral triangle. However, for any $\varepsilon > 0$, can you produce a polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is contained within the ε-neighborhood of an equilateral triangle of unit side length? (The ε-neighborhood of a subset of the plane is the set of points that are within a distance of ε of some point in the subset.)

19. By Problem 17, there is no polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is a regular n-gon, where n is a positive integer greater than 2. However, for any $\varepsilon > 0$, can you produce a polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is contained within the ε-neighborhood of a regular n-gon with unit side length?

20. By Problem 17, there is no polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is the boundary of a semicircle (that is, a 180° arc of a circle together with the diameter connecting its endpoints). However, for any $\varepsilon > 0$, can you produce a polynomial $p(x, y)$ such that the graph of $p(x, y) = 0$ is contained within the ε-neighborhood of the boundary of a semicircle of unit radius?

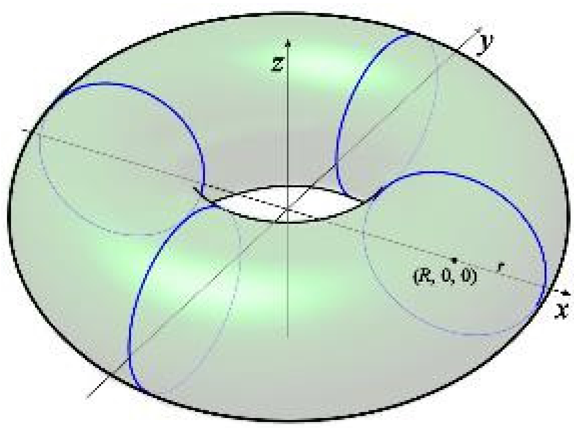
3D

Let's move on to the *xyz*-coordinate space.

21. Produce equations whose graphs are each of the following objects.

a. A plane
b. A single point
c. A line
d. A sphere
e. An infinite cylinder
f. An infinite cone

22. Let $R > r > 0$ be constants. A torus can be constructed by taking a circle of radius r and center $(R, 0, 0)$ in the xz-coordinate plane, and then rotating this circle about the z-axis. Produce an equation whose graph is this torus.



23. Produce an equation whose graph is the surface of a cube.

24. Produce an equation whose graph consists of two interlocking circles.

25. For each of the Platonic solids (regular tetrahedron, cube, regular octahedron, regular dodecahedron, and regular icosahedron) with edge length 1 and for any $\varepsilon > 0$, produce a polynomial equation $p(x, y, z) = 0$ whose graph is contained within the ε-neighborhood of the boundary of the solid.

Complex Numbers

So far, we've considered equations whose variables stand only for real numbers. Working with complex numbers opens up an exciting new world. If we think of (x, y) as a pair of complex numbers, then the *xy*-complex coordinate plane has four real dimensions. In this situation, the graph of an equation typically corresponds to a surface.

26. See if you can puzzle out the shape of the surface corresponding to the graph of the equation $y^2 = x^3 - x$, where x and y are variables that represent complex numbers.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 28 - Meet 5 March 4, 2021	Mentors:	Talia Blum, Cecilia Esterman, Kimberly Hadaway, Adeline Hillier, Jenny Kaufmann, Bridget Li, Rebecca Nelson, Kate Pearce, AnaMaria Perez, Nehar Poddar, Gisela Redondo, Vievie Romanelli, Savannah Tynan, Emma Wang, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng
	Visitor:	Daina Taimina, Cornell University

A knitted model of the hyperbolic plane showing three lines through a point all parallel to a 4th line. Image courtesy of Daina Taimina.

Daina Taimina is a retired adjunct associate professor of mathematics at Cornell University. She was born in Latvia and is a pioneer of crocheting hyperbolic spaces.

In 1997, she saw a fragile paper model of the hyperbolic plane (after an idea of Beltrami in 1868). She thought she could make more durable models by knitting them, and she did.

In a hyperbolic plane, circles of radius r have circumferences greater than $2\pi r$. If two lines cross each other at right angles, instead of forming a flat plane or both curving together as they would on the surface of a sphere, in hyperbolic geometry, they would curve in opposite directions, such as on the surface of a Pringles potato chip. Daina illustrated this by crossing her two index fingers at right angles so that they curved opposite to each other.

In the Euclidean plane, through a point outside a line, there is a unique line parallel to the given line. However, in a hyperbolic plane, through a point outside a line, there are infinitely many lines that do not intersect the given line. Daina's knitted models can be used to illustrate this face beautifully (see the picture above).

To knit hyperbolic planes, the idea is to create expanding rings, where the number of stitches in each ring grows according to a specific formula. Because the number of stiches grows exponentially with the size of the model, it can take a lot of time to create. Shown at right is a photo of Daina with a model that she worked on over a period of 2 years. It is the largest model she has created so far.

Courtesy of Daina Taimina

If you would like to read more about hyperbolic spaces and learn how to create your own models of them, Daina has written a wonderful book together with David Henderson called *Experiencing Geometry* which can be downloaded for free from: projecteuclid.org/euclid.bia/1598805325.

Daina's work has led her to world's outside of math, such as with her creation *White Cloud*, which was exhibited in Riga, the capital of Latvia.

Session 28 - Meet 6 March 11, 2021	Mentors:	Talia Blum, Cecilia Esterman, Adeline Hillier, Kate Pearce, AnaMaria Perez, Gisela Redondo, Vievie Romanelli, Savannah Tynan, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng

A 6-sided die is a classic way to insert randomness into a board game and other scenarios. Suppose you want 5 distinct outcomes of equal probability. Can you think of a way to make a physical 5-sided die? How would you determine that it is fair? Can you make a die that doesn't necessarily have 5-sides but can be used to produce 5 distinct, equally likely outcomes with just a single toss?

Session 28 - Meet 7 March 18, 2021	Mentors:	Talia Blum, Cecilia Esterman, Michelle Li, Rebecca Nelson, Kate Pearce, Nehar Poddar, Gisela Redondo, Vievie Romanelli, Savannah Tynan, Emma Wang, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng

In discussing the optimal shape for a cat carrier, one group started a discussion of shapes and shapes with holes. It quickly became apparent that it was already quite a trick to precisely define what a hole is! How would you define a hole for a 3-dimensional shape? What about a 2-dimensional shape? How many holes does a donut have? A coffee mug? A tee shirt? What object in your room has the most holes (based on your definition)?

Session 28 - Meet 8 April 1, 2021	Mentors:	Talia Blum, Cecilia Esterman, Adeline Hillier, Jenny Kaufmann, Bridget Li, Kate Pearce, AnaMaria Perez, Nehar Poddar, Vievie Romanelli, Savannah Tynan, Jane Wang, Rebecca Whitman, Angelina Zhang, Rachel Zheng

A group of members continues examining $3 \times 3 \times 3$ tic-tac-toe variants. The thought of extending to different dimensions led to the question of what tic-tac-toe might look like in one dimension, i.e. played on a number line with players marking integer points. For example one could still keep the idea of XXX as the winning pattern and ask if it's possible for X to win if O tries to prevent it. Can you think of any patterns that will always result in X winning?

Session 28 - Meet 9 April 8, 2021	Mentors:	Talia Blum, Cecilia Esterman, Adeline Hillier, Jenny Kaufmann, Rebecca Nelson, Kate Pearce, AnaMaria Perez, Gisela Redondo, Vievie Romanelli, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng

Some members have become interested in magic squares, which are $n \times n$ arrays of distinct numbers where the sum of numbers in any row, column, or main diagonal is the same. They decided to make a magic square using their birthdate, for example if their birthdate was May 15, 2010, they used 5, 15, 20, 10 as the top row in a 4×4 magic square. Can you complete such a magic square? Can you make a magic square using your birthdate? What if you were given an arbitrary date, could you come up with a way to find the remaining 9 values? Are there any restrictions on what dates can be completed to a magic square?

Session 28 - Meet 10 April 15, 2021	Mentors:	Cecilia Esterman, Adeline Hillier, Bridget Li, Rebecca Nelson, AnaMaria Perez, Gisela Redondo, Vievie Romanelli, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng

Sudoku is a favorite puzzle of some of our members. Their interest has led them to ask some interesting questions including “How many Sudoku boards are possible?” and “What is the minimum number of starting clues required?” For the first question this led to a discussion of what exactly should be counted – the end result, i.e. how many ways are there to arrange the numbers 1-9 so that the board is filled according to the rules of Sudoku? – or the starting position, i.e. how many Sudoku puzzles exist? For the second question members decided to start with smaller boards such as the 4×4 Sudoku grid. Have you ever tried to make a Sudoku puzzle? Have you ever thought about writing a computer program to solve Sudoku puzzles? What Sudoku related questions do you have?

Session 28 - Meet 11 April 29, 2021	Mentors:	Cecilia Esterman, Adeline Hillier, Bridget Li, Rebecca Nelson, Kate Pearce, AnaMaria Perez, Gisela Redondo, Vievie Romanelli, Savannah Tynan, Jane Wang, Karissa Wenger, Rebecca Whitman, Angelina Zhang, Rachel Zheng

A topic that has prompted quite a bit of deep discussion and exploration is the connection between math and music. There are many avenues one could explore in terms of this connection, from Fourier analysis to group theory. One particular avenue that caught a member’s interest was looking at a topological representation of pitch classes. In this representation hexagons were used to tile the plane, each hexagon representing a single note. Together 12 hexagonal notes arranged, in 4 rows of 3, formed a larger pattern that repeated itself throughout the space. As you left each outer side of this repeating pattern you would go to a tile on the opposite side. In this way each side could be identified with another side and then one could imagine gluing these sides together to form a closed surface. Can you figure out what that surface would look like? A similar question involves gluing opposite sides of a square together, what does the final object look like? What if you took an octagon? Can you see any patterns or relationships between how many sides your original polygon starts with and what the final object looks like?

Calendar

Session 27: (all dates in 2020)

September	10	Start of the twenty-seventh session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	

Session 28: (all dates in 2021)

January	28	Start of the twenty-eighth session!
February	4	
	11	
	18	No meet
	25	
March	4	Daina Taimina, Cornell University
	11	
	18	
	25	No meet
April	1	
	8	
	15	
	22	No meet
	29	
May	6	

Girls' Angle has run over 150 Math Collaborations. Math Collaborations are fun, fully collaborative, math events that can be adapted to a variety of group sizes and skill levels. We now have versions where all can participate remotely. We have now run four such "all-virtual" Math Collaboration. If interested, contact us at girlsangle@gmail.com. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

____-

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is \$20/meet for members and \$30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory
Yaim Cooper, Institute for Advanced Study
Julia Elisenda Grigsby, professor of mathematics, Boston College
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, assistant dean and director teaching & learning, Stanford University
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Bjorn Poonen, professor of mathematics, MIT
Liz Simon, graduate student, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, associate professor, University of Washington
Karen Willcox, Director, Oden Institute for Computational Engineering and Sciences, UT Austin
Lauren Williams, professor of mathematics, Harvard University

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________